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On Cartesian Product of Sequence Spaces Related to Metric Space Defined by Orlicz Functions

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ABSTRACT: In this article we have introduced the Cartesian product sequence spaces $\ell_\infty \times \ell_\infty(M,d)$, $c\times c(M,d)$ and $c_0\times c_0(M,d)$ related to metric space (X,d) defined by the Orlicz function. We have established different properties like solidity, completeness etc. and also we obtain some inclusion results involving the spaces $\ell_\infty \times \ell_\infty(M,d)$, $c \times c(M,d)$ and $c_0 \times c_0(M,d)$.

Key Words: Cartesian product, Metric space, Orlicz function, p-absolutely summable sequences, Completeness, Cone metric space.

Contents

1. Introduction

Throughout the article $w\times w$, $\ell_\infty\times\ell_\infty$, $c\times c$ and $c_0\times c_0$ denote the spaces of *all*, *bounded*, *convergent* and *null* Cartesian product sequences respectively with elements in X, where (X,d) denotes a metric space and let θ be the zero element of $X\times X$.

An Orlicz function is a function $M:[0,\infty]\to[0,\infty]$, which is continuous, non-decreasing, convex with $M(0)=0$, $M(x)>0$, for $x>0$, and $M(x)\to\infty$, $x\to\infty$.

An Orlicz function is a function M is said to satisfy Δ_2-condition for all values of K, if there exists a constant $K>0$, such that $M(Kx)\le KM(x)$, for all $K\ge 0$. The Δ_2-condition is equivalent to $M(Lx)\le LM(x)$, for all $x>0$ and for $L>1$.

If the convexity of the Orlicz function is replaced by $M(x+y)\le M(x)+M(y)$, then this function is called as *modulus function*, introduced by Nakano [4] and later investigated by Ruckle [8], Maddox [3] and others.

Remark 1.1 *An Orlicz function satisfies the inequality $M(\lambda x)\le\lambda M(x)$, for all λ with $0<\lambda<1$.*

2. Definitions and Background

Lindenstrauss and Tzafriri [2] used the notion of Orlicz function and introduced the sequence space

$$\ell_M=\left\{(x_k)\in w:\sum_{k=1}^{\infty}M\left(\frac{|x_k|}{\rho}\right)<\infty,\text{for some}\rho>0\right\}.$$

The space ℓ_M with the norm

$$||(x_k)||=inf\left\{\rho>0:\sum_{k=1}^{\infty}M\left(\frac{|x_k|}{\rho}\right)\le 1\right\},$$

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becomes a Banach space, which is called an Orlicz sequence space. The space ℓ_M is closely related to the space ℓ_p, which is an Orlicz sequence space with $M(x) = x^p$, for $1 \le p \le \infty$.
Sequence spaces has been studied from different aspects in recent years, by Tripathy et al. [9], Different classes of sequence spaces defined by Orlicz functions have been introduced and their properties have been investigated by Et et al. [1], Nath and Tripathy [5,6,7], Tripathy et al. [10], Tripathy and Dutta [11], Tripathy and Dutta [12], Tripathy and Gosawami [13], Tripathy and Mahanta [14,15] and many others.

Let (X, d) be a metric space. In this article we introduce the following Cartesian product sequence spaces in a metric space (X, d).

$$\ell_\infty \times \ell_\infty(M,d) = \left\{ \overline{x} = (x_k, y_k) \in w \times w : \sup_k M\left(\frac{d((x_k,y_k),\theta)}{\rho}\right) < \infty, \text{ for some } \rho > 0 \right\};$$

$$c \times c(M,d) = \{ \overline{x} = (x_k, y_k) \in w \times w : M\left(\frac{d((x_k,y_k),(L,K))}{\rho}\right) \to 0, \text{ as } k \to \infty, \text{ for some } \rho > 0 \text{ and } L, K \in X \};$$

$$c_0 \times c_0(M,d) = \left\{ \overline{x} = (x_k, y_k) \in w \times w : M\left(\frac{d((x_k,y_k),\theta)}{\rho}\right) \to 0, \text{ as } k \to \infty, \text{ for some } \rho > 0 \right\}.$$

