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Properties of Soft ω-Homeomorphisms in Soft Topological Spaces

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ABSTRACT: In this article, we define soft ω-closed maps, soft ω-open maps, soft ω-irresolute maps and soft ω-homeomorphisms in soft topological spaces. We further study the properties of these maps with appropriate examples.

Key Words: Soft ω-open map, soft ω-closed map, soft ω-irresolute map, soft ω-continuous map, soft ω-homeomorphism.

Contents

1. Introduction

To solve the complicated real life problems in engineering, social science and other fields, several theories like fuzzy set theory [15], theory of interval mathematics [14] etc. have been introduced. But all these theories had some drawbacks. To avoid these drawbacks or inadequacy of the parameterization tool, Molodtsov [9] used an adequate parameterization. He initiated the basic notion of "soft set theory" in 1999 and presented the first result of the theory. He has attracted many researchers to work on this theory. Maji et al. [8] applied this theory in 2003, to solve problems in decision making.

The idea of "soft topological spaces" were studied by Shabir and Naz [12]. Later, Hussain and Ahmad [4] studied the "properties of soft topological spaces". Meanwhile, Aras and Sonmez [1] discussed the "properties of soft continuous mappings". The "properties of soft semi-open sets and soft semi-closed sets" were discussed by Chen [2]. Later, in 2019 Ghour [3] introduced soft open maps, soft closed maps and soft homeomorphisms in soft topological spaces. Then, Kandil et al. [6] studied on "generalizations of soft closed sets and their operation approaches in soft topological spaces". Sundaram and John [13] introduced ω-closed sets in topology. Motivating from his idea, Paul [10] proposed "soft ω-closed sets in soft topological spaces". Inspiring from all these ideas especially [5], we define soft ω-open maps, soft ω-closed maps, soft ω-irresolute maps and soft ω-bomeomorphisms in "soft topological spaces". We establish the properties of these maps with some appropriate examples.

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2. Preliminaries

In this section, we discuss some well known basic definitions which are helpful while proving our main results. Throughout this paper, we shall denote I_U by universal set, Δ by parameter set and $(I_U, \tilde{\tau}, \Delta)$ by "soft topological space".

Definition 2.1 *[12] "Suppose I_U be an initial universal set, Δ be a parameter set and $\mathcal{P}(I_U)$ is the power set of I_U. A pair (M_Δ, Δ) is called a soft set over I_U, if M_Δ is a mapping given by $M_\Delta : \Delta \to \mathcal{P}(I_U)$."*

Definition 2.2 *[9] "For two soft sets (M_{Δ_a}, Δ_a) and (N_{Δ_b}, Δ_b) over a common universe I_U, where Δ_a ad Δ_b are subsets of Δ, then (M_{Δ_a}, Δ_a) is a soft subset of (N_{Δ_b}, Δ_b) if*
(i) $\Delta_a \subseteq \Delta_b$ and
(ii) for all $\delta_{a_1} \in \Delta_a$, $M_{\Delta_a}(\delta_{a_1})$ and $N_{\Delta_b}(\delta_{a_1})$ are identical approximations.
We write $(M_{\Delta_a}, \Delta_a)\tilde{\subseteq}(N_{\Delta_b}, \Delta_b)$."

Remark 2.1 *[9] "(M_{Δ_a}, Δ_a) is soft superset of (N_{Δ_b}, Δ_b), if (N_{Δ_b}, Δ_b) is a soft subset of (M_{Δ_a}, Δ_a). We denote it by $(M_{\Delta_a}, \Delta_a)\tilde{\supseteq}(N_{\Delta_b}, \Delta_b)$."*

Definition 2.3 *[9] "Two soft sets (M_{Δ_a}, Δ_a) and (N_{Δ_b}, Δ_b) over a common universe I_U are equal if (M_{Δ_a}, Δ_a) is a soft subset of (N_{Δ_b}, Δ_b) and (N_{Δ_b}, Δ_b) is soft subset of (M_{Δ_a}, Δ_a)."*

Definition 2.4 *[9] "Let (M_Δ, Δ) be a soft set over I_U, then (M_Δ, Δ) is*
(1) null soft set denoted by $\tilde{\phi}$ if for all $\delta_1 \in \Delta$, $M_\Delta(\delta_1) = \phi$.
(2) absolute soft set denoted by $\tilde{I_U}$ if for all $\delta_1 \in \Delta, M_\Delta(\delta_1) = I_U$."

Definition 2.5 *[12] "The union of two soft sets (M_{Δ_a}, Δ_a) and (N_{Δ_b}, Δ_b) over the common universe I_U is the soft set (H_{Δ_c}, Δ_c), where $\Delta_c = \Delta_a \cup \Delta_b$ and for all $\delta \in \Delta_c$,*

$$H_{\Delta_c}(\delta) = \begin{cases} M_{\Delta_a}(\delta) & \text{if} \quad \delta \in \Delta_a - \Delta_b \\ N_{\Delta_b}(\delta) & \text{if} \quad \delta \in \Delta_b - \Delta_a \\ M_{\Delta_a}(\delta) \cup N_{\Delta_b} & \text{if} \quad \delta \in \Delta_a \cap \Delta_b \end{cases}$$

We write $(M_{\Delta_a}, \Delta_a)\tilde{\cup}(N_{\Delta_b}, \Delta_b) = (H_{\Delta_c}, \Delta_c)$."

Definition 2.6 *[12] "The intersection (H_{Δ_c}, Δ_c) of two soft sets (M_{Δ_a}, Δ_a) and (N_{Δ_b}, Δ_b) over a common universe I_U, denoted by $(M_{\Delta_a}, \Delta_a) \tilde{\cap} (N_{\Delta_b}, \Delta_b)$, is defined as $\Delta_c = \Delta_a \cap \Delta_b$ and*
$H_{\Delta_c}(\delta) = M_{\Delta_a}(\delta) \cap N_{\Delta_b}(\delta)$ for all $\delta \in \Delta_c$."

Definition 2.7 *[12] "The relative complement of a soft set (M_Δ, Δ) denoted by $(M_\Delta, \Delta)^c$ and is defined by $(M_\Delta, \Delta)^c = (M^c_\Delta, \Delta)$ where $M^c_\Delta : \Delta \to \mathcal{P}(I_U)$ is a mapping defined by $M^c_\Delta(\delta) = I_U - M_\Delta(\delta)$ for all $\delta \in \Delta$."*

Definition 2.8 *[12] "Let I_U be an initial universal set, Δ be the non-empty set of parameters and $\tilde{\tau}$ be the collection of soft sets over I_U, then $\tilde{\tau}$ is a soft topology on I_U, if*
(1) $\tilde{\phi}$, $\tilde{I_U} \tilde{\in} \tilde{\tau}$,
(2) union of any number of soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$,
(3) intersection of any two soft sets in $\tilde{\tau}$ belongs to $\tilde{\tau}$.
Then, the triplet $(I_U, \tilde{\tau}, \Delta)$ is called a soft topological space over I_U. The members of $\tilde{\tau}$ are called soft open sets and complements of them are called soft closed sets in I_U."

Definition 2.9 *[12] "Let $(I_U, \tilde{\tau}, \Delta)$ be a soft topological space and (M_Δ, Δ) be a soft set over I_U, then the soft closure of (M_Δ, Δ), denoted by $\overline{(M_\Delta, \Delta)}$ is defined as the intersection of all soft closed supersets of (M_Δ, Δ)."*

Remark 2.2 *[12] "If (M_Δ, Δ) is soft closed set, then $\overline{(M_\Delta, \Delta)} = (M_\Delta, \Delta)$."*

Definition 2.10 *"Let $(I_U, \tilde{\tau}, \Delta)$ be a soft topological space and (M_Δ, Δ) be a soft set over I_U, then the soft interior of (M_Δ, Δ), denoted by $(M_\Delta, \Delta)^\circ$ is defined as the union of all soft open subsets of (M_Δ, Δ)."*

Remark 2.3 *[12] "If (M_Δ, Δ) is soft open set, then $(M_\Delta, \Delta)^\circ = (M_\Delta, \Delta)$."*

Definition 2.11 *[7] "Suppose (I_U, Δ) and (I_V, Δ^*) be soft classes. If $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ be mappings. Then, a mapping $(\mathcal{F}, \Upsilon) : (I_U, \Delta) \to (I_V, \Delta^*)$ is defined as : for a soft set (M_{Δ_1}, Δ_1) in (I_U, Δ), $[(\mathcal{F}, \Upsilon)\{(M_{\Delta_1}, \Delta_1)\}, \Delta_1^*]$, $\Delta_1^* = \Upsilon(\Delta_1)\tilde{\subseteq}\Delta^*$ is a soft set in (I_V, Δ^*) given by*

$$[(\mathcal{F}, \Upsilon)\{(M_{\Delta_1}, \Delta_1)\}](\delta^*) = \mathcal{F}[\bigcap_{\delta \in \Upsilon^{-1}(\delta^*) \cap \Delta_1^*} M(\delta)] \text{ for } \delta^* \in \Delta_1^* \tilde{\subseteq} \Delta^*,$$

$[(\mathcal{F}, \Upsilon)\{(M_{\Delta_1}, \Delta_1)\}, \Delta_1^*]$ is called soft image of soft set (M_{Δ_1}, Δ_1). If $\Delta_1^* = \Delta^*$, then we shall write by $[(\mathcal{F}, \Upsilon)\{(M_{\Delta_1}, \Delta_1)\}]$."

Definition 2.12 *[7] "Suppose $(\mathcal{F}, \Upsilon) : (I_U, \Delta) \to (I_V, \Delta^*)$ is a mapping and $(N_{\Delta_1^*}, \Delta_1^*)$ be a soft set in (I_V, Δ^*), where $\Delta_1^* \tilde{\subseteq} \Delta^*$. Let $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ be mappings. Then, $[(\mathcal{F}, \Upsilon)^{-1}\{(N_{\Delta_1^*}, \Delta_1^*)\}, \Delta_1]$, $\Delta_1 = \Upsilon^{-1}(\Delta_1^*)$ is a soft set in (I_U, Δ) defined as*

$$[(\mathcal{F}, \Upsilon)^{-1}\{(N_{\Delta_1^*}, \Delta_1^*)\}](\delta) = \mathcal{F}^{-1}\{N_{\Delta_1^*}\{\Upsilon(\delta)\}\} \text{ for } \delta \in \Delta_1 \tilde{\subseteq} \Delta.$$

$[(\mathcal{F}, \Upsilon)^{-1}\{(M_{\Delta_1^*}, \Delta_1^*)\}, \Delta_1]$ is called soft inverse image of soft set $(N_{\Delta_1^*}, \Delta_1^*)$."

