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Interval-Valued Fuzzy b-Irresolute Mappings

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ABSTRACT: In this paper, we introduce the concepts of IVF b- irresolute mappings and IVF b-irresolute open mappings and in- vestigate some characterizations for them on the interval-valued fuzzy topological spaces.

Key Words: IVF b-open, IVF b-closed, IVF b-interior, IVF b-closure.

Contents

1. Introduction

The concept of a fuzzy subset was introduced and studied by L. A. Zadeh [6] in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. C. L. Chang [3] introduced and studied fuzzy topological spaces in 1968 as a generalization of topological spaces. Many researchers like this concept and many others have contributed to the development of fuzzy topological spaces. M. B. Gorzalczany [4] introduced the concept of interval-valued fuzzy set which is a generalization of fuzzy sets. T. K. Mondal and S. K. Samantha [5] introduced the topology of interval valued fuzzy sets. In this paper, we introduce the concepts of IVF b- irresolute mappings and IVF b-irresolute open mappings and in- vestigate some characterizations for them on the interval-valued fuzzy topological spaces.

2. Preliminaries

Let $D[0,1]$ be the set of all closed subintervals of the unit interval $[0,1]$. The elements of $D[0,1]$ are generally denoted by capital letters $M, N, ...$, and note that $M = [M^L, M^U]$, where M^L and M^U are the lower and the upper end points respectively. Especially, we denote $\mathbf{0} = [0,0]$, $\mathbf{1} = [1,1]$, and $\mathbf{a} = [a,a]$ for every $a \in (0,1)$. We also note that (i) $(\forall M, N \in D[0,1])(M = N \Leftrightarrow M^L = N^L, M^U = N^U)$. (ii) $(\forall M, N \in D[0,1])(M \subseteq N \Leftrightarrow M^L \subseteq N^L, M^U \subseteq N^U)$. For every $M \in D[0,1]$, the complement of M, denoted by M^c, is defined by $M^c = 1 - M = [1 - M^U, 1 - M^L]$. Let X be a nonempty set. A mapping $A : X \to D[0,1]$ is called an interval-valued fuzzy set (briefly, an IVF set) in X. For each $x \in X$, $A(x)$ is a closed interval whose lower and upper end points are denoted by $A(x)^L$ and $A(x)^U$, respectively. For any $[a,b] \in D[0,1]$, the IVF set whose value is the interval $[a,b]$ for all $x \in X$ is denoted by $\widetilde{[a,b]}$. In particular, for any $a \in [0,1]$, the IVF set whose value is $\mathbf{a} = [a,a]$ for all $x \in X$ is denoted by simply $\widetilde{a}$. For a point $p \in X$ and for $[a,b] \in D[0,1]$ with $b > 0$, the IVF set which takes the value $[a,b]$ at p and $\mathbf{0}$ elsewhere in X is called an interval-valued fuzzy point (briefly, an IVF point) and is denoted by $[a,b]_p$. In particular, if $b = a$, then it is also denoted by a_p. Denote by $IVF(X)$ the set of all IVF sets in X. For every $\mathcal{A}, \mathcal{B} \in IVF(X)$, we define $\mathcal{A} = \mathcal{B} \Leftrightarrow (\forall x \in X)([\mathcal{A}(x)]^L = [\mathcal{B}(x)]^L$ and $[\mathcal{A}(x)]^U = [\mathcal{B}(x)]^U)$, $\mathcal{A} \subset \mathcal{B} \Leftrightarrow (\forall x \in X)([\mathcal{A}(x)]^L \subseteq [\mathcal{B}(x)]^L$ and $[\mathcal{A}(x)]^U \subseteq [\mathcal{B}(x)]^U)$. The complement $\mathcal{A}^c$ of $\mathcal{A}$ is defined by $[\mathcal{A}^c(x)]^L = 1 - [\mathcal{A}(x)]^U$ and $[\mathcal{A}^c(x)]^U = 1 - [\mathcal{A}(x)]^L$ for all $x \in X$. For a family of IVF sets $\{\mathcal{A}_i : i \in A\}$, where Λ is an index set, the union $G = \bigcup_{i\in\Lambda} \mathcal{A}_i$ and the intersection $F = \bigcap_{i\in\Lambda} \mathcal{A}_i$ are defined by $(\forall x \in X)([G(x)]^L = \sup_{i\in\Lambda}[\mathcal{A}_i(x)]^L, [G(x)]^U = \sup_{i\in\Lambda}[\mathcal{A}_i(x)]^U)$,

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$(\forall x \in X)([G(x)]^L = \inf_{i\in\Lambda}[\mathcal{A}_i(x)]^L, [G(x)]^U = \inf_{i\in\Lambda}[\mathcal{A}_i(x)]^U)$, respectively. Let $f : X \to Y$ be a mapping and let $\mathcal{A}$ be an IVF set in X. Then the image of $\mathcal{A}$ under f, denoted by $f(\mathcal{A})$, is defined as follows: $[f(\mathcal{A})(y)]^L = \begin{cases} \sup\limits_{y=f(x)} [\mathcal{A}(x)]^L & f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise}, \end{cases}$ $[f(\mathcal{A})(y)]^U = \begin{cases} \sup\limits_{y=f(x)} [\mathcal{A}(x)]^U & f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise}. \end{cases}$ for all $y \in Y$. Let $\mathcal{B}$ be an IVF set in Y. Then the inverse image of $\mathcal{B}$ under f, denoted by $f^{-1}(\mathcal{B})$, is defined as follows: $(\forall x \in X)([f^{-1}(\mathcal{B})(x)]^L = [\mathcal{B}(f(x))]^L,\ \ [f^{-1}(\mathcal{B})(x)]^U = [\mathcal{B}(f(x))]^U)$.

Definition 2.1 *[5] A family τ of IVF sets in X is called an interval-valued fuzzy topology (briefly, IVF topology) for X if it satisfies:*

1. $\mathbf{0}, \mathbf{1} \in \tau$,
2. $\mathcal{A}, \mathcal{B} \in \tau \Rightarrow \mathcal{A} \cap \mathcal{B} \in \tau$,
3. $\mathcal{A}_i \in \tau,\ i \in \Lambda \Rightarrow \bigcup\limits_{i\in\Lambda} \mathcal{A}_i \in \tau$.

Every member of τ is called an IVF open set. An IVF set $\mathcal{A}$ in X is called an IVF closed set if the complement of $\mathcal{A}$ is an IVF open set, that is, $\mathcal{A}^c \in \tau$. Moreover, (X, τ) is called an interval-valued fuzzy topological space (briefly, IVF topological space).