Definition 2.1 *A sequence space E is said to be solid (or normal), if $(\alpha_k x_k) \in E$, whenever $(x_k) \in E$, for all sequence (α_k) of scalars such that $|\alpha_k| \le 1$, for all $k \in N$.*

Definition 2.2 *A sequence space is said to be symmetric, if $(x_k) \in E$ implies $\left(x_{\pi(k)}\right) \in E$, where $\pi(k)$ is a permutation of the elements of N.*

Definition 2.3 *A sequence space E is said to be monotone, if E contains the canonical pre images of all its step spaces.*

Note. From the existing literature and the above definitions, it is clear that a class of sequences is solid implies it is monotone.

3. Main Results

In this section we establish the results of this paper.

Theorem 3.1 *The classes of sequences $Z(M,d)$, for $Z = \ell_\infty \times \ell_\infty$, $c \times c$, $c_0 \times c_0$ are linear spaces.*

Proof: We establish the result for the case $\ell_\infty \times \ell_\infty(M,d)$ and the other cases can be established following similar techniques. Let, $\overline{x}_1 = (x_k, y_k)$, $\overline{x}_2 = (u_k, v_k) \in \ell_\infty \times \ell_\infty(M,d)$. Then, we have,

$$\sup_k M\left(\frac{d((x_k,y_k),\theta)}{\rho_1}\right) < \infty, \text{ for some } \rho_1 > 0,$$

and

$$\sup_k M\left(\frac{d((u_k,v_k),\theta)}{\rho_2}\right) < \infty, \text{ for some } \rho_2 > 0.$$

Let, $\alpha, \beta \in C$ be scalar and $\rho = max(2|\alpha|\rho_1, 2|\beta|\rho_2)$. Then, we have

$$\begin{aligned}
\sup_k M\left(\frac{\alpha d((x_k,y_k),\theta) + \beta d((u_k,v_k),\theta)}{\rho}\right) &\le \sup_k M\left(\frac{\alpha d((x_k,y_k),\theta)}{\rho} + \frac{\beta d(u_k,v_k)}{\rho}\right) \\
&= \sup_k M\left(\alpha\frac{d((x_k,y_k),\theta)}{\rho} + \beta\frac{d((u_k,v_k),\theta)}{\rho}\right) \\
&\le \frac{1}{2}\sup_k M\left(\frac{d((x_k,y_k),\theta)}{\rho_1}\right) + \frac{1}{2}\sup_k M\left(\frac{d((u_k,v_k),\theta)}{\rho_2}\right) \\
&< \infty.
\end{aligned}$$

$\Rightarrow (\alpha(x_k, y_k) + \beta(u_k, v_k)) \in \ell_\infty \times \ell_\infty(M, d)$.
i.e. $(\alpha\overline{x}_1 + \beta\overline{x}_2) \in \ell_\infty \times \ell_\infty(M, d)$.
Thus, $\ell_\infty \times \ell_\infty(M, d)$ is a linear space . □

Theorem 3.2 *The spaces $Z(M, d)$, for $Z = \ell_\infty \times \ell_\infty$, $c \times c$ and $c_0 \times c_0$ are normed spaces, normed by*

$$f((x_k, y_k)) = inf \left\{\rho > 0 : \sup_k M\left(\frac{d((x_k, y_k), \theta)}{\rho}\right) \leq 1\right\}.$$

Proof: Let (X, d) be a metric space. Let, λ be a scalar, then we have,

$$\begin{aligned} f(\lambda(x_k, y_k)) &= inf \left\{r > 0 : \sup_k M\left(\frac{d((\lambda x_k, \lambda y_k), \theta)}{\rho}\right) \leq 1\right\} \\ &= |\lambda| inf \left\{\rho > 0 : \sup_k M\left(\frac{d((x_k, y_k), \theta)}{\rho}\right) \leq 1\right\}, \text{ where } \rho = \frac{r}{|\lambda|}. \\ &= |\lambda| f((x_k, y_k)). \end{aligned}$$

Let, (x_k, y_k), $(u_k, v_k) \in \ell_\infty \times \ell_\infty(M, d)$.

