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The exact solitary and periodic wave solutions of the loaded nonlinear evolution equations via the functional variable method

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ABSTRACT: In this article, we investigated new travelling wave solutions for the loaded modified Korteweg-de Vries-Kadomtsev-Petviashvili equation via the functional variable method. The performance of this method is reliable and effective and gives the exact solitary and periodic wave solutions. All solutions of this equation have been examined and three dimensional graphics of the obtained solutions have been drawn by using the Matlab program. The graphical representations of some obtained solutions are demonstrated to better understand their physical features. The exact solutions have its great importance to reveal the internal mechanism of the physical phenomena. This method presents a wider applicability for handling nonlinear wave equations.

Key Words: Kadomtsev-Petviashvili equation, periodic wave, hyperbolic function, loaded modified Korteweg-de Vries-Kadomtsev-Petviashvili equation, solitary wave.

Contents

1. Introduction

Mathematical modeling of most real-life problems usually yields functional equations, such as ordinary differential equations, partial differential equations, fractional equations, integral equations, and so on. Many nonlinear realistic physicalphenomena can be described by integrodifferential equations. These equations arise in several fields of science, such as fluid dynamics, physics of plasmas, biological models, nonlinear optics, chemical kinetics, quantum mechanics, ecological systems, electricity, ocean, and sea, and many others. One of the most important nonlinear evolution equation is Korteweg De Vries (KdV) equation.

The KdV equation firstly observed by John Scott Russell in experiments, and then Lord Rayleigh and Joseph Boussinesq studied it theoretically. Finally in 1895, Korteweg and De Vries formulated model equation for the shallow water waves of long wave length and small amplitude. The KdV equation

$$u_t + 6uu_x + u_{xxx} = 0, \tag{1.1}$$

has many connections to several branches of physics. The eq. (1.1) is especially important due to the potential application of different properties of electrostatic waves in the development of new theories

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of chemical physics, space environments, plasma physics, fluid dynamics, astrophysics, optical physics, nuclear physics, geophysics, dusty plasma, fluid mechanics and different other fields of applied physics [1,2,3,4,5,6,7].

In recent years, to study electrostatic waves specially to discuss different properties of solitary waves is the field of soliton dynamics has played a significant role for many researchers and have received a considerable attention of them. In 1973, Schamel introduced during the study of ion-acoustic nonlinear solitary waves because of the trapping of electrons [1,2] which is expressed in the following basic form

$$u_t + 6u^2 u_x + u_{xxx} = 0. \tag{1.2}$$

Schamel investigated the dependence of asymptotic behaviors of ion-acoustic waves of small with finite amplitude on the number of resonant electrons, in the result of which he presented after some derivation a Kdv type equation with a strong nonlinearity given as in dimension less form.

In 1970, Boris Kadomtsev and Vladimir Petviashvili originate the Kadomtsev-Petviashvili (KP) equation. The two researchers derived the equation that now bears their name as a model to study the evolution of long ion-acoustic waves of small amplitude propagating in plasmas under the effect of long transverse perturbations. In the absence of transverse dynamics, this problem is described by the KdV equation. The KP equation was soon widely accepted as a natural extension of the classical KdV equation to two spatial dimensions, and was later derived as a model for surface and internal water waves by Ablowitz and Segur (1979), and in nonlinear optics by Pelinovsky, Stepanyants and Kivshar (1995), as well as in other physical settings.

The KP equation is a nonlinear partial differential equation in two spatial and one temporal coordinate which describes the evolution of nonlinear, long waves of small amplitude with slow dependence on the transverse coordinate. There are two distinct versions of the KP equation, which can be written in normalized form as follows:

$$(u_t + 6uu_x + u_{xxx})_x + u_{yy} = 0. \tag{1.3}$$

The eq. (1.3) equation has been used extensively as a model for two-dimensional shallow water waves [8,9,10] and ion-acoustic waves in plasmas [11]. More recently, it has been obtained as a reduced model in ferromagnetics, Bose-Einstein condensation and string theory. The KP equation is still used as a classical model for developing and testing of new mathematical techniques, e.g. in problems of well-posedness in non-classical function spaces [12], in applications of the dynamical system methods for water waves [13], and in the variational theory of existence and stability of energy minimizers [14].

Using the idea of Kadomtsev and Petviashvili, who relaxed the restriction that the waves be strictly onedimensional in the KdV equation, leads to the (2+1)-dimensional modified KdV-KP equation [17,20]. A general form of the modified Korteweg-de Vries-Kadomtsev-Petviashvili (modified KdV-KP) equation is

$$(u_t + \alpha u_x - 6\beta u^2 u_x + u_{xxx})_x + u_{yy} = 0, \tag{1.4}$$

where $u(x, y, t)$ is an unknown function, $x \in R$, $y \in R$, $t \geq 0$, α and β are any constants.