Definition 2.2 *[5] For an IVF set $\mathcal{A}$ in an IVF topological space (X, τ), the IVF closure and the IVF interior of $\mathcal{A}$, denoted by $\mathrm{Cl}(\mathcal{A})$, $\mathrm{Int}(\mathcal{A})$, respectively, are defined as $\mathrm{Cl}(\mathcal{A}) = \cap\{\mathcal{B} \in I^X : \mathcal{B}$ is IVF closed and $\mathcal{A} \subset \mathcal{B}\}$, $\mathrm{Int}(\mathcal{A}) = \cup\{\mathcal{B} \in I^X : \mathcal{B}$ is IVF open and $\mathcal{B} \subset \mathcal{A}\}$, respectively. Note that $\mathrm{Int}(\mathcal{A})$ is the largest IVF open set which is contained in $\mathcal{A}$, and that $\mathcal{A}$ is IVF open if and only if $\mathcal{A} = \mathrm{Int}(\mathcal{A})$.*

Definition 2.3 *[1] An IVF set $\mathcal{A}$ of an IVF topological space (X, τ) is said to be b-open if $\mathcal{A} \subseteq \mathrm{Int}(\mathrm{Cl}(\mathcal{A})) \cup \mathrm{Cl}(\mathrm{Int}(\mathcal{A}))$.*

The complement of an IVF b-open set is called an IVF b-closed set.

Definition 2.4 *[1] For an IVF set $\mathcal{A}$ in an IVF topological space (X, τ), we define the following*

1. $b\,\mathrm{Cl}(\mathcal{A}) = \cap\{\mathcal{B} \in I^X : \mathcal{B}$ *is IVF b-closed and* $\mathcal{A} \subset \mathcal{B}\}$,
2. $b\,\mathrm{Int}(\mathcal{A}) = \cup\{\mathcal{B} \in I^X : \mathcal{B}$ *is IVF b-open and* $\mathcal{B} \subset \mathcal{A}\}$.

Definition 2.5 *[2] A map $f : (X, \tau) \to (Y, \sigma)$ is said to be IVF b-continuous if for every IVF open set $\mathcal{A}$ in Y, $f^{-1}(\mathcal{A})$ is IVF b-open in X.*

3. Interval-valued fuzzy b-irresolute mappings

Definition 3.1 *A map $f : (X, \tau) \to (Y, \sigma)$ is said to be IVF b-irresolute if for every IVF b-open set $\mathcal{A}$ in Y, $f^{-1}(\mathcal{A})$ is IVF b-open in X.*

Proposition 3.1 *Every IVF b-irresolute mapping is IVF b-continuous.*

The converse of the above Proposition is not true as seen from the following example.

Example 3.1 *Let $\mathcal{A}, \mathcal{B}, \mathcal{C}$ be IVF sets in $I = [0, 1]$ defined by*

$$\mathcal{A}(x) = \begin{cases} \left[\frac{1}{3}x, \frac{2}{3}x\right] & 0 \le x \le \frac{1}{2} \\ \left[-\frac{1}{2}x + \frac{5}{12}, -\frac{2}{3}x + \frac{2}{3}\right] & \frac{1}{2} \le x \le \frac{5}{6} \\ \left[0, -\frac{2}{3}x + \frac{2}{3}\right] & \frac{5}{6} \le x \le 1 \end{cases}$$

$$\mathcal{B}(x) = \begin{cases} \left[\frac{5}{6}x + \frac{1}{6}, x + \frac{1}{6}\right] & 0 \le x \le \frac{5}{6} \\ \left[\frac{5}{6}x + \frac{1}{6}, 1\right] & \frac{5}{6} \le x \le 1 \end{cases}$$

$$\mathcal{C}(x) = \left[\frac{1}{3}x, \frac{1}{2}\right], 0 \leq x \leq 1$$

for all $x \in I$. Then $\tau = \{\boldsymbol{0}, \mathcal{A}, \mathcal{C}, \boldsymbol{1}\}$ and $\sigma = \{\boldsymbol{0}, \mathcal{A}, \boldsymbol{1}\}$ are IVF topologies for I. It is clear that $\mathcal{B}$ is an IVF b-open set in (X, σ) but not IVF b-open set in (X, τ). Hence the identity mapping $f : (X, \tau) \to (X, \sigma)$ is an IVF b-continuous but it is not IVF b-irresolute.

Theorem 3.1 *For a map $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is IVF b-irresolute.*
2. *$f^{-1}(\mathcal{B})$ is IVF b-closed for each IVF b-closed set $\mathcal{B}$ of Y.*
3. *$f(b\,\mathrm{Cl}(\mathcal{A})) \subset b\,\mathrm{Cl}(f(\mathcal{A}))$ for each $\mathcal{A} \in I^X$.*
4. *$b\,\mathrm{Cl}(f^{-1}(\mathcal{B})) \subset f^{-1}(b\,\mathrm{Cl}(\mathcal{B}))$ for each $\mathcal{B} \in I^Y$.*
5. *$f^{-1}(b\,\mathrm{Int}(\mathcal{B})) \subset b\,\mathrm{Int}(f^{-1}(\mathcal{B}))$ for each $\mathcal{B} \in I^Y$.*

Proof: (1) $\Rightarrow$ (2): It is obvious.
(2) $\Rightarrow$ (3): For any $\mathcal{A} \in I^X$, since $b\,\mathrm{Cl}(f(\mathcal{A}))$ is an IVF b-closed set in Y, by (2), $f^{-1}(b\,\mathrm{Cl}(f(\mathcal{A})))$ is IVF b-closed and $\mathcal{A} \subset f^{-1}(b\,\mathrm{Cl}(f(\mathcal{A})))$. Thus we have $b\,\mathrm{Cl}(\mathcal{A}) \subset b\,\mathrm{Cl}(f^{-1}(f(\mathcal{A}))) \subset f^{-1}(b\,\mathrm{Cl}(f(\mathcal{A})))$. Then $f(b\,\mathrm{Cl}(\mathcal{A})) \subset b\,\mathrm{Cl}(f(\mathcal{A}))$.
(3) $\Rightarrow$ (4): For any $\mathcal{B} \in I^Y$, from (3), we have $f(b\,\mathrm{Cl}(f^{-1}(\mathcal{B}))) \subset b\,\mathrm{Cl}(f(f^{-1}(\mathcal{B}))) \subset b\,\mathrm{Cl}(\mathcal{B})$. Hence $b\,\mathrm{Cl}(f^{-1}(\mathcal{B})) \subset f^{-1}(b\,\mathrm{Cl}(\mathcal{B}))$.
(4) $\Rightarrow$ (5): For any $\mathcal{B} \in I^Y$, from (4), it follows $f^{-1}(b\,\mathrm{Int}(\mathcal{B})) = \boldsymbol{1} - (f^{-1}(b\,\mathrm{Cl}(\boldsymbol{1} - \mathcal{B}))) \subset \boldsymbol{1} - b\,\mathrm{Cl}(f^{-1}(\boldsymbol{1} - \mathcal{B})) = b\,\mathrm{Int}(f^{-1}(\mathcal{B}))$. Hence, we have $f^{-1}(b\,\mathrm{Int}(\mathcal{B})) \subset b\,\mathrm{Int}(f^{-1}(\mathcal{B}))$.
(5) $\Rightarrow$ (1): Let $\mathcal{A}$ be an IVF b-open set of Y. By (5), $f^{-1}(\mathcal{A}) = f^{-1}(b\,\mathrm{Int}(\mathcal{A})) \subset b\,\mathrm{Int}(f^{-1}(\mathcal{A}))$. Hence $f^{-1}(\mathcal{A})$ is an IVF b-open set. Therefore, f is IVF b-irresolute. □