Then, for some $\rho_1 > 0, \rho_2 > 0$, we have

$$f((x_k, y_k)) = inf \left\{\rho_1 > 0 : \sup_k M\left(\frac{d((x_k, y_k), \theta)}{\rho_1}\right) \leq 1\right\}$$

and

$$f((u_k, v_k)) = inf \left\{\rho_2 > 0 : \sup_k M\left(\frac{d((u_k, v_k), \theta)}{\rho_2}\right) \leq 1\right\}.$$

Let, $\rho = \rho_1 + \rho_2$. Then we have,
$\sup_k M\left(\frac{d(x_k, y_k) + d(u_k, v_k)}{\rho}\right)$
$\leq \frac{\rho_1}{\rho_1 + \rho_2} \sup_k M\left(\frac{d((x_k, y_k), \theta)}{\rho_1}\right) + \frac{\rho_2}{\rho_1 + \rho_2} \sup_k M\left(\frac{d((u_k, v_k), \theta)}{\rho_2}\right) \leq 1.$
Since, $\rho_1 > 0$ and $\rho_2 > 0$, so we have,

$$\begin{aligned} f((x_k, y_k) + (u_k, v_k)) &= inf \left\{\rho = \rho_1 + \rho_2 > 0 : \sup_k M\left(\frac{d((x_k, y_k), \theta) + d((u_k, v_k), \theta)}{\rho}\right) \leq 1\right\} \\ &\leq inf \left\{\rho_1 > 0 : \sup_k M\left(\frac{d(x_k, y_k)}{\rho_1}\right) \leq 1\right\} + inf \left\{\rho_2 > 0 : \sup_k M\left(\frac{d(u_k, v_k)}{\rho_2}\right) \leq 1\right\} \\ &= f((x_k, y_k)) + f((u_k, v_k)). \end{aligned}$$

Hence, f is a semi-norm on $Z(M, d)$, for $Z = \ell_\infty \times \ell_\infty$. Similarly, we can prove for $Z = c \times c$ and $c_0 \times c_0$. □

Proposition 3.1 *The sequence space $\ell_\infty \times \ell_\infty(M, d)$, is symmetric.*

Proof: Let $\overline{x} = (x_k, y_k) \in \ell_\infty \times \ell_\infty(M, d)$. Then, for some $\rho > 0$, we have,

$$\sup_k M\left(q\left(\frac{d((x_k, y_k), \theta)}{\rho}\right)\right) < \infty.$$

Let, $\overline{y} = (u_k, v_k)$ be a arrangement of (x_k, y_k).Then we have, $x_i = u_{p_i}$ and $y_i = v_{q_i}$, for all $i \in N$. We have,

$$\sup_k M\left(q\left(\frac{d((u_k, v_k), \theta)}{\rho}\right)\right) < \infty, \text{ for some } \rho > 0$$

$\Rightarrow \overline{y} = (u_k, v_k) \in \ell_\infty \times \ell_\infty(M, d)$.
Hence, $\ell_\infty \times \ell_\infty(M, d)$ is symmetric. □

Proposition 3.2 *The classes of sequence $Z(M,d)$, for $Z = \ell_\infty \times \ell_\infty$ and $c_0 \times c_0$ are solid and hence are monotone, but the class of sequences $c \times c(M,d)$ is neither solid nor monotone.*

Proof: Let (α_k) be a sequence of scalars such that $|\alpha_k| \leq 1$, for all $k \in N$. Let, $\overline{x} = (x_k, y_k) \in Z(M,q)$. The proof for $Z = \ell_\infty \times \ell_\infty$ and $c_0 \times c_0$ are obvious in view of the following inequality,

$$M\left(q\left(\frac{d(\alpha_k x_k, \alpha_k y_k)}{\rho}\right)\right) \leq M\left(q\left(\frac{d(x_k, y_k)}{\rho}\right)\right), \text{ for all } k \in N.$$

