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Pairwise Neutrosophic b-Locally Open Set in Neutrosophic Bi-topological space

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ABSTRACT: The main aim of this article is to procure the notion of pairwise neutrosophic locally open set (in short PNLO-set), pairwise neutrosophic locally closed set (in short PNLC-set), pairwise neutrosophic b-locally open set(in short PN-b-LO-set), pairwise neutrosophic b-locally closed set (in short PN-b-LC-set), PN-b-LO*-set, PN-b-LC*-set, PN-b-LO**-set and PN-b-LC**-set via neutrosophic bi-topological space (in short NBTS), and investigate several properties of these classes of sets. Besides, we establish some remarks, lemmas, theorems, etc. on these classes of sets via NBTS. Further, we furnish few illustrative examples.

Key Words: PN-b-LO-Set, PN-b-LO*-Set, PN-b-LO**-Set, NT-Space, NBTS.

Contents

1. Introduction

In the year 1998, Smarandache [29] presented the notion of neutrosophic set (in short N-Set) as a generalization of fuzzy set (in short F-Set) theory and intuitionistic F-Set (in short IF-Set) theory. Till now, many researchers around the globe applied the notion of N-Set and its extensions in the area of theoretical research ([4], [5], [6], [9], [10], [11], [12], [14], [17], [18], [20], [21], [22], [24], [30], [32], etc.) as well as in the area of multi-criteria decision-making strategy ([7], [8], [19], etc.). In the year 2012, Salama and Alblowi [26] grounded the idea of neutrosophic topology (in short N-topology) on N-Sets, and established the concept of neutrosophic topological space (in short NT-space) as an extension of fuzzy topological space [2] and intuitionistic fuzzy topological space [3]. Afterwards, Salama and Alblowi [27] studied the notion of generalized N-Set and generalized NT-space. In the year 2014, Salama et al. [28] established the notion of neutrosophic closed set and neutrosophic continuous function via NT-space. Thereafter, the concept of neutrosophic semi open (in short NSO) function via NT-space was studied by Arokiarani et al. [1] in the year 2017. Afterwards, Rao and Srinivasa [25] grounded the notion of neutrosophic pre open (in short NPO) set and neutrosophic pre closed (in short NPC) set via NT-space. The concept of neutrosophic semi-closed set and neutrosophic semi-open set via NT-spaces was first grounded by Iswaraya and Bageerathi [15] in the year 2016. The idea of generalized neutrosophic closed sets via NT-spaces was presented by Dhavaseelan and Jafari [12]. Dhavaseelan et al. [13] established the notion of neutrosophic α^m-continuity via NT-space. In the year 2018, Sreeja and Sarankumar [30] established the notion of generalized α-closed sets via NT-space. The notion of neutrosophic generalized closed sets via NT-space was first grounded by Pushpalatha and Nandhini [24] in the year 2019. Afterwards, Ebenanjar et al. [14] established the notion of neutrosophic b-open (in short N-b-O) set in NT-spaces. In the year 2020, Page and Imran [22] studied the notion of neutrosophic generalized homeomorphism via NT-spaces. The idea of neutrosophic generalized b-closed sets via NT-spaces was first grounded by Maheswari et al. [18]. Afterwards, Maheswari and Chandrasekar [17] studied the concept of neutrosophic gb continuity via NT-spaces. Later on, the concept of generalized N-b-O sets via NT-spaces was presented by Das and Pramanik [4]. Das and Pramanik [5] further grounded the concept of neutrosophic ϕ-open sets and neutrosophic ϕ-continuous functions via NT-space. Later on, the idea of neutrosophic simply b-open sets via NT-space was grounded by Das and Tripathy [10]. Recently, Das and Tripathy [11] presented the notion of neutrosophic b-locally open set via NT-space. In the year 1963, Kelly [16] introduced the concept of bitopological space. In the year 2011, Tripathy and Sarma [33] introduced the notion of b-locally open sets via bitopological spaces. The idea of pairwise b-locally open function

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and pairwise b-locally closed function via bitopological spaces was presented by Tripathy and Sarma [34]. In the year 2013, Tripathy and Sarma [35] further grounded the notion of weakly b-continuous mapping via bi-topological spaces. In the year 2017, Tripathy and Debnath [31] presented the notion of fuzzy b-locally open set via fuzzy bitopological spaces. Later on, Paul et al. [23] studied the concept of locally closed set in fuzzy bitopological spaces. Afterwards, Tripathy and Sarma [36] presented the notion of pairwise generalized b-R_0 spaces via bi-topological spaces. In the year 2019, Ozturk and Ozkan [21] grounded the idea of neutrosophic bi-topological space (in short NBTS) by extending the concept of fuzzy bi-topological spaces. Later on, Das and Tripathy [9] presented the idea of pairwise N-b-O sets via NBTSs, and studied their different properties. In the year 2020, Mwchahary and Basumatary [20] further studied the notion of NBTSs. Later on, Tripathy and Das [32] presented the notion of pairwise neutrosophic b-continuous mapping via NBTSs.

Research gap: No study on PNLO-set, PNLC-set, PN-b-LO-set, PN-b-LC-set, PN-b-LO*-set, PN-b-LC*set, PN- b-LO**-set and PN-b-LC**-set via NBTSs has been reported in the recent literature.

Motivation: To fill the research gap, we introduce the notion of PNLO-set, PNLC-set, PN-b-LO-set, PN-b-LC-set, PN-b-LO*-set, PN-b-LC*set, PN- b-LO**-set and PN-b-LC**-set via NBTSs.

The remaining part of the article has been divided into the following sections:

Section-2 presents some relevant preliminaries and definitions on NS, NT-space and NBTS. In section-3, we introduce the concept of PNLO-set, PNLC-set, PN-b-LO-set, PN-b-LC-set, PN-b-LO*-set, PN-b-LC*set, PN-b-LO**-set and PN-b-LC**-set via NBTSs. Besides, we introduce the notion of PNL-C-mapping, PN-b-L-Cmapping, PN-b-L*-C-mapping and PN-b-L**-C-mapping via NBTS. Further, we formulate some interesting results in the form of theorems, propositions, remarks, etc. via NBTSs. Finally, in section-4, we conclude the paper by stating some future directions of research.

2. Some Relevant Results:

In this section, we provide some basic definitions and results on N-Set, NT-space and NBTS those are very helpful for the preparation of the main results of this article.

Definition 2.1 *[29] An N-Set Y over a fixed set W is defined as follows:*

$$Y = \{(e, T_Y(e), I_Y(e), F_Y(e)) : e \in W\},$$

where T_Y, I_Y and F_Y denotes the truth, indeterminacy and false membership functions respectively from W to the unit interval [0,1] such that

$$0 \leq T_Y(e) + I_Y(e) + F_Y(e) \leq 3, \forall e \in W.$$

Example 2.1 Suppose that $W = \{e, d\}$ be a fixed set. Then, Y=$\{(e, 0.6, 0.6, 0.6),\ (d, 0.5, 0.3, 0.3)\}$ is an N-Set over W.

