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3-Prime Near-rings involving right n-derivations

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ABSTRACT: This paper explores the commutativity of 3-prime near ring $\mathcal{M}$ when it admits a right n-derivation in which some algebraic identities are satisfied, specifically on semigroup ideals of $\mathcal{M}$

Key Words: 3-prime near ring, semigroup ideal, right n-derivation, left multiplier, right multiplier, multiplier.

Contents

1. Introduction

$\mathcal{M}$ always denotes a left near ring with commutative center $Z(R)$. Recall that $\mathcal{M}$ is called 2-torsion free if $2x = 0$ implies $x = 0$ for all $x \in \mathcal{M}$ and usually $\mathcal{M}$ will be 3-prime, if for each $x, y \in \mathcal{M}, x\mathcal{M}y = \{0\}$ implies $x = 0$ or $y = 0$. A nonempty subset $\mathcal{A}$ of $\mathcal{M}$ is called semigroup left ideal (resp. semigroup right ideal) if $\mathcal{M}\mathcal{A} \subseteq \mathcal{A}$ (resp. $\mathcal{A}\mathcal{M} \subseteq \mathcal{A}$) and $\mathcal{A}$ will be labeled a semigroup ideal even though it satisfies the criteria for both a semigroup left ideal and a semigroup right ideal. We suggest the reader to Pilz [19] for further information conceming the near-rings. Let $\mathcal{S}$ be any mapping from $\mathcal{M}$ into itself, for any pair of elements $x, y \in \mathcal{M}$, we define $[x, y]_\delta = x\delta(y) - yx$ and $(x \circ y)_\delta = x\delta(y) + yx$, in particular $[x, y]_1 = [x, y]$ and $(x \circ y)_1 = x \circ y$.

There are a number of studies that make the claim that 3-prime near-rings with certain restricted mappings exhibit behavior similar to that of rings (see [1-19] where further references can be found). An additive mapping $\mathcal{S}$ from $\mathcal{M}$ into itself is said to be right (resp. left) multiplier of $\mathcal{M}$ if $\mathcal{S}(xy) = x\mathcal{S}(y)(or, \mathcal{S}(xy) = \mathcal{S}(x)y)$ for each $x, y \in \mathcal{M}$. If $\mathcal{S}$ is both left and right multiplier then will be called multiplier. Several authors studied the commutativity of prime and semiprime of near-ring $\mathcal{M}$, which satisfy suitable algebraic conditions on appropriate subset of the near-rings. For instance, we mention to [16], where more references can be found.

There is a great deal of work regarding of symmetric bi-derivation, permuting triderivation, n-derivations, generalized n-derivations, $(\sigma, \tau) - n$-derivation, two sided $\alpha-$ n-derivations in near-rings. The authors Oztüurk and Park, in [17] and [18], were the first to present the ideas of symmetric bi-derivation, permuting triderivation, and permuting n-derivation in the context of near-rings. Ashrafe and Siddeeque [1] investigate the process of permuting n-derivations and identify some of the features that are involved. In the year 2016, Majeed and the author [11] came up with a novel idea for the near-ring that they named right n-derivation, and they acquired fresh findings that are significant for academics working in this area. In addition to this, the author [10] presented the idea of generalized right n-derivation, demonstrating that 3-prime near-rings that satisfy some identities involving generalized right n-derivations are commutative rings. This was accomplished by showing that these identities involve generalized right n-derivations. Recall that a left near-ring $\mathcal{M}$ is called zero-symmetric if $0x = 0$, for all $x \in \mathcal{M}$, in [3] the author proved that when a near-ring $\mathcal{M}$ admits a right n-derivation, then $\mathcal{M}$