Theorem 3.2 *A bijective map $f : (X, \tau) \to (Y, \sigma)$ is IVF b-irresolute if and only if $b\,\mathrm{Int}(f(\mathcal{A})) \subset f(b\,\mathrm{Int}(\mathcal{A}))$ for each $\mathcal{A} \in I^X$.*

Proof: Suppose that f is IVF b-irresolute. For any $\mathcal{A} \in I^X$, since $f^{-1}(b\,\mathrm{Int}(f(\mathcal{A})))$ is IVF b-open, from Theorem 3.1 and injectivity, it follows $f^{-1}(b\,\mathrm{Int}(f(\mathcal{A}))) \subset b\,\mathrm{Int}(f^{-1}(f(\mathcal{A}))) = b\,\mathrm{Int}(\mathcal{A})$. And from surjectivity of f, $b\,\mathrm{Int}(f(\mathcal{A})) = f(f^{-1}(b\,\mathrm{Int}(f(\mathcal{A})))) \subset f(b\,\mathrm{Int}(\mathcal{A}))$. For the converse, let $\mathcal{B}$ be an IVF b-open set of Y. From the hypothesis and surjectivity of f, it follows $f(b\,\mathrm{Int}(f^{-1}(\mathcal{B}))) \supset b\,\mathrm{Int}(f(f^{-1}(\mathcal{B}))) = b\,\mathrm{Int}(\mathcal{B}) = \mathcal{B}$. Since f is injective, $b\,\mathrm{Int}(f^{-1}(\mathcal{B})) \supset f^{-1}(\mathcal{B})$. Then $b\,\mathrm{Int}(f^{-1}(\mathcal{B})) = f^{-1}(\mathcal{B})$. Hence f is IVF b-irresolute. □

Theorem 3.3 *For a map $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is IVF b-irresolute.*
2. *$\mathrm{Cl}(\mathrm{Int}(f^{-1}(\mathcal{B}))) \cap \mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathcal{B}))) \subseteq b\,\mathrm{Cl}(f^{-1}(\mathcal{B}))$ for any $\mathcal{B} \in I^Y$.*
3. *$b\,\mathrm{Int}(f^{-1}(\mathcal{B})) \subseteq \mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathcal{B}))) \cup \mathrm{Cl}(\mathrm{Int}(f^{-1}(\mathcal{B})))$ for any $\mathcal{B} \in I^Y$.*
4. *$f(\mathrm{Cl}(\mathrm{Int}(\mathcal{A})) \cap \mathrm{Int}(\mathrm{Cl}(\mathcal{A}))) \subseteq b\,\mathrm{Int}(f(\mathcal{A}))$ for every $\mathcal{A} \in I^X$.*

Proof: (1) $\Rightarrow$ (2): Let $\mathcal{B} \in I^Y$. Then $b\,\mathrm{Cl}(\mathcal{B})$ is an IVF b-closed set of Y. By (1), $f^{-1}(b\,\mathrm{Cl}(\mathcal{B}))$ is an IVF b-closed set in X. Hence $f^{-1}(b\,\mathrm{Cl}(\mathcal{B})) \supseteq (\mathrm{Int}(\mathrm{Cl}(f^{-1}(b\,\mathrm{Cl}(\mathcal{B})))) \cap \mathrm{Cl}(\mathrm{Int}(f^{-1}(b\,\mathrm{Cl}(\mathcal{B}))))) \supseteq$ $\mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathcal{B}))) \cap \mathrm{Cl}(\mathrm{Int}(f^{-1}(\mathcal{B}))))$.
(2) $\Rightarrow$ (3): Let $\mathcal{B} \in I^Y$. Then $1 - \mathcal{B} \in I^Y$. By (2),

$$\begin{array}{rcl} f^{-1}(b\,\mathrm{Cl}(1-\mathcal{B})) & \supseteq & \mathrm{Int}(\mathrm{Cl}(f^{-1}(1-\mathcal{B}))) \cap \mathrm{Cl}(\mathrm{Int}(f^{-1}(1-\mathcal{B})))) \\ 1 - f^{-1}(b\,\mathrm{Int}(\mathcal{B})) & \supseteq & 1 - (\mathrm{Cl}(\mathrm{Int}(f^{-1}(\mathcal{B}))) \cup \mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathcal{B})))) \\ f^{-1}(b\,\mathrm{Int}(\mathcal{B})) & \subseteq & (\mathrm{Cl}(\mathrm{Int}(f^{-1}(\mathcal{B}))) \cup \mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathcal{B})))). \end{array}$$

(3) $\Rightarrow$ (4): Let $\mathcal{A} \in I^Y$. Let us put $\mathcal{B} = f(\mathcal{A})$, then $\mathcal{A} \subseteq f^{-1}(\mathcal{B})$. According to the assumption, $1-(\mathrm{Int}(\mathrm{Cl}(1-\mathcal{A}))\cup\mathrm{Cl}(\mathrm{Int}(1-\mathcal{A})) \subseteq 1-(\mathrm{Int}(\mathrm{Cl}(f^{-1}(1-\mathcal{B})))\cup\mathrm{Cl}(\mathrm{Int}(f^{-1}(1-\mathcal{B})))) \subseteq 1-(f^{-1}(b\,\mathrm{Int}(1-\mathcal{B})))$. Thus, $\mathrm{Cl}(\mathrm{Int}(\mathcal{A})) \cap \mathrm{Int}(\mathrm{Cl}(\mathcal{A})) \subseteq \mathrm{Cl}(\mathrm{Int}(f^{-1}(\mathcal{B}))) \cap \mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathcal{B}))) \subseteq f^{-1}(b\,\mathrm{Cl}(\mathcal{B}))$. So $f(\mathrm{Cl}(\mathrm{Int}(\mathcal{B})) \cap \mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathcal{B}))) \subseteq f(f^{-1}(b\,\mathrm{Cl}(\mathcal{B}))) \subseteq b\,\mathrm{Cl}(\mathcal{A}) = b\,\mathrm{Cl}(f(\mathcal{A}))$.
(4) $\Rightarrow$ (1): Let $\mathcal{B}$ be any IVF b-closed set of Y. So $f(\mathrm{Cl}(\mathrm{Int}(f^{-1}(\mathcal{B})))\cap\mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathcal{B})))) \subseteq b\,\mathrm{Int}(f(f^{-1}(\mathcal{B}))) \subseteq b\,\mathrm{Int}(\mathcal{B}) = \mathcal{B}$, $(\mathrm{Cl}(\mathrm{Int}(f^{-1}(\mathcal{B}))) \cap \mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathcal{B})))) \subseteq f^{-1}(f(\mathrm{Cl}(\mathrm{Int}(f^{-1}(\mathcal{B}))) \cap \mathrm{Int}(\mathrm{Cl}(f^{-1}(\mathcal{B}))))) \subseteq f^{-1}(\mathcal{B})$. Thus, $f^{-1}(\mathcal{B})$ is an IVF b-closed set of X; hence f is IVF b-irresolute. □

