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# Fixed Point Results in Partial Fuzzy Metric Spaces

S. Sowndrarajan, Vishal Gupta* and M. Jeyaraman

ABSTRACT: $\digamma_\Psi$-contractive mapping in Partial Fuzzy Metric Space (PFMS) is defined and basic results are established. Sequentially convergent and sub sequentially convergent are also defined for PFMS, then generalizations of fixed point theorems of Kannan and Chatterjea are proved in the setting of PFMS along with suitable examples.

Key Words: Fixed point, $\digamma_\Psi$-contraction, partial fuzzy metric, Kannan fixed point theorem.

## Contents



## 1. Introduction

Zadeh [23] presented Fuzzy Sets (FS), which play a vital role in science and engineering. Kramosil and Michaelek [10] defined the notion of Fuzzy Metric Space (FMS) in 1975. George and Veeramani [5] redefined the concept of FMS in association with triangular norm. Many researchers have worked on FMS and have given many of its characteristics and Fixed Point Theorems (FPT). Medical image processing, decision making and imaging signal processing are some of the applications of FMS. The Partial Metric Space (PMS) defined by Matthews [12] is a generalization of the concept of a metric space in which points can have "self-spacing" and proved Banach contraction mapping theorem and some basic properties. Then, Oltra, Valero and Altun et al. [16] gave generalizations of the results of Matthews. Also,he established the first fixed point theorem, which he renamed as the partial contraction mapping theorem. Shaban Sedghi, Nabi and Altun [18] introduced PFMS and established some FPT in its setting. Many authors have proved FPT of Kannan and Chatterjea in FMS and PMS [11,13,21].Other authors also worked in the similar directions, are mentioned in [1,2,4,6,7,8,9,14,15,17,20,22].

In this manuscript, following the understanding of Sedghi et al, we will study the relationship between a PMS and FMS. Next, we prove the generalization of FPT of Kannan and Chatterjea in establishing PFMS.

## 2. Preliminaries

**Definition 2.1** *[12] A partial metric on a non empty set $\Sigma$ is a function $p : \Sigma \times \Sigma \to \mathbb{R}_+$ such that for all $\xi, \nu, \omega \in \Sigma$,*
*(i) $\xi = \nu$ if and only if $p(\xi, \xi) = p(\xi, \nu) = p(\nu, \nu)$,*
*(ii) $p(\xi, \xi) \le p(\xi, \nu)$,*
*(iii) $p(\xi, \nu) = p(\nu, \xi)$,*
*(iv) $p(\xi, \omega) \le p(\xi, \nu) + p(\nu, \omega) - p(\nu, \nu)$.*
*The pair $(\Sigma, p)$ is called a PMS. For every partial metric $p$ on $\Sigma$, the function $p : \Sigma \times \Sigma \to \mathbb{R}_+$ on family of p-open balls is defined by*

$$p^s(\xi, \nu) = 2p(\xi, \nu) - p(\xi, \xi) - p(\nu, \nu)$$

* Corresponding author

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*is a usual metric on* $\Sigma$.

**Definition 2.2** *[18] A PFMS is a function* $\mathcal{P}_{\mathcal{M}} : \Sigma \times \Sigma \times (0, \infty) \to [0, 1]$ *so that, for all* $\xi, \nu, \omega \in \Sigma$ *and* $\sigma, \varsigma > 0$

(i) (PM-1) $\xi = \nu \Leftrightarrow \mathcal{P}_{\mathcal{M}}(\xi, \xi, \sigma) = \mathcal{P}_{\mathcal{M}}(\xi, \nu, \sigma) = \mathcal{P}_{\mathcal{M}}(\nu, \nu, \sigma)$,
(ii) (PM-2) $\mathcal{P}_{\mathcal{M}}(\xi, \xi, \sigma) \geq \mathcal{P}_{\mathcal{M}}(\xi, \nu, \sigma)$,
(iii) (PM-3) $\mathcal{P}_{\mathcal{M}}(\xi, \nu, \sigma) = \mathcal{P}_{\mathcal{M}}(\nu, \xi, \sigma)$,
(iv) (PM-4) $\mathcal{P}_{\mathcal{M}}(\xi, \nu, \max\{\sigma, \varsigma\}) * \mathcal{P}_{\mathcal{M}}(\omega, \omega, \max\{\lambda, \varsigma\}) \geq \mathcal{P}_{\mathcal{M}}(\xi, \omega, \sigma) * P_M(\omega, \nu, \varsigma)$,
(v) (PM-5) $\mathcal{P}_{\mathcal{M}}(\xi, \nu, .) : (0, \infty) \to [0, 1]$ *is continuous.*