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will be zero symmetric. According to [11] a right derivation has been defined to be: an additive mapping d from $\mathcal{M}$ into itself satisfying $d(xy)=d(x)y+d(y)x$, for each $x,y\in\mathcal{M}$ and n-additive mapping $d:\underbrace{\mathcal{M}\times\mathcal{M}\times\ldots\times\mathcal{M}}_{n\text{-times}}\rightarrow\mathcal{M}$ is said to be right n-derivation of $\mathcal{M}$ if the following equations hold for each $x_1,x_1{}',x_2,x_2{}',\ldots,x_{\mathrm{n}},x_{\mathrm{n}}{}'\in\mathcal{M}$ ':

$$\begin{gathered}
d\left(x_1x_1{}',x_2,\ldots,x_n\right)=d\left(x_1,x_2,\ldots,x_n\right)x_1'+d\left(x_1{}',x_2,\ldots,x_n\right)x_1\\
d\left(x_1,x_2x_2{}',\ldots,x_n\right)=d\left(x_1,x_2,\ldots,x_n\right)x_2'+d\left(x_1,x_2{}',\ldots,x_n\right)x_2\\
\vdots\\
d\left(x_1,x_2,\ldots,x_nx_n'\right)=d\left(x_1,x_2,\ldots,x_n\right)x_n'+d\left(x_1,x_2,\ldots,x_n'\right)x_n
\end{gathered}$$

For example, Let $\mathcal{S}$ be a left near-ring, define
$\mathcal{M}=\left\{\left(\begin{array}{ccc}0&x&y\\0&0&z\\0&0&0\end{array}\right),x,y,z,0\in\mathcal{S}\right\}$, $d:\mathcal{M}\rightarrow\mathcal{M}$ and $d_1:\underbrace{\mathcal{M}\times\mathcal{M}\times\ldots\times\mathcal{M}}_{n\text{-times}}\rightarrow\mathcal{M}$ such that:
$d\left(\begin{array}{ccc}0&x&y\\0&0&z\\0&0&0\end{array}\right)=\left(\begin{array}{ccc}0&0&0\\0&0&z\\0&0&0\end{array}\right)$

$$d_1\left(\left(\begin{array}{ccc}0&x_1&y_1\\0&0&z_1\\0&0&0\end{array}\right),\left(\begin{array}{ccc}0&x_2&y_2\\0&0&z_2\\0&0&0\end{array}\right),\ldots,\left(\begin{array}{ccc}0&x_{\mathrm{n}}&y_{\mathrm{n}}\\0&0&z_n\\0&0&0\end{array}\right)\right)=\left(\begin{array}{ccc}0&0&0\\0&0&z_1z_2\ldots z_{\mathrm{n}}\\0&0&0\end{array}\right)$$

After that, it is possible to establish beyond a reasonable doubt that d is a right derivation (which is neither a left derivation nor a derivation) and d_1 is a right n-derivation which is not n-derivation.
We will give new essential results in this field; more specifically, we will consider right n-derivation on a near-ring and show that 3-prime near-rings satisfying some identities involving right n-derivations and semigroup ideals are commutative rings. These are some of the most recent and important results in this field. The prior research that was carried out served as a source of inspiration for these discoveries. In point of fact, our findings extend and expand upon a number of well-established theorems on near-rings, which is a very exciting development.
From now, $\mathcal{M}$ is 3 -prime near-ring.

2. Preliminaries

Our starting point will be the subsequent lemmas that are required in order to develop the proofs of our primary findings. In [6], there is evidence that bolsters the validity of the first three lemmas.

Lemma 2.1 **(a)** *If $z\in\mathcal{Z}(\mathcal{M})$ and x is any element of $\mathcal{M}$ such that xz or $zx\in\mathcal{Z}(\mathcal{M})$, then $x\in\mathcal{Z}(\mathcal{M})$.*

(b) *If $\mathcal{Z}(\mathcal{M})$ contains a nonzero element z for which $z+z\in\mathcal{Z}(\mathcal{M})$, then $\mathcal{M}$ is abelian.*

Lemma 2.2 *If $\mathcal{Z}(\mathcal{M})$ contains a nonzero semigroup left ideal or semigroup right ideal, then $\mathcal{M}$ is a commutative ring*

Lemma 2.3 *Let $\mathcal{A}$ be a nonzero semigroup ideal of $\mathcal{M}$ and $x,y\in\mathcal{M}$.*

(a) *If $x\mathcal{A}=\{0\}$ or $\mathcal{A}x=\{0\}$, then $x=0$.*

(b) *If $x\mathcal{A}y=\{0\}$, then either $x=0$ or $y=0$.*

Lemma 2.4 *Let $\mathcal{A}$ be nonzero semigroup ideal of $\mathcal{M}$ and δ is any mapping from $\mathcal{M}$ into itself,*