Theorem 3.4 *If $f : (X, \tau) \to (Y, \sigma)$ is IVF b-irresolute mapping, then $f^{-1}(B) \subseteq b\,\mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(\mathcal{B})) \cup \mathrm{Cl}(\mathrm{Int}(\mathcal{B}))))$ for each IVF b-open set $\mathcal{B}$ of Y.*

Proof: Let $\mathcal{B}$ be an IVF b-open set of Y. Then $f^{-1}(\mathcal{B}) \subseteq f^{-1}(Int(\mathrm{Cl}(\mathcal{B})) \cup \mathrm{Cl}(\mathrm{Int}(\mathcal{B})))$. Since $f^{-1}(\mathcal{B})$ is an an IVF b-open set of X, we have $f^{-1}(\mathcal{B}) \subseteq b\,\mathrm{Int}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(\mathcal{B})) \cup \mathrm{Cl}(\mathrm{Int}(\mathcal{B}))))$. □

Definition 3.2 *An IVF set $\mathcal{A}$ of an IVF topological space (X, τ) is said to be an IVF b-neighbourhood of an IVF point M_x if there exists an IVF b-open set $\mathcal{B}$ of X such that $M_x \in \mathcal{B} \subset \mathcal{A}$.*

Theorem 3.5 *For a map $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is IVF b-irresolute,*
2. *for any IVF point x_α of X and any $\mathcal{B}$ is IVF b-open in Y containing $f(x_\alpha)$, there exists an IVF b-open set $\mathcal{A}$ of X containing x_α such that $\mathcal{A} \subseteq f^{-1}(\mathcal{B})$,*
3. *for any IVF point x_α of X and any $\lfloor$ is IVF b-open in Y containing $f(x_\alpha)$, there exists an IVF b-open set $\mathcal{A}$ of X containing x_α such that $f(\mathcal{A}) \subseteq \mathcal{B}$,*
4. *For every IVF point M_x in X and every IVF b-neighbourhood $\mathcal{B}$ of $f(M_x)$, $f^{-1}(\mathcal{B})$ is an IVF b-neighbourhood of M_x.*

Proof: (1) $\Rightarrow$ (2): Let f be IVF b-irresolute. Let x_α be an IVF point of X and let $\mathcal{B}$ be IVF b-open in Y containing $f(x_\alpha)$. Then $x_\alpha \in f^{-1}(\mathcal{B}) = b\,\mathrm{Int}(f^{-1}(\mathcal{B}))$. The result follows for $\mathcal{A} = b\,\mathrm{Int}(f^{-1}(\mathcal{B}))$.
(2) $\Rightarrow$ (3): It follows from the relation $f(\mathcal{A}) \subseteq f(f^{-1}(\mathcal{B})) \subseteq \mathcal{B}$.
(3) $\Rightarrow$ (1): Let $\mathcal{B}$ be IVF b-open in Y and let x_α be an IVF point of X such that $x_\alpha \in f^{-1}(\mathcal{B})$. Then $f(x_\alpha) \in \mathcal{B}$. According to the assumption, there exists an IVF b-open set $\mathcal{A}$ of X containing x_α such that $f(\mathcal{A}) \subseteq \mathcal{B}$. Then $x_\alpha \in \mathcal{A} \subseteq f^{-1}(f(\mathcal{A})) \subseteq f^{-1}(\mathcal{B})$ and $x_\alpha \in \mathcal{A} = b\,\mathrm{Int}(\mathcal{A}) \subseteq b\,\mathrm{Int}(f^{-1}(\mathcal{B})$. Since x_α is an arbitrary fuzzy point and $f^{-1}(\mathcal{B})$ is the union of all fuzzy points which belong in $f^{-1}(\mathcal{B})$, $f^{-1}(\mathcal{B}) \subseteq b\,\mathrm{Int}\, f^{-1}(\mathcal{B})$. Hence f is fuzzy b-irresolute.
(3) $\Rightarrow$ (4): Let M_x be an IVF point in X and $\mathcal{B}$ be an IVF b-neighbourhood of $f(M_x)$. Then there exists an IVF b-open set $\mathcal{C}$ of Y such that $f(M_x) \in \mathcal{C} \subseteq \mathcal{B}$. By (3), there exists an IVF b-open set $\mathcal{A}$ of X such that $M_x \in \mathcal{A}$ and $f(\mathcal{A}) \subseteq C \subseteq \mathcal{B}$. Thus $M_x \in \mathcal{A} \subseteq f^{-1}(f(\mathcal{A})) \subseteq f^{-1}(\mathcal{C}) \subseteq f^{-1}(\mathcal{B})$, and so $f^{-1}(\mathcal{B})$ is an IVF b-neighbourhood of M_x.
(4) $\Rightarrow$ (3): Let M_x be an IVF point in X and $\mathcal{B}$ an IVF b-open set of Y with $f(M_x) \in \mathcal{B}$. Then $\mathcal{B}$ is an IVF b-neighbourhood of $f(M_x)$. By (4), there exists an IVF b-open set $\mathcal{D}$ of X such that $M_x \in \mathcal{D} \subseteq f^{-1}(\mathcal{B})$. Then $f(M_x) \in f(\mathcal{D}) \subseteq f(f^{-1}(\mathcal{B})) \subseteq \mathcal{B}$, and thus (3) is valid. □

Theorem 3.6 *If $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ are IVF b-irresolute mappings, then $g \circ f : (X, \tau) \to (Z, \eta)$ is IVF an b-irresolute mapping.*

Proof: Straightforward. □

Corollary 3.1 *If $f : (X, \tau) \to (Y, \sigma)$ is an IVF b-irresolute mapping and $g : (Y, \sigma) \to (Z, \eta)$ is an IVF b-continuous mapping, then $g \circ f : (X, \tau) \to (Z, \eta)$ is an IVF b-irresolute mapping.*

Definition 3.3 *An IVF set $\mathcal{A}$ of an IVF topological space (X,τ) is said to be IVF b-compact if for every IVF b-open cover $\mathcal{A} = \{\mathcal{A}_i \in I^X : i \in J\}$ of $\mathcal{A}$, there exists $J_0 = \{1,2,3,...n\} \subset J$ such that $\mathcal{A} \subset \underset{i\in J_0}{\cup} \mathcal{A}_i$.*