The second part of the proof follows from the fact, "a sequence space is solid implies is monotone". The class of sequences $X = c \times c(M,d)$, is neither solid nor monotone follows from the following example. □

Example 3.1 Consider the Orlicz function M defined by
$M(\overline{x}) = |(\overline{x})|^p, p \geq 1$, where the symbol "$|.|$" represents the Euclidean distance from the origin.
and $q(\overline{x}) = \sup_k |(x_k, y_k)|$, for $\overline{x} = (x_k, y_k) \in c \times c$.
Define the sequence $\overline{x} = (x_k, y_k)$ by $\overline{x} = (x_k, y_k) = (1,1)$, for all $k \in N$.
Consider the sequence (α_k) of scalars defined by $\alpha_k = (-1)^k$, for all $k \in N$.
Then,$(x_k, y_k) \in Z(M,d)$, but $(\alpha_k(x_k, y_k)) \notin Z(M,d)$, for $Z = c \times c$ and $(c \times c)^B$.
Hence, the class of sequences $c \times c(M,q)$ is not solid.
Therefore, $c \times c(M,q)$ is not monotone.

Theorem 3.3 *Let (X,d) be a complete metric space, then the spaces $Z(M,d)$, for $Z = \ell_\infty \times \ell_\infty$, $c \times c$ and $c_0 \times c_0$ are complete metric spaces with respect to the metric f defined by*

$$f((x_k, y_k), \theta) = inf\left\{\rho > 0 : \sup_k \left(\frac{d((x_k, y_k), \theta)}{\rho}\right) \leq 1\right\}.$$

Proof: We prove the theorem for the space $c \times c(M,d)$ and the proof for the other cases can be established following similar technique.
Let $\overline{x}^i = (x_k^i, y_k^i)$ be a Cauchy sequence in $c \times c(M,d)$. We have to show the followings:

(*i*) $(x_k^i, y_k^i) \to (x_k, y_k)$, as $i \to \infty$, for each $k \in N$

(*ii*) $(x_i, y_i) \to (x, y)$, as $i \to \infty$, where $\lim_{k\to\infty}(x_k^i, y_k^i) = (x_i, y_i)$, for each $i \in N$

(*iii*) $(x_k, y_k) \to (x, y)$ (relative to M).
Let $\varepsilon > 0$ be given. For a fixed $\overline{x}_0 > 0$, choose $r > 0$ such that

$M\left(\frac{r\overline{x}_0}{3}\right) \geq 1$ and $n_0 \in N$ be such that

$f((x_k^i, y_k^i) - (x_k^j, y_k^j)) < \frac{\varepsilon}{r\overline{x}_0}$, for all $i, j \geq n_0$.

By the definition of f we have,

$$\sup_k M\left(\frac{d((x_k^i, y_k^i), (x_k^j, y_k^j))}{f((x_k^i, y_k^i) - (x_k^j, y_k^j))}\right) \leq 1 \leq M\left(\frac{r\overline{x}_0}{3}\right), \text{ for all } i, j \geq n_0$$

$$\Rightarrow M\left(\frac{d((x_k^i, y_k^i), (x_k^j, y_k^j))}{f((x_k^i, y_k^i) - (x_k^j, y_k^j))}\right) \leq M\left(\frac{r\overline{x}_0}{3}\right), \text{for all } i, j \geq n_0$$

$$\Rightarrow (d((x_k^i, y_k^i), (x_k^j, y_k^j))) < \frac{r\overline{x}_0}{3}.\frac{\varepsilon}{r\overline{x}_0}$$

$$= \frac{\varepsilon}{3}, \text{ for all } i, j \geq n_0. \quad (3.1)$$

Therefore, (x_k^i, y_k^i) is a Cauchy sequence in $X \times X$, for all $k \in N$. Since, X is complete, so there exists $(x_k, y_k) \in X \times X$, such that

$d(x_k^i, y_k^i) \to d(x_k, y_k)$, as $k \to \infty$, for each $k \in N$.

