



Spatial Binary Multiset Topological Relations

Rakhal Das^{a*} and Binod Chandra Tripathy^b and Bindia Goswami^c

ABSTRACT: In this study, we define binary space and spatial binary multiset topological relations. We take advantage of spatial binary relations and binary space partition tree characteristics. We define a topology for spatial multisets and examine their many attributes.

Key Words: Spatial topology, spatial data, geographical information system, binary space partition, spatial multiset.

Contents

1 Introduction	1
2 Preliminary and Definitions:	2
2.1 Formation of spatial multiset:	3
2.2 Defining Spatial Topology	4
2.3 A Simple Example	5
3 Main Results	6
4 Spatial Multiset Topology	8
5 Conclusion	8

1. Introduction

The importance of topology in Geographic Information Science (GIS) is indisputable for the digital world. Topology has been entered formally into the dictionary of Geographical Information Systems through a book by McDonnell and Kemp [10]. Many applications have been done using the properties of metric and topological spaces in GIS [5] Billen et al. [1] Chen and Zhou [3]. The objects of geography have their own form, position, and appearance. Therefore, modeling such a set of geographic objects requires adequate tools in geometry and topology. There are many properties that two geographic objects may have in common. We study the common property and use it successfully. The network analysis can be extended to all types of objects in Geographic Information Systems (GIS), as long as a rule of relationship is given for setting up an interconnected relationship or topology. Furthermore, most of the existing studies adopted a graph theoretic approach for setting up networks, and the thing-to-thing relationships are constrained by a direct relationship. For instance, two disjoint areas are usually considered to have no relationships.

The binary space partition tree is a geometric data structure obtained by a recursive partitioning scheme called binary space partition (for short, BSP) over a set of input objects. The space is partitioned along a hyperplane into two half-spaces, and then either half-space is partitioned recursively until every sub-problem contains only a trivial fraction of the input objects. The concept of BSP emerged in the computer graphics community in the seventies. It was originally designed to assist efficient hidden-surface removal algorithms for moving viewpoints, but it has later found widespread applications in many areas of computational and combinatorial geometry.

* Corresponding author

2010 *Mathematics Subject Classification*: 91B72, 62M30, 11F23, 14J80.

Submitted March 10, 2022. Published May 23, 2025

In this article, we use the abstract relationship to solve the criminal case. This paper developed a model that can help set up thing-to-thing relationships (for things that have no obvious direct relationships) to explore the structural properties of individual objects of a geographic system as well as the geographic system as a whole.

We established this article based on spatial relationships between spatial objects. We have used multiset theory and the binary space partition tree method. Many researchers have worked on spatial relations; Tang et. al. worked on “Topological Relations Between Fuzzy Regions in a Fuzzy Topological Space” [13]. Csaba [4] worked on Binary Space Partitions. Many researchers have worked on multiset theory Blizard [2], Girish and John [6, 7, 8], Jakaria [9] worked on “Note on multiset topologies” Shraavan and Tripathy [11, 12, 14] they have investigated many interesting results in multiset theory. We have applied the theory of multiset and other spatial relations in this article. Spatial fuzzy topological space studied by Das [18]

This paper is divided into five separate sections. Section 1 is the introduction, and Section 2 is the basic definition and preliminary idea of the article. In Section 3, we define some new definitions and work formulas for our work. In Section 4, we give an example for solving the criminal case. In Section 5, we discuss the conclusion of this article.

2. Preliminary and Definitions:

In this section, we provide some basic definitions and principal ideas related to spatial topology and GIS.

The main objective of this article is to define a multiset spatial relationship. Since similar types of objects can be present in many places at a time, it is essential to define a proper multiset spatial relationship for GIS. Using the binary space partition tree property, the objects can be easily distinguished and analyzed.