The eq. (1.4) was investigated to used to model a variety of nonlinear phenomena. The exact travelling wave solutions of the eq. (1.4) have been studied by many authors [15,16,17,18,19,20].

In this article, we consider the loaded modified KdV-KP equation

$$(u_t + \alpha u_x - 6\beta u^2 u_x + u_{xxx})_x + u_{yy} + \gamma(t)u(0, 0, t)u_{xx} = 0, \tag{1.5}$$

where $u(x, y, t)$ is an unknown function, $x \in R$, $y \in R$, $t \geq 0$, α and β are any constants, $\gamma(t)$ is the given real continuous function.

Many powerful and direct methods have been developed to find special solutions of nonlinear evolution equations such as, Weierstrass elliptic function method [21], Jacobi elliptic function expansion method [22], tanh-function method [23], inverse scattering transform method [24], functional variable method [25,26,27,28,29,30], Hirota method [31], Backlund transform method [32], exp-function method [33], G/G' expansion method [34,35], truncated Painleve expansion method [36], extended tanh-method [37] and the homogeneous balance method [38] are used for searching the exact solutions.

We construct exact travelling wave solutions of the loaded modified KdV-KP equation by the functional variable method. The performance of this method is reliable and effective and gives the exact solitary wave solutions and periodic wave solutions. The traveling wave solutions obtained via this method are expressed by hyperbolic functions and the trigonometric functions. The graphical representations of some obtained solutions are demonstrated to better understand their physical features. This method presents a wider applicability for handling nonlinear wave equations.

It is known that the loaded differential equations contain some of the traces of an unknown function. In [39,40,41,42,43,44,45], the term of "loaded equation" was used for the first time, the most general definitions of the loaded differential equation were given and also a detailed classifications of the differential loaded equations as well as their numerous applications were presented. A complete description of solutions of the nonlinear loaded equations and their applications can be found in papers [46,47,48,49].

2. Description of the functional variable method

Consider nonlinear evolution equations with independent variables x, y and t is of the form

$$F(u, u_x, u_y, u_t, u_{xx}, u_{tt}, u_{yy}, u_{xy}, u_{xt}, u_{yt}...) = 0, \tag{2.1}$$

where F is a polynomial in $u = u(x, y, t)$ and its partial derivatives. In [50], Zerarka and others have summarized the functional variable method in the following.

Step 1. We use the wave transformation

$$\xi = px + qy - kt, \tag{2.2}$$

where p and q are constants, k is the speed of the traveling wave.

Next, we can introduce the following transformation for a travelling wave solution of eq. (2.1)

$$u(x, y, t) = u(\xi), \tag{2.3}$$

and the chain

$$\frac{\partial u}{\partial x} = p\frac{du}{d\xi}, \frac{\partial u}{\partial y} = q\frac{du}{d\xi}, \frac{\partial u}{\partial t} = -k\frac{du}{d\xi}, ... \tag{2.4}$$

Using eq. (2.3) and (2.4), the nonlinear partial differential eq. (2.1) can be transformed into an ordinary differential equation of the form

$$P(u,\, u', u'', u''', ...) = 0, \tag{2.5}$$

where P is a polynomial in $u(\xi)$ and its total derivatives, $u' = \frac{du}{d\xi}$.

Step 2. Then we make a transformation in which the unknown function u is considered as a functional variable in the form

$$u' = F(u), \tag{2.6}$$

then, the solution can be found by the relation

$$\int \frac{du}{F(u)} = \xi + \xi_0, \tag{2.7}$$

here ξ_0 is a constant of integration which is set equal to zero for convenience. Some successive differentiations of u in terms of F are given as

$$\begin{aligned} u'' &= \tfrac{dF(u)}{du}\tfrac{du}{d\xi} = \tfrac{dF(u)}{du}F(u) = \tfrac{1}{2}\tfrac{d\left(F^2(u)\right)}{du}, \\ u''' &= \tfrac{1}{2}\tfrac{d^2\left(F^2(u)\right)}{du^2}\sqrt{F^2(u)}, \\ u^{(IV)} &= \tfrac{1}{2}\left[\tfrac{d^3\left(F^2(u)\right)}{du^3}F^2(u) + \tfrac{d^2\left(F^2(u)\right)}{du^2}\tfrac{d\left(F^2(u)\right)}{du}\right], \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned} \tag{2.8}$$

Step 3. The ordinary differential eq. (2.3) can be reduced in terms of u, F and its derivatives upon using the expressions of eq. (2.7) into eq. (2.1) gives

$$H(u, \frac{dF(u)}{du}, \frac{d^2F(u)}{du^2}, \frac{d^3F(u)}{du^3}, ...) = 0. \tag{2.9}$$

The key idea of this particular form eq. (2.9) is of special interest because it admits analytical solutions for a large class of nonlinear wave type equations. After integration, eq. (2.9) provides the expression of F and this, together with eq. (2.6), give appropriate solutions to the original problem.