**Example 2.1** *[18] Let* $\Sigma$ *be a non empty set and* $\xi \star \nu = \xi\nu$ *for all* $\xi, \nu \in \Sigma$. *Let* $\mathcal{P}_{\mathcal{M}} : \Sigma \star \Sigma \star (0, \infty) \to [0, 1]$ *be a mapping defined by*

$$\mathcal{P}_{\mathcal{M}}(\xi, \nu, \lambda) = \frac{\lambda}{\lambda + \mathcal{P}(\xi, \nu)},$$

*then* $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ *is a PFMS induced by standard metric. Also note that* $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ *is not a FMS.*
*In FMS* $(\Sigma, \mathcal{M}, \star)$ *the function* $\mathcal{M}(\xi, \nu, .) : (0, \infty) \to [0, 1]$ *is a non - decreasing for all* $\xi, \nu \in \Sigma$, *but in a PFMS* $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ *the function* $\mathcal{P}_{\mathcal{M}}(\xi, \nu, .) : (0, \infty) \to [0, 1]$ *may not be non-decreasing for all* $\xi, \nu \in \Sigma$.
*Next, we present an example to show that* $\mathcal{P}_{\mathcal{M}}(\xi, \nu, .) : (0, \infty) \to [0, 1]$ *may not be non-decreasing for all* $\xi, \nu \in \Sigma$.

**Example 2.2** *If* $\Sigma = \mathcal{R}$, $\xi \star \nu = \min\{\xi, \nu\}$ *for all* $\xi, \nu \in [0, 1]$.
*Consider a mapping* $\mathcal{P}_{\mathcal{M}} : \Sigma \times \Sigma \times (0, \infty) \to [0, 1]$ *defined by*

$$\mathcal{P}_{\mathcal{M}}(\xi, \nu, \lambda) = \begin{cases} 2e^{-\lambda}, & if \quad \xi = \nu \\ \frac{2}{3}e^{-\lambda}, & if \quad \xi \neq \nu \end{cases}$$

*It is easy to see that* $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ *is a PFMS.*
*Also, it is not complicated to verify that* $\mathcal{P}_{\mathcal{M}}(\xi, \nu, .) : (0, \infty) \to [0, 1]$ *is a decreasing function.*

**Definition 2.3** *Let* $(\Sigma, d)$ *be a metric space. A mapping* $\digamma : \Sigma \to \Sigma$ *is said to be graph closed (sub sequentially convergent), if for every sequence* $\{\xi_n\}$ *we have* $\lim_{n\to\infty} \digamma\xi_n = \alpha$, *so that* $\digamma(\beta) = \alpha$, *for some* $\beta \in \Sigma$.

**Definition 2.4** *Consider* $(\Sigma, \mu)$ *be a metric space and* $\Psi, G : \Sigma \to \Sigma$ *be two maps. The function* $\Psi$ *is called* $G_\Psi$ *- contraction, if there exists* $0 \leq \lambda < \infty$ *so that* $\forall\ \xi, \nu \in \Sigma$,

$$\Xi(\mu(\digamma\Psi\xi, \digamma\Psi\nu, t)) \geq \lambda\Xi(\mu(\digamma\xi, \digamma\nu, t)),$$

*where the function* $\Xi : [0, \infty) \to [0, \infty)$ *is non - decreasing continuous and* $\Xi^{-1}(0) = 0$. *Also, map* $G$ *is graph closed and one to one.*

**Definition 2.5** *Consider* $(\Sigma, \mu)$ *be a metric space and* $G : \Sigma \to \Sigma$ *and* $\{\nu_n\}$ *be a sequence in* $\Sigma$,

(i) $G$ *is called sequentially convergent if* $\{G\nu_n\}$ *converges, then* $\{\nu_n\}$ *also converges.*

(ii) $G$ *is called sub sequentially convergent if* $\{G\nu_n\}$ *converges, then* $\{\nu_n\}$ *has a convergent subsequence.*

*For example, the function* $G\xi = \xi$ *is sequentially convergent on the metric space* $(\mathbb{R}, |.|)$.