Lemma 2.1 [7] "Let $(\mathcal{F}, \Upsilon) : (I_U, \Delta) \to (I_V, \Delta^*)$, $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ be soft mappings. Then, for soft set (M_{Δ_1}, Δ_1), (N_{Δ_2}, Δ_2) and a family of soft sets (F_{Δ_i}, Δ_i) in the soft class (I_U, Δ), we have
(i) $(\mathcal{F}, \Upsilon)(\tilde{\phi}) = \tilde{\phi}$.
(ii) $(\mathcal{F}, \Upsilon)(\tilde{I_U}) \tilde{\subseteq} \tilde{I_V}$.
(iii) $(\mathcal{F}, \Upsilon)\{(M_{\Delta_1}, \Delta_1) \tilde{\cup} (N_{\Delta_2}, \Delta_2)\} = (\mathcal{F}, \Upsilon)(M_{\Delta_1}, \Delta_1) \tilde{\cup} (\mathcal{F}, \Upsilon)(N_{\Delta_2}, \Delta_2)$.
In general, $(\mathcal{F}, \Upsilon)[\tilde{\cup}_i\{F_{\Delta_i}, \Delta_i\}] = \tilde{\cup}_i(\mathcal{F}, \Upsilon)\{F_{\Delta_i}, \Delta_i\}$.
(iv) $(\mathcal{F}, \Upsilon)\{(M_{\Delta_1}, \Delta_1) \tilde{\cap} (N_{\Delta_2}, \Delta_2)\} \tilde{\subseteq} (\mathcal{F}, \Upsilon)(M_{\Delta_1}, \Delta_1) \tilde{\cap} (\mathcal{F}, \Upsilon)(N_{\Delta_2}, \Delta_2)$.
In general, $(\mathcal{F}, \Upsilon)[\cap_i\{F_{\Delta_i}, \Delta_i\}] \tilde{\subseteq} \cap_i (\mathcal{F}, \Upsilon)\{F_{\Delta_i}, \Delta_i\}$.
(v) If $(M_{\Delta_1}, \Delta_1) \tilde{\subseteq} (N_{\Delta_2}, \Delta_2)$, then $(\mathcal{F}, \Upsilon)(M_{\Delta_1}, \Delta_1) \tilde{\subseteq} (\mathcal{F}, \Upsilon)(N_{\Delta_2}, \Delta_2)$."

Lemma 2.2 [7] "Let $(\mathcal{F}, \Upsilon)^{-1} : (I_V, \Delta^*) \to (I_U, \Delta)$ be a soft mapping. Then, for soft set $(M_{\Delta_1^*}, \Delta_1^*)$, $(N_{\Delta_2^*}, \Delta_2^*)$ and a family of soft sets $(F_{\Delta_i^*}, \Delta_i^*)$ in the soft class (I_V, Δ^*), we have
(i) $(\mathcal{F}, \Upsilon)^{-1}(\tilde{\phi}) = \tilde{\phi}$.
(ii) $(\mathcal{F}, \Upsilon)^{-1}(\tilde{I_V}) = \tilde{I_U}$.
(iii) $(\mathcal{F}, \Upsilon)^{-1}\{(M_{\Delta_1^*}, \Delta_1^*) \tilde{\cup} (N_{\Delta_2^*}, \Delta_2^*)\} = (\mathcal{F}, \Upsilon)^{-1}(M_{\Delta_1^*}, \Delta_1^*) \tilde{\cup} (\mathcal{F}, \Upsilon)^{-1}(N_{\Delta_2^*}, \Delta_2^*)$.
In general, $(\mathcal{F}, \Upsilon)^{-1}[\cup_i\{F_{\Delta_i^*}, \Delta_i^*\}] = \cup_i(\mathcal{F}, \Upsilon)^{-1}\{F_{\Delta_i^*}, \Delta_i^*\}$.
(iv) $(\mathcal{F}, \Upsilon)^{-1}\{(M_{\Delta_1^*}, \Delta_1^*) \tilde{\cap} (N_{\Delta_2^*}, \Delta_2^*)\} = (\mathcal{F}, \Upsilon)^{-1}(M_{\Delta_1^*}, \Delta_1^*) \tilde{\cap} (\mathcal{F}, \Upsilon)^{-1}(N_{\Delta_2^*}, \Delta_2^*)$.
In general, $(\mathcal{F}, \Upsilon)^{-1}[\cap_i\{F_{\Delta_i^*}, \Delta_i^*\}] = \cap_i(\mathcal{F}, \Upsilon)^{-1}\{F_{\Delta_i^*}, \Delta_i^*\}$.
(v) If $(M_{\Delta_1^*}, \Delta_1^*) \tilde{\subseteq} (N_{\Delta_2^*}, \Delta_2^*)$, then $(\mathcal{F}, \Upsilon)^{-1}(M_{\Delta_1^*}, \Delta_1^*) \tilde{\subseteq} (\mathcal{F}, \Upsilon)^{-1}(N_{\Delta_2^*}, \Delta_2^*)$."

Definition 2.13 *"A soft map $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is*
(i) soft continuous map [7] if inverse image of soft closed set over I_V is soft closed over I_U.
(ii) soft closed map [3] if image of soft closed set over I_U is soft closed set over I_V.
(iii) soft open map [3] if image of soft open set over I_U is soft open set over I_V.
(iv) soft irresolute map [6] if image of soft semi-open set over I_V is soft semi-open over I_U.
(v) soft homeomorphism [1] if $(\mathcal{F}, \Upsilon)$ is bijective, soft continuous and $(\mathcal{F}, \Upsilon)^{-1}$ is soft continuous mapping."

Definition 2.14 *[11] "Let (W_Δ^0, Δ) be a soft set over I_U. Then, (W_Δ^0, Δ) is **soft ω-open set** if for any soft semi-closed set (C_Δ, Δ) contained in (W_Δ^0, Δ), we have $(C_\Delta, \Delta) \tilde{\subseteq} (W_\Delta^0, \Delta)^\circ$. The set of all soft ω-open sets is denoted by $G_{s\omega}(\tilde{I_U})$."*

Proposition 2.1 [11] If $(W_\Delta^0, \Delta) \in \tilde{\tau}$, then $(W_\Delta^0, \Delta) \in G_{s\omega}(\tilde{I_U})$. But if $(W_\Delta^0, \Delta) \in G_{s\omega}(\tilde{I_U})$, then it is not necessary that $(W_\Delta^0, \Delta) \in \tilde{\tau}$.

Definition 2.15 *[11] "Consider a soft set (W_Δ, Δ) over I_U. Then, the soft ω-interior of (W_Δ, Δ), denoted by $\{(W_\Delta, \Delta)\}^\circ_\omega$, is defined as the union of all soft ω-open subsets of (W_Δ, Δ) i.e., $\{(W_\Delta, \Delta)\}^\circ_\omega = \tilde{\cup}\, \{(F_\Delta, \Delta) : (W_\Delta, \Delta) \tilde{\supseteq} (F_\Delta, \Delta), (W_\Delta, \Delta) \tilde{\in} G_{s\omega}(\tilde{I_U})\}$."*

Remark 2.4 *"[11] (i) $(W_\Delta, \Delta) \tilde{\supseteq} \{(W_\Delta, \Delta)\}^\circ_\omega$.*
(ii)$\{(W_\Delta, \Delta)\}^\circ_\omega \tilde{\in} G_{s\omega}(\tilde{I_U})$."

Lemma 2.3 [11] "$\{(W_\Delta, \Delta)\}^\circ_\omega$ is the largest soft ω-open set contained in (W_Δ, Δ)."

Lemma 2.4 [11] "A soft set $(W_\Delta, \Delta) \tilde{\in} G_{s\omega}(\tilde{I_U})$ if and only if $\{(W_\Delta, \Delta)\}^\circ_\omega = (W_\Delta, \Delta)$."

Lemma 2.5 [11] "Consider a soft topological space $(I_U, \tilde{\tau}, \Delta)$, (W_Δ, Δ) and (W'_Δ, Δ) are soft sets over I_U, then
(i) $\{\tilde{\phi}\}^\circ_\omega = \tilde{\phi}$ and $\{\tilde{I_U}\}^\circ_\omega = \tilde{I_U}$.
(ii) $[\{(W_\Delta, \Delta)\}^\circ_\omega]^\circ_\omega = \{(W_\Delta, \Delta)\}^\circ_\omega$.
(iii) If $(W_\Delta, \Delta) \tilde{\subseteq} (W'_\Delta, \Delta)$, then $\{(W_\Delta, \Delta)\}^\circ_\omega \tilde{\subseteq} \{(W'_\Delta, \Delta)\}^\circ_\omega$.
(iv)$\{(W_\Delta, \Delta)\}^\circ_\omega \tilde{\cup} \{(W'_\Delta, \Delta)\}^\circ_\omega \tilde{\subseteq} \{(W_\Delta \cup W'_\Delta, \Delta)\}^\circ_\omega$.
(v)$\{(W_\Delta \cap W'_\Delta, \Delta)\}^\circ_\omega = \{(W_\Delta, \Delta)\}^\circ_\omega \tilde{\cap} \{(W'_\Delta, \Delta)\}^\circ_\omega$.
(vi) $(W_\Delta, \Delta)^\circ \tilde{\subseteq} \{(W_\Delta, \Delta)\}^\circ_\omega$."

Definition 2.16 *[11] "Consider a soft set (W_Δ, Δ) over I_U. Then, (W_Δ, Δ) is soft ω-closed set if for any soft semi-open set (O_Δ, Δ) containing (W_Δ, Δ), we have $\overline{(W_\Delta, \Delta)} \tilde{\subseteq} (O_\Delta, \Delta)$. The collection of all soft ω-closed sets is denoted by $F_{s\omega}(\tilde{I_U})$."*

Proposition 2.2 [11] If $(W_\Delta, \Delta) \tilde{\in} \tilde{\tau}^c$, then $(W_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_U})$.

Definition 2.17 *[11] "Consider a soft set (W_Δ, Δ) over I_U. Then, the soft ω-closure of (W_Δ, Δ), denoted by $\{\overline{(W_\Delta, \Delta)}\}_\omega$, is defined as the intersection of all soft ω-closed supersets of (W_Δ, Δ) i.e., $\{\overline{(W_\Delta, \Delta)}\}_\omega = \tilde{\cap}\, \{(F_\Delta, \Delta) : (W_\Delta, \Delta) \tilde{\subseteq} (F_\Delta, \Delta), (W_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_U})\}$."*

Remark 2.5 *[11] "(i) $(W_\Delta, \Delta) \tilde{\subseteq} \{\overline{(W_\Delta, \Delta)}\}_\omega$.*
(ii)$\{\overline{(W_\Delta, \Delta)}\}_\omega \tilde{\in} F_{s\omega}(\tilde{I_U})$."

Lemma 2.6 [11] "$\{\overline{(W_\Delta, \Delta)}\}_\omega$ is the smallest soft ω-closed set containing (W_Δ, Δ)."

Lemma 2.7 [11] "A soft set $(W_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_U})$ if and only if $\{\overline{(W_\Delta, \Delta)}\}_\omega = (W_\Delta, \Delta)$."