Definition 2.1 *A binary space partition tree is a recursive partition scheme for an input set of pairwise interior disjoint objects in R^d , $d \in N$. If the input contains two full-dimensional objects or a lower-dimensional object, we partition the space by a hyper plane h and recursively apply two binary space partitions for the objects clipped in each of the two open half-spaces of h . If the input is at most one full-dimensional object (and no lower-dimensional object), we stop. The partition algorithm naturally corresponds to a binary tree: Every node corresponds to the input of*

a recursive call of the BSP: The root corresponds to the initial input set, and the two children of a non-leaf node correspond to the inputs of its two sub-problems. The BSP tree data structure is based on this binary tree: Every leaf stores at most one full-dimensional object which is the input of the corresponding sub-problem; and every non-leaf node stores the splitting hyperplane and the (lower-dimensional) objects of the corresponding sub-problem that lie on the splitting hyperplane. As a convention, the non-leaf nodes store only k -dimensional fragments of k -dimensional objects lying on the splitting hyperplane in R^d , $0 \leq k \leq d$. For example, if a splitting hyperplane h crosses an input segment s then the point $h \cap s$ is never stored. Figure-1 depicts an example of a recursive partitioning and corresponding BSP tree for four input objects.

Definition 2.2 *A binary search tree (BST) is a binary tree where each node has a comparable key (and an associated value) and satisfies the restriction that the key in any node is larger than the keys in all nodes in that node’s left sub-tree and smaller than the keys in all nodes in that node’s right sub-tree.*

The binary space partitioning tree method is very effective in determining visible relationships among a static group of 3D polygons as seen from an arbitrary viewpoint. These are the following procedures to make a binary space partition:

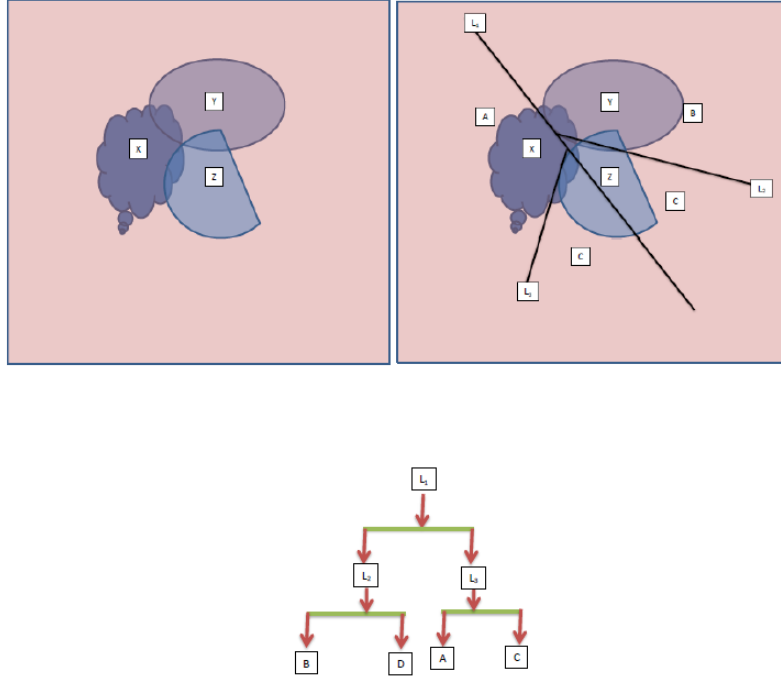


Figure 1: Three connected objects are analyzed by binary space partition.

1. Select any polygon as the root of the tree i.e. the 1st partition plane.
2. Partition the object space into two half-spaces (inside and outside of the partition plane determined by the normal to the plane); Some objects lie in the front half-space while the others are in the back half-space in the partition plane.
3. If a polygon is intersected by the partition plane, split it into two polygons so that they belong to different half-spaces.
4. Select line polygon in the roots front as the left node or child and another in the roots back as the right node or child
5. Recursively subdivide spaces, considering the plane of the children's as a partition plane, until a subspace contains a single polygon.

We will use some of its partial properties in this article.