**Definition 2.6** *[3] Let* $(\mathcal{X}, d)$ *be a metric space and* $\mathcal{T} : \mathcal{X} \to \mathcal{X}$.

(i) $\mathcal{T}$ *is said to be sequentially convergent if for every sequence* $\{\acute{y}_n\}$, *if* $\{\mathcal{T}\acute{y}_n\}$ *is convergent the* $\{\acute{y}_n\}$ *is also convergent.*

(ii) $\mathcal{T}$ *is said to be subsequentially convergent if for every sequence* $\{\acute{y}_n\}$, *if* $\{\mathcal{T}\acute{y}_n\}$ *is convergent the* $\{\acute{y}_n\}$ *has a convergent subsequence.*

For Example, the functions $\mathcal{T}\acute{x} = \acute{x}$ is sequentially convergent on the metric space $(\mathbb{R}, |.|)$. The mapping $\mathcal{T}\acute{x} = \acute{x}^2$ is not sequentially convergent on the metric space $(\mathbb{R}, |.|)$, but it is subsequentially convergent.

## 3. Main Results

This section mainly uses FPT of Kannan and Chatterjea to introduce and generalize $\digamma_\Psi$-type fixed point theorems in PFMS.

**Theorem 3.1** *If $(\Sigma, \mathcal{P}_\mathcal{M}, \star)$ be a complete PFMS, $\digamma, \Psi : \Sigma \to \Sigma$ such that $\digamma$ is one to one and subsequently convergent. If $\xi, \nu \in \Sigma$ and for all $\lambda \in [0,1]$,*

$$\Xi(\mathcal{P}_\mathcal{M}(\digamma\Psi\xi, \digamma\Psi\nu, \lambda) \geq \lambda\Xi(\mathcal{P}_\mathcal{M}(\digamma\xi, \digamma\nu, \lambda)), \tag{3.1.1}$$

*where $\Xi : [0,\infty) \to [0,\infty)$ such that*

*(i) $\Xi$ is non decreasing,*

*(ii) $\Xi$ is continuous,*

*(iii) $\Xi(\xi) = 0$ iff $\xi = 0$.*

*Then $\Psi$ possess only one fixed point in $\Sigma$.*

*Proof.* Consider $\xi_0 \in \Sigma$. Take $\xi_n = \Psi\xi_{n-1} = \Psi^n\xi_0$, $n = 1, 2, \ldots$

$$\begin{aligned}\Xi(\mathcal{P}_\mathcal{M}(\digamma\xi_n, \digamma\xi_{n+1}, \lambda)) &= \Xi(\mathcal{P}_\mathcal{M}(\digamma\Psi\xi_{n-1}, \digamma\Psi\xi_n, \lambda)) \\ &\geq \lambda\Xi(\mathcal{P}_\mathcal{M}(\digamma\xi_{n-1}, \digamma\xi_n, \lambda)) \\ &\;\vdots \\ &\geq \lambda^{n-1}\Xi(\mathcal{P}_\mathcal{M}(\digamma\xi_0, \digamma\xi_1, \lambda)).\end{aligned} \tag{3.1.2}$$

Consider $\forall m, n \in \mathbb{N}$ where $m > n$, we get

$$\begin{aligned}\Xi(\mathcal{P}_\mathcal{M}(\digamma\xi_n, \digamma\xi_m, \lambda)) &= \Xi(\mathcal{P}_\mathcal{M}(\digamma\Psi^n\xi_0, \digamma\Psi^m\xi_0, \lambda)) \\ &\geq \lambda^n\Xi(\mathcal{P}_\mathcal{M}(\digamma\xi_0, \digamma\Psi^{m-n}\xi_0, \lambda)).\end{aligned} \tag{3.1.3}$$

Taking $m, nn \to \infty$ in (3.1.3), we get

$$\Xi(\mathcal{P}_M(\digamma\xi_n, \digamma\xi_m, \lambda) \to 0 \quad as \quad m, n \to \infty.$$

Since, $\Xi$ is continuous, we obtain

$$\lim_{m,n\to\infty} \mathcal{P}_\mathcal{M}(\digamma\xi_n, \digamma\xi_m, \lambda) = 0. \tag{3.1.4}$$