Lemma 2.8 [11] "Consider a soft topological space $(I_U, \tilde{\tau}, \Delta)$, (W_Δ, Δ) and (W'_Δ, Δ) are soft sets over I_U, then
(i) $\{\overline{\tilde{\phi}}\}_\omega = \tilde{\phi}$ and $\{\overline{\tilde{I_U}}\}_\omega = \tilde{I_U}$.
(ii) $[\overline{\{\overline{(W_\Delta, \Delta)}\}_\omega}]_\omega = \{\overline{(W_\Delta, \Delta)}\}_\omega$.
(iii) If $(W_\Delta, \Delta) \tilde{\subseteq} (W'_\Delta, \Delta)$, then $\{\overline{(W_\Delta, \Delta)}\}_\omega \tilde{\subseteq} \{\overline{(W'_\Delta, \Delta)}\}_\omega$.
(iv)$\{\overline{(W_\Delta \cup W'_\Delta, \Delta)}\}_\omega = \{\overline{(W_\Delta, \Delta)}\}_\omega \tilde{\cup} \{\overline{(W'_\Delta, \Delta)}\}_\omega$.
(v)$\{\overline{(W_\Delta \cap W'_\Delta, \Delta)}\}_\omega \tilde{\subseteq} \{\overline{(W_\Delta, \Delta)}\}_\omega \tilde{\cap} \{\overline{(W'_\Delta, \Delta)}\}_\omega$.
(vi) $\{\overline{(W_\Delta, \Delta)}\}_\omega \tilde{\subseteq} \overline{(W_\Delta, \Delta)}$."

3. Soft ω-Closed Maps

In this section, we first introduce a soft ω-closed map and then study the properties of this with the help of some examples.

Definition 3.1 *A soft map $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is **soft ω-closed** if for every $(A_\Delta, \Delta) \tilde{\in} \tilde{\tau}^c$, we have $(\mathcal{F}, \Upsilon)(A_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_V})$.*

Example 3.1 Let $I_U = \{\eta_1, \eta_2\}$ and $I_V = \{v_1, v_2\}$ be the universal sets, $\Delta = \{\delta_1, \delta_2\}$ and $\Delta^* = \{\delta_1^*, \delta_2^*\}$ be the non-empty parameter sets such that $(I_U, \tilde{\tau}, \Delta)$ and $(I_V, \tilde{\tau^*}, \Delta^*)$ be the soft topological spaces where $\tilde{\tau} = \{\tilde{\phi},\ \tilde{I_U},\ \{(\delta_1, \{\eta_1\}), (\delta_2, \{\eta_2\})\}\}$ and $\tilde{\tau}^* = \{\tilde{\phi},\ \tilde{I_V}\}$. Suppose $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft map where $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ is defined as $\mathcal{F}(\eta_i) = v_j$, $i, j = 1, 2$ and $i \neq j$ and $\Upsilon(\delta_i) = \delta_j^*$, $i, j = 1, 2$ and $i \neq j$. Then $(\mathcal{F}, \Upsilon)$ is soft ω-closed map.

Theorem 3.1 Every soft closed mapping is soft ω-closed mapping.
Proof: Let $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft closed mapping and $(A_\Delta, \Delta) \tilde{\in} \tilde{\tau}^c$. Then, $(\mathcal{F}, \Upsilon)(A_\Delta, \Delta) \tilde{\in} \tilde{\tau^*}^c$ and by Proposition 2.2, we have $(\mathcal{F}, \Upsilon)(A_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_V})$. Hence, $(\mathcal{F}, \Upsilon)$ is soft ω-closed map.

Example 3.2 *Every soft ω-closed map is not soft closed map by Example 3.1.*

The following example shows that the composition of two soft ω-closed maps need not be a soft ω-closed map:

Example 3.3 *Let $I_U = \{\eta_1, \eta_2\}$, $I_V = \{v_1, v_2\}$ and $I_W = \{w_1, w_2\}$ be the universal sets, $\Delta = \{\delta_1, \delta_2\}$, $\Delta^* = \{\delta_1^*, \delta_2^*\}$ and $\zeta = \{\zeta_1, \zeta_2\}$ be the parameter sets such that $(I_U, \tilde{\tau}, \Delta)$, $(I_V, \tilde{\tau^*}, \Delta^*)$ and $(I_W, \tilde{\sigma}, \zeta)$ be soft topological spaces where $\tilde{\tau} = \{\tilde{\phi},\ \tilde{I_U},\ \{(\delta_1, \{\eta_1\}), (\delta_2, \phi)\}$, $\tilde{\tau}^* = \{\tilde{\phi},\ \tilde{I_V}\}$ and $\tilde{\sigma} = \{\tilde{\phi}, \tilde{I_W}, \{(\zeta_1, \{w_2\}), (\zeta_2, \{w_1\})\}\}$. Suppose $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft map where $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ is defined as $\mathcal{F}(\eta_i) = v_i$, $i = 1, 2$ and $\Upsilon(\delta_j) = \delta_j^*$, $j = 1, 2$.*
Suppose $(g, \Phi) : (I_V, \tilde{\tau}^*, \Delta^*) \to (I_W, \tilde{\sigma}, \zeta)$ is a soft map where $g : I_V \to I_W$ and $\Phi : \Delta^* \to \zeta$ is defined as $g(v_i) = w_i$, $i = 1, 2$ and $\Phi(\delta_j^*) = \zeta_j$, $j = 1, 2$. Then, clearly $(\mathcal{F}, \Upsilon)$ and (g, Φ) are soft ω-closed maps but the composition map $(g, \Phi) \bigodot (\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_W, \tilde{\sigma}, \zeta)$ is not soft ω-closed map as we have a soft closed set $(A_\Delta, \Delta) = \{(\delta_1, \{\eta_2\}), (\delta_2, I_U)\}$ in $(I_U, \tilde{\tau}, \Delta)$ but $(g, \Phi) \bigodot (\mathcal{F}, \Upsilon)\{(A_\Delta, \Delta)\}$ is not soft ω-closed set in $(I_W, \tilde{\sigma}, \zeta)$.

Theorem 3.2 If $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is a soft closed mapping and $(g, \Phi) : (I_V, \tilde{\tau}^*, \Delta^*) \to (I_W, \tilde{\sigma}, \delta)$ is soft ω-closed mapping, then the composition mapping $(g, \Phi) \bigodot (\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_W, \tilde{\sigma}, \zeta)$ is soft ω-closed mapping.
Proof: Let $(A_\Delta, \Delta) \tilde{\in} \tilde{\tau}^c$. As $(\mathcal{F}, \Upsilon)$ is soft closed map, thus $(\mathcal{F}, \Upsilon)(A_\Delta, \Delta) \tilde{\in} \tilde{\tau^*}^c$ and since (g, Φ) is soft ω-closed map, therefore $(g, \Phi)\{(\mathcal{F}, \Upsilon)(A_\Delta, \Delta)\} \tilde{\in} F_{s\omega}(\tilde{I_W})$. Thus, $(g, \Phi) \bigodot (\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_W, \tilde{\sigma}, \zeta)$ is soft ω-closed mapping.

Example 3.4 *If in above thm, $(\mathcal{F}, \Upsilon)$ is soft ω-closed map and (g, Φ) is soft closed map, then $(g, \Phi) \bigodot (\mathcal{F}, \Upsilon)$ need not be soft ω-closed map by Example 3.3.*

Now, we define a soft ω-continuous map in soft topological spaces to discuss its relation with soft ω-closed map.

Definition 3.2 *A map $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is **soft ω-continuous** if for every $(A_{\Delta^*}, \Delta^*) \tilde{\in} \tilde{\tau^*}^c$, we have $(\mathcal{F}, \Upsilon)^{-1}(A_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_U})$.*

Example 3.5 Let $I_U = \{\eta_1, \eta_2\}$ and $I_V = \{v_1, v_2\}$ be the universal sets, $\Delta = \{\delta_1, \delta_2\}$ and $\Delta^* = \{\delta_1^*, \delta_2^*\}$ be the parameter sets such that $(I_U, \tilde{\tau}, \Delta)$ and $(I_V, \tilde{\tau^*}, \Delta^*)$ be soft topological spaces where $\tilde{\tau} = \{\tilde{\phi},\ \tilde{I_U},\ \{(\delta_1, \{\eta_2\}), (\delta_2, I_U)\}\}$ and $\tilde{\tau}^* = \{\tilde{\phi},\ \tilde{I_V},\ \{(\delta_1^*, \{v_1\}), (\delta_2^*, \{v_2\})\}\}$. Suppose $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft map where $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ is defined as $\mathcal{F}(\eta_i) = v_i$, $i = 1, 2$ and $\Upsilon(\delta_i) = \delta_j^*$, $i, j = 1, 2$ and $i \neq j$. Then, $(\mathcal{F}, \Upsilon)$ is soft ω-continuous map.

Theorem 3.3 Every soft continuous map is soft ω-continuous.
Proof: Let $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau^*}, \Delta^*)$ be a soft continuous map and $(A_{\Delta^*}, \Delta^*) \tilde{\in} \tilde{\tau^*}^c$.
As $(\mathcal{F}, \Upsilon)$ is soft continuous map, thus $(\mathcal{F}, \Upsilon)^{-1}(A_{\Delta^*}, \Delta^*) \tilde{\in} \tilde{\tau}^c$ and using Proposition 2.2, we have $(\mathcal{F}, \Upsilon)^{-1}(A_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_U})$. Thus, $(\mathcal{F}, \Upsilon)$ is soft ω-continuous map.

Theorem 3.4 If $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is soft continuous surjective mapping and $(g, \Phi) : (I_V, \tilde{\tau}^*, \Delta^*) \to (I_W, \tilde{\sigma}, \zeta)$ is soft map such that the composition map $(g, \Phi) \bigodot (\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_W, \tilde{\sigma}, \zeta)$ is soft ω-closed, then (g, Φ) is soft ω-closed map.
Proof: Let $(B_{\Delta^*}, \Delta^*) \tilde{\in} \tilde{\tau^*}^c$. Since $(\mathcal{F}, \Upsilon)$ is soft continuous map, therefore $(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*) \tilde{\in} \tilde{\tau}^c$ and as $(g, \Phi) \bigodot (\mathcal{F}, \Upsilon)$ is soft ω-closed map, thus $(g, \Phi) \bigodot (\mathcal{F}, \Upsilon)\{(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*)\} \tilde{\in} F_{s\omega}(\tilde{I_W})$. But $(\mathcal{F}, \Upsilon)$ is soft surjective map, thus $(g, \Phi) \bigodot (\mathcal{F}, \Upsilon)\{(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*)\} = (g, \Phi)(B_{\Delta^*}, \Delta^*)$ i.e.,
$(g, \Phi)(B_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_W})$. Hence, (g, Φ) is soft ω-closed map.