2.1. Formation of spatial multiset:

Consider a 3D polygon as spatial data after using a binary space partitioning tree (BSPT) we get some spatial data with environmental GIS. Suppose we get n pollution sources whose impact areas are identified, through a buffer operation, as a polygon layer. The m polluted zones likely overlap each other. This constitutes a map layer, denoted by $Y = (y_j : j = 1, 2, 3, \dots, m)$. To assess the pollution impact on a set of locations with another map layer $X = (x_i : i = 1, 2, 3, \dots, n)$, it is not sufficient to just examine which location is within which pollution zones. We should take a step further to put all locations within an interconnected context (a network view) using the concept of spatial topology. For instance, location x_1 is out of the pollution zone of y_1 or x_1 is in the pollution zone of y_1 , therefore the binary relation between 0 or 1. This is a binary relation table between the sets X and Y , as follows:

Y/X	x_1	x_2	x_3	...	x_n
y_1	1 or 0	1 or 0	1 or 0	...	1 or 0
y_2	1 or 0	1 or 0	1 or 0	...	1 or 0
y_3	1 or 0	1 or 0	1 or 0	...	1 or 0
...
y_m	1 or 0	1 or 0	1 or 0	...	1 or 0

$X = \{x_1/m_1, x_2/m_2, x_3/m_3, \dots, x_n/m_n\}$ Here, $0 \leq m_i \leq m$.

$Y = \{y_1/n_1, y_2/n_2, y_3/n_3, \dots, y_n/n_m\}$ Here, $0 \leq n_i \leq n$.

Here, $[X]^m$ and $[Y]^n$ represents the multiset where

Spatial Here, we introduce a model to identify spatial relationships of geographic objects via common objects. To introduce the model, let us start with an example from social networks. Two persons, A and B , are not acquainted in a general social sense as colleagues or friends, but they can be considered as “adjacent”, as both, for instance, belong to the same geographic information community. The community or organization people belong to is a context that keeps people “adjacent”. Extending the case into a geographic context, we can say that, for instance, two houses are adjacent if they are both situated along the same street or within the same district. We can further state that a pair of objects sharing more common objects is more “adjacent”. Now let us turn to the formal model for identifying such spatial relationships. The model aims to represent spatial relationships of various objects as a simplified complex. To this end, some formal definitions are also presented in the context of GIS.

2.2. Defining Spatial Topology

The definition of a map layer according to set theory. A map layer is defined as a set of spatial objects at a certain scale in a database or on a map. For example, $M = \{A_1, A_2, \dots, A_n\}$ or $M\{A_i : i = 1, 2, \dots, n\}$ denotes a map layer (using a capital letter) that consists of multiple objects (using small letters). The objects can be put into four categories: point, line, area, and volume objects in terms of basic graphic primitives. Note that the definition of objects must be appropriate with respect to the modeling purpose. For instance, a street layer can be considered a set of interconnected street segments, or an interconnected named street depending on the modeling purpose (Jiang and Claramunt 2004); a city layer can be represented as a point or area object depending on the representation scale.

A spatial topology (τ) is defined as a subset of the Cartesian product of two map layers, L and M , denoted by $\tau = L \times M$. To set up a spatial topology, we need to examine the relationship of every pair of objects from one map layer to another. As distinct from topological relationships based on possible intersections of internal, external, and boundary of spatially extended objects (Egenhofer 1991), we simply take a binary relationship. That is, if an object 1 is within, or intersects, another object m , we say there is a relationship $\lambda = (1, m)$, otherwise no relationship, $\lambda = \emptyset$ [Note: the pair $(1, m)$ is ordered, and $(m, 1)$ represents an inverse relation denoted by $\lambda - 1$]. The relationship can be simply expressed as “an object has a relationship to a contextual object.” If a set of primary objects shares a common contextual object, we say the set of objects is adjacent or proximate. Thus, two types of map layers can be distinguished: the primary layer for the primary objects, with which a spatial topology is to be explored, and the contextual layer, whose objects constitute a context for the primary objects. It is important to note that the contextual layer can be given in a rather abstract way with a set of features (instead of map objects). This way, the relationship between the primary to contextual objects can be expressed as “an object has certain features.” For the sake of convenience and with the notation $\tau = L \times M$, we refer to the first letter as the primary layer and the second as the contextual layer. The notion of spatial topology presents a network view as to how the primary objects become interconnected via the contextual objects. A spatial topology can be represented as a simplicial complex. Before examining the representation, we turn to the definition of simplicial complex (Atkin 1977). A simplicial complex is a collection of relevant simplices. Let us assume the elements of a set A form simplices (or polyhedra, denoted by σ_d where d is the dimension of the simplex); and the elements of a set B form vertices according to the binary relation λ , indicating that a pair of elements (a_i, b_j) from the two different sets A and B , $a_i \in A$ and $b_j \in B$, are related. The simplicial complex can be denoted as $KA(B; \lambda)$. In general, each simplex is expressed as a