Thus, we see that $\{\digamma\xi_n\}$ is a Cauchy sequence in a complete PFMS $(\Sigma, \mathcal{P}_M, \star)$. So, there exists $\mu \in \Sigma$ such that $\{\digamma\xi_n\}$ converges to $\mu \in \Sigma$. Since, $\digamma$ is subsequently convergent, then there exists an $\omega \in \Sigma$ such that

$$\lim_{\lambda\to\infty} \mathcal{P}_\mathcal{M}(\xi_{n(\lambda)}, \omega, \lambda) = \mathcal{P}_\mathcal{M}(\omega, \omega, \lambda).$$

Also, $\digamma$ is continuous and $\xi_{n(\lambda)} \to \omega$, therefore

$$\lim_{\lambda\to\infty} \digamma\xi_{n(\lambda)} = \digamma\omega \quad and \quad \lim_{\lambda\to\infty} \mathcal{P}_\mathcal{M}(\digamma\xi_{n(\lambda)}, \omega, \lambda) = \mathcal{P}_\mathcal{M}(\omega, \omega, \lambda).$$

Since, $\{\digamma\xi_{n(\lambda)}\}$ is a subsequence of $\{\digamma\xi_n\}$, so we get $\digamma\omega = \mu$.
Also,

$$d^s(\digamma\xi_n, \digamma\omega, \lambda) = 2\mathcal{P}_\mathcal{M}(\digamma\xi_n, \digamma\omega, \lambda) - \mathcal{P}_\mathcal{M}(\digamma\xi_n, \digamma\xi_n, \lambda) - \mathcal{P}_\mathcal{M}(\digamma\omega, \digamma\omega, \lambda). \tag{3.1.5}$$

Let $n \to \infty$ in (3.1.5), we have

$$\lim_{n\to\infty} d^s(\digamma\xi_n, \digamma\omega, \lambda) = 0.$$

Consider, definition(2.6)(ii) and (3.1.5) we hold

$$\lim_{n\to\infty} \mathcal{P}_{\mathcal{M}}(F\xi_n, F\omega, \lambda) = \lim_{m,n\to\infty} \mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_m, \lambda) = \mathcal{P}_{\mathcal{M}}(F\omega, F\omega, \lambda) = 0.$$

Now, we prove that $\omega \in \Sigma$ is a fixed point of $f$. Since $\Xi$ is continuous

$$\begin{aligned}\Xi(\mathcal{P}_{\mathcal{M}}(F\Psi\omega, F\Psi\xi_{n+1}, \lambda)) &= \Xi(\mathcal{P}_{\mathcal{M}}(F\Psi\omega, F\Psi\xi_n, \lambda)) \\ &\geq \lambda\Xi(\mathcal{P}_{\mathcal{M}}(F\omega, F\xi_n, \lambda)).\end{aligned} \tag{3.1.6}$$

Let $n \to \infty$ in (3.1.6), we obtain

$$\Xi(\mathcal{P}_{\mathcal{M}}(F\Psi\omega, F\omega, \lambda) \geq 0,$$

this implies that $\mathcal{P}_{\mathcal{M}}(F\Psi\omega, F\omega, \lambda) = 0$ and hence $F\Psi\omega = F\omega$. Since $F$ is one to one, then we get $\Psi\omega = \omega$. Assume that, $\nu$ is another fixed point of $\Psi$ then we have $\Psi\omega = \nu$ and

$$\begin{aligned}\Xi(\mathcal{P}_{\mathcal{M}}(F\omega, F\nu, \lambda)) &= \Xi(\mathcal{P}_{\mathcal{M}}(F\Psi\omega, F\Psi\nu, \lambda)) \\ &\geq \lambda\Xi(\mathcal{P}_{\mathcal{M}}(F\omega, F\nu, \lambda)),\end{aligned} \tag{3.1.7}$$

which is a contradiction unless $\mathcal{P}_{\mathcal{M}}(F\omega, F\nu, \lambda) = 0$. Thus $F\omega = F\nu$.
On substituting $\{n(\lambda)\}$ in place of $\{n\}$ and considering $F$ to be sequentially convergent, we have

$$\lim_{n\to\infty} \xi_n = \omega.$$

Hence, it is proved that $\{\omega_n\}$ converges to the fixed point of $\Psi$.