$\Rightarrow x_k^i \to x_k$ and $\Rightarrow y_k^i \to y_k$, as $k \to \infty$, for each $k \in N$.

$\Rightarrow (x_k^i, y_k^i) \to (x_k, y_k)$, as $k \to \infty$, for each $k \in N$.

(i) We have,

$\lim_{k\to\infty} (x_k^i, y_k^i) = (x_i, y_i)$, for each $i \in N$.

$\Rightarrow \lim_{k\to\infty} x_k^i = x_i$ and $\lim_{k\to\infty} y_k^i = y_i$, for each $i \in N$.

$\Rightarrow \lim_{k\to\infty} d(x_k^i, y_k^i) = d(x_i, y_i)$.
Then, for each $i \in N$ and for a given $\varepsilon > 0$,

$$M\left(\frac{d((x_k^i, y_k^i), d(x_i, y_i))}{r}\right) \leq \sup_k M\left(\frac{\varepsilon}{3r}\right),$$

for all $k \in N$, for each $i \in N$ and some $r > 0$,

$$\Rightarrow d((x_k^i, y_k^i), d(x_i, y_i)) < \frac{\varepsilon}{3}, \tag{3.2}$$

for all $k \in N$, for each $i \in N$ and by continuity of M.
Let, $i, j \geq n_0$ and $k \in N$.
Then we have,

$d((x_i, y_i), (x_j, y_j))$

$\leq d((x_k^i, y_k^i), (x_i, y_i)) + d((x_k^i, y_k^i), (x_k^j, y_k^j)) + d((x_k^j, y_k^j), (x_j, y_j))$

$< \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon$, by (1) and (2).

Hence, (x_i, y_i) is a Cauchy sequence in $X \times X$, which is complete.

Thus, (x_i, y_i) converges in X and let $\lim_{i\to\infty} d(x_i, y_i) = d(x, y)$.

$\Rightarrow \lim_{i\to\infty} x_i = x$ and $lim_{i\to\infty} y_i = y$.
$\Rightarrow \lim_{i\to\infty} (x_i, y_i) = (x, y)$.
(ii) For a given $\varepsilon > 0$, let $i \geq m_0$ and $r > 0$ be chosen such that $M\left(\frac{\varepsilon}{r}\right) < \varepsilon_1$.
From (3.2) we have,
$q(d(x_k^i, y_k^i) - d(x_i, y_i)) < \frac{\varepsilon}{3}$.
By (3.1) we have, $q(d(x_k, y_k) - d(x_k^i, y_k^i)) < \frac{\varepsilon}{3}$, for all $i \geq m_0$.
Again by (3.2) we get, $q(d(x_i, y_i) - d(x, y)) < \frac{\varepsilon}{3}$, for all $i \geq m_0$. Hence, for all $i \geq m_0$, we have

$q(d(x_k, y_k) - d(x, y)) \leq d((x_k, y_k), (x_k^i, y_k^i)) + d((x_k^i, {y_k}^i), (x_i, y_i)) + d((x_i, y_i), (x, y))$

$\Rightarrow d((x_k, y_k), (x, y)) < \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon$.

$\Rightarrow M\left(\frac{d((x_k, y_k), (x, y))}{r}\right) \leq M\left(\frac{\varepsilon}{r}\right) = \varepsilon_1$, for $r > 0$ and $k \in N$.
Therefore, $M\left(\frac{d((x_k, y_k), d(x, y))}{r}\right) \to 0$, as $k \to \infty$, for some $r > 0$.
Hence, $(x_k, y_k) \in c \times c(M, d)$. Thus, $c \times c(M, d)$ is a complete semi-normed space. □

Theorem 3.4 *Let M_1 and M_2 be two Orlicz functions. Then we have,*

(i) $Z(M_2, q) \subseteq Z(M_1 \circ M_2, q)$.