Definition 2.2 *[29] Suppose that W be a fixed set. Then, the null N-set (0_W) and whole N-set (1_W) over W are defined as follows:*

$$0_W = \{(e, 0, 0, 1) : e \in W\} \& 1_W = \{(e, 1, 0, 0) : e \in W\}.$$

Clearly, $0_W \subseteq Y \subseteq 1_W$, for every N-Set Y over W.

The N-Sets 0_W and 1_W also has three other representations. They are given as follows:
(i) $0_W = \{(e, 0, 0, 0) : e \in W\} \& 1_W = \{(e, 1, 1, 1) : e \in W\}$;
(ii) $0_W = \{(e, 0, 1, 0) : e \in W\} \& 1_W = \{(e, 1, 0, 1) : e \in W\}$;
(iii) $0_W = \{(e, 0, 1, 1) : e \in W\} \& 1_W = \{(e, 1, 1, 0) : e \in W\}$.

Definition 2.3 *[29] Let $Y = \{(e, T_Y(e), I_Y(e), F_Y(e)) : e \in W\}$ and $U = \{(e, T_U(e), I_U(e), F_U(e)) : e \in W\}$ be any two N-Sets over a fixed set W. Then,*
(i) Y^c=$\{(e,\ 1 - T_Y(e),\ 1 - I_Y(e), 1 - F_Y(e)) : e \in W\}$ *and* U^c=$\{(e,\ 1 - T_U(e), 1 - I_U(e), 1 - F_U(e))$: $e \in W\}$;
(ii) $Y \subseteq U$ *iff* $T_Y(e) \leq T_U(e), I_Y(e) \geq I_U(e), F_Y(e) \geq F_U(e), \forall e \in W$;
(iii) $Y \cap U = \{(e, \min\{T_Y(e), T_U(e)\}, \max\{I_Y(e), I_U(e)\}, \max\{F_Y(e), F_U(e)\}) : e \in W\}$;
(iv) $Y \cup U = \{(e, \max\{T_Y(e), T_U(e)\}, \min\{I_Y(e), I_U(e)\}, \min\{F_Y(e), F_U(e)\}) : e \in W\}$.

Definition 2.4 *[26] A collection τ of N-Sets over a fixed set W is called an neutrosophic topology (in short N-topology) on W if and only if the following axioms hold:*
(i) $0_W, 1_W \in \tau$;
(ii) $Y_1, Y_2 \in \tau \Rightarrow Y_1 \cap Y_2 \in \tau$;
(iii) $\{Y_i : i \in \Delta\} \subseteq \tau \Rightarrow \cup_{i\in\Delta} Y_i \in \tau$.
Then, the structure (W, τ) is called an NT-space. Suppose that $Y \in \tau$, then Y is called an neutrosophic open set (in short NOS) and its complement i.e., Y^c is called an neutrosophic closed set (in short NCS) in (W, τ).

Definition 2.5 *[11] Let (W, τ) be an NT-space. An N-Set G over W is called an neutrosophic locally open (in short NLO) set if $G = H \cup K$, where H is an neutrosophic open set and K is an neutrosophic closed set in (W, τ).*

Definition 2.6 *[11] In an NT-space (W, τ), an N-Set G is called an neutrosophic b-locally open (in short N-b-LO) set if there exist an neutrosophic b-open set H and an neutrosophic b-closed set K in W such that $G = H \cup K$.*

Definition 2.7 *[11] In an NT-space (W, τ), an N-Set G is called an N-b-LO* set if $G = H \cup K$, where H is an neutrosophic b-open set and K is an neutrosophic closed set in (W, τ).*

Definition 2.8 *[11] In an NT-space (W, τ), an N-Set G is called an N-b-LO** set if $G = H \cup K$, where H is an neutrosophic open set and K is an neutrosophic b-closed set in (W, τ).*

Definition 2.9 *[21] Assume that (W, τ_1) and (W, τ_2) be two different NT-spaces. Then, the triplet (W, τ_1, τ_2) is called an neutrosophic bi-topological space (in short NBTS).*

Definition 2.10 *[9] Assume that (W, τ_1, τ_2) be an NBTS. Then, an N-Set Y is called τ_{ij} N-b-O set if and only if $Y \subseteq N^i_{cl} N^j_{int}(Y) \cup N^j_{int} N^i_{cl}(Y)$.*

Definition 2.11 *[9] Assume that (W, τ_1, τ_2) be an NBTS. Then, an N-Set Y is called τ_{ij} NSO set if and only if $Y \subseteq N^i_{cl} N^j_{int}\ (Y)$.*

Definition 2.12 *[9] Assume that (W, τ_1, τ_2) be an NBTS. Then, an N-Set Y is called τ_{ij} NPO set if and only if $Y \subseteq N^j_{int} N^i_{cl}(Y)$.*

Theorem 2.1 *[9] Assume that (W, τ_1, τ_2) be an NBTS. Then,*
(i) every NOS in $(W, \tau_j)\,(j = 1, 2)$ is a τ_{ij} NSO set in (W, τ_1, τ_2);
(ii) every NOS in $(W, \tau_j)\,(j = 1, 2)$ is a τ_{ij} NPO set in (W, τ_1, τ_2).

Definition 2.13 *[9] Assume that (W, τ_1, τ_2) be an NBTS. Then, an N-Set Y over W is called a pairwise NOS (in short PNOS) in the NBTS (W, τ_1, τ_2) if there exist an NOS Y_1 in τ_1 and another NOS Y_2 in τ_2 such that $Y = Y_1 \cup Y_2$.*

Definition 2.14 *[9] Assume that (W, τ_1, τ_2) be an NBTS. Then, an N-Set Y is called a pairwise NSO set (in short PNSO-set) in the NBTS (W, τ_1, τ_2) if $Y = U \cup V$, where U is a τ_{ij} NSO set and V is a τ_{ji} NSO set in (W, τ_1, τ_2).*

Definition 2.15 *[9] Assume that (W, τ_1, τ_2) be an NBTS. Then, an N-Set Y is called a pairwise NPO set (in short PNPO-set) in the NBTS (W, τ_1, τ_2) if $Y = U \cup V$, where U is a τ_{ij} NPO set and V is a τ_{ji} NPO set in (W, τ_1, τ_2).*

Definition 2.16 *[9] Suppose that (W, τ_1, τ_2) be an NBTS. Then, an N-Set Y is said to be a pairwise N-b-O set (in short PN-b-OS) in (W, τ_1, τ_2) if $Y = H \cup V$, where H is a τ_{ij} N-b-O set and V is a τ_{ji} N-b-O set in (W, τ_1, τ_2).*
In that case, its complement i.e., Y^c is said to be a pairwise neutrosophic b-closed set (in short PN-b-CS) in (W, τ_1, τ_2).

Theorem 2.2 *[9] Suppose that (W, τ_1, τ_2) be an NBTS. Then,*
(i) every PNOS is a PNSO-set;
(ii) every PNOS is a PNPO-set.

Theorem 2.3 *[9] In an NBTS (W, τ_1, τ_2),*
(i) every τ_{ij} N-b-O set is a PN-b-OS;
(ii) every PNSO-set is also a PN-b-OS;
(iv) every PNPO-set is also a PN-b-OS.