**Corollary 3.1** *Let $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ be a complete PFMS and $\Psi$ be a self mapping on $\Sigma$ into itself. If $\alpha \in [0,1)$ and $\xi, \nu \in \Sigma$,*

$$\mathcal{P}_{\mathcal{M}}(\Psi\xi, \Psi\nu, \lambda) \geq \alpha\mathcal{P}_{\mathcal{M}}(\xi, \nu, \lambda), \tag{3.2.1}$$

*then $\Psi$ has a unique fixed point.*

**Theorem 3.2** *Consider, $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ be a PFMS which is complete and $F, \Psi : \Sigma \to \Sigma$ such that $F$ is one to one, continuous and subsequently convergent. If $\xi, \nu \in \Sigma$ and for all $\beta \in [0, \frac{1}{2})$, we have*

$$\Xi(\mathcal{P}_{\mathcal{M}}(F\Psi\xi, F\Psi\nu, \lambda)) \geq \beta[\Xi(\mathcal{P}_{\mathcal{M}}(F\xi, F\Psi\xi, \lambda)) + \Xi(\mathcal{P}_{\mathcal{M}}(F\nu, F\Psi\nu, \lambda))], \tag{3.3.1}$$

*where $\Xi : [0,\infty) \to [0,\infty)$ such that*

*(i) $\Xi$ is non decreasing,*

*(ii) $\Xi$ is continuous,*

*(iii) $\Xi(\xi) = 0$ iff $\xi = 0$.*

*Then, $\Psi$ possess a unique fixed point in $\Sigma$.*

*Proof.* Consider $\xi_0 \in \Sigma$. Let $\xi_n = \Psi\xi_{n-1} = \Psi^{n\xi_0}$, n = 1,2, ...

$$\begin{aligned}\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_{n+1}, \lambda)) &= \Xi(\mathcal{P}_{\mathcal{M}}(F\Psi f\xi_{n-1}, F\Psi\xi_n, \lambda)) \\ &\geq \beta[\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_{n-1}, F\xi_n, \lambda)) + \Xi(\mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_{n+1}, \lambda))],\end{aligned} \tag{3.3.2}$$

therefore we get,

$$\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_{n+1}, \lambda) \geq \frac{\beta}{1-\beta}\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_{n-1}, F\xi_n, \lambda)$$

$$\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_{n+1}, \lambda) \geq \left(\frac{\beta}{1-\beta}\right)^n \Xi(\mathcal{P}_{\mathcal{M}}(F\xi_0, F\xi_1, \lambda). \tag{3.3.3}$$

Taking $n \to \infty$ in (3.3.3), we get

$$\Xi(\mathcal{P}_M(F\xi_n, F\xi_{n+1}, \lambda) \to 0 \ \ as \ \ m, n \to \infty.$$

Consider $\forall\ m, n \in \mathbb{N}$ where $m > n$, we get

$$\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_{n+1}, \lambda)) \geq \left(\frac{\beta}{1-\beta}\right)^n \Xi(\mathcal{P}_{\mathcal{M}}(F\xi_0, Ff^{m-n}\xi_1, \lambda)). \tag{3.3.4}$$

Taking $m, n \to \infty$ in (3.3.4), we have

$$\Xi(\mathcal{P}_M(F\xi_n, F\xi_m, \lambda) \to 0 \ \ as \ \ m, n \to \infty.$$

So, we obtain $\mathcal{P}_M(F\xi_n, F\xi_m, \lambda) \to 0$ as $m, n \to \infty$. Since, $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ is complete PFMS, we obtain $\{F\xi_n\}$ is a Cauchy sequence, so there exist $\omega \in \Sigma$ such that $\{F\xi_n\}$ converges to $F\xi \in \Sigma$ and $\xi_{n(\lambda)} \to \omega$, then we get

$$\lim_{\lambda\to\infty} F\xi_{n(\lambda)} = F\omega \ \ and \ \ \lim_{\lambda\to\infty} \mathcal{P}_{\mathcal{M}}(F\xi_{n(\lambda)}, F\omega, \lambda)) = \mathcal{P}_{\mathcal{M}}(F\omega, F\omega, \lambda)).$$

Now, we prove that $\omega \in \acute{X}$ is a fixed point of $f$. Indeed, we have

$$\begin{aligned} \Xi(\mathcal{P}_{\mathcal{M}}(F\Psi\omega, F\xi_{n+1}, \lambda)) &= \Xi(\mathcal{P}_{\mathcal{M}}(F\Psi\omega, F\xi_n, \lambda)) \\ &\geq \beta[\Xi(\mathcal{P}_{\mathcal{M}}(F\omega, F\Psi\omega, \lambda) + \Xi(\mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_{n+1}, \lambda)]. \end{aligned} \tag{3.3.5}$$