Theorem 3.7 *Let $F : (X,\tau) \to (Y,\sigma)$ be an IVF b-irresolute mapping. If $\mathcal{A}$ is an IVF b-compact set in X, then $f(\mathcal{A})$ is IVF b-compact in Y.*

Proof: Let $\{\mathcal{B}_i \in I^X : i \in J\}$ be an IVF b-open cover of $f(\mathcal{A})$. Then $\{f^{-1}(\mathcal{B}_i) : i \in J\}$ is an IVF b-open cover of $\mathcal{A}$ in X. By the definition of IVF b-compactness, there exists $J_0 = \{1,2,3,...n\} \subset J$ such that $\mathcal{A} \subset \underset{i\in J_0}{\cup} f^{-1}(\mathcal{B}_i)$. Then $f(\mathcal{A}) \subset f(\underset{i\in J_0}{\cup} f^{-1}(\mathcal{B}_i)) = \underset{i\in J_0}{\cup} f(f^{-1}(\mathcal{B}_i)) \subset \underset{i\in J_0}{\cup} \mathcal{B}_i$. Then $f(\mathcal{A}) \subset \underset{i\in J_0}{\cup} \mathcal{B}_i$. Hence $f(\mathcal{A})$ is IVF b-compact in Y. □

Definition 3.4 *A map $f : (X,\tau) \to (Y,\sigma)$ is called an IVF b-irresolute open (resp. IVF b-irresolute closed) mapping if for every IVF b-open (resp. IVF b-closed) set $\mathcal{A}$ in X, $f(\mathcal{A})$ is IVF b-open (resp. IVF b-closed) in Y.*

Theorem 3.8 *For a map $f : (X,\tau) \to (Y,\sigma)$, the following are equivalent:*

1. *f is IVF b-irresolute open.*
2. *$f(b\,\mathrm{Int}(\mathcal{A})) \subset b\,\mathrm{Int}(f(\mathcal{A}))$ for $\mathcal{A} \in I^X$.*
3. *$b\,\mathrm{Int}(f^{-1}(\mathcal{B})) \subset f^{-1}(b\,\mathrm{Int}(\mathcal{B}))$ for $\mathcal{B} \in I^Y$.*
4. *For $\mathcal{B} \in I^Y$ and each IVF b-closed set $\mathcal{A}$ of X with $f^{-1}(\mathcal{B}) \subset \mathcal{A}$, there exists an IVF b-closed set $\mathcal{C}$ of Y such that $\mathcal{B} \subset \mathcal{C}$ and $f^{-1}(\mathcal{C}) \subset \mathcal{A}$.*

Proof: (1) $\Rightarrow$ (2): For $\mathcal{A} \in I^X$, $f(b\,\mathrm{Int}(\mathcal{A}))$

$$\begin{aligned} &= f(\cup\{\mathcal{B} \in I^X : \mathcal{B} \subset \mathcal{A}, \mathcal{B}\ is\ IVF\ b-open\ in\ X\}) \\ &= \cup\{f(\mathcal{B}) \in I^Y : f(\mathcal{B}) \subset f(\mathcal{A}), f(\mathcal{B})\ is\ IVF\ b-open\ in\ Y\} \\ &\subset \cup\{U \in I^Y : \mathcal{U} \subset f(\mathcal{A}), \mathcal{U}\ is\ IVF\ b-open\ in\ Y \in I^X\} \\ &= b\,\mathrm{Int}(f(\mathcal{A})). \end{aligned}$$

(2) $\Rightarrow$ (3): For $\mathcal{B} \in I^Y$, from (2) it follows that $f(b\,\mathrm{Int}(f^{-1}(\mathcal{B}))) \subset b\,\mathrm{Int}(f(f^{-1}(\mathcal{B}))) \subset b\,\mathrm{Int}(\mathcal{B})$. Hence $b\,\mathrm{Int}(f^{-1}(\mathcal{B})) \subset f^{-1}(b\,\mathrm{Int}(\mathcal{B}))$.
(3)$\Rightarrow$ (4): Let $\mathcal{A}$ be an IVF b-closed set of X with $f^{-1}(\mathcal{B}) \subset \mathcal{A}$ for $\mathcal{B} \in I^Y$. Since $\mathbf{1} - \mathcal{A} \subset \mathbf{1} - f^{-1}(\mathcal{B}) = f^{-1}(\mathbf{1}-\mathcal{B})$, $b\,\mathrm{Int}(\mathbf{1}-\mathcal{A}) = \mathbf{1}-\mathcal{A} \subset b\,\mathrm{Int}(f^{-1}(\mathbf{1}-\mathcal{B}))$. By (3), $\mathbf{1}-\mathcal{A} \subset b\,\mathrm{Int}(f^{-1}(\mathbf{1}-\mathcal{B})) \subset f^{-1}(b\,\mathrm{Int}(\mathbf{1}-\mathcal{B}))$. Thus $\mathcal{A} \supset \mathbf{1} - (f^{-1}(b\,\mathrm{Int}(1-\mathcal{B}))) = f^{-1}(\mathbf{1} - b\,\mathrm{Int}(\mathbf{1}-\mathcal{B})) = f^{-1}(b\,\mathrm{Cl}(\mathcal{B}))$. Now set $\mathcal{C} = b\,\mathrm{Cl}(\mathcal{B})$. Then $\mathcal{C}$ is an IVF b-closed set of Y such that $\mathcal{B} \subset \mathcal{C}$ and $f^{-1}(\mathcal{C}) \subset \mathcal{A}$.
(4) $\Rightarrow$ (1): Let $\mathcal{A}$ be an IVF b-open set of X. Then $f^{-1}(\mathbf{1} - f(\mathcal{A})) = \mathbf{1} - f^{-1}(f(\mathcal{A})) \subset \mathbf{1} - \mathcal{A}$ and $\mathbf{1} - \mathcal{A}$ is IVF b-closed. By (4), there exists an IVF b-closed set $\mathcal{C}$ such that $\mathbf{1} - f(\mathcal{A}) \subset \mathcal{C}$ and $f^{-1}(\mathcal{C}) \subset \mathbf{1} - \mathcal{A}$. It implies $\mathbf{1} - \mathcal{C} \subset f(\mathcal{A})$ and $f(\mathcal{A}) \subset f(\mathbf{1} - f^{-1}(\mathcal{C})) = f(f^{-1}(\mathbf{1} - \mathcal{C})) \subset \mathbf{1} - \mathcal{C}$. Hence $f(\mathcal{A})$ is an IVF b-open set in Y. □