Throughout the article, we denote the followings:
(a) τ_i-neutrosophic open set in (W, τ_1, τ_2) = neutrosophic open set in the NT-space (W, τ_i), $i = 1, 2$;
(b) τ_i-neutrosophic closed set in (W, τ_1, τ_2) = neutrosophic closed set in the NT-space (W, τ_i), $i = 1, 2$.

3. Pairwise Neutrosophic b-Locally Open Set:

In this section, we procure the notion of PNLO-set, PN-b-LO-set, PN-b-LO*-set and PN-b-LO**-set via NBTSs, and established several interesting results on them.

Definition 3.1 *Suppose that (W, τ_1, τ_2) be an NBTS. Then, an N-Set Y over W is called a pairwise neutrosophic locally open set (in short PNLO-set) if $Y = A \cup B$, where A is a τ_1-neutrosophic open set and B is a τ_2-neutrosophic closed set in (W, τ_1, τ_2). In that case, the complement of Y i.e., Y^c is called a pairwise neutrosophic locally closed set (in short PNLC-set) in (W, τ_1, τ_2).*

Example 3.1 Let $W = \{e, d\}$. Then, τ_1=$\{0_N,\ 1_N,\ A_1,\ A_2\}$, τ_2=$\{0_N,\ 1_N,\ B_1,\ B_2\}$ be two different N-topologies defined on W, where A_1=$\{(e, 0.4, 0.5, 0.4),\ (d, 0.7, 0.2, 0.3)\}$, A_2=$\{(e, 0.8, 0.4, 0.1),\ (d, 0.8, 0.2, 0.2)\}$, B_1=$\{(e, 0.8, 0.1, 0.2),\ (d, 0.6, 0.2, 0.3)\}$, B_2=$\{(e, 0.4, 0.3, 0.4),\ (d, 0.4, 0.3, 0.5)\}$. Therefore, (W, τ_1, τ_2) is an NBTS. Clearly, both the N-Sets Y_1=$A_1 \cup (B_2)^c$=$\{(e, 0.6, 0.5, 0.4),\ (d, 0.7, 0.2, 0.3)\}$ and Y_2=$A_2 \cup (B_1)^c$=$\{(e, 0.8, 0.4, 0.1),\ (d, 0.8, 0.2, 0.2)\}$ are PNLO-sets in (W, τ_1, τ_2), and the N-Sets $(Y_1)^c$=$\{(e, 0.4, 0.5, 0.6),\ (d, 0.3, 0.8, 0.7)\}$ and $(Y_2)^c$=$\{(e, 0.2, 0.6, 0.9),\ (d, 0.2, 0.8, 0.8)\}$ are PNLC-sets in (W, τ_1, τ_2).

Remark 3.1 *In an NBTS (W, τ_1, τ_2), the null N-Set 0_N and whole N-Set 1_N are both PNLO-set and PNLC-set.*

Theorem 3.1 *Let (W, τ_1, τ_2) be an NBTS. If Y and R be any two PNLO-sets, then $Y \cup R$ is also a PNLO-set in (W, τ_1, τ_2).*

Proof: Let Y and R be two PNLO-sets in (W, τ_1, τ_2). Then, there exist two τ_1-neutrosophic open sets Y_1, R_1 and two τ_2-neutrosophic closed sets Y_2, R_2 such that $Y = Y_1 \cup Y_2$ and $R = R_1 \cup R_2$. Now, we have $Y \cup R = (Y_1 \cup Y_2) \cup (R_1 \cup R_2) = (Y_1 \cup R_1) \cup (Y_2 \cup R_2)$, where $(Y_1 \cup R_1)$ is a τ_1-neutrosophic open set and $(Y_2 \cup R_2)$ is a τ_2-neutrosophic closed set in (W, τ_1, τ_2). Hence, $Y \cup R$ is a PNLO-set in (W, τ_1, τ_2). □

Definition 3.2 *Suppose that (W, τ_1, τ_2) be an NBTS. Then, an N-Set Y over W is called a pairwise neutrosophic b-locally open set (in short PN-b-LO-set) if $Y = A \cup B$, where A is a τ_1-neutroisophic b-open set and B is a τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). In that case, the complement of Y i.e., Y^c is called a pairwise neutrosophic b-locally closed set (in short PN-b-LC-set) in (W, τ_1, τ_2).*

Example 3.2 Let us consider an NBTS (W, τ_1, τ_2) as shown in Example 3.1. Clearly, $P_1 = \{(e, 0.5, 0.6, 0.7),\ (d, 0.6, 0.7, 0.8)\}$ is a τ_1-neutroisophic b-open set and P_2=$\{(e, 0.1, 0.6, 0.6), (d, 0.3, 0.8, 0.6)\}$ is a τ_2-neutroisophic b closed set in (W, τ_1, τ_2). Then, the N-Set P=$P_1 \cup P_2$=$\{(e, 0.5, 0.6, 0.6),\ (d, 0.6, 0.7, 0.6)\}$ is a PN-b-LO-set in (W, τ_1, τ_2) and its complement i.e., P^c=$\{(e, 0.5, 0.4, 0.4),\ (d, 0.4, 0.3, 0.4)\}$ is a PN-b-LC-set in (W, τ_1, τ_2).

Remark 3.2 *In an NBTS (W, τ_1, τ_2), the null N-Set 0_N and whole N-Set 1_N are both PN-b-LO-set and PN-b-LC-set.*

Theorem 3.2 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y and R be any two PN-b-LO-sets, then $Y \cup R$ is also a PN-b-LO-set in (W, τ_1, τ_2).*

Proof: Assume that Y and R be any two PN-b-LO-sets in (W, τ_1, τ_2). Then, there exist two τ_1-neutrosophic b-open sets Y_1, R_1 and two τ_2-neutrosophic b-closed sets Y_2, R_2 such that $Y = Y_1 \cup Y_2$ and $R = R_1 \cup R_2$. Now, we have $Y \cup R = (Y_1 \cup Y_2) \cup (R_1 \cup R_2) = (Y_1 \cup R_1) \cup (Y_2 \cup R_2)$, where $(Y_1 \cup R_1)$ is a τ_1-neutrosophic b-open set and $(Y_2 \cup R_2)$ is a τ_2-neutrosophic b-open set in (W, τ_1, τ_2). Hence, $Y \cup R$ is a PN-b-LO-set in (W, τ_1, τ_2). □

Definition 3.3 *Suppose that (W, τ_1, τ_2) be an NBTS. Then, an N-Set Y over W is called a PN-b-LO*-set if there exist a τ_1-neutrosophic b-open set A and a τ_2-neutrosophic closed set B in (W, τ_1, τ_2) such that $Y = A \cup B$.*

Example 3.3 Suppose (W, τ_1, τ_2) be an NBTS as shown in Example 3.1. Then, $P_1 = \{(e, 0.5, 0.6, 0.7), (d, 0.6, 0.7, 0.8)\}$ is a τ_1-neutrosophic b-open set and $C = (B_1)^c = \{(e, 0.2, 0.9, 0.8), (d, 0.4, 0.8, 0.7)\}$ is a τ_2-neutrosophic closed set in (W, τ_1, τ_2). Hence, D = P$_1$ ∪ C $= \{(e, 0.5, 0.6, 0.7), (d, 0.6, 0.7, 0.7)\}$ is a PN-b-LO*-set in (W, τ_1, τ_2), and its complement i.e., $D^c = \{(e, 0.5, 0.4, 0.3), (d, 0.4, 0.3, 0.3)\}$ is a PN-b-LC*-set in (W, τ_1, τ_2).