Letting $n \to \infty$ in (3.3.5), we get

$$\Xi(\mathcal{P}_{\mathcal{M}}(F\Psi\omega, F\omega, \lambda)) \geq \beta\Xi(\mathcal{P}_{\mathcal{M}}(F\omega, F\Psi\omega, \lambda)). \tag{3.3.6}$$

The above inequality is contradiction unless $\mathcal{P}_{\mathcal{M}}(F\omega, F\Psi\omega, \lambda) = 0$. Therefore, $F\omega = F\Psi\omega$. Since, $F$ is one to one mapping, we obtain $\Psi\omega = \omega$. Hence, we showed that $\omega \in \Sigma$ is a fixed point of $\Psi$. The uniqueness can be easily proved on the similar lines to the theorem (3.1).

**Corollary 3.2** *Let $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ be a complete PFMS and $F, \Psi : \Sigma \to \Sigma$ such that $F$ is one to one, continuous and subsequently convergent. If $\xi, \nu \in \Sigma$ and for all $\beta \in [0, \frac{1}{2})$, we have*

$$\mathcal{P}_{\mathcal{M}}(F\Psi f\xi, F\Psi\nu, \lambda) \geq \beta[\mathcal{P}_{\mathcal{M}}(F\xi, F\Psi\xi, \lambda) + \mathcal{P}_{\mathcal{M}}(F\nu, F\Psi\nu, \lambda)]. \tag{3.4.1}$$

*Then $f$ has a unique fixed point in $\Sigma$.*

**Corollary 3.3** *Let $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ be a complete PFMS and $F : \Sigma \to \Sigma$ be a self mapping. If $\xi, \nu \in \Sigma$ and for all $\beta \in [0, \frac{1}{2})$, we have*

$$\Xi(\mathcal{P}_{\mathcal{M}}(\Psi\xi, F\Psi\nu, \lambda) \geq \beta[\Xi(\mathcal{P}_{\mathcal{M}}(\xi, \Psi\xi, \lambda)) + \Xi(\mathcal{P}_{\mathcal{M}}(\nu, \Psi\nu, \lambda))]. \tag{3.5.1}$$

*Then $\Psi$ has a unique fixed point in $\Sigma$.*

**Theorem 3.3** *Let $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ be a complete PFMS and $F, \Psi : \Sigma \to \Sigma$ such that $F$ is one to one, continuous and subsequently convergent. If $\xi, \nu \in \Sigma$ and for all $\delta \in [0, \frac{1}{2})$, we have*

$$\Xi(\mathcal{P}_{\mathcal{M}}(F\Psi\xi, F\Psi\nu, \lambda)) \geq \delta[\Xi(\mathcal{P}_{\mathcal{M}}(F\xi, F\Psi\nu, \lambda)) + \Xi(\mathcal{P}_{\mathcal{M}}(F\nu, F\Psi\xi, \lambda))], \tag{3.6.1}$$

*where $\Xi : [0, \infty) \to [0, \infty)$ such that*

*(i) $\Xi$ is non decreasing,*

*(ii) $\Xi$ is continuous,*

*(iii) $\Xi^{-1}(0) = 0$.*

*Then $\Psi$ possess a unique fixed point in $\Sigma$.*

*Proof.* Let $\xi_0 \in \Sigma$. Consider $\xi_n = \Psi\xi_{n-1} = \Psi^n\xi_0$, n = 1,2,.... Also,
$\mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_n, \lambda) \geq \mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_{n+1}, \lambda)$

$$\begin{aligned}\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_{n+1}, \lambda) &= \Xi(\mathcal{P}_{\mathcal{M}}(F\Psi\xi_{n-1}, F\Psi\xi_n, \lambda)\\ &\geq \delta[\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_{n-1}, F\xi_{n+1}, \lambda) + \Xi(\mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_n, \lambda)]\\ &\geq \delta[\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_{n-1}, F\xi_{n+1}, \lambda) + \Xi(\mathcal{P}_{\mathcal{M}}(F\xi_{n+1}, F\xi_n, \lambda)],\end{aligned} \tag{3.6.2}$$

therefore we have,

$$\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_n, F\xi_{n+1}, \lambda) \geq \left(\frac{\delta}{1-\delta}\right)\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_{n-1}, F\xi_{n+1}, \lambda)).$$