A soft map $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ which is soft ω-continuous but not soft ω-closed map, can be seen from the following example:

Example 3.6 *Let $I_U=\{\eta_1,\eta_2,\eta_3\}$ and $I_V=\{v_1,v_2,v_3\}$ be the universal sets, $\Delta=\{\delta_1,\delta_2\}$ and $\Delta^*=\{\delta_1^*,\delta_2^*\}$ be the parameter sets such that $(I_U,\tilde{\tau},\Delta)$ and $(I_V,\tilde{\tau}^*,\Delta^*)$ be soft topological spaces where $\tilde{\tau}=\{\tilde{\phi},\ \tilde{I_U},\ \{(\delta_1,\{\eta_1,\eta_2\}),(\delta_2,\{\eta_1,\eta_2\})\}\}$ and $\tilde{\tau}^*=\{\tilde{\phi},\ \tilde{I_V},\{(\delta_1^*,\{v_1\}),(\delta_2^*,\{v_1\})\}\}$. Suppose $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ be a soft map where $\mathcal{F}:I_U\to I_V$ and $\Upsilon:\Delta\to\Delta^*$ is defined as $\mathcal{F}(\eta_1)=v_1$, $\mathcal{F}(\eta_2)=v_1$ and $\mathcal{F}(\eta_3)=v_2$ and $\Upsilon(\delta_i)=\delta_i^*$, $i=1,2$. Then, $(\mathcal{F},\Upsilon)$ is soft ω-continuous map but not soft ω-closed map.*

Theorem 3.5 A soft mapping $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ is soft ω-closed if and only if for every soft set (A_Δ,Δ) over I_U, we have

$$\{\overline{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)}\}_\omega\ \tilde{\subseteq}\ (\mathcal{F},\Upsilon)\{\overline{(A_\Delta,\Delta)}\}$$

Proof: Suppose $(\mathcal{F},\Upsilon)$ is soft ω-closed mapping. As $\overline{(A_\Delta,\Delta)}\ \tilde{\in}\ \tilde{\tau}^c$, thus $(\mathcal{F},\Upsilon)\{\overline{(A_\Delta,\Delta)}\}\ \tilde{\in}\ F_{s\omega}(\tilde{I_V})$. Now,

$$\begin{aligned} & (A_\Delta,\Delta)\ \tilde{\subseteq}\ \overline{(A_\Delta,\Delta)} \\ \Rightarrow\qquad & (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}\ \tilde{\subseteq}\ (\mathcal{F},\Upsilon)\{\overline{(A_\Delta,\Delta)}\} \end{aligned}$$

Thus, $(\mathcal{F},\Upsilon)\{\overline{(A_\Delta,\Delta)}\}$ is soft ω-closed set containing $(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}$. But we have $\{\overline{(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}}\}_\omega$ is the smallest soft ω-closed set containing $(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}$.
Thus,

$$\{\overline{(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}}\}_\omega\ \tilde{\subseteq}\ (\mathcal{F},\Upsilon)\{\overline{\{(A_\Delta,\Delta)\}}\}.$$

Conversely, let $(A_\Delta,\Delta)\ \tilde{\in}\ \tilde{\tau}^c$. We have to prove that $(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}\ \tilde{\in}\ F_{s\omega}(\tilde{I_V})$.
By given hypothesis, we have

$$\begin{aligned} & \{\overline{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)}\}_\omega\ \tilde{\subseteq}\ (\mathcal{F},\Upsilon)\{\overline{(A_\Delta,\Delta)}\} \\ \Rightarrow\qquad & \{\overline{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)}\}_\omega\ \tilde{\subseteq}\ (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\} \end{aligned}$$

But we always have

$$\begin{aligned} & (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}\ \tilde{\subseteq}\ \{\overline{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)}\}_\omega \\ \Rightarrow\qquad & (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}=\{\overline{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)}\}_\omega \end{aligned}$$

Thus, $(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}\ \tilde{\in}\ F_{s\omega}(\tilde{I_V})$ and hence $(\mathcal{F},\Upsilon)$ is soft ω-closed map.

Theorem 3.6 A soft map $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ is soft ω-closed if and only if for every soft set (B_{Δ^*},Δ^*) over I_V and for each soft set $(A_\Delta,\Delta)\ \tilde{\in}\ \tilde{\tau}$ such that $(\mathcal{F},\Upsilon)^{-1}(B_{\Delta^*},\Delta^*)\ \tilde{\subseteq}\ (A_\Delta,\Delta)$, there exists a soft set $(G_{\Delta^*},\Delta^*)\ \tilde{\in}\ G_{s\omega}(\tilde{I_V})$ containing (B_{Δ^*},Δ^*) such that $(\mathcal{F},\Upsilon)^{-1}(G_{\Delta^*},\Delta^*)\ \tilde{\subseteq}\ (A_\Delta,\Delta)$.
Proof: Let $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ be a soft ω-closed mapping, (B_{Δ^*},Δ^*) be a soft set over I_V and $(A_\Delta,\Delta)\ \tilde{\in}\ \tilde{\tau}$ be such that $(\mathcal{F},\Upsilon)^{-1}(B_{\Delta^*},\Delta^*)\ \tilde{\subseteq}\ (A_\Delta,\Delta)$. Then, $(A_\Delta^c,\Delta)\ \tilde{\in}\ \tilde{\tau}^c$ and as $(\mathcal{F},\Upsilon)$ is soft ω-closed map. Thus, $(\mathcal{F},\Upsilon)(A_\Delta^c,\Delta)\ \tilde{\in}\ F_{s\omega}(\tilde{I_V})$.
This implies that $\{(\mathcal{F},\Upsilon)(A_\Delta^c,\Delta)\}^c=(G_{\Delta^*},\Delta^*)\ \tilde{\in}\ G_{s\omega}(\tilde{I_V})$ containing (B_{Δ^*},Δ^*) such that $(\mathcal{F},\Upsilon)^{-1}(G_{\Delta^*},\Delta^*)\ \tilde{\subseteq}\ (A_\Delta,\Delta)$.
Conversely, let $(F_\Delta,\Delta)\ \tilde{\in}\ \tilde{\tau}^c$. Then, $(\mathcal{F},\Upsilon)^{-1}\{(\mathcal{F},\Upsilon)(F_\Delta,\Delta)\}^c\ \tilde{\subseteq}\ (F_\Delta^c,\Delta)$ and $(F_\Delta^c,\Delta)\ \tilde{\in}\ \tilde{\tau}$ i.e., we have $(B_{\Delta^*},\Delta^*)=\{(\mathcal{F},\Upsilon)(F_\Delta,\Delta)\}^c$ and $(A_\Delta,\Delta)=(F_\Delta^c,\Delta)$. Thus, by given hypothesis, there exists a soft set $(G_{\Delta^*},\Delta^*)\ \tilde{\in}\ G_{s\omega}(\tilde{I_V})$ containing (B_{Δ^*},Δ^*) such that $(\mathcal{F},\Upsilon)^{-1}(G_{\Delta^*},\Delta^*)\tilde{\subseteq}(A_\Delta,\Delta)$ i.e., we have

$$\begin{aligned} & (\mathcal{F},\Upsilon)^{-1}(G_{\Delta^*},\Delta^*)\ \tilde{\subseteq}\ (A_\Delta,\Delta)=(F_\Delta^c,\Delta) \\ \Rightarrow\qquad & (F_\Delta,\Delta)\ \tilde{\subseteq}\ \{(\mathcal{F},\Upsilon)^{-1}(G_{\Delta^*},\Delta^*)\}^c \qquad ...(1) \end{aligned}$$

Now,

$$\begin{aligned} & \{(\mathcal{F},\Upsilon)(F_\Delta,\Delta)\}^c \tilde{\subseteq} (G_{\Delta^*},\Delta^*) \\ \Rightarrow \quad & (G^c_{\Delta^*},\Delta^*) \tilde{\subseteq} (\mathcal{F},\Upsilon)(F_\Delta,\Delta) \\ & \tilde{\subseteq} (\mathcal{F},\Upsilon)\{(\mathcal{F},\Upsilon)^{-1}(G_{\Delta^*},\Delta^*)\}^c \quad \text{(Using (1))} \\ & \tilde{\subseteq} (G^c_{\Delta^*},\Delta^*) \\ \Rightarrow \quad & (\mathcal{F},\Upsilon)(F_\Delta,\Delta) = (G^c_\Delta,\Delta^*) \end{aligned}$$

As $(G^c_{\Delta^*},\Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_V})$, thus $(\mathcal{F},\Upsilon)(F_\Delta,\Delta) \tilde{\in} F_{s\omega}(\tilde{I_V})$ and hence $(\mathcal{F},\Upsilon)$ is soft ω-closed map.

4. Soft ω-Open Maps

Corresponding to a soft ω-closed map, we define a soft ω-open map and discuss various properties of it with the help of some examples.

Definition 4.1 *A soft map $(\mathcal{F},\Upsilon) : (I_U,\tilde{\tau},\Delta) \to (I_V,\tilde{\tau}^*,\Delta^*)$ is **soft ω-open** if for every soft set $(A_\Delta,\Delta) \tilde{\in} \tilde{\tau}$, we have $(\mathcal{F},\Upsilon)(A_\Delta,\Delta) \tilde{\in} G_{s\omega}(\tilde{I_V})$.*

Example 4.1 Example 3.1 is an example of soft ω-open map.

Theorem 4.1 Every soft open mapping is soft ω-open. But the converse is not true by Example 3.1.

Example 4.2 *The composition of two soft ω-open maps is not necessary soft ω-open by Example 3.3.*

Theorem 4.2 If $(\mathcal{F},\Upsilon) : (I_U,\tilde{\tau},\Delta) \to (I_V,\tilde{\tau}^*,\Delta^*)$ is a soft open mapping and $(g,\Phi) : (I_V,\tilde{\tau}^*,\Delta^*) \to (W,\tilde{\sigma},\delta)$ is soft ω-open map, then the composition map $(g,\Phi)\bigodot(\mathcal{F},\Upsilon) : (I_U,\tilde{\tau},\Delta) \to (W,\tilde{\sigma},\delta)$ is soft ω-open.

Example 4.3 *If in above thm, $(\mathcal{F},\Upsilon)$ is soft ω-open map and (g,Φ) is soft open map, then $(g,\Phi)\bigodot(\mathcal{F},\Upsilon)$ need not be soft ω-open map by Example 3.3.*

Theorem 4.3 If $(\mathcal{F},\Upsilon) : (I_U,\tilde{\tau},\Delta) \to (I_V,\tilde{\tau}^*,\Delta^*)$ is soft continuous surjective map and $(g,\Phi) : (I_V,\tilde{\tau}^*,\Delta^*) \to (I_W,\tilde{\sigma},\zeta)$ is soft map such that the composition map $(g,\Phi)\bigodot(\mathcal{F},\Upsilon) : (I_U,\tilde{\tau},\Delta) \to (I_W,\tilde{\sigma},\zeta)$ is soft ω-open, then (g,Φ) is soft ω-open map.

Now, we prove a thm which is a characterization of soft ω-open set.