Remark 3.3 *In an NBTS $(W,\ \tau_1, \tau_2)$, the null N-Set (0_N) and whole N-Set (1_N) are both PN-b-LO*-set and PN-b-LC*-set.*

Theorem 3.3 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y and R be any two PN-b-LO*-sets, then $Y \cup R$ is also a PN-b-LO*-set in (W, τ_1, τ_2).*

Proof: Assume that Y and R be any two PN-b-LO*-sets in an NBTS (W, τ_1, τ_2). Then, there exist two τ_1-neutrosophic b-open sets Y_1, R_1 and two τ_2-neutrosophic closed sets Y_2, R_2 such that $Y = Y_1 \cup Y_2$ and $R = R_1 \cup R_2$. Now, we have $Y \cup R = (Y_1 \cup Y_2) \cup (R_1 \cup R_2) = (Y_1 \cup R_1) \cup (Y_2 \cup R_2)$, where $(Y_1 \cup R_1)$ is a τ_1-neutrosophic b-open set and $(Y_2 \cup R_2)$ is a τ_2-neutrosophic closed set in (W, τ_1, τ_2). Hence, $Y \cup R$ is a PN-b-LO*-set in (W, τ_1, τ_2). □

Definition 3.4 *Suppose that (W, τ_1, τ_2) be an NBTS. Then, an N-Set Y over W is called a PN-b-LO**-set if there exist a τ_1-neutrosophic open set A and a τ_2-neutrosophic b-closed set B of (W, τ_1, τ_2) such that $Y = A \cup B$.*

Example 3.4 Suppose (W, τ_1, τ_2) be an NBTS as shown in Example 3.1. Clearly, $A_1 = \{(e, 0.4, 0.5, 0.4), (d, 0.7, 0.2, 0.3)\}$ is a τ_1-neutrosophic open set and $P_2 = \{(e, 0.1, 0.6, 0.6), (d, 0.3, 0.8, 0.6)\}$ is a τ_2-neutrosophic b closed set in (W, τ_1, τ_2). Hence, $E = A_1 \cup P_2 = \{(e, 0.4, 0.5, 0.4), (d, 0.7, 0.2, 0.3)\}$ is a PN-b-LO**-set in (W, τ_1, τ_2), and its complement i.e., $D^c = \{(e, 0.6, 0.5, 0.6), (d, 0.3, 0.8, 0.7)\}$ is a PN-b-LC**-set in (W, τ_1, τ_2).

Remark 3.4 *In an NBTS (W, τ_1, τ_2), the null N-Set (0_N) and whole N-Set (1_N) are both PN-b-LO**-set and PN-b-LC**-set.*

Theorem 3.4 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y and R be any two PN-b-LO**-sets, then $Y \cup R$ is also a PN-b-LO**-setin (W, τ_1, τ_2).*

Proof: Assume that Y and R be any two PN-b-LO**-sets in an NBTS (W, τ_1, τ_2). Then, there exist two τ_1-neutrosophic open sets Y_1, R_1 and two τ_2-neutrosophic b-closed sets Y_2, R_2 such that $Y = Y_1 \cup Y_2$ and $R = R_1 \cup R_2$. Now, we have $Y \cup R = (Y_1 \cup Y_2) \cup (R_1 \cup R_2) = (Y_1 \cup R_1) \cup (Y_2 \cup R_2)$, where $(Y_1 \cup R_1)$ is a τ_1-neutrosophic open set and $(Y_2 \cup R_2)$ is a τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). Hence, $Y \cup R$ is a PN-b-LO**-set in (W, τ_1, τ_2). □

Theorem 3.5 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y is a PNLO-set in (W, τ_1, τ_2), then it is also a PN-b-LO set in (W, τ_1, τ_2).*

Proof: Suppose that (W, τ_1, τ_2) be an NBTS. Let Y be a PNLO-set in (W, τ_1, τ_2). So there exists a τ_1-neutrosophic open set A and τ_2-neutrosophic closed set B such that $Y = A \cup B$. Since every τ_1-neutrosophic open set is a τ_1-neutrosophic b-open set, so A is a τ_1-neutrosophic b-open set in (W, τ_1, τ_2). Similarly, since every τ_2-neutrosophic closed set is a τ_2-neutrosophic b-closed set, so B is a τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). Thus, we have $Y = A \cup B$, where A is a τ_1-neutrosophic b-open set and B is a τ_2-neutrosophic closed set in (W, τ_1, τ_2). Hence, Y is a PN-b-LO-set in (W, τ_1, τ_2). □

Remark 3.5 *If Y is a PNLC-set in (W, τ_1, τ_2), then it is also a PN-b-LC-set in (W, τ_1, τ_2).*

Remark 3.6 *The converse of Theorem 3.5 may not be true in general. This follows from the following example.*

Example 3.5 Let us consider an NBTS (W, τ_1, τ_2) as shown in Example 3.1. Clearly, the N-set $P = \{(e, 0.5, 0.6, 0.6), (d, 0.6, 0.7, 0.6)\}$ is a PN-b-LO-set in (W, τ_1, τ_2). But, it is not a PNLO-set in (W, τ_1, τ_2).

Theorem 3.6 *If Y is a PNLO-set in an NBTS (W, τ_1, τ_2), then it is also a PN-b-LO*-set in (W, τ_1, τ_2).*

Proof: Let Y be a PNLO-set in an NBTS (W, τ_1, τ_2). So there exists a τ_1-neutrosophic open set A and τ_2-neutrosophic closed set B such that $Y = A \cup B$. Since every τ_1-neutrosophic open set is a τ_1 neutrosophic b-open set, so A is a τ_1-neutrosophic b-open set in (W, τ_1, τ_2). Thus, we have $Y = A \cup B$, where A is a τ_1-neutrosophic b-open set and B is a τ_2-neutrosophic closed set in (W, τ_1, τ_2). Hence, Y is a PN-b-LO*-set in (W, τ_1, τ_2). □

Remark 3.7 *If Y is a PNLC-set in (W, τ_1, τ_2), then it is also a PN-b-LC*-set in (W, τ_1, τ_2).*

Remark 3.8 *The converse of Theorem 3.6 may not be true in general. This follows from the following example.*

Example 3.6 Let us consider an NBTS (W, τ_1, τ_2) as shown in Example 3.1. Clearly, the N-set $D = \{(e, 0.5, 0.6, 0.7), (d, 0.6, 0.7, 0.7)\}$ is an PN-b-LO*-set in (W, τ_1, τ_2), which is shown in Example 3.3. But, D is not a PNLO-set in (W, τ_1, τ_2).