Theorem 3.9 *A map $f : (X,\tau) \to (Y,\sigma)$ is IVF b-irresolute open if and only if $f(b\,\mathrm{Int}(\mathcal{A})) \subset \mathrm{Int}(\mathrm{Cl}(f(\mathcal{A}))) \cup \mathrm{Cl}(\mathrm{Int}(f(\mathcal{A}))$ for each $\mathcal{A} \in I^X$.*

Proof: Let $\mathcal{A} \in I^X$. Then $f(b\,\mathrm{Int}(\mathcal{A}))$ is IVF b-open in Y. Hence $f(b\,\mathrm{Int}(\mathcal{A})) \subset \mathrm{Int}(\mathrm{Cl}(f(b\,\mathrm{Int}(\mathcal{A}))) \cup \mathrm{Cl}(\mathrm{Int}(f(b\,\mathrm{Int}(\mathcal{A})) \subset \mathrm{Int}(\mathrm{Cl}(f(\mathcal{A}))) \cup \mathrm{Cl}(\mathrm{Int}(f(\mathcal{A}))$. □

Theorem 3.10 *A map $f : (X,\tau) \to (Y,\sigma)$ satisfies $f(\mathrm{Cl}(\mathrm{Int}(\mathcal{A})) \cup \mathrm{Int}(\mathrm{Cl}(\mathcal{A}))) \subset \mathrm{Cl}(\mathrm{Int}(f(\mathcal{A}))) \cup \mathrm{Int}(\mathrm{Cl}(f(\mathcal{A})))$ for each IVf b-open set $\mathcal{A} \in I^X$, then f is IVF b-irresolute open.*

Proof: Let $\mathcal{A}$ be an IVF b-open set of X. Then $\mathcal{A} \subset \mathrm{Cl}(\mathrm{Int}(\mathcal{A})) \cup \mathrm{Int}(\mathrm{Cl}(\mathcal{A}))$. By assumption, $f(\mathcal{A}) \subset f(\mathrm{Cl}(\mathrm{Int}(\mathcal{A})) \cup \mathrm{Int}(\mathrm{Cl}(\mathcal{A}))) \subset \mathrm{Cl}(\mathrm{Int}(f(\mathcal{A}))) \cup \mathrm{Int}(\mathrm{Cl}(f(\mathcal{A})))$; hence $f(\mathcal{A})$ is an IVF b-open set of Y. Hence f is IVF b-irresolute open. □

Corollary 3.2 *A bijective map $f : (X, \tau) \to (Y, \sigma)$ is IVF b-irresolute open if and only if $b\,\mathrm{Cl}(f(\mathcal{A})) \subset f(b\,\mathrm{Cl}(\mathcal{A}))$ for each $\mathcal{A} \in I^X$.*

Theorem 3.11 *A bijective map $f : (X, \tau) \to (Y, \sigma)$ is IVF b-irresolute closed if and only if $f^{-1}(b\,\mathrm{Cl}(\mathcal{A})) \subset b\,\mathrm{Cl}(f^{-1}(\mathcal{A}))$ for each $\mathcal{A} \in I^Y$.*

Proof: It is similarly proved from Theorem 3.8. □

Theorem 3.12 *If $f : (X, \tau) \to (Y, \sigma)$ is an IVF b-irresolute open map, then for each $\mathcal{B} \in I^Y$, $f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathcal{B})) \cap f^{-1}(\mathrm{Int}(\mathrm{Cl}(\mathcal{B})) \subseteq b\,\mathrm{Cl}(f^{-1}(\mathcal{B}))$.*

Proof: Let $\mathcal{B} \in I^Y$. Then $b\,\mathrm{Cl}(f^{-1}(\mathcal{B}))$ be an IVF b-closed set in X. From Theorem 3.8 (4), it follows that there exists an IVF b-closed set $\mathcal{C}$ of Y such that $\mathcal{B} \subseteq \mathcal{C}$ and $f^{-1}(\mathcal{C}) \subseteq b\,\mathrm{Cl}(f^{-1}(\mathcal{B})))$. Thus $f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathcal{B}))) \cap f^{-1}(\mathrm{Int}(\mathrm{Cl}(\mathcal{B}))) \subseteq f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathcal{C})) \cap \mathrm{Int}(\mathrm{Cl}(\mathcal{C}))) \subseteq f^{-1}(\mathcal{C}) \subseteq b\,\mathrm{Cl}(f^{-1}(\mathcal{B}))$. □

Theorem 3.13 *If $f : (X, \tau) \to (Y, \sigma)$ is a bijective map such that $f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathcal{B})) \cap f^{-1}(\mathrm{Int}(\mathrm{Cl}(\mathcal{B})) \subseteq b\,\mathrm{Cl}(f^{-1}(\mathcal{B}))$ for each $\mathcal{B} \in I^Y$, then f is an IVF b-irresolute open map.*

Proof: Let $\mathcal{A}$ be an IVF b-open set of X. Then from the given condition, $f^{-1}(\mathrm{Cl}(\mathrm{Int}(f(\bar{1} - \mathcal{A}))) \cap f^{-1}(\mathrm{Int}(\mathrm{Cl}(f(\bar{1} - \mathcal{A}))) \subseteq b\,\mathrm{Cl}(f^{-1}(f(\bar{1} - \mathcal{A}))) = b\,\mathrm{Cl}(\bar{1} - \mathcal{A}) = \bar{1} - \mathcal{A}$, and so $\mathrm{Cl}(\mathrm{Int}(f(\bar{1} - \mathcal{A}))) \cap \mathrm{Int}(\mathrm{Cl}(f(\bar{1} - \mathcal{A}))) \subseteq f(\bar{1} - \mathcal{A})$, which shows that $f(\bar{1} - \mathcal{A})$ is an IVF b-closed set of Y. Since f is bijective, $f(\mathcal{A})$ is an IVF b-open set of Y, hence f is an IVF b-iresolute open map. □

Theorem 3.14 *Foa a mapping $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is IVF b-irresolute closed.*
2. *$b\,\mathrm{Cl}(f(\mathcal{A})) \subseteq f(b\,\mathrm{Cl}(\mathcal{A}))$ for each $\mathcal{A} \in I^X$.*
3. *If f is bijective, then $f^{-1}(b\,\mathrm{Cl}(\mathcal{B})) \subseteq b\,\mathrm{Cl}(f^{-1}(\mathcal{A}))$ for each $\mathcal{B} \in I^Y$.*