q -dimensional geometrical figure with $q + 1$ vertex. The collection of all the simplices forms the simplicial complex. For every relation λ , it is feasible to consider the conjugate relation $\lambda - 1$, by reversing the relations between two sets A and B by transposing the original incidence matrix. The conjugate structure is denoted as $KB(A; \lambda - 1)$.

A spatial topology can be represented as a simplicial complex, where the simplices are primary objects and the vertices are contextual objects. Formally, the simplicial complex for the spatial topology $\tau = L \times M$ is denoted by $KL(M; \lambda)$, where L represents the primary layer and M the contextual layer, the relation between a primary object and contextual object $\lambda = (1, m)$. A spatial topology can be represented as an incidence matrix Λ , where the columns represent objects with a primary layer and the rows represent objects with a contextual layer. Formally, it is represented as follows: extend and enhance the existing ones for more advanced spatial analysis and modelling. In this respect, the simplicial complex representation provides a powerful tool for exploring the structural properties of spatial topology. Before introducing the structural analysis, we take a look at a simple example.

2.3. A Simple Example

Let us assume, with environmental GIS, four pollution sources whose impact areas are identified, through a buffer operation, as a polygon layer. It is likely that the three polluted zones overlap each other. This constitutes a map layer, denoted by $M = (y|j = 1, 2, 3, 4)j$. To assess the pollution impact on a set of locations with another map layer $N = (x_i|i = 1, 2, \dots, 7)$ it is not sufficient to just examine which location is within which pollution zones. We should take a step further to put all locations within an interconnected context (a network view) using the concept of spatial topology. For instance, location x_1 is out of the pollution zone of y_1 , but it may get polluted through x_5 and x_5 , assuming the kind of pollution is transmittable. Only under the network view are we able to investigate the pollution impact thoroughly.

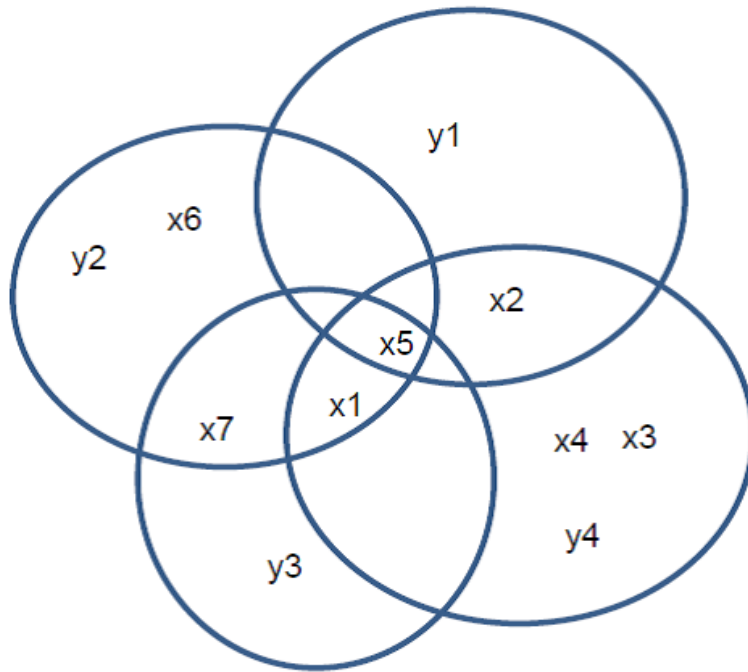


Figure 2: Three connected objects are analyzed by binary space partition.

From the above figure, the spatial topology can be represented as an incidence matrix as follows:

$$\Lambda = \begin{pmatrix} . & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \\ y_1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ y_2 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ y_3 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ y_4 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}.$$

3. Main Results

In this section, we establish the main results of this article.