Theorem 3.7 *If Y is a PNLO-set in an NBTS (W, τ_1, τ_2), then it is also a PN-b-LO**-set in (W, τ_1, τ_2).*

Proof: Suppose that (W, τ_1, τ_2) be an NBTS. Let Y be a PNLO-set in (W, τ_1, τ_2). So there exists a τ_1-neutrosophic open set A and τ_2-neutrosophic closed set B such that $Y = A \cup B$. Since every τ_2-neutrosophic closed set is a τ_2 neutrosophic b-closed set, so B is a τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). Thus, we have $Y = A \cup B$, where A is a τ_1-neutrosophic open set and B is a τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). Hence, Y is a PN-b-LO**-set in (W, τ_1, τ_2). □

Remark 3.9 *If Y is a PNLC-set in (W, τ_1, τ_2), then it is also a PN-b-LC**-set in (W, τ_1, τ_2).*

Remark 3.10 *The converse of Theorem 3.7 may not be true in general. This follows from the following example.*

Example 3.7 Let us consider an NBTS (W, τ_1, τ_2) as shown in Example 3.1. Suppose that $Q = \{(e, 0.7, 0.5, 0.4),\ (d0.8, 0.2, 0.3)\}$ be an N-set defined over W. Now, one can express $Q = P \cup A_1$, where $P = \{(e, 0.7, 0.7, 0.7),\ (d, 0.8, 0.8, 0.8)\}$ is a τ_2-neutrosophic b-closed set and A_1 is a τ_1-neutrosophic open set in (W, τ_1, τ_2). Hence, the N-set Q is a PN-b-LO**-set in (W, τ_1, τ_2). But, it is not a PNLO-set in (W, τ_1, τ_2).

Theorem 3.8 *Every PN-b-LO*-set in (W, τ_1, τ_2) is also a PN-b-LO-set in (W, τ_1, τ_2).*

Proof: Suppose that Y be a PN-b-LO*-set in (W, τ_1, τ_2). So, there exists a τ_1-neutrosophic b-open set A and a τ_2 neutrosophic closed set B such that $Y = A \cup B$. Since, every τ_2-neutrosophic closed set is a τ_2-neutrosophic b-closed set in (W, τ_1, τ_2), so B is a τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). Thus, we have $Y = A \cup B$, where A is a τ_1 neutrosophic b-open set and B is a τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). Therefore, Y is a PN-b-LO-set in (W, τ_1, τ_2). □

Remark 3.11 *The converse of Theorem 3.8 is not always true. It follows from the following example.*

Example 3.8 Let us consider an NBTS (W, τ_1, τ_2) as shown in Example 3.1. Then, $P = \{(e, 0.5, 0.6, 0.6), (d, 0.6, 0.7, 0.6)\}$ is a PN-b-LO-set in (W, τ_1, τ_2), which is shown in Example 3.2. But, it is not a PN-b-LO*-set in (W, τ_1, τ_2).

Theorem 3.9 *Suppose that Y be an neutrosophic sub-set of an NBTS (W, τ_1, τ_2). If Y is a PN-b-LO**-set in (W, τ_1, τ_2), then it is a PN-b-LO-set in (W, τ_1, τ_2).*

Proof: Suppose that Y be a PN-b-LO**-set in (W, τ_1, τ_2). So, there exists a τ_1-neutrosophic open set A and a τ_2-neutrosophic b-closed set B such that $Y = A \cup B$. Since, every τ_1-neutrosophic open set is a τ_1-neutrosophic b-open set in (W, τ_1, τ_2), so A is a τ_1-neutrosophic b-open set in (W, τ_1, τ_2). Thus, we have $Y = A \cup B$, where A is a τ_1-neutrosophic b-open set and B is a τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). Therefore, Y is a PN-b-LO-set in (W, τ_1, τ_2). □

Remark 3.12 *The converse of Theorem 3.9 is not always true. It follows from the following example.*

Example 3.9 Let us consider an NBTS (W, τ_1, τ_2) as shown in Example 3.1. Then, $P = \{(e, 0.5, 0.6, 0.6), (d, 0.6, 0.7, 0.6)\}$ is a PN-b-LO-set in (W, τ_1, τ_2), which is shown in Example 3.2. But, it is not a PN-b-LO**-set in (W, τ_1, τ_2).

Theorem 3.10 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y is a PNLO-set and R is a τ_1-neutrosophic b-open set in (W, τ_1, τ_2), then $Y \cup R$ is also a PN-b-LO-set in (W, τ_1, τ_2).*

Proof: Let Y be a PNLO-set and R be a τ_1-neutrosophic b-open set in an NBTS (W, τ_1, τ_2). Since Y is a PNLO-set, so there exists a τ_1-neutrosophic open set Y_1 and a τ_2-neutrosophic closed set Y_2 such that $Y = Y_1 \cup Y_2$. Since, every τ_1-neutrosophic open set is also a τ_1-neutrosophic b-open set, so Y_1 is a τ_1-neutrosophic b-open set in (W, τ_1, τ_2). Therefore, we have $Y \cup R = (Y_1 \cup Y_2) \cup R = (Y_1 \cup R) \cup Y_2$. Since, the union of two τ_1-neutrosophic b-open sets in (W, τ_1, τ_2) is also a τ_1-neutrosophic b-open set, so $Y_1 \cup R = E$ (say) is a τ_1-neutrosophic b-open set in (W, τ_1, τ_2). Hence, $Y \cup R = E \cup Y_2$, where E is a τ_1-neutrosophic b-open set and Y_2 is a τ_2-neutrosophic closed set in (W, τ_1, τ_2). □

Lemma 3.1 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y is a PNLO-set and R is a τ_1-neutrosophic open set in (W, τ_1, τ_2), then $Y \cup R$ is also a PN-b-LO-set in (W, τ_1, τ_2).*

Theorem 3.11 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y is a PNLO-set and R is both τ_1-neutrosophic b-open set and τ_2-neutrosophic b-closed set in (W, τ_1, τ_2), then $Y \cap R$ is also a PN-b-LO-set in (W, τ_1, τ_2).*

Proof: Suppose that (W, τ_1, τ_2) be an NBTS. Suppose that Y be a PNLO-set and R be both τ_1-neutrosophic b-open set and τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). Since Y is a PNLO-set, so there exists a τ_1-neutrosophic open set Y_1 and a τ_2-neutrosophic closed set Y_2 such that $Y = Y_1 \cup Y_2$. Now, we have $Y \cap R = (Y_1 \cup Y_2) \cap R = (Y_1 \cap R) \cup (Y_2 \cap R)$. Since, R is both τ_1-neutrosophic b-open set and τ_2-neutrosophic b-closed set in (W, τ_1, τ_2), then $(Y_1 \cap R) = E$ (say) is τ_1-neutrosophic b-open set and $(Y_2 \cap R) = Q$ (say) is τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). Therefore,we have $Y \cap R = E \cup Q$, where E is a τ_1-neutrosophic b-open set and Q is a τ_2-neutrosophic b-closed set. Hence, $Y \cap R$ is a PN-b-LO-set in (W, τ_1, τ_2). □