(a) *If $[v,u]_\delta\in\mathcal{Z}(\mathcal{M})$ for any $v,u\in\mathcal{A}$, then either $\mathcal{M}$ is commutative ring or $\delta(\mathcal{A})\subseteq\mathcal{Z}(\mathcal{M})$*

(b) *If δ is additive mapping and $(v\circ u)_\delta\in\mathcal{Z}(\mathcal{M})$ for any $v,u\in\mathcal{A}$, then either $\mathcal{M}$ is commutative ring or $\delta(-\mathcal{A})\subseteq\mathcal{Z}(\mathcal{M})$.*

Proof: (a) Suppose that $[v, u]_\delta \in Z(\mathcal{M})$ for any $v, u \in \mathcal{A}$, then

$$(v\delta(u) - uv) \in Z(\mathcal{M}) \ for\ any\ v, u \in \mathcal{A} \tag{2.1}$$

Replace v by uv in (2.1) to get $u(v\delta(u) - uv) \in \mathcal{Z}(\mathcal{M})$ for any $v, u \in \mathcal{A}$, using Lemma 2.1 (a) lastly implies

$$either\ \mathrm{u} \in \mathcal{Z}(\mathcal{M})\ or\ v\delta(u) = uv\ for\ any\ v, u \in \mathcal{A} \tag{2.2}$$

If there $\mathrm{u}_0 \in \mathcal{A}$ such that $\mathrm{u}_0 \in \mathcal{Z}(\mathcal{M})$, replace u in (2.1) by u_0, we obtain $v\delta(\mathrm{u}_0) - \mathrm{u}_0 v \in \mathcal{Z}(\mathcal{M})$, putting mv, where m in $\mathcal{M}$ instead of v in last relation implies $m(v\delta(\mathrm{u}_0) - \mathrm{u}_0 v) \in \mathcal{Z}(\mathcal{M})$ and using Lemma 2.1 (a) forces either $\mathcal{M} \subseteq \mathcal{Z}(\mathcal{M})$ or $v\delta(\mathrm{u}_0) = \mathrm{u}_0 v$, hence (2.2) becomes: either $\mathcal{M}$ is a commutative ring or $v\delta(u) = uv$ for any $\mathrm{u}, v \in \mathcal{A}$. If $v\delta(u) = uv$ for any u, $v \in \mathcal{A}$, replace v by vt in last equation and use it to get $vt\delta(u) = uvt = v\delta(u)t$ for any $t \in \mathcal{M}, \mathrm{u}, v \in \mathcal{A}$. Which means $\mathcal{A}[\delta(u), \mathrm{t}] = \{0\}$ for any $t \in \mathcal{M}, \mathrm{u} \in \mathcal{A}$. Using Lemma 2.3 (a) lastly we find $\delta(u) \in \mathcal{Z}(\mathcal{M})$.
(b) Using the same arguments in the proof of (a), we can get the desired result.

Lemma 2.5 *Let $\mathcal{A}$ be a semigroup ideal of $\mathcal{M}$ and δ is a left (or right) multiplier of $\mathcal{M}$ such that $\delta(\mathcal{A}) \subseteq Z(\mathcal{M})$ or $\delta(-\mathcal{A}) \subseteq Z(\mathcal{M})$, then $\mathcal{M}$ is a commutative ring.*

Proof: Suppose that δ is a left multiplier of $\mathcal{M}$ and $\delta(\mathcal{A}) \subseteq \mathcal{Z}(\mathcal{M})$, then $\delta(ut) = \delta(u)\mathrm{t} \in \mathcal{Z}(\mathcal{M})$ for all u in $\mathcal{A}$ and t in $\mathcal{M}$ using Lemma 2.1 (a) to get $\delta(u) = 0$ for all u in $\mathcal{A}$ or $\mathcal{M}$ is a commutative ring , it is obvious that the first case leads to $\delta = 0$ (a contradiction) and the proof is complete. We can use the same way when δ is a right multiplier.

Now, suppose that δ is a left multiplier of $\mathcal{M}$ and $\delta(-\mathcal{A}) \subseteq \mathcal{Z}(\mathcal{M})$, then $\delta(-tu) = \delta(t(-u) = \delta(t)(-\mathrm{u}) \in \mathcal{Z}(\mathcal{M})$ for all u in $\mathcal{A}$ and