Definition 3.1 A spatial data set X , is defined as the set of elements from which spatial msets are constructed. The spatial mset space $[X]^n$ is the set of all spatial msets whose elements are from spatial set X such that no element occurs more than n times. Throughout this article, we denote a spatial multiset drawn from the multiset space $[X]^n$ by M

Definition 3.2 An spatial mset S_M drawn from the set spatial X is represented by a count function S_M or $C_{S_M} : X \rightarrow N$, where S_N represents the set of non-negative integers.

Here $C_{S_M}(x)$ is the number of occurrences of the element x in the spatial mset M drawn from the spatial set $X = \{x_1, x_2, \dots, x_n\}$ as $S_M = \{m_1/x_1, m_2/x_2, \dots, m_n/x_n\}$ where n_i is the number of occurrences of the element $x_i, i = 1, 2, \dots, n$ in the spatial mset S_M . The elements which are not included in the spatial mset S_M have zero count.

Consider two spatial msets S_M and S_N drawn from a spatial set X . The following operations are defined on the spatial msets.

Definition 3.3 Let $[X]^n$ be a spatial mset space. Let $x \in X$ and $\{S_{M_1}, S_{M_2}, S_{M_3}, \dots\}$ be a collection of spatial sub-msets drawn from $[X]^n$. Then the following operations are possible under arbitrary collections of spatial msets.

1. The Union is defined by $\cup_{i \in I} S_{M_i} = \{C_{S_M}(x)/x : C_{S_M}(x) = \max C_{S_{M_i}}(x) : i \in I, \text{ for all } x \in X\}$.
2. The intersection is defined by $\cap_{i \in I} S_{M_i} = \{C_{M(x)}/x : C_{M(x)} = \min\{C_{S_{M_i}}(x) : i \in I\}, \text{ for all } x \in X\}$.
3. The mset complement is defined by $S_{M^c} = S_Z \ominus S_M = \{C_{M^c}(x)/x : C_{M^c}(x) = C_Z(x) - C_M(x), \text{ for all } x \in X\}$.

Here S_Z is drawn from $[X]^n$ which has the maximum multiplicity of the elements that occur but may not all same. We define here two other operations for the non-empty spatial multiset called spatial S_{Mpo} operation and S_{mmpo} operations.

Definition 3.4 Let us consider two spatial multiset $S_M = \{m_1/x_1, m_2/x_2, \dots, m_n/x_n\}$ and $S_K = \{k_1/x_1, k_2/x_2, \dots, k_n/x_n\}$ define on universal set $[X]^n$. Now we define the above two operations S_{Mpo} and S_{mmpo} in the following way:

$$\begin{aligned} S_{Mpo}(S_M, S_K) \\ = \{C_{S_{Mpo}}(x)/x : \text{for all } x \in X\}. \end{aligned}$$

Where,

$$C_{S_{Mpo}}(x) = \begin{cases} LCM\{m_i, k_i\}, & \text{if } m_i \text{ and } k_i \text{ both are non zero} \\ 0, & \text{if least one of } m_i \text{ and } k_i \text{ are zero} \end{cases}$$

$$\begin{aligned} S_{mmpo}(S_M, S_K) \\ = \{C_{S_{mmpo}}(x)/x : \text{for all } x \in X\}. \end{aligned}$$

Where,

$$C_{S_{mmpo}}(x) = \begin{cases} HCF\{m_i, k_i\}, & \text{if } m_i \text{ and } k_i \text{ both are non zero} \\ p_i, & p_i = \text{maximum } \{m_i, k_i\}. \end{cases}$$

Example 3.1 Let $X = \{x, y, z, t, v\}$ be a spatial multiset. Let $S_M = \{3/x, 2/y, 4/z, 6/t, 5/v\}$ and $S_K = \{2/x, 4/y, 8/z, 5/t, 5/v\}$. Then the S_{Mpo} and S_{mmpo} operations between S_M and S_K
 $S_{Mpo}(S_M, S_K) = \{6/x, 4/y, 8/z, 30/t, 5/v\}$
 $S_{mmpo}(S_M, S_K) = \{1/x, 2/y, 4/z, 1/t, 5/v\}$

Definition 3.5 If S_M be a spatial mset define on a universal set X . Then the support of spatial multi set S_M is a spatial crisp set, which is denoted by $*S$ and define by $*S = \{x \in X : C_{S_M}(x) > 0\}$.