3. Solutions of the loaded modified Korteweg-de Vries-Kadomtsev-Petviashvili equation

We will show how to find the exact solution of the loaded modified KdV-KP by the functional variable method. Using the wave variable

$$u(x,y,t)=u(\xi),\ \xi=x+y-kt, \tag{3.1}$$

that will convert eq. (1.5) to an ordinary differential equation

$$\left(-ku'+\alpha u'-6\beta u^2u'+u'''\right)'+u''+\gamma(t)u(0,0,t)u''=0. \tag{3.2}$$

Integrating twice eq. (3.2) with respect to ξ, and put the constant of integration zero, we have

$$u''=2\beta u^3+(k-\alpha-1-\gamma(t)u(0,0,t))\,u. \tag{3.3}$$

Following eq. (2.8), it is easy to deduce from eq. (3.3) an expression for the function $F(u)$

$$\frac{1}{2}\frac{d\left(F^2(u)\right)}{du}=2\beta u^3+(k-\alpha-1-\gamma(t)u(0,0,t))\,u. \tag{3.4}$$

Integrating eq. (3.4) and setting the constant of integration to zero yields

$$F(u)=u\sqrt{\beta}\sqrt{u^2-\mu}, \tag{3.5}$$

where $\mu(t)=\frac{\gamma(t)u(0,t)+\alpha+1-k}{\beta}$. From eq. (2.6) and eq. (3.5) we deduce that

$$\frac{du}{u\sqrt{u^2-\mu}}=\sqrt{\beta}d\xi. \tag{3.6}$$

After integrating eq. (3.6), with zero constant of integration, we have following exact solution

$$u(x,y,t)=\frac{\sqrt{\frac{\gamma(t)u(0,0,t)+\alpha+1-k}{\beta}}}{\cos(\sqrt{\gamma(t)u(0,0,t)+\alpha+1-k}(x+y-kt))}. \tag{3.7}$$

It is obvious that the function $u(0,0,t)$ can be easily found based on expression (3.7).

We have several types of travelling wave solutions of the loaded modified KdV-KP (1.5) as follows:
1) When $\sqrt{\gamma(t)u(0,0,t)+\alpha+1-k}>0$, $\beta>0$, we have the periodic wave solution

$$u(x,y,t)=\frac{\sqrt{\frac{\gamma(t)u(0,0,t)+\alpha+1-k}{\beta}}}{\cos(\sqrt{\gamma(t)u(0,0,t)+\alpha+1-k}(x+y-kt))}. \tag{3.8}$$

2) When $\sqrt{\gamma(t)u(0,0,t)+\alpha+1-k}<0$, $\beta<0$, we have the solitary wave solution

$$u(x,y,t)=\frac{\sqrt{\frac{\gamma(t)u(0,0,t)+\alpha+1-k}{\beta}}}{\cosh(\sqrt{\gamma(t)u(0,0,t)+\alpha+1-k}(x+y-kt))}. \tag{3.9}$$

Now, by choosing free parametrs we will write the travelling wave solutions of the loaded modified KdV-KP equation in the simple form which can be used for the graphical illustrations.

If $k=-1$, $\beta=1$, $\alpha=2$ and $\gamma(t)=t^2$, then we have

$$u(x,y,t)=\frac{t\sqrt{u(0,0,t)}}{cos(t(x+y+t)\sqrt{u(0,0,t)})}, \tag{3.10}$$

where

$$u(0,0,t) = \left(\frac{\sqrt[3]{27t^3 + 3\sqrt{3}\sqrt{27t^6 - 8}}}{3t^2} + \frac{2}{t^2 \sqrt[3]{27t^3 + 3\sqrt{3}\sqrt{27t^6 - 8}}} \right)^2. \tag{3.11}$$