Lemma 3.2 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y is a PNLO-set and R is both τ_1-neutrosophic open set and τ_2-neutrosophic closed set in (W, τ_1, τ_2), then $Y \cap R$ is also a PN-b-LO-set in (W, τ_1, τ_2).*

Theorem 3.12 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y is a PN-b-LO-set and R is a τ_1-neutrosophic b-open set in (W, τ_1, τ_2), then $Y \cup R$ is also a PN-b-LO-set in (W, τ_1, τ_2).*

Proof: Suppose that (W, τ_1, τ_2) be an NBTS. Assume that Y be a PN-b-LO-set and R be a τ_1-neutrosophic b-open set in (W, τ_1, τ_2). Since Y is a PN-b-LO-set, so there exists a τ_1-neutrosophic b-open set Y_1 and a τ_2-neutrosophic b-closed set Y_2 such that $Y = Y_1 \cup Y_2$. Now, we have $Y \cup R = (Y_1 \cup Y_2) \cup R = (Y_1 \cup R) \cup Y_2$. Since, the union of two τ_1 neutrosophic b-open sets in (W, τ_1, τ_2) is also a τ_1-neutrosophic b-open set, so $Y_1 \cup R = S$ (say) is a τ_1-neutrosophic b-open set in (W, τ_1, τ_2). Hence, $Y \cup R = S \cup Y_2$, where S is a τ_1-neutrosophic b-open set and Y_2 is a τ_2-neutrosophic closed set in (W, τ_1, τ_2). □

Lemma 3.3 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y is a PN-b-LO-set and R is a τ_1-neutrosophic open set in (W, τ_1, τ_2), then $Y \cup R$ is also a PN-b-LO-set in (W, τ_1, τ_2).*

Theorem 3.13 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y is a PN-b-LO-set and R is both τ_1-neutrosophic b-open set and τ_2-neutrosophic b-closed set in (W, τ_1, τ_2), then $Y \cap R$ is also a PN-b-LO-set in (W, τ_1, τ_2).*

Proof: Suppose that (W, τ_1, τ_2) be an NBTS. Suppose that Y be a PN-b-LO-setand R be both τ_1-neutrosophic b open set and τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). Since Y is a PN-b-LO-set, so there exists a τ_1 neutrosophic b-open set Y_1 and a τ_2-neutrosophic b-closed set Y_2 such that $Y = Y_1 \cup Y_2$. Now, we have $Y \cap R = (Y_1 \cup Y_2) \cap R = (Y_1 \cap R) \cup (Y_2 \cap R)$. Since, R is both τ_1-neutrosophic b-open set and τ_2-neutrosophic b-closed set in $((W, \tau_1, \tau_2)$, so $(Y_1 \cap R) = E$ (say) is a τ_1-neutrosophic b-open set and $(Y_2 \cap R) = Q$ (say) is a τ_2-neutrosophic b-closed set in (W, τ_1, τ_2). Therefore, we have $Y \cap R = E \cup Q$, where E is a τ_1-neutrosophic b-open set and Q is a τ_2 neutrosophic b-closed set. Hence, $Y \cap R$ is a PN-b-LO-set in (W, τ_1, τ_2). □

Lemma 3.4 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y is a PN-b-LO-setand R is both τ_1-neutrosophic open set and τ_2-neutrosophic closed set in (W, τ_1, τ_2), then $Y \cap R$ is also a PN-b-LO-set in (W, τ_1, τ_2).*

Theorem 3.14 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y be a PN-b-LO*-set and R be a τ_1-neutrosophic open set in (W, τ_1, τ_2), then $Y \cup R$ is also a PN-b-LO*-set in (W, τ_1, τ_2).*

Proof: Suppose that (W, τ_1, τ_2) be an NBTS. Assume that Y be a PN-b-LO*-set and R be a τ_1-neutrosophic open set in (W, τ_1, τ_2). Since Y is a PN-b-LO*-set, so there exists a τ_1-neutrosophic b-open set Y_1 and a τ_2 neutrosophic closed set Y_2 such that $Y = Y_1 \cup Y_2$. Now, we have $Y \cup R = (Y_1 \cup Y_2) \cup R = (Y_1 \cup R) \cup Y_2$. Since, the union of two τ_1-neutrosophic b-open sets in (W, τ_1, τ_2) is also a τ_1-neutrosophic b-open set, so $Y_1 \cup R = S$ (say) is a τ_1-neutrosophic b-open set in (W, τ_1, τ_2). Hence, $Y \cup R = S \cup Y_2$, where S is a τ_1-neutrosophic b-open set and Y_2 is a τ_2-neutrosophic closed set in (W, τ_1, τ_2). Therefore, $Y \cup R$ is a PN-b-LO*-set in (W, τ_1, τ_2). □

Theorem 3.15 *Suppose that (W, τ_1, τ_2) be an NBTS. If Y be a PN-b-LO**-set and R be a τ_1- neutrosophic open set in (W, τ_1, τ_2), then $Y \cup R$ is a PN-b-LO**-set in (W, τ_1, τ_2).*

Proof: Suppose that (W, τ_1, τ_2) be an NBTS. Assume that Y be a PN-b-LO**-set and R be a τ_1-neutrosophic open set in (W, τ_1, τ_2). Since Y is a PN-b-LO**-set, so there exists a τ_1-neutrosophic open set Y_1 and a τ_2 neutrosophic b-closed set Y_2 such that $Y = Y_1 \cup Y_2$. Now, we have $Y \cup R = (Y_1 \cup Y_2) \cup R = (Y_1 \cup R) \cup Y_2$. Since, the union of two τ_1-neutrosophic open sets in (W, τ_1, τ_2) is also a τ_1-neutrosophic open set, so $Y_1 \cup R = S$ (say) is a τ_1 neutrosophic open set in $(W, \tau_1, \tau_2\ )$. Hence, $Y \cup R = S \cup Y_2$, where S is a τ_1-neutrosophic open set and Y_2 is a τ_2 neutrosophic b-closed set in (W, τ_1, τ_2). Therefore, $Y \cup R$ is a PN-b-LO**-set in (W, τ_1, τ_2). □

Theorem 3.16 *Suppose that (W, τ_1, τ_2) be an NBTS. Assume that Y be an N-set in (W, τ_1, τ_2). Then, Y is a PN-b-LO*-set if and only if $Y = R \cup \tau_2$-int (Y), for some τ_1-neutrosophic b-closed set R.*

Proof: Suppose that (W, τ_1, τ_2) be an NBTS. Assume that Y be a PN-b-LO*-set in (W, τ_1, τ_2). Then, Y=$R \cup E$, where R is a τ_1-neutrosophic b-open set and E is a τ_2-neutrosophic closed set in (W, τ_1, τ_2).
Since, $R \subseteq Y$ and τ_2-int $(Y) \subseteq Y$, we have $R\cup$ τ_2-int $(Y) \subseteq Y$........................(1)
Further, since τ_2-int $(Y) \subseteq E$, therefore, $R \cup \tau_2$-int $(Y) \subseteq R \cup E = Y$...........(2)
From eq. (1) and eq. (2), we have, $Y = R \cup \tau_2 - \text{int}(Y)$.
Conversely, let R be a τ_1-neutrosophic b-open set. Clearly, τ_1-int (Y) is a τ_2-neutrosophic open set. Thus there exist a τ_1-neutrosophic b-open set R and a τ_2-neutrosophic open set τ_2-int (Y) such that $Y = R \cup \tau_2$-int (Y). Hence, Y is a PN-b-LO*-set. □