Definition 3.6 If X be the collection of all spatial objects set then the following definition

1. A spatial mset S_M is said to be an empty spatial set if for all $x \in X$, $C_{S_M}(x) = 0$.
2. Two spatial sub-mset S_M and S_N define on a universal set X are said to be equal if $C_{S_M}(x) = C_{S_N}(x)$ for all $x \in X$.
3. Two spatial sub-mset S_M and S_N define on a universal set X are said to be spatial sub-mset if $C_{S_N}(x) \leq C_{S_M}(x)$ for all $x \in X$.

Proposition 3.1 For any two spatial multiset we have the following proposition :

1. The union of two spatial mset is also a spatial mset.
2. The intersection of two spatial mset is also a spatial mset.

Proof: Since spatial multiset is also a multiset define on a universal set X . So, by the definition 3.3 the above two propositions easily follow. \square

Theorem 3.1 For any two spatial non-empty multisets S_M and S_N defined on a universal set X the following result holds:

$$S_M \cup S_N \subseteq S_{Mpo}(S_M, S_N). \\ S_{mmpo}(S_M, S_N) \subseteq S_M \cap S_N.$$

Proof: (1) Let $m/x \in S_M$ and $n/x \in S_N$ for any $x \in X$. We have for any two non-negative integers m and n $Max\{m, n\} \leq LCM\{m, n\}$, where LCM is refer to List Common Multiple. Since the union of two spatial multisets is also a spatial multiset. The result is obvious. (Using definition 3.4 and proposition 3.1(1)).

The converse of the theorem is not true. For this, we will provide the following example as follows.

Consider, $X = \{x, y, z, v\}$ be a spatial multi-universal set. We have $S_M = \{2/x, 5/y, 4/z, 1/v\}$ and $S_K = \{7/x, 2/y, 5/z, 1/v\}$. Then the union and S_{Mpo} operations between S_M and S_K is as follows $S_{Mpo}(S_M, S_K) = \{14/x, 10/y, 20/z, 1/v\}$ and $S_M \cup S_N = \{7/x, 5/y, 5/z, 1/v\}$. Clearly $S_{Mpo}(S_M, S_K) \not\subseteq S_M \cup S_N$. (2) We know that for any two non-negative integers m and n $H.C.F\{m, n\} \leq Min\{m, n\}$, where H.C.F is refer to Highest Common Factor. We know that the intersection of two spatial multisets is also a spatial multiset. Using definition 3.4(2) and proposition 3.8(2) we get the result.

The converse of the theorem is not true. For this, we will provide the following example as follows.

Consider, $X = \{x, y, z, v\}$ be a spatial multi set. We have, $S_M = \{4/x, 1/y, 7/z, 2/v\}$ and $S_K = \{4/x, 5/y, 3/z, 4/v\}$. The intersection and S_{mmpo} operations between S_M and S_K is as follows: $S_{mmpo}(S_M, S_K) = \{4/x, 1/y, 1/z, 2/v\}$ and $S_M \cap S_N = \{4/x, 1/y, 3/z, 2/v\}$. Clearly, $S_M \cap S_N \not\subseteq S_{mmpo}(S_M, S_K)$. Hence the results.

4. Spatial Multiset Topology

Definition 4.1 Let S_M be any non-empty spatial mset defined on a universal set X and τ be the collection of subsets of the multiset M . The pair (S_M, τ) is said to be a spatial multiset topological space if the following property holds:

1. $\emptyset, S_M \in \tau$.
2. If $S_K, S_N \in \tau$ then $S_K \cap S_N \in \tau$.
3. If $S_{K_i}, i \in \Lambda \in \tau$ then, $\cup_{i \in \Lambda} S_{K_i} \in \tau$.

Definition 4.2 A spatial sub-mset S_N of spatial multiset topological space (S_M, τ) defined on $[X]^w$ is said to be an open spatial mset, if $S_N \in \tau$ and closed if $S_{N^c} \in \tau$.