Theorem 4.4 A soft map $(\mathcal{F},\Upsilon) : (I_U,\tilde{\tau},\Delta) \to (I_V,\tilde{\tau}^*,\Delta^*)$ is soft ω-open if and only if for any soft set (A_Δ,Δ) over I_U, we have $\{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)\}^\circ_\omega \tilde{\supseteq} (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)^o\}$.
Proof: First, suppose that $(\mathcal{F},\Upsilon)$ is soft ω-open map and (A_Δ,Δ) is any soft set over I_U. As $(A_\Delta,\Delta)^\circ \tilde{\in} \tilde{\tau}$, thus $(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}^\circ \tilde{\in} G_{s\omega}(\tilde{I_V})$.
Now,

$$\begin{aligned} & (A_\Delta,\Delta)^\circ \tilde{\subseteq} (A_\Delta,\Delta) \\ \Rightarrow \quad & (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}^\circ \tilde{\subseteq} (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\} \end{aligned}$$

This implies that $(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}^\circ$ is the soft ω-open set contained in $(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}$. But we have $\{(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}^\circ_\omega$ is the largest soft ω-open set contained in $(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}$. Thus, $\{(\mathcal{F},\Upsilon)(A,\Delta)\}^\circ_\omega \tilde{\supseteq} (\mathcal{F},\Upsilon)\{(A,\Delta)^o\}$.
Conversely, let $(A_\Delta,\Delta) \tilde{\in} \tilde{\tau}$ and we have to show that $(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\} \tilde{\in} G_{s\omega}(\tilde{I_V})$.
By given hypothesis, we have

$$\begin{aligned} & \{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)\}^\circ_\omega \tilde{\supseteq} (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)^o\} \\ \Rightarrow \quad & \{(\mathcal{F},\Upsilon)(A,\Delta)\}^\circ_\omega \tilde{\supseteq} (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\} \end{aligned}$$

But we always have $\{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)\}^\circ_\omega \tilde{\subseteq} (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}$. Thus, $\{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)\}^\circ_\omega = (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}$ and hence $(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\} \tilde{\in} G_{s\omega}(\tilde{I_V})$. Hence, $(\mathcal{F},\Upsilon)$ is soft ω-open map.

Theorem 4.5 A soft map $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is soft ω-open if and only if for every soft set (B_{Δ^*}, Δ^*) in $\tilde{I_V}$ and for each soft set $(A_\Delta, \Delta) \tilde{\in} \tilde{\tau^c}$ such that $(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*) \tilde{\subseteq} (A_\Delta, \Delta)$, there exists a soft set $(G_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_V})$ containing (B_{Δ^*}, Δ^*) such that $(\mathcal{F}, \Upsilon)^{-1}(G_{\Delta^*}, \Delta^*) \tilde{\subseteq} (A_\Delta, \Delta)$.

Theorem 4.6 A soft map $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is soft ω-open if and only if for any soft set (B_{Δ^*}, Δ^*) over I_V, we have

$$(\mathcal{F}, \Upsilon)^{-1}\{\overline{(B_{\Delta^*}, \Delta^*)}\}_\omega \tilde{\subseteq} \overline{(\mathcal{F}, \Upsilon)^{-1}\{(B_{\Delta^*}, \Delta^*)\}}$$

Proof: Let $(\mathcal{F}, \Upsilon)$ be a soft ω-open map.Then, for any soft set (B_{Δ^*}, Δ^*) over I_V, we have $(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*) \tilde{\subseteq} \{\overline{(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*)}\}$. By above thm, for any soft set (B_{Δ^*}, Δ^*) of I_V and $(A_\Delta, \Delta) = \overline{(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*)} \tilde{\in} \tilde{\tau^c}$, there exists a soft set $(G_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_V})$ such that $(B_{\Delta^*}, \Delta^*) \tilde{\subseteq} (G_{\Delta^*}, \Delta^*)$ and

$$\begin{aligned} & (\mathcal{F}, \Upsilon)^{-1}(G_{\Delta^*}, \Delta^*) \tilde{\subseteq} (A_\Delta, \Delta) \\ \Rightarrow \quad & (\mathcal{F}, \Upsilon)^{-1}(G_{\Delta^*}, \Delta^*) \tilde{\subseteq} \{\overline{(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*)}\} \end{aligned}$$

Now,

$$\begin{aligned} & (B_{\Delta^*}, \Delta^*) \tilde{\subseteq} (G_{\Delta^*}, \Delta^*) \\ \Rightarrow \quad & \{\overline{(B_{\Delta^*}, \Delta^*)}\}_\omega \tilde{\subseteq} \{\overline{(G_{\Delta^*}, \Delta^*)}\}_\omega = (G_{\Delta^*}, \Delta^*) \\ \Rightarrow \quad & (\mathcal{F}, \Upsilon)^{-1}\{\overline{(B_{\Delta^*}, \Delta^*)}\}_\omega \tilde{\subseteq} (\mathcal{F}, \Upsilon)^{-1}(G_{\Delta^*}, \Delta^*) \tilde{\subseteq} \{\overline{(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*)}\} \end{aligned}$$

Conversely, let (B_{Δ^*}, Δ^*) be any soft subset of I_V and $(A_\Delta, \Delta) \tilde{\in} \tilde{\tau^c}$ containing $(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*)$. If we take $(G_{\Delta^*}, \Delta^*) = \{\overline{(B_{\Delta^*}, \Delta^*)}\}_\omega$. Then, (G_{Δ^*}, Δ^*) is soft ω-closed set and $(B_{\Delta^*}, \Delta^*) \tilde{\subseteq} (G_{\Delta^*}, \Delta^*)$. By given assumption,

$$\begin{aligned} (\mathcal{F}, \Upsilon)^{-1}(G_{\Delta^*}, \Delta^*) &= (\mathcal{F}, \Upsilon)^{-1}\{\overline{(B_{\Delta^*}, \Delta^*)}\}_\omega \\ &\tilde{\subseteq} \{\overline{(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*)}\} \\ &\tilde{\subseteq} \overline{(A_\Delta, \Delta)} = (A_\Delta, \Delta). \end{aligned}$$

Thus, by above thm, $(\mathcal{F}, \Upsilon)$ is soft ω-open map.

Remark 4.1 *(i) A soft ω-closed map need not imply soft ω-open map.*
Example: Let $I_U = \{\eta_1, \eta_2, \eta_3\}$ and $I_V = \{v_1, v_2\}$ be the universal sets, $\Delta = \{\delta_1, \delta_2\}$ and $\Delta^* = \{\delta_1^*, \delta_2^*\}$ be the parameter sets such that $(I_U, \tilde{\tau}, \Delta)$ and $(I_V, \tilde{\tau}^*, \Delta^*)$ be soft topological spaces where $\tilde{\tau} = \{\tilde{\phi}, \tilde{I_U}, \{(\delta_1, \{\eta_1, \eta_2\}), (\delta_2, \{\eta_2, \eta_3\})\}\}$ and $\tilde{\tau}^* = \{\tilde{\phi}, \tilde{I_V}, \{(\delta_1^*, \{v_1\}), (\delta_2^*, I_V)\}\}$. Suppose $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft map where $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ is defined as $\mathcal{F}(\eta_1) = v_1$, $\mathcal{F}(\eta_2) = v_2$ and $\mathcal{F}(\eta_3) = v_2$ and $\Upsilon(\delta_i) = \delta_i^*$, $i = 1, 2$. Then, $(\mathcal{F}, \Upsilon)$ is soft ω-closed map but not soft ω-open as we have a soft set $(A_\Delta, \Delta) = \{(\delta_1, \{\eta_1, \eta_2\}), (\delta_2, \{\eta_2, \eta_3\})\} \tilde{\in} \tilde{\tau}$ such that $(\mathcal{F}, \Upsilon)(A_\Delta, \Delta) = \{(\delta_1^*, \{v_1, v_2\}), (\delta_2^*, \{v_2\})\} \tilde{\notin} G_{s\omega}(\tilde{I_V})$.
(ii) A soft ω-open map need not imply soft ω-closed map.
Example: Let $I_U = \{\eta_1, \eta_2\}$ and $I_V = \{v_1, v_2, v_3\}$ be the universal sets, $\Delta = \{\delta_1, \delta_2\}$ and $\Delta^* = \{\delta_1^*, \delta_2^*\}$ be the parameter sets such that $(I_U, \tilde{\tau}, \Delta)$ and $(I_V, \tilde{\tau}^*, \Delta^*)$ be soft topological spaces where $\tilde{\tau} = \{\tilde{\phi}, \tilde{I_U}, \{(\delta_1, \phi), (\delta_2, \{\eta_2\})\}\}$ and $\tilde{\tau}^* = \{\tilde{\phi}, \tilde{I_V}, \{(\delta_1^*, \{v_1, v_2\}), (\delta_2^*, \{v_1, v_2\})\}\}$. Suppose $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft map where $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ is defined as $\mathcal{F}(\eta_1) = v_1$ and $\mathcal{F}(\eta_2) = v_2$ and $\Upsilon(\delta_i) = \delta_i^*$, $i = 1, 2$. Then, $(\mathcal{F}, \Upsilon)$ is soft ω-open map but not soft ω-closed map as we have a soft closed set $\tilde{I_U}$ such that $(\mathcal{F}, \Upsilon)(\tilde{I_U}) = \{(\delta_1^*, \{v_1, v_2\}), (\delta_2^*, \{v_1, v_2\})\}$ is not soft ω-closed set in I_V.

The following example shows that there exists a soft map which is neither soft ω-open nor soft ω-closed

Example 4.4 *Let $I_U = \{\eta_1, \eta_2\}$ and $I_V = \{v_1, v_2\}$ be the universal sets, $\Delta = \{\delta_1, \delta_2\}$ and $\Delta^* = \{\delta_1^*, \delta_2^*\}$ be the parameter sets such that $(I_U, \tilde{\tau}, \Delta)$ and $(I_V, \tilde{\tau}^*, \Delta^*)$ be soft topological spaces where $\tilde{\tau} = \{\tilde{\phi},\ \tilde{I_U},\ \{(\delta_1, \{\eta_1\}), (\delta_2, \{\eta_2\})\}\}$ and $\tilde{\tau}^* = \{\tilde{\phi},\ \tilde{I_V}, \{(\delta_1^*, \{v_2\}), (\delta_2^*, \{v_1\})\}\}$. Suppose $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft map where $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ is defined as $\mathcal{F}(\eta_i) = v_i$ and $\Upsilon(\delta_i) = \delta_i^*$, $i = 1, 2$. Then, $(\mathcal{F}, \Upsilon)$ is neither soft ω-open map nor soft ω-closed map.*

Example 4.5 *A soft map which is both soft ω-open and soft ω-closed map can be seen from Example 3.1.*

Definition 4.2 *A soft map $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is called **soft ω-bicontinuous map** if it is both soft ω-open and soft ω-continuous map.*

A soft map $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ which is soft ω-continuous but not soft ω-open map can be seen from the following example:

Example 4.6 *Let $I_U = \{\eta_1, \eta_2, \eta_3\}$ and $I_V = \{v_1, v_2, v_3\}$ be the universal sets, $\Delta = \{\delta_1, \delta_2\}$ and $\Delta^* = \{\delta_1^*, \delta_2^*\}$ be the parameter sets such that $(I_U, \tilde{\tau}, \Delta)$ and $(I_V, \tilde{\tau}^*, \Delta^*)$ be soft topological spaces where $\tilde{\tau} = \{\tilde{\phi},\ \tilde{I_U},\ \{(\delta_1, \{\eta_1\}), (\delta_2, \{\eta_2\})\},\ \{(\delta_1, \{\eta_3\}), (\delta_2, \{\eta_3\})\},\ \{(\delta_1, \{\eta_1, \eta_3\}), (\delta_2, \{\eta_2, \eta_3\})\}\}$ and $\tilde{\tau}^* = \{\tilde{\phi},\ \tilde{I_V}, \{(\delta_1^*, \{v_2, v_3\}), (\delta_2^*, \{v_2, v_3\})\}\}$. Suppose $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft map where $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ is defined as $\mathcal{F}(\eta_1) = v_1$, $\mathcal{F}(\eta_2) = v_1$ and $\mathcal{F}(\eta_3) = v_2$ and $\Upsilon(\delta_i) = \delta_i^*$, $i = 1, 2$. Then, $(\mathcal{F}, \Upsilon)$ is soft ω-continuous map but not soft ω-open map.*

Theorem 4.7 Let $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a bijective soft map, then the following statements are equivalent:
(i) $(\mathcal{F}, \Upsilon)^{-1} : (I_V, \tilde{\tau}^*, \Delta^*) \to (I_U, \tilde{\tau}, \Delta)$ is soft ω-continuous.
(ii) $(\mathcal{F}, \Upsilon)$ is soft ω-open map.
(iii) $(\mathcal{F}, \Upsilon)$ is soft ω-closed map.
Proof: $(i) \Rightarrow (ii)$
Let $(G_\Delta, \Delta) \tilde{\in} \tilde{\tau}$. As $(\mathcal{F}, \Upsilon)^{-1}$ is soft ω-continuous, thus

$$\{(\mathcal{F}, \Upsilon)^{-1}\}^{-1}(G_\Delta, \Delta) = (\mathcal{F}, \Upsilon)(G_\Delta, \Delta) \tilde{\in} G_{s\omega}(\tilde{I_V}) \qquad [\because (\mathcal{F}, \Upsilon) \text{ is bijective soft map}]$$

Thus, $(\mathcal{F}, \Upsilon)$ is soft ω-open map.
$(ii) \Rightarrow (iii)$
Let $(F_\Delta, \Delta) \tilde{\in} \tilde{\tau}^c$. Then, $(F_\Delta^c, \Delta) \tilde{\in} \tilde{\tau}$. But $(\mathcal{F}, \Upsilon)$ is soft ω-open map. This implies that $(\mathcal{F}, \Upsilon)(F_\Delta^c, \Delta) \tilde{\in} G_{s\omega}(\tilde{I_V})$ i.e.,

$$\{(\mathcal{F}, \Upsilon)(F_\Delta^c, \Delta)\}^c = (\mathcal{F}, \Upsilon)(F_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_V}) \qquad [\because (\mathcal{F}, \Upsilon) \text{ is bijective soft map}]$$

Thus, $(\mathcal{F}, \Upsilon)$ is soft ω-closed map.
$(iii) \Rightarrow (i)$
Let $(F_\Delta, \Delta) \tilde{\in} \tilde{\tau}^c$. As $(\mathcal{F}, \Upsilon)$ is soft ω-closed map, thus $(\mathcal{F}, \Upsilon)(F_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_V})$. But $(\mathcal{F}, \Upsilon)(F_\Delta, \Delta) = \{(\mathcal{F}, \Upsilon)^{-1}\}^{-1}(F_\Delta, \Delta)$ i.e., $\{(\mathcal{F}, \Upsilon)^{-1}\}^{-1}(F_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_V})$. Hence, $(\mathcal{F}, \Upsilon)^{-1}$ is soft ω-continuous map.

5. Soft ω-Irresolute Maps

In this section, we define a soft ω-irresolute map in soft topological spaces and study its relation with soft irresolute map. We further study some properties of it with the help of some examples.

Definition 5.1 *A soft map $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is **soft ω-irresolute** if for every soft set $(A_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_V})$, we have $(\mathcal{F}, \Upsilon)^{-1}(A_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_U})$.*

Example 5.1 Let $I_U = \{\eta_1, \eta_2\}$ and $I_V = \{v_1, v_2\}$ be universal sets, $\Delta = \{\delta_1, \delta_2\}$ and $\Delta^* = \{\delta_1^*, \delta_2^*\}$ be the parameter sets such that $(I_U, \tilde{\tau}, \Delta)$ and $(I_V, \tilde{\tau}^*, \Delta^*)$ be soft topological spaces where $\tilde{\tau} = \{\tilde{\phi},\ \tilde{I_U}\}$ and $\tilde{\tau}^* = \{\tilde{\phi},\ \tilde{I_V}, \{(\delta_1^*, \{v_1\}), (\delta_2^*, \{v_2\})\}\}$. Suppose $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft map where $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ is defined as $\mathcal{F}(\eta_i) = v_i$, $i = 1, 2$ and $\Upsilon(\delta_j) = \delta_j^*$, $j = 1, 2$. Then, $(\mathcal{F}, \Upsilon)$ is soft ω-irresolute map.

Remark 5.1 *(i) Soft irresolute map need not imply soft ω-irresolute map.*
Example: Let $I_U = \{\eta_1, \eta_2\}$ and $I_V = \{\upsilon_1, \upsilon_2\}$ be the universal sets, $\Delta = \{\delta_1, \delta_2\}$ and $\Delta^* = \{\delta_1^*, \delta_2^*\}$ be the parameter sets such that $(I_U, \tilde{\tau}, \Delta)$ and $(I_V, \tilde{\tau^*}, \Delta^*)$ be soft topological spaces where $\tilde{\tau} = \{\tilde{\phi}, \tilde{I_U}, \{(\delta_1, \{\eta_1\}), (\delta_2, \phi)\}\}$ and $\tilde{\tau}^* = \{\tilde{\phi}, \tilde{I_V}\}$. Suppose $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft map where $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ is defined as $\mathcal{F}(\eta_i) = \upsilon_i$, $i = 1, 2$ and $\Upsilon(\delta_j) = \delta_j^*$, $j = 1, 2$. Then, $(\mathcal{F}, \Upsilon)$ is soft irresolute map but not soft ω-irresolute map as we have a soft ω-closed set $(A_{\Delta^*}, \Delta^*) = \{(p_1^*, \{\upsilon_1\}), (p_2^*, \{\upsilon_2\})\}$ in $\tilde{I_V}$ such that $(\mathcal{F}, \Upsilon)^{-1}(A_{\Delta^*}, \Delta^*) = \{(\delta_1, \{\eta_1\}), (\delta_2, \{\eta_2\})\}$ is not soft ω-closed in $\tilde{I_U}$.
(ii) Soft ω-irresolute map need not imply soft irresolute map by Example 5.1.

Theorem 5.1 A soft map $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is soft ω-irresolute if for every soft set $(B_{\Delta^*}, \Delta^*) \tilde{\in} G_{s\omega}(\tilde{I_V})$, we have $(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*) \tilde{\in} G_{s\omega}(\tilde{I_U})$.
Proof: Let $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft ω-irresolute map and $(B_{\Delta^*}, \Delta^*) \tilde{\in} G_{s\omega}(\tilde{I_V})$.
Then, $(B^c_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_V})$ and thus $(\mathcal{F}, \Upsilon)^{-1}(B^c_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_U})$ i.e.,
$\{(\mathcal{F}, \Upsilon)^{-1}(B^c_{\Delta^*}, \Delta^*)\}^c \tilde{\in} G_{s\omega}(\tilde{I_U})$ and hence $(\mathcal{F}, \Upsilon)^{-1}(B_{\Delta^*}, \Delta^*) \tilde{\in} G_{s\omega}(\tilde{I_U})$.

Theorem 5.2 If $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is soft ω-irresolute map, then the soft map is soft ω-continuous.
Proof: Let $(A_{\Delta^*}, \Delta^*) \tilde{\in} \tilde{\tau^*}^c$. By Proposition 2.2, we have $(A_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_V})$. But $(\mathcal{F}, \Upsilon)$ is soft ω-irresolute map, thus $(\mathcal{F}, \Upsilon)^{-1}(A_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_U})$. Hence, $(\mathcal{F}, \Upsilon)$ is soft ω-continuous map.

Remark 5.2 *The converse of above thm is not true, in general by example in Remark 5.1.*

Theorem 5.3 If $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is soft bijective, soft closed and soft irresolute map, then $(\mathcal{F}, \Upsilon)^{-1} : (I_V, \tilde{\tau^*}, \Delta^*) \to (I_U, \tilde{\tau}, \Delta)$ is soft ω-irresolute map.
Proof: Let $(A_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_U})$ and we prove that $\{(\mathcal{F}, \Upsilon)^{-1}\}^{-1}(A_\Delta, \Delta) = (\mathcal{F}, \Upsilon)(A_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_V})$. For this, consider a soft semi-open set (O_{Δ^*}, Δ^*) over I_V such that

$$\begin{aligned} & (\mathcal{F}, \Upsilon)(A_\Delta, \Delta) \tilde{\subseteq} (O_{\Delta^*}, \Delta^*) \\ \Rightarrow \quad & (A_\Delta, \Delta) \tilde{\subseteq} (\mathcal{F}, \Upsilon)^{-1}(O_{\Delta^*}, \Delta^*). \end{aligned}$$

Now, $(\mathcal{F}, \Upsilon)$ is given to be soft irresolute map, thus $(\mathcal{F}, \Upsilon)^{-1}(O_{\Delta^*}, \Delta^*)$ is soft semi-open set over I_U and as $(A_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_U})$, thus

$$\begin{aligned} & \overline{(A_\Delta, \Delta)} \tilde{\subseteq} (\mathcal{F}, \Upsilon)^{-1}(O_{\Delta^*}, \Delta^*) \\ \Rightarrow \quad & (\mathcal{F}, \Upsilon)\{\overline{(A_\Delta, \Delta)}\} \tilde{\subseteq} (O_{\Delta^*}, \Delta^*). \end{aligned}$$

Since $(\mathcal{F}, \Upsilon)$ is soft closed map and $\overline{(A_\Delta, \Delta)} \tilde{\in} \tilde{\tau^c}$, thus $(\mathcal{F}, \Upsilon)\{\overline{(A_\Delta, \Delta)}\} \tilde{\in} \tilde{\tau^*}^c$ and by using 2.2, we have $(\mathcal{F}, \Upsilon)\{\overline{(A_\Delta, \Delta)}\} \tilde{\in} F_{s\omega}(\tilde{I_V})$. Therefore,

$$\begin{aligned} & \overline{[(\mathcal{F}, \Upsilon)\{\overline{(A_\Delta, \Delta)}\}]} \tilde{\subseteq} (O_{\Delta^*}, \Delta^*) \\ \Rightarrow \quad & \overline{[(\mathcal{F}, \Upsilon)\{(A_\Delta, \Delta)\}]} \tilde{\subseteq} (O_{\Delta^*}, \Delta^*). \end{aligned}$$

Thus, $(\mathcal{F}, \Upsilon)(A_\Delta, \Delta) \tilde{\in} F_{s\omega}(\tilde{I_V})$ and hence $(\mathcal{F}, \Upsilon)^{-1} : (I_V, \tilde{\tau^*}, \Delta^*) \to (I_U, \tilde{\tau}, \Delta)$ is soft ω-irresolute map.

Theorem 5.4 The composition of two soft ω-irresolute maps is again a soft ω-irresolute map.
Proof: Let $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ and $(g, \Phi) : (I_V, \tilde{\tau}^*, \Delta^*) \to (I_W, \tilde{\sigma}, \zeta)$ be two soft ω-irresolute maps. We prove that $(g, \Phi) \bigodot (\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_W, \tilde{\sigma}, \zeta)$ is also a soft ω-irresolute map. For this, consider a soft set $(C_\zeta, \zeta) \tilde{\in} F_{s\omega}(\tilde{I_W})$. As (g, Φ) is soft ω-irresolute map, thus $(g, \Phi)^{-1}(C_\zeta, \zeta) \tilde{\in} F_{s\omega}(\tilde{I_V})$. Now, $(\mathcal{F}, \Upsilon)$ is soft ω-irresolute map, thus
$(\mathcal{F}, \Upsilon)^{-1}\{(g, \Phi)^{-1}(C_\zeta, \zeta)\} \tilde{\in} F_{s\omega}(\tilde{I_U})$ i.e., $[(g, \Phi) \bigodot (\mathcal{F}, \Upsilon)]^{-1}\{(C_\zeta, \zeta)\} \tilde{\in} F_{s\omega}(\tilde{I_U})$.
Hence, $(g, \Phi) \bigodot (\mathcal{F}, \Upsilon)$ is soft ω-irresolute map.

Theorem 5.5 If $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ be any soft map and $(g,\Phi):(I_V,\tilde{\tau}^*,\Delta^*)\to(I_W,\tilde{\sigma},\zeta)$ be a soft ω-irresolute injective map such that $(g,\Phi)\bigodot(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_W,\tilde{\sigma},\zeta)$ is soft ω-closed map, then $(\mathcal{F},\Upsilon)$ is soft ω-closed map.
Proof: Let $(A_\Delta,\Delta)\tilde{\in}\tilde{\tau}^c$. Since $(g,\Phi)\bigodot(\mathcal{F},\Upsilon)$ is soft ω-closed map, therefore $(g,\Phi)\bigodot(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}\tilde{\in}F_{s\omega}(\tilde{I_W})$. But (g,Φ) is given as soft ω-irresolute map. Thus, $(g,\Phi)^{-1}[(g,\Phi)\bigodot(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}]\tilde{\in}F_{s\omega}(\tilde{I_U})$. But (g,Φ) is injective soft map, thus $(g,\Phi)^{-1}[(g,\Phi)\bigodot(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)\}]=(\mathcal{F},\Upsilon)(A_\Delta,\Delta)$ i.e., $(\mathcal{F},\Upsilon)(A_\Delta,\Delta)\tilde{\in}F_{s\omega}(\tilde{I_U})$. Hence, $(\mathcal{F},\Upsilon)$ is soft ω-closed map.

Theorem 5.6 A soft map $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ is soft ω-irresolute if and only if for every soft set (F_{Δ^*},Δ^*) over I_V, we have $(\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}^\circ_\omega\tilde{\subseteq}\{(\mathcal{F},\Upsilon)^{-1}(F_{\Delta^*},\Delta^*)\}^\circ_\omega$.
Proof: Let $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ be a soft ω-irresolute map. As $\{(F_{\Delta^*},\Delta^*)\}^\circ_\omega\tilde{\in}G_{s\omega}(\tilde{I_V})$. Thus, $(\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}^\circ_\omega\tilde{\in}G_{s\omega}(\tilde{I_U})$. Thus,

$$\begin{aligned}
& (\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}^\circ_\omega=[(\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}^\circ_\omega]^\circ_\omega\\
& \qquad\tilde{\subseteq}[(\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}]^\circ_\omega\\
\Rightarrow\quad & (\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}^\circ_\omega\tilde{\subseteq}\{(\mathcal{F},\Upsilon)^{-1}(F_{\Delta^*},\Delta^*)\}^\circ_\omega
\end{aligned}$$

Conversely, let $(G_{\Delta^*},\Delta^*)\tilde{\in}G_{s\omega}(\tilde{I_V})$, then $\{(G_{\Delta^*},\Delta^*)\}^\circ_\omega=(G_{\Delta^*},\Delta^*)$. By given condition, we have

$$\begin{aligned}
(\mathcal{F},\Upsilon)^{-1}\{(G_{\Delta^*},\Delta^*)\}^\circ_\omega &\tilde{\subseteq}\{(\mathcal{F},\Upsilon)^{-1}(G_{\Delta^*},\Delta^*)\}^\circ_\omega\\
(\mathcal{F},\Upsilon)^{-1}\{(G_{\Delta^*},\Delta^*)\} &\tilde{\subseteq}\{(\mathcal{F},\Upsilon)^{-1}(G_{\Delta^*},\Delta^*)\}^\circ_\omega
\end{aligned}$$

But we have

$$\{(\mathcal{F},\Upsilon)^{-1}(G_{\Delta^*},\Delta^*)\}^\circ_\omega\tilde{\subseteq}(\mathcal{F},\Upsilon)^{-1}\{(G_{\Delta^*},\Delta^*)\}$$

Thus, $\{(\mathcal{F},\Upsilon)^{-1}(G_{\Delta^*},\Delta^*)\}^\circ_\omega=(\mathcal{F},\Upsilon)^{-1}\{(G_{\Delta^*},\Delta^*)\}$ i.e., $(\mathcal{F},\Upsilon)^{-1}(G_{\Delta^*},\Delta^*)\tilde{\in}G_{s\omega}(\tilde{I_U})$ and hence $(\mathcal{F},\Upsilon)$ is soft ω-irresolute map.

Theorem 5.7 Let $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ be a soft map. Then, the following statements are equivalent:
(i) $(\mathcal{F},\Upsilon)$ is soft ω-irresolute map.
(ii) $(\mathcal{F},\Upsilon)\{\overline{(G_\Delta,\Delta)}_\omega\}\tilde{\subseteq}\{\overline{(\mathcal{F},\Upsilon)(G_\Delta,\Delta)}\}_\omega$.
(iii) $\{\overline{(\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}}\}_\omega\tilde{\subseteq}(\mathcal{F},\Upsilon)^{-1}\{\overline{(F_{\Delta^*},\Delta^*)}_\omega\}$.
Proof: (i) $\Rightarrow$ (ii)
Let $(\mathcal{F},\Upsilon)$ be a soft ω-irresolute map and (G_Δ,Δ) be any soft set over I_U. As $\{\overline{(\mathcal{F},\Upsilon)(G_\Delta,\Delta)}\}_\omega\tilde{\in}F_{s\omega}(\tilde{I_V})$, thus $(\mathcal{F},\Upsilon)^{-1}\{\overline{(\mathcal{F},\Upsilon)(G_\Delta,\Delta)}\}_\omega\tilde{\in}F_{s\omega}(\tilde{I_U})$. Also,

$$(G_\Delta,\Delta)\tilde{\subseteq}(\mathcal{F},\Upsilon)^{-1}\{(\mathcal{F},\Upsilon)(G_\Delta,\Delta)\}\tilde{\subseteq}(\mathcal{F},\Upsilon)^{-1}\{\overline{(\mathcal{F},\Upsilon)(G_\Delta,\Delta)}\}_\omega.$$

Thus, $(\mathcal{F},\Upsilon)^{-1}\{\overline{(\mathcal{F},\Upsilon)(G_\Delta,\Delta)}\}_\omega\tilde{\in}F_{s\omega}(\tilde{I_U})$ containing (G_Δ,Δ) but we have $\{\overline{(G_\Delta,\Delta)}\}_\omega$ is the smallest soft ω-closed set containing (G_Δ,Δ). Hence,

$$\begin{aligned}
& \{\overline{(G_\Delta,\Delta)}\}_\omega\tilde{\subseteq}(\mathcal{F},\Upsilon)^{-1}\{\overline{(\mathcal{F},\Upsilon)(G_\Delta,\Delta)}\}_\omega\\
\Rightarrow\quad & (\mathcal{F},\Upsilon)\{\overline{(G_\Delta,\Delta)}_\omega\}\tilde{\subseteq}\{\overline{(\mathcal{F},\Upsilon)(G_\Delta,\Delta)}\}_\omega
\end{aligned}$$

(ii) $\Rightarrow$ (iii)
Let (F_{Δ^*},Δ^*) be any soft set over I_V, then $(G_\Delta,\Delta)=(\mathcal{F},\Upsilon)^{-1}(F_{\Delta^*},\Delta^*)$ is a soft set over I_U. By given condition (ii), we have

$$\begin{aligned}
& (\mathcal{F},\Upsilon)\{\overline{(G_\Delta,\Delta)}_\omega\}\tilde{\subseteq}\{\overline{(\mathcal{F},\Upsilon)(G_\Delta,\Delta)}\}_\omega\\
\Rightarrow\quad & (\mathcal{F},\Upsilon)\{\overline{(\mathcal{F},\Upsilon)^{-1}(F_{\Delta^*},\Delta^*)}_\omega\}\tilde{\subseteq}\{\overline{(\mathcal{F},\Upsilon)(\mathcal{F},\Upsilon)^{-1}(F_{\Delta^*},\Delta^*)}\}_\omega\tilde{\subseteq}\overline{(F_{\Delta^*},\Delta^*)}_\omega\\
\Rightarrow\quad & \{\overline{(\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}}\}_\omega\tilde{\subseteq}(\mathcal{F},\Upsilon)^{-1}\{\overline{(F_{\Delta^*},\Delta^*)}_\omega\}
\end{aligned}$$

(iii) $\Rightarrow$(i)
Let $(F_{\Delta^*}, \Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_V})$. By given condition (iii), we have

$$\begin{aligned} & \{\overline{(\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}}\}_\omega \tilde{\subseteq} (\mathcal{F},\Upsilon)^{-1}\{\overline{(F_{\Delta^*},\Delta^*)}_\omega\} \\ \Rightarrow \quad & \{\overline{(\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}}\}_\omega \tilde{\subseteq} (\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}. \end{aligned}$$

But we always have

$$\begin{aligned} & (\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\} \tilde{\subseteq} \{\overline{(\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}}\}_\omega \\ \Rightarrow \quad & (\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\} = \{\overline{(\mathcal{F},\Upsilon)^{-1}\{(F_{\Delta^*},\Delta^*)\}}\}_\omega. \end{aligned}$$

Thus, $(\mathcal{F},\Upsilon)^{-1}(F_{\Delta^*},\Delta^*) \tilde{\in} F_{s\omega}(\tilde{I_U})$. Hence, $(\mathcal{F},\Upsilon)$ is soft ω-irresolute map.

6. Soft ω-Homeomorphisms

This section contains soft ω-homeomorphism and its relation with soft homeomorphism. We further study some properties of it with appropriate examples.

Definition 6.1 *A bijective soft map $(\mathcal{F},\Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is **soft ω-homeomorphism** if $(\mathcal{F},\Upsilon)$ is soft ω-bicontinuous maps.*

Example 6.1 *Example 3.1 is an example of soft ω-homeomorphism.*

Using Theorem 4.7, there is an alternate definition of soft ω-homeomorphism, which is defined below:

Definition 6.2 *A bijective soft map $(\mathcal{F},\Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is **soft ω-homeomorphism** if $(\mathcal{F},\Upsilon)$ and $(\mathcal{F},\Upsilon)^{-1}$ both are soft ω-continuous maps.*

A bijective soft map $(\mathcal{F},\Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ which is soft ω-continuous but not soft ω-homeomorphism, can be seen from the following example:

Example 6.2 *Let $I_U = \{\eta_1, \eta_2, \eta_3\}$ and $I_V = \{v_1, v_2, v_3\}$ be the universal sets, $\Delta = \{\delta_1, \delta_2\}$ and $\Delta^* = \{\delta_1^*, \delta_2^*\}$ be the parameter sets such that $(I_U, \tilde{\tau}, \Delta)$ and $(I_V, \tilde{\tau^*}, \Delta^*)$ be soft topological spaces where $\tilde{\tau} = \{\tilde{\phi}, \tilde{I_U}, \{(\delta_1, \{\eta_1, \eta_3\}), (\delta_2, \eta_2)\}\}$ and $\tilde{\tau}^* = \{\tilde{\phi}, \tilde{I_V}, \{(\delta_1^*, \{v_1\}), (\delta_2^*, v_2)\}\}$. Suppose $(\mathcal{F},\Upsilon) : (I_U, \tilde{\tau}, \Delta)$ $\to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft map where $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ is defined as $\mathcal{F}(\eta_i) = v_i$, $i = 1, 2, 3$ and $\Upsilon(\delta_j) = \delta_j^*$, $j = 1, 2$. Then, $(\mathcal{F},\Upsilon)$ is bijective soft ω-continuous map but not soft ω-homeomorphism.*

A bijective soft map $(\mathcal{F},\Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ which is soft ω-open but not soft ω-homeomorphism, can be seen from the following example:

Example 6.3 *Let $I_U = \{\eta_1, \eta_2, \eta_3\}$ and $I_V = \{v_1, v_2, v_3\}$ be the universal sets, $\Delta = \{\delta_1, \delta_2\}$ and $\Delta^* = \{\delta_1^*, \delta_2^*\}$ be the parameter sets such that $(I_U, \tilde{\tau}, \Delta)$ and $(I_V, \tilde{\tau^*}, \Delta^*)$ be soft topological spaces where $\tilde{\tau} = \{\tilde{\phi}, \tilde{I_U}, \{(\delta_1, \{\eta_1\}), (\delta_2, \eta_1)\}\}$ and $\tilde{\tau}^* = \{\tilde{\phi}, \tilde{I_V}, \{(\delta_1^*, \{v_1\}), (\delta_2^*, v_1)\}, \{(\delta_1^*, \{v_1, v_2\}), (\delta_2^*, \{v_1, v_2\})\}\}$. Suppose $(\mathcal{F},\Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a soft map where $\mathcal{F} : I_U \to I_V$ and $\Upsilon : \Delta \to \Delta^*$ is defined as $\mathcal{F}(\eta_i) = v_i$, $i = 1, 2, 3$ and $\Upsilon(\delta_j) = \delta_j^*$, $j = 1, 2$. Then, $(\mathcal{F},\Upsilon)$ is bijective soft ω-open map but not soft ω-homeomorphism.*

Theorem 6.1 *If a bijective soft map $(\mathcal{F},\Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is such that for every soft set (B_Δ, Δ) over I_U, we have*

$$(\mathcal{F},\Upsilon)\{\overline{(B_\Delta,\Delta)}_\omega\} = \overline{\{(\mathcal{F},\Upsilon)(B_\Delta,\Delta)\}},$$

then $(\mathcal{F},\Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ is soft ω-homeomorphism.
Proof: *Let (B_Δ, Δ) be any soft closed set over I_U, then we have*

$$\begin{aligned} & (\mathcal{F},\Upsilon)\{\overline{(B_\Delta,\Delta)}_\omega\} = \overline{\{(\mathcal{F},\Upsilon)(B_\Delta,\Delta)\}} \\ \Rightarrow \quad & (\mathcal{F},\Upsilon)\{\overline{(B_\Delta,\Delta)}_\omega\} \tilde{\subseteq} \overline{\{(\mathcal{F},\Upsilon)(B_\Delta,\Delta)\}}. \end{aligned}$$

Thus, we have, $(\mathcal{F},\Upsilon)$ is soft ω-continuous map.
Now, we prove that the soft map is soft ω-closed map. As (B_Δ,Δ) is soft closed set over I_U, thus (B_Δ,Δ) is soft ω-closed set over I_U. By given hypothesis, we have

$$\begin{aligned} & (\mathcal{F},\Upsilon)\{\overline{(B_\Delta,\Delta)}_\omega\} = \overline{\{(\mathcal{F},\Upsilon)(B_\Delta,\Delta)\}} \\ \Rightarrow\quad & (\mathcal{F},\Upsilon)\{\overline{(B_\Delta,\Delta)}\} = \overline{\{(\mathcal{F},\Upsilon)(B_\Delta,\Delta)\}}. \end{aligned}$$

This implies that $(\mathcal{F},\Upsilon)\{\overline{(B_\Delta,\Delta)}\}$ is soft closed set over I_V and hence it is soft ω-closed set. Therefore, the soft map $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ is soft ω-closed map and being a bijective soft ω-continuous and soft ω-closed map, it is soft ω-homeomorphism.

If in above thm, a soft map $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ is soft ω-homeomorphism, then it is not necessary that it satisfies the given condition. This can be seen by the following example:

Example 6.4 *Consider two universal sets $I_U=\{\eta_1,\eta_2,\eta_3\}$ and $I_V=\{v_1,v_2,v_3\}$, two paramter sets $\Delta=\{\delta_1,\delta_2\}$ and $\Delta^*=\{\delta_1^*,\delta_2^*\}$ such that $(I_U,\tilde{\tau},\Delta)$ and $(I_V,\tilde{\tau^*},\Delta^*)$ be soft topological spaces where $\tilde{\tau} = \{\tilde{\phi},\ \tilde{I_U},\{(\delta_1,\{\eta_1\}),(\delta_2,\eta_1)\},\{(\delta_1,\{\eta_2,\eta_3\}),(\delta_2,\eta_2,\eta_3)\}\}$ and $\tilde{\tau}^*=\{\tilde{\phi},\ \tilde{I_V},\{(\delta_1^*,\{v_1,v_2\}),\ (\delta_2^*,\{v_1,v_2\})\}$, $\{(\delta_1^*,\{v_3\}),(\delta_2^*,v_3)\}\}$. Suppose $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ be a soft map where $\mathcal{F}:I_U\to I_V$ and $\Upsilon:\Delta\to\Delta^*$ is defined as $\mathcal{F}(\eta_1)=v_3$,$\mathcal{F}(\eta_2)=v_2$, and $\mathcal{F}(\eta_3)=v_1$ and $\Upsilon(\delta_j)=\delta_j^*$, $j=1,2$. Then, $(\mathcal{F},\Upsilon)$ is soft ω-homeomorphism but*

$$(\mathcal{F},\Upsilon)\{\overline{(B_\Delta,\Delta)}_\omega\} \neq \overline{\{(\mathcal{F},\Upsilon)(B_\Delta,\Delta)\}}.$$

As there exists a soft set $(B_\Delta,\Delta)=\{(\delta_1,\{\eta_1,\eta_3\}),(\delta_2,\eta_1,\eta_2)\}$ such that $(\mathcal{F},\Upsilon)\{\overline{(B_\Delta,\Delta)}_\omega\}=(\delta_1,\{\eta_1,\eta_3\})$, $(\delta_2,\eta_1,\eta_2)\}$ but $\overline{\{(\mathcal{F},\Upsilon)(B_\Delta,\Delta)\}}=\tilde{I_V}$.

Theorem 6.2 *If a bijective soft map $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ is such that for every soft set (A_Δ,Δ) over I_U, we have*

$$(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)^\circ_\omega\} = \{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)\}^\circ_\omega,$$

then $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ is soft ω-homeomorphism.
Proof: *Let (A_Δ,Δ) be any soft open set over I_U, then we have*

$$\begin{aligned} & (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)^\circ_\omega\} = \{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)\}^\circ_\omega \\ \Rightarrow\quad & (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)^\circ_\omega\} \tilde{\supseteq} \{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)\}^\circ_\omega. \end{aligned}$$

Thus, we have $(\mathcal{F},\Upsilon)$ is soft ω-continuous map.
Now, we prove that the soft map is soft ω-open map. As (A_Δ,Δ) is soft open set over I_U, thus (A_Δ,Δ) is soft ω-open set over I_U. By given hypothesis, we have

$$\begin{aligned} & (\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)^\circ_\omega\} = \{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)\}^\circ_\omega \\ \Rightarrow\quad & (\mathcal{F},\Upsilon)(A_\Delta,\Delta) = \{(\mathcal{F},\Upsilon)(B_\Delta,\Delta)\}^\circ_\omega. \end{aligned}$$

This implies that $(\mathcal{F},\Upsilon)(A_\Delta,\Delta)$ is soft open set over I_V and hence it is soft ω-open set. Therefore, the soft map $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ is soft ω-open map and being a bijective soft ω-continuous and soft ω-open map, it is soft ω-homeomorphism.

If in above thm, a soft map $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ is soft ω-homeomorphism, then it is not necessary that it satisfies the given condition. This can be seen by the following example:

Example 6.5 *In Example 6.4, the soft map $(\mathcal{F},\Upsilon):(I_U,\tilde{\tau},\Delta)\to(I_V,\tilde{\tau}^*,\Delta^*)$ is soft ω-homeomorphism but*

$$(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)^\circ_\omega\} \neq \{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)\}^\circ_\omega.$$

As there exists a soft set $(A_\Delta,\Delta)=\{(\delta_1,\{\eta_2\}),(\delta_2,\{\eta_3\})\}$ such that $(\mathcal{F},\Upsilon)\{(A_\Delta,\Delta)^\circ_\omega\}=\{(\delta_1,\{\eta_2\}),(\delta_2,\{\eta_3\})\}$ but $\{(\mathcal{F},\Upsilon)(A_\Delta,\Delta)\}^\circ_\omega=\tilde{\phi}$.

Theorem 6.3 *Every soft homeomorphism is soft ω-homeomorphism.*
Proof: *The proof follows from the Theorem 3.3.*

Example 6.6 *The converse of above thm is not true in general by Example 3.1.*

Theorem 6.4 *Let $(\mathcal{F}, \Upsilon) : (I_U, \tilde{\tau}, \Delta) \to (I_V, \tilde{\tau}^*, \Delta^*)$ be a bijective soft map, then the following statements are equivalent:*
(i) $(\mathcal{F}, \Upsilon)$ is soft ω-homeomorphism.
(ii) $(\mathcal{F}, \Upsilon)$ is soft ω-continuous and soft ω-open map.
(iii) $(\mathcal{F}, \Upsilon)$ is soft ω-continuous and soft ω-closed map.
Proof: *Proof is obvious from Definition and Theorem 4.7.*

Example 6.7 *The composition of two soft ω-homeomorphisms need not be soft ω-homeomorphism by Example 4.2.*

7. Concluding Remarks

" Soft set theory" is a wide mathematical aid for handling vagueness and uncertainty. In this paper, some basic concepts of soft sets and soft topological spaces are considered. We defined soft ω-open maps, soft ω-closed maps, soft ω-irresolute maps and soft ω-homeomorphisms in "soft topological spaces". We further discussed some properties of these maps with suitable examples.

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