Theorem 3.17 *Suppose that (W, τ_1, τ_2) be an NBTS. Assume that Y be an N-set in (W, τ_1, τ_2). Then, Y is a PN-b-LO**-set if and only if $Y = R \cup \tau_1$-int (Y), for some τ_2-neutrosophic b-closed set R.*

Proof: Assume that Y be a PN-b-LO**-set in an NBTS (W, τ_1, τ_2). Then, $Y = R \cup E$, where R is a τ_2-neutrosophic b-open set and E is a τ_1-neutrosophic closed set in (W, τ_1, τ_2).
Since, $R \subseteq Y$ and τ_1-int $(Y) \subseteq Y$, we have $R \cup \tau_1$-int $(Y) \subseteq Y$..................(3)
Further, since τ_1-int $(Y) \subseteq E$, therefore, $R \cup \tau_1$-int $(Y) \subseteq R \cup E = Y$.........(4)
From eq. (3) and eq. (4), we have, $Y = R \cup \tau_1$-int (Y).
Conversely, let R be a τ_2-neutrosophic b-open set. Clearly, τ_2-int (Y) is a τ_1-neutrosophic open set. Thus there exist a τ_2-neutrosophic b-open set R and a τ_1-neutrosophic open set τ_1-int (Y) in (W, τ_1, τ_2) such that $Y = R \cup \tau_1$-int (Y). Hence, Y is a PN-b-LO*-set. □

Definition 3.5 *Suppose that (W, τ_1, τ_2) and (M, δ_1, δ_2) be two NBTSs. Then, an one to one and onto mapping $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is called as*
(i) pairwise neutrosophic locally continuous mapping (in short PNL-C-mapping) if and only if $\xi^{-1}(L)$ is a PNLO-set in W, whenever L is a PNLO-set in M.
(ii) pairwise neutrosophic b-locally continuous mapping (in short PN-b-L-C-mapping) if and only if $\xi^{-1}(L)$ is a PN-b-LO-set in W, whenever L is a PNLO-set in M.
(iii) pairwise neutrosophic b-locally star continuous mapping (in short PN-b-L-C-mapping) if and only if $\xi^{-1}(L)$ is a PN-b-LO*-set in W, whenever L is a PNLO-set in M.*
*(iv) pairwise neutrosophic b-locally double star mapping (in short PN-b-L**-C-mapping) if and only if $\xi^{-1}(L)$ is a PN-b-LO**-set in W, whenever L is a PNLO-set in M.*

Theorem 3.18 *Every PNL-C-mapping is a PN-b-L-C-mapping.*

Proof: Suppose that $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ be a PNL-C-mapping. Let L be a PNLO-set in M. Since, $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a PNL-C-mapping, so $\xi^{-1}(L)$ is a PNLO-set in W. Further, since every PNLO-set in W is also a PN-b-LO-set in W, so $\xi^{-1}(L)$ is a PN-b-LO-set in (W, τ_1, τ_2). Therefore, $\xi^{-1}(L)$ is a PN-b-LO-set in W, whenever L is a PNLO-set in M. Hence, the mapping $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a PN-b-L-C-mapping. □

Theorem 3.19 *Every PNL-C-mapping is also a PN-b-L*-C-mapping.*

Proof: Suppose that $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ be a PNL-C-mapping. Let L be a PNLO-set in M. Since, $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a PNL-C-mapping, so $\xi^{-1}(L)$ is a PNLO-set in W. Further, since every PNLO-set in W is also a PN-b-LO*-set in W, so $\xi^{-1}(L)$ is a PN-b-LO*-set in (W, τ_1, τ_2). Therefore, $\xi^{-1}(L)$ is a PN-b-LO*-set in W, whenever L is a PNLO-set in M. Hence, the mapping $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a PN-b-L*-C-mapping. □

Theorem 3.20 *Every PNL-C-mapping is also a PN-b-L**-C-mapping.*

Proof: Suppose that $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ be a PNL-C-mapping. Let L be a PNLO-set in M. Since, $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a PNL-C-mapping, so $\xi^{-1}(L)$ is a PNLO-set in W. Further, since every PNLO-set in W is also a PN-b-LO**-set in W, so $\xi^{-1}(L)$ is a PN-b-LO**-set in (W, τ_1, τ_2). Therefore, $\xi^{-1}(L)$ is a PN-b-LO**-set in W, whenever L is a PNLO-set in M. Hence, the mapping $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a PN-b-L**-C-mapping. □

Theorem 3.21 *Every PN-b-L*-C-mapping is a PN-b-L-C-mapping.*

Proof: Suppose that $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ be a PN-b-L*-C-mapping. Let L be a PNLO-set in M. Since, $\xi : (W,\ \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a PN-b-L*-C-mapping, so $\xi^{-1}(L)$ is a PN-b-LO* in W. Further, since every PN-b-LO* in W is also a PN-b-LO-set in W, so $\xi^{-1}(L)$ is a PN-b-LO-set in (W, τ_1, τ_2). Therefore, $\xi^{-1}(L)$ is a PN-b-LO-set in W, whenever L is a PN-b-LO* in M. Hence, the mapping $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a PN-b-L-C-mapping. □

Theorem 3.22 *Every PN-b-L**-C-mapping is a PN-b-L-C-mapping.*

Proof: Suppose that $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ be a PN-b-L**-C-mapping. Let L be a PNLO-set in M. Since, $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a PN-b-L**-C-mapping, so $\xi^{-1}(L)$ is a PN-b-LO** in W. Further, since every PN-b-LO**-set in W is also a PN-b-LO-set in W, so $\xi^{-1}(L)$ is a PN-b-LO-set in $(W, \tau_1, \tau_2\ )$. Therefore, $\xi^{-1}(L)$ is a PN-b-LO-set in W, whenever L is a PN-b-LO**-set in M. Hence, the mapping $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a PN-b-L-C-mapping. □

Theorem 3.23 *Let $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ and $\zeta : (M, \delta_1, \delta_2) \to (X, \theta_1, \theta_2)$ be two PNL-C-mappings. Then, the composition mapping $\zeta \circ \xi : (W, \tau_1, \tau_2) \to (X, \theta_1, \theta_2)$ is also a PNL-C-mapping.*