(ii) $Z(M_1, q) \cap Z(M_2, q) \subseteq Z(M_1 + M_2, q)$,

for $Z = \ell_\infty \times \ell_\infty, c \times c$ *and* $c_0 \times c_0$.

Proof: (i) We establish the inclusion relation for the case $Z = c_0 \times c_0$, the other cases can be proved following similar technique.

Let $\varepsilon > 0$ be given.

Since, M_1 is continuous, so there exist $\delta > 0$ such that $M_1(\delta) = \varepsilon$.

Let $(x_k, y_k) \in c_0 \times c_0(M_2, d)$.

Therefore, $M_2\left(\frac{d((x_k,y_k),\theta)}{\rho}\right) \to 0$, as $k \to \infty$, for some $\rho > 0$.

$\Rightarrow M_2\left(\frac{d((x_k,y_k),\theta)}{\rho}\right) < \delta$.

$\Rightarrow M_1\left(M_2\left(\frac{d((x_k,y_k),\theta)}{\rho}\right)\right) < M_1(\delta)$.

$\Rightarrow (M_1 \circ M_2)\left(\frac{d((x_k,y_k),\theta)}{\rho}\right) < \varepsilon$.

$\Rightarrow (M_1 \circ M_2)\left(\frac{d((x_k,y_k),\theta)}{\rho}\right) \to 0$, $k \to \infty$, for some $\rho > 0$.

$\Rightarrow (x_k, y_k) \in c_0 \times c_0(M_1 \circ M_2, d)$.

$\Rightarrow c_0 \times c_0(M_2, d) \subseteq c_0 \times c_0(M_1 \circ M_2, d)$.

Hence, $Z(M_2, d) \subseteq Z(M_1 \circ M_2, d)$ for $Z = c_0 \times c_0, c \times c, \ell_\infty \times \ell_\infty$.

(ii) We prove the result for the case $Z = c \times c$.

The other cases can be established following similar technique.

Let, $(x_k, y_k) \in c \times c(M_1, d) \cap c \times c(M_2, d)$

$\Rightarrow (x_k, y_k) \in c \times c(M_1, d)$ and $(x_k, y_k) \in c \times c(M_2, d)$.

Let, $\varepsilon > 0$ be given. Then, we have

$M_1\left(\frac{d((x_k,y_k),(L,K))}{\rho_1}\right) \to 0$, for all $k \in N$, for some $\rho_1 > 0$ and

$M_2\left(\frac{d((x_k,y_k),(L,K)}{\rho_2})\right) \to 0$, for all $k \in N$, for some $\rho_2 > 0$

$\Rightarrow M_1\left(\frac{d(x_k,y_k)-(L,K))}{\rho_1}\right) < \frac{\varepsilon}{2}$, for all $k \in N$, for some $\rho_1 > 0$ and

$M_2\left(q\left(\frac{d(x_k,y_k)-d(L,K)}{\rho_2}\right)\right) < \frac{\varepsilon}{2}$, for all $k \in N$, for some $\rho_2 > 0$.

Let, $\rho = max\{\rho_1, \rho_2\}$.

Then, for $k \in N$ and $\rho > 0$, we have,

$(M_1 + M_2)\left(\frac{d((x_k,y_k),(L,K))}{\rho}\right)$
$\leq M_1\left(\frac{d((x_k,y_k),(L,K))}{\rho}\right) + M_2\left(\frac{d((x_k,y_k),(L,K))}{\rho}\right)$
$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$
$\Rightarrow (M_1 + M_2)\left(\frac{d((x_k,y_k),(L,K))}{\rho}\right) < \varepsilon.$
$\Rightarrow (M_1 + M_2)\left(\frac{d((x_k,y_k),(L,K))}{\rho}\right) \to 0$, for all $k \to \infty$, for some $\rho > 0$
$\Rightarrow (x_k, y_k) \in c \times c(M_1 + M_2, q)$
Thus, $c \times c(M_1, q) \cap c \times c(M_2, q) \subseteq c \times c(M_1 + M_2, q)$.
Hence, we have $Z(M_1, q) \cap Z(M_2, q) \subseteq Z(M_1 + M_2, q)$, for $Z = c \times c, c_0 \times c_0, \ell_\infty \times \ell_\infty$. □

We state the following result without proof, which can be established using standard technique.