Proof: (1) ⇔ (2): Let $\mathcal{A} \in I^X$. Then $b\,\mathrm{Cl}(\mathcal{A})$ is an IVF b-closed set in X. Since f is IVF b-closed, $f(b\,\mathrm{Cl}(\mathcal{A}))$ is IVF b-closed in Y. Since $f(\mathcal{A}) \subseteq f(b\,\mathrm{Cl}(\mathcal{A}))$, $b\,\mathrm{Cl}(f(\mathcal{A})) \subseteq b\,\mathrm{Cl}(f(\mathrm{Cl}(\mathcal{A}))) = f(b\,\mathrm{Cl}(\mathcal{A}))$. Conversely, let $\mathcal{A}$ be an IVF b-closed set in X. Then $b\,\mathrm{Cl}(\mathcal{A}) = \mathcal{A}$ and $f(\mathcal{A}) \in I^Y$. By (2), $b\,\mathrm{Cl}(f(\mathcal{A})) \subseteq f(b\,\mathrm{Cl}(\mathcal{A})) = f(\mathcal{A})$. So we have, $f(\mathcal{A}) \subseteq b\,\mathrm{Cl}(f\mathcal{A})) \subseteq f(\mathcal{A})$ and hence $f(\mathcal{A}) = b\,\mathrm{Cl}(f(\mathcal{A}))$. Then $f(\mathcal{A})$ is IVF b-closed in Y; hence f is IVF b-irresolute closed.
(2) ⇔ (3): Let $\mathcal{B} \in I^Y$. Then $f^{-1}(\mathcal{B}) \in I^X$. Since f is on-to, $b\,\mathrm{Cl}(\mathcal{B}) = b\,\mathrm{Cl}(f(f^{-1}(\mathcal{B})) \subseteq f(b\,\mathrm{Cl}(f^{-1}(\mathcal{B})))$. Since f is one-to-one, $f^{-1}(b\,\mathrm{Cl}(\mathcal{B})) \subseteq f^{-1}(f(b\,\mathrm{Cl}(f^{-1}(\mathcal{B})))) = b\,\mathrm{Cl}(f^{-1}(\mathcal{B}))$. Conversely, let $\mathcal{A} \in I^X$. Then $f(\mathcal{A}) \in I^Y$. Since f is one-to-one, $f^{-1}(b\,\mathrm{Cl}(f(\mathcal{A}))) \subseteq b\,\mathrm{Cl}(f^{-1}f(\mathcal{A})) = b\,\mathrm{Cl}(\mathcal{A})$. Since f is on-to, we have $b\,\mathrm{Cl}(f(\mathcal{A})) = f(f^{-1}(b\,\mathrm{Cl}(f(\mathcal{A})))) \subseteq f(b\,\mathrm{Cl}(\mathcal{A}))$. □

Theorem 3.15 *A map $f : (X, \tau) \to (Y, \sigma)$ is IVF b-irresolute closed if and only if $\mathrm{Int}(\mathrm{Cl}(f(\mathcal{A}))) \cap \mathrm{Cl}(\mathrm{Int}(f(\mathcal{A})) \subset f(b\,\mathrm{Cl}(\mathcal{A}))$ for each $\mathcal{A} \in I^X$.*

Theorem 3.16 *A map $f : (X, \tau) \to (Y, \sigma)$ satisfies $f(\mathrm{Cl}(\mathrm{Int}(\mathcal{A})) \cap \mathrm{Int}(\mathrm{Cl}(\mathcal{A}))) \subset \mathrm{Cl}(\mathrm{Int}(f(\mathcal{A}))) \cap \mathrm{Int}(\mathrm{Cl}(f(\mathcal{A})))$ for each IVF b-closed set $\mathcal{A} \in I^X$, then f is IVF b-irresolute closed.*

Theorem 3.17 *For a bijective map $f : (X, \tau) \to (Y, \sigma)$, the following statements hold:*

1. *f is IVF b-irresolute open if, and only if it is IVF b-irresolute closed;*

2. *f is IVF b-irresolute open (closed) if, and only if f^{-1} is IVF b-irresolute.*

Proof: (1). Clear.
(2). It follows from the relation $(f^{-1})^{-1}(\mathcal{A}) = f(\mathcal{A})$ for each $\mathcal{A} \in I^X$. □

Theorem 3.18 *For a bijective map $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is IVF b-irresolute closed.*
2. *$f^{-1}(b\,\mathrm{Cl}(\mathcal{B})) \subseteq b\,\mathrm{Cl}(f^{-1}(\mathcal{B}))$ for each $\mathcal{B} \in I^Y$.*
3. *f is IVF b-irresolute open.*
4. *f^{-1} is IVF b-irresolute.*

Proof: (1) $\Leftrightarrow$ (2): For each $\mathcal{B} \in I^Y$, by (1) and Theorem 3.14 (2), we have $f(b\,\mathrm{Cl}((f^{-1}\mathcal{B}))) \supseteq b\,\mathrm{Cl}(ff^{-1}(\mathcal{B})) = b\,\mathrm{Cl}(\mathcal{B})$. Since f is injective, $b\,\mathrm{Cl}(f^{-1}(\mathcal{B})) = f^{-1}(f(b\,\mathrm{Cl}(f^{-1}(\mathcal{B})))) \supseteq f^{-1}(b\,\mathrm{Cl}(\mathcal{B}))$. Conversely, from (2), put $\mathcal{B} = f(\mathcal{A})$ for each $\mathcal{A} \in I^X$. Since f is injective, $f^{-1}(b\,\mathrm{Cl}(f(\mathcal{A}))) \subseteq b\,\mathrm{Cl}(f^{-1}(f(\mathcal{A}))) = b\,\mathrm{Cl}(\mathcal{A})$. Since f is surjective, $b\,\mathrm{Cl}(f(\mathcal{A})) \subseteq f(b\,\mathrm{Cl}(\mathcal{A}))$. From Theorem 3.14 (2), f is IVF b-irresolute closed.
(2) $\Leftrightarrow$ (3): Clearly, it is proved from $f^{-1}(b\,\mathrm{Cl}(\mathcal{B})) \subseteq b\,\mathrm{Cl}(f^{-1}(\mathcal{B})) \Leftrightarrow f^{-1}(1 - b\,\mathrm{Int}(1 - \mathcal{B})) \subseteq 1 - b\,\mathrm{Int}(1 - f^{-1}(\mathcal{B})) \Leftrightarrow 1 - f^{-1}(b\,\mathrm{Int}(1 - \mathcal{B})) \subseteq 1 - b\,\mathrm{Int}(f^{-1}(1 - \mathcal{B})) \Leftrightarrow f^{-1}(b\,\mathrm{Int}(1 - \mathcal{B})) \supseteq b\,\mathrm{Int}(f^{-1}(1 - \mathcal{B}))$.
(2) $\Leftrightarrow$ (4): Follows Theorem 3.17, it is trivial. □

Theorem 3.19 *A map $f : (X, \tau) \to (Y, \sigma)$ is IVF b-irresolute closed if, and only if for each fuzzy set $\mathcal{B}$ of Y and each IVF b-open set $\mathcal{A}$, $f^{-1}(\mathcal{B}) \subset \mathcal{A}$, there exists an IVF b-open set $\mathcal{C}$ such that $\mathcal{B} \subset \mathcal{C}$ and $f^{-1}(\mathcal{C}) \subset \mathcal{A}$.*