Proof: Suppose that $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ and $\zeta : (M, \delta_1, \delta_2) \to (X, \theta_1, \theta_2)$ be two PNL-C-mappings. Let L be a PNLO-set in (X, θ_1, θ_2). Since, $\zeta : (M, \delta_1, \delta_2) \to (X, \theta_1, \theta_2)$ is a PNL-C-mapping, so $\zeta^{-1}(L)$ is a PNLO-set in $(M, \delta_1,\ \delta_2)$. Further, since $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a PNL-C-mapping, so $\xi^{-1}\left(\zeta^{-1}(L)\right) = (\zeta \circ \xi)^{-1}(L)$ is a PNLO-set in (W, τ_1, τ_2). Therefore, $(\zeta \circ \xi)^{-1}(L)$ is a PNLO-set in (W, τ_1, τ_2) whenever L is a PNLO-set in (X, θ_1, θ_2). Hence, the composition mapping $\zeta \circ \xi : (W, \tau_1, \tau_2) \to (X, \theta_1, \theta_2)$ is also a PNL-C-mapping. □

Theorem 3.24 *Let $\xi : (W, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ be a PN-b-L-C-mapping, and $\zeta : (M, \delta_1, \delta_2) \to (X, \theta_1, \theta_2)$ be a PNL-C-mapping. Then, the composition mapping $\zeta \circ \zeta : (W, \tau_1, \tau_2) \to (X, \theta_1, \theta_2)$ is also a PN-b-L-C-mapping.*

Proof: Suppose that $\xi : (W, \tau_1, \tau_2) \rightarrow (M, \delta_1, \delta_2)$ be a PN-*b*-L-C-mapping, and $\zeta : (M, \delta_1, \delta_2) \rightarrow (X, \theta_1, \theta_2)$ be a PNLC-mapping. Let L be a PNLO-set in (X, θ_1, θ_2). Since, $\zeta : (M, \delta_1, \delta_2) \rightarrow (X, \theta_1, \theta_2)$ is a PNL-C-mapping, so $\zeta^{-1}(L)$ is a PNLO-set in (M, δ_1, δ_2). Further, since $\xi : (W, \tau_1, \tau_2) \rightarrow (M, \delta_1, \delta_2)$ is a PN-*b*-L-C-mapping, so $\xi^{-1}\left(\zeta^{-1}(L)\right) = (\zeta \circ \xi)^{-1}(L)$ is a PN-b-LO-set in (W, τ_1, τ_2). Therefore, $(\zeta \circ \xi)^{-1}(L)$ is a PN-b-LO-set in (W, τ_1, τ_2) whenever L is a PNLO-set in (X, θ_1, θ_2). Hence, the composition mapping $\zeta \circ \xi : (W, \tau_1, \tau_2) \rightarrow (X, \theta_1, \theta_2)$ is a PN-*b*-L-C-mapping. □

Theorem 3.25 *Let $\xi : (W, \tau_1, \tau_2) \rightarrow (M, \delta_1, \delta_2)$ be a PN-b-L*-C-mapping, and $\zeta : (M, \delta_1, \delta_2) \rightarrow (X, \theta_1, \theta_2)$ be a PNLC-mapping. Then, the composition mapping $\zeta \circ \xi: (W, \tau_1, \tau_2) \rightarrow (X, \theta_1, \theta_2)$ is also a PN-b-L*-C-mapping.*

Proof: Suppose that $\xi : (W, \tau_1, \tau_2) \rightarrow (M, \delta_1, \delta_2)$ be a PN-*b*-L*-C-mapping, and $\zeta : (M, \delta_1, \delta_2) \rightarrow (X, \theta_1, \theta_2)$ be a PNLC-mapping. Let L be a PNLO-set in (X, θ_1, θ_2). Since, $\zeta : (M, \delta_1, \delta_2) \rightarrow (X, \theta_1, \theta_2)$ is a PNL-C-mapping, so $\zeta^{-1}(L)$ is a PNLO-set in (M, δ_1, δ_2). Further, since $\xi : (W, \tau_1, \tau_2) \rightarrow (M, \delta_1, \delta_2)$ is a PN-b-L*-C-mapping, so $\xi^{-1}\left(\zeta^{-1}(L)\right) = (\zeta \circ \xi)^{-1}(L)$ is a PN-*b*-LO*-set in (W, τ_1, τ_2). Therefore, $(\zeta \circ \xi)^{-1}(L)$ is a PN-*b*-LO*-set in (W, τ_1, τ_2) whenever L is a PNLO-set in (X, θ_1, θ_2).Hence, the composition mapping $\zeta \circ \xi : (W, \tau_1, \tau_2) \rightarrow (X, \theta_1, \theta_2)$ is a PN-*b*-L*-C-mapping. □

Theorem 3.26 *Let $\xi : (W, \tau_1, \tau_2) \rightarrow (M, \delta_1, \delta_2)$ be a PN-b-L**-C-mapping, and $\zeta : (M, \delta_1, \delta_2) \rightarrow (X, \theta_1, \theta_2)$ be a PNLC-mapping. Then, the composition mapping $\zeta \circ \xi : (W, \tau_1, \tau_2) \rightarrow (X, \theta_1, \theta_2)$ is a PN-b-L**-C-mapping.*

Proof: Suppose that $\xi : (W, \tau_1, \tau_2) \rightarrow (M, \delta_1, \delta_2)$ be a PN-*b*-L**-C-mapping, and $\zeta : (M, \delta_1, \delta_2) \rightarrow (X, \theta_1, \theta_2)$ be a PNL-C-mapping. Let L be a PNLO-set in (X, θ_1, θ_2). Since, $\zeta : (M, \delta_1, \delta_2) \rightarrow (X, \theta_1, \theta_2)$ is a PNL-C-mapping, so $\zeta^{-1}(L)$ is a PNLO-set in (M, δ_1, δ_2). Further, since $\xi : (W, \tau_1, \tau_2) \rightarrow (M, \delta_1, \delta_2)$ is a PN-*b*-L**-C-mapping, so $\xi^{-1}\left(\zeta^{-1}(L)\right) = (\zeta \circ \xi)^{-1}(L)$ is a PN-*b*-LO**-set in (W, τ_1, τ_2). Therefore, $(\zeta \circ \xi)^{-1}(L)$ is a PN-*b*-LO**-set in (W, τ_1, τ_2) whenever L is a PNLO-set in (X, θ_1, θ_2). Hence, the composition mapping $\zeta \circ \xi : (W, \tau_1, \tau_2) \rightarrow (X, \theta_1, \theta_2)$ is a PN-*b*-L**-C-mapping. □

4. Conclusions

In this article, we have introduced the notion of PN-*b*-LO-set, PN-*b*-LC-set, PN-*b*-LO*-set, PN-*b*-LC*-set, PN-*b*-LO**-set and PN-*b*-LC**-set via NBTSs. By defining PN-*b*-LO-set, PN-*b*-LC-set, PN-*b*-LO*-set, PN-*b*-LC*-set, PN-*b*-LO**-set and PN-*b*-LC**-set, we have established several interesting results via NBTSs, and furnished few illustrative examples. In the future, we hope that the proposed notions can also be applied in the field of Quadripartitioned Neutrosophic Topological Space, Pentapartitioned Neutrosophic Topological Space, Rough Pentapartitioned Neutrosophic Topological Space, Neutrosophic Tri-topological Space, Neutrosophic Soft Bitopological Space, etc.

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