Theorem 3.5 *Let M be an Orlicz function, (X, d) be a metric space, then $Z(d) \subseteq Z(M, d)$, for $Z = \ell_\infty \times \ell_\infty, c \times c$ and $c_0 \times c_0$.*

Conclusion. In this article we have introduced cartesian product of bounded, convergent and null cartesian product sequence spaces defined by Orlicz functions in cone metric spaces. We have investigated their different algebraic and topological properties and established some inclusion results. The work done can be applied for introducing new classes of sequences and studying their different properties.

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Declarations

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References

1. M. Et, P.Y. Lee and B.C. Tripathy, *Strongly almost $(V, \lambda)(\Delta^r)$-summable sequences defined by Orlicz function.* Hokkaido Math. J., 35, 197-213,(2006).
2. J. Lindenstrauss and L. Tzafriri, *On Orlicz sequence spaces.* Israel J. Math., 10, 379-390,(1971).
3. I. J. Maddox, *sequence spaces defined by a modulus.* Math. Proc. Camb. Phil. Soc., 100, 161-166,(1986).
4. H. Nakano, *Concave Modulars.* J. Math. Soc. Jap., 5, 29-49,(1953).
5. P.K. Nath and B.C. Tripathy, *Convergent complex uncertain sequences defined by Orlicz function.* Ann.Univ, Craiova, Math. Comp. Sci. Ser., 46(1), 139-149, (2019).
6. P.K. Nath and B.C. Tripathy, *Statistical convergence of complex uncertain sequences defined by Orlicz function.* Proyecciones J. Math., 39(2), 301-315,(2020).
7. P.K Nath and B.C. Tripathy, *On paranormed type p-absolutely summable uncertain sequence spaces defined by Orlicz functions.* Commun. Korean Math. Soc. 36(1), 121-134,(2021). https://doi.org/10.4134/CKMS.c200174.
8. W. H. Ruckle, *FK-spaces in which the sequence of coordinate vectors is bounded.* Canad. J. Math., 25, 973-978,(1973).
9. B.C. Tripathy, R. Dey and N.R. Das, *Ordered vector valued statistically convergent sequence space.* Afr. Mat., 26, 433-441, (2015).
10. B.C. Tripathy, N.L. Braha and A.J. Dutta, *A new class of fuzzy sequences related to the ℓ_p space defined by Orlicz function.* J. Intel Fuzzy Syst,, 26, 1273-1278,(2014).
11. B.C. Tripathy and A.J. Dutta, *Statistically pre-Cauchy fuzzy real-valued sequences defined by Orlicz function.* Proyecciones J. Math., 33(3), 235-243,(2014).
12. B.C. Tripathy and H. Dutta, *On some lacunary difference sequence spaces defined by a sequence of Orlicz functions and q-lacunary Δ_n^m-statistical convergence.* Anal. Sti. Univ. Ovid., Seria Mat., 20(1), 417-430,(2012).
13. B.C. Tripathy and R. Goswami, *Vector valued multiple sequences defined by Orlicz functions.* Boletim da Sociedade Paranaense de Matemática, 33(1), 67-79,(2015).

14. B.C. Tripathy and S. Mahanta, *On a class of generalized lacunary difference sequence spaces defined by Orlicz function.* Acta Math. Applicata Sinica (Eng. Ser.), 20(2), 231-238,(2004).

15. B.C. Tripathy and S. Mahanta, *On a class of difference sequences related to the ℓ^p space defined by Orlicz functions.* Math. Slovaca, 57(2), 171-178,(2007).

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