If $k = -1$, $\beta = -1$, $\alpha = 2$ and $\gamma(t) = -t^2$, then we have

$$u(x,y,t) = \frac{t\sqrt{u(0,0,t)}}{cosh(t(x+y+t)\sqrt{u(0,0,t)})}, \tag{3.12}$$

where

$$u(0,0,t) = \left(\frac{\sqrt[3]{27t^3 + 3\sqrt{3}\sqrt{27t^6 + 8}}}{3t^2} + \frac{2}{t^2 \sqrt[3]{27t^3 + 3\sqrt{3}\sqrt{27t^6 + 8}}} \right)^2. \tag{3.13}$$

4. Graphical representation of the solutions

We have presented some graphs of solitary and periodic waves constructed by taking suitable values of the involved unknown parameters to visualize the underlying mechanism to the original physical phenomena. Using mathematical software Matlab, three-dimensional plots of the obtained solutions have been shown in Figure 1 and Figure 2. Solitary and periodic wave solutions represent an important type of solutions for nonlinear partial differential equations as many nonlinear partial differential equations have been found to have a variety of solitary wave solutions. The solitary wave solutions obtained in this article are encouraging, applicable, and could be helpful in analyzing long wave propagation on the surface of a fluid layer under the action of gravity, iron sound waves in plasma, and vibrations in a nonlinear string.

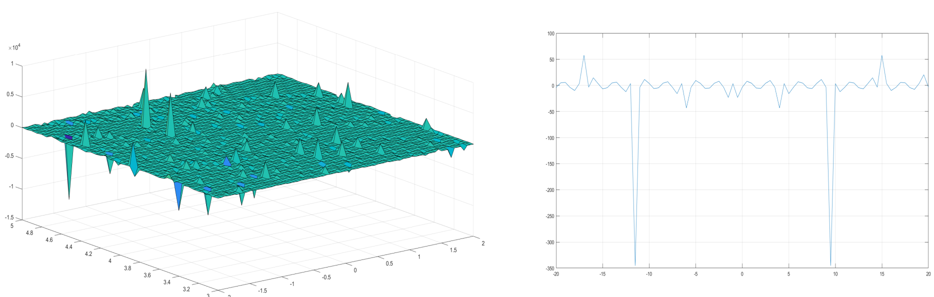

Figure 1: Periodic wave solution of the loaded modified KdV-KP equation for $k = -1$, $\beta = 1$, $\alpha = 2$ and $\gamma(t) = t^2$ and $y = 0$

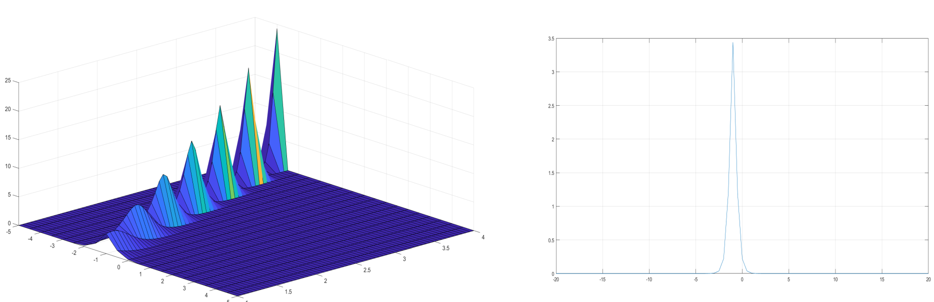

Figure 2: Solitary wave solution of the loaded modified KdV-KP equation for $k = -1$, $\beta = -1$, $\alpha = 2$ and $\gamma(t) = -t^2$ and $y = 0$

5. Conclusion

Some new traveling wave solutions have been successfully used to obtain several traveling wave solutions of the loaded modified KdV-KP by the functional variable method. The exactness of the obtained results is studied by using software Matlab. The received solutions with free parameters may be important to explain some physical phenomena. The advantage of method is give more solution such as soliton solutions and periodic solutions than other popular analytical methods. We have shown that, this method can provide a useful way to efficiently find the exact structures of solutions to a variety of nonlinear wave equations. The functional variable method is flexible, reliable and straightforward to find solutions of some nonlinear evolution equations arising in engineering and science.

6. Data availability statement

The original contributions presented in the study are included in the article/supplementary material, further inquiries can be directed to the corresponding author/s.

7. Author contributions

Bazar Babajanov, Fakhriddin Abdikarimov and Alisher Babajonov conceived of the presented idea. Bazar Babajanov developed the theory and performed the computations. Fakhriddin Abdikarimov and Alisher Babajonov verified the methods. All authors discussed the results and contributed to the final manuscript. Fakhriddin Abdikarimov contributed to the article and approved the submitted version.

8. Conflict of interest

The authors declare that they have no conflict of interest.

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