Consider, for all $m(\lambda), n(\lambda) \in \mathbb{N}$ where $m(\lambda) > n(\lambda)$, we get

$$\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_{m(\lambda)}, F\xi_{n(\lambda)}, \lambda)) \geq \left(\frac{\delta}{1-\delta}\right)^{n(\lambda)}\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_{m(\lambda)-n(\lambda)}, F\xi_{n(\lambda)}, \lambda)). \tag{3.6.3}$$

As $F$ converges subsequently, thus there exists $\omega \in \Sigma$, so that

$$\lim_{\lambda\to\infty} \mathcal{P}_{\mathcal{M}}(\xi_{n(\lambda)}, \omega, \lambda) = \lim_{\lambda\to\infty} \mathcal{P}_{\mathcal{M}}(\omega, \omega, \lambda).$$

Let $\lambda \to \infty$ in (3.6.3), we obtain that

$$\Xi(\mathcal{P}_{\mathcal{M}}(F\xi_{m(\lambda)}, F\xi_{n(\lambda)}, \lambda)) \to 0 \;\; as \;\; \lambda \to \infty. \tag{3.6.4}$$

The inequality (3.6.4) implies that

$$\mathcal{P}_{\mathcal{M}}(F\xi_{m(\lambda)}, F\xi_{n(\lambda)}, \lambda)) = 0.$$

Therefore, we obtain $\{F\xi_n\}$ is a Cauchy sequence in a complete PFMS $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ and there exist $\omega \in \Sigma$ such that $\omega$ is a unique fixed point of $\Psi$.

**Corollary 3.4** *Let $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ be a complete PFMS and $F, \Psi : \Sigma \to \Sigma$ such that $F$ is one to one, continuous and subsequently convergent. If $\xi, \nu \in \Sigma$ and for all $\delta \in [0, \frac{1}{2})$, we have*

$$\mathcal{P}_{\mathcal{M}}(F\Psi\xi, F\Psi\nu, \lambda) \geq \delta[\mathcal{P}_{\mathcal{M}}(F\xi, F\Psi\nu, \lambda) + \mathcal{P}_{\mathcal{M}}(F\nu, F\Psi\xi, \lambda)].$$

*Then, $\Psi$ has a unique fixed point in $\Sigma$. The iterative sequence $\{\Psi^n\xi_0\}$ converges to the fixed point.*

**Corollary 3.5** *Let $(\Sigma, \mathcal{P}_{\mathcal{M}}, \star)$ be a complete PFMS and $\Psi : \Sigma \to \Sigma$ such that $F$ is one to one, continuous and subsequently convergent. If $\xi, \nu \in \Sigma$ and for all $\delta \in [0, \frac{1}{2})$, we have*

$$\Xi(\mathcal{P}_{\mathcal{M}}(\Psi\xi, \Psi\nu, \lambda)) \geq \delta[\Xi(\mathcal{P}_{\mathcal{M}}(\xi, \Psi\nu, \lambda)) + \Xi(\mathcal{P}_{\mathcal{M}}(\nu, \Psi\xi, \lambda))],$$

*where $\Xi : [0, \infty) \to [0, \infty)$ is non decreasing continuous and $\Xi^{-1}(0) = \{0\}$, then $\Psi$ possess a unique fixed point in $\Sigma$.*

## 4. Conclusion

In this manuscript, we have proved $F_\Psi$-contractive mappings in PFMS and proved suitable examples. Generalizations of FPT of Kannan and Chatterjea are extended to PFMS.

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*S. Sowndrarajan,*

*PG and Research Department of Mathematics,*

*Arignar Anna Govt. Arts College, Namakkal,*

*Affiliated to Periyar University, Salem,*

*Tamil Nadu, India.*

*E-mail address:* sowndariitm@gmail.com

*and*

*Vishal Gupta,*

*Department of Mathematics,*

*MMEC, Maharishi Markandeshwar (Deemed to be University),*

*Mullana, Haryana, India.*

*E-mail address:* vishal.gmn@gmail.com, vgupta@mmumullana.org

*and*

*M. Jeyaraman,*
*PG and Research Department of Mathematics,*
*Raja Doraisingam Govt. Arts College, Sivagangai,*
*Affiliated to Alagappa University, Karaikudi,*
*Tamil Nadu, India.*
*E-mail address:* jeya.math@gmail.com