Theorem 4.1 The following results hold for spatial multiset topology.

1. Arbitrary intersection of open spatial mset is an open spatial mset.
2. Arbitrary union of open spatial mset is an open spatial mset.
3. Union of two closed spatial mset is a closed spatial mset.
4. A spatial Spatial multiset topology is a Hausdorff space.

5. Conclusion

In this article, we introduce a new type of multiset, which is called a spatial multiset. We use the BSP tree method and the spatial data analysis method. The application of this field is very clear. We can use this theory for topological modeling, decision-making problems, analyzing the progress of a country, and solving rubbery and criminal case problems using GPS tolls and spatial mset. Many researchers can get interest in this application-based area.

Conflict of Interest: The authors declare that they have no conflict of interest.

Authors Contribution: All the authors have made equal contributions to the preparation of this article.

References

1. R. Bilen, S. Zlatanova, P. Mathonet and F. Boniver, *The Dimensional Model: A Framework to Distinguish Spatial Relationships*. In Advances in Spatial Data Handling, 10th International Symposium on Spatial Data Handling, edited by D. Richardson and P. van Oosterom, Ottawa, July 8(12) (2002), 285–298.
2. W. Blizard, *Multiset theory*. Notre Dame Jour. Formal Logic. 30 (1989), 36–66.
3. S.P. Chen and C.H. Zhou. *Conformity Information Source*. Earth Information Science 3 (2003) 1–3.
4. D.T. Csaba, *Binary Space Partitions: Recent Developments*” *Combinatorial and Computational Geometry*. MSRI Publications 52 (2005), 529–556.
5. M.T. David *Topology Revisited: Representing Spatial Relations*. International Journal of Geographical Information Science 15(8) (2001), 689–705
6. K.P. Girish and S.J. John, *On multiset topologies*. Theory Appl. Math. Comp. Sci. 2 (2012), 37–52.
7. K.P. Girish and S.J. John, *Relations and functions in multiset context*. Inf. Sci., 179(6) (2009), 758–768.
8. K.P. Girish and S.J. John, *Multiset topologies induced by multiset relations*. Information Sciences, 188 (2012), 298–313.
9. A. Jakaria, *Note on multiset topologies*. Annals Fuzzy Math. Inform. 10(5) (2015), 825–827.
10. R. Mc Donnell and K. Kemp, *International GIS Dictionary*. New York: Wiley. (1995).
11. K. Shravan and B.C. Tripathy, *Multiset ideal topological spaces and local functions*. Proyecciones Journal of Mathematics. 37(4) (2018) 699–711.
12. K. Shravan and B.C. Tripathy, *Multiset mixed topological space*. Soft Comput., 23 (2019), 9801–9805.

13. X. Tang, W. Kainz and H. Wang, *Topological Relations Between Fuzzy Regions in a Fuzzy Topological Space*. International Journal of Applied Earth Observation and Geoinformation 12 (2010), 151–165.
14. B.C. Tripathy and G.C. Ray, *On mixed fuzzy topological spaces and countability*. Soft Computing, 16(10) (2012), 1691-1695.
15. B.C. Tripathy and G.C. Ray, *Mixed fuzzy ideal topological spaces*. Appl. Math. Comput., 220 (2013), 602-607.
16. B.C. Tripathy, R. Das, *Multiset Mixed Topological Space* Transactions of A. Razmadze Mathematical Institute, 177(1) (2023), 119–123.
17. R. Das, *Generalized multiset and multiset ideal continuous function* Applied Sciences, 2022, Vol 24, p98.
18. R. Das, *Spatial fuzzy topological space* Proyecciones (Antofagasta), 2022, vol 41(4), 999-1013.

^aDepartment of Mathematics; The ICFAI University, Agartala-799210, Tripura, India. ^bDepartment of Mathematics, Tripura University, Agartala-799022, India. ^cAssistant Professor, Swami Vivekananda Institute of Science & Technology Dakshin Gobindapur, P.S Sonarpur, Kolkata 700145.
 E-mail address: `rakhaldas95@gmail.com`
 E-mail address: `tripathybc@gmail.com`
 E-mail address: `pujagoswami156@gmail.com`