Proof: Let $\mathcal{B} \in I^Y$ and let $\mathcal{A}$ be IVF b-open such that $f^{-1}(\mathcal{B}) \subset \mathcal{A}$. Then $f(\mathbf{1} - \mathcal{A})$ is IVF b-closed. We put $\mathcal{C} = \mathbf{1} - f(\mathbf{1} - \mathcal{A})$. Then $\mathcal{C}$ is IVF b-open, $\mathcal{B} \subset \mathcal{C}$ and $f^{-1}(\mathcal{C}) = f^{-1}(\mathbf{1} - f(\mathbf{1} - \mathcal{A})) \subset f^{-1}f(\mathcal{C}) \subset \mathcal{C}$. Conversely, let $\mathcal{A}$ be IVF b-closed. Then $\mathbf{1} - \mathcal{A}$ is IVF b-open and $\mathbf{1} - \mathcal{A} \supset f^{-1}(\mathbf{1} - f(\mathcal{A}))$. According to the assumption there exists an IVF b-open set $\mathcal{C}$ such that $\mathbf{1} - f(\mathcal{A}) \subset \mathcal{C}$ and $f^{-1}(\mathcal{C}) \subset \mathbf{1} - \mathcal{A}$. Hence, $f(\mathcal{A}) = \mathbf{1} - \mathcal{C}$ is IVF b-closed. □

The proof of the following Theorems are follows from Theorem 3.1, Theorem 3.8 and Theorem 3.9.

Theorem 3.20 *A map $f : (X, \tau) \to (Y, \tau)$ is IVF b-irresolute closed and IVF b-irresolute if, and only if $f(b\,\mathrm{Cl}(\mathcal{A})) = b\,\mathrm{Cl}(f(\mathcal{A}))$ for each $\mathcal{A} \in I^X$.*

Theorem 3.21 *A map $f : (X, \tau) \to (Y, \tau)$ is IVF b-irresolute open and IVF b-irresolute if, and only if $f^{-1}(b\,\mathrm{Cl}(\mathcal{A})) = b\,\mathrm{Cl}(f^{-1}(\mathcal{A}))$ for each $\mathcal{A} \in I^Y$.*

Theorem 3.22 *A map $f : (X, \tau) \to (Y, \tau)$ is IVF b-irresolute open and IVF b-irresolute if, and only if $f^{-1}(b\,\mathrm{Int}(\mathcal{A})) = b\,\mathrm{Int}(f^{-1}(\mathcal{A}))$ for each $\mathcal{A} \in I^Y$.*

Theorem 3.23 *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be mappings. Then the following statements are true:*

1. *If f and g are IVF b-irresolute open (closed), then $g \circ f$ is IVF b-irresolute open (closed).*
2. *If $g \circ f$ is IVF b-irresolute and g is IVF b-irresolute open (closed) and injective, then f is IVF b-irresolute.*
3. *If $g \circ f$ is IVF b-irresolute open (closed) and g is IVF b-irresolute and injective, then f is IVF b-irresolute open (closed).*

4. *If $g \circ f$ is IVF b-irresolute and f is IVF b-irresolute open (closed) and surjective, then g is IVF b-irresolute.*

5. *If $g \circ f$ is IVF b-irresolute open (closed) and f is IVF b-irresolute and surjective, then g is IVF b-irresolute open (closed).*

Proof: Follows from the respective definitions. □

Definition 3.5 *A bijective mapping $f : (X, \tau) \to (Y, \sigma)$ is called a IVF b-homeomorphism if both f and f^{-1} are IVF b-irresolute.*

Theorem 3.24 *For a bijective map $f : (X, \tau) \to (Y, \sigma)$, the following statements are equivalent:*

1. *f is a IVF b-homeomorphism;*
2. *f^{-1} is a IVF b-homeomorphism;*
3. *f and f^{-1} are IVF b-irresolute open (closed);*
4. *f is IVF b-irresolute continuous and IVF b-irresolute open (closed);*
5. *$f(b\,\mathrm{Cl}(\mathcal{A})) = b\,\mathrm{Cl}(f(\mathcal{A}))$ for each $\mathcal{A} \in I^X$;*
6. *$f(b\,\mathrm{Int}(\mathcal{A})) = b\,\mathrm{Int}(f(\mathcal{A}))$ for each $\mathcal{A} \in I^X$;*
7. *$f^{-1}(b\,\mathrm{Int}(\mathcal{B}) = b\,\mathrm{Int}(f^{-1}(\mathcal{B}))$ for each $\mathcal{B} \in I^Y$;*
8. *$b\,\mathrm{Cl}(f^{-1}(\mathcal{B})) = f^{-1}(b\,\mathrm{Cl}(\mathcal{B}))$ for each $\mathcal{B} \in I^Y$.*

Proof: (1) $\Rightarrow$ (2): It follows immediately from the definition of an IVF b-homeomorphism and the relation $(f^{-1})^{-1} = f$.
(2) $\Rightarrow$ (3): It follows from Theorem 3.17.
(3) $\Rightarrow$ (4): It follows from Theorem 3.17.
(4) $\Rightarrow$ (5): It follows from Theorem 3.17 and Theorem 3.20.
(5) $\Rightarrow$ (6): Let $\mathcal{B} \in I^Y$. Then $f(b\,\mathrm{Int}(\mathcal{B})) = \bar{1} - (f(b\,\mathrm{Cl}(\bar{1} - \mathcal{B}))) = \bar{1} - (b\,\mathrm{Cl}(f(\bar{1} - \mathcal{B}))) = b\,\mathrm{Int}(f(\mathcal{B}))$.
(6) $\Rightarrow$ (7): Let $\mathcal{B} \in I^X$. According to the assumption $f(b\,\mathrm{Int}(f^{-1}(\mathcal{B}))) = b\,\mathrm{Int}(f(f^{-1}(\mathcal{B}))) = b\,\mathrm{Int}(\mathcal{B})$. Thus $f^{-1}(f(b\,\mathrm{Int}(f^{-1}(\mathcal{B}))) = f^{-1}(b\,\mathrm{Int}(\mathcal{B}))$. Hence $b\,\mathrm{Int}(f^{-1}(\mathcal{B})) = f^{-1}(b\,\mathrm{Int}(\mathcal{B}))$.
(7) $\Rightarrow$ (8): Let $\mathcal{A} \in I^Y$. Then $b\,\mathrm{Cl}(f^{-1}(\mathcal{A})) = \bar{1} - (f^{-1}(b\,\mathrm{Int}(\bar{1} - \mathcal{A}))) = \bar{1} - (b\,\mathrm{Int}(f^{-1}(\bar{1} - \mathcal{A}))) = f^{-1}(b\,\mathrm{Cl}(\mathcal{A}))$.
(8) $\Rightarrow$ (1): It follows from Theorem 3.17 and Theorem 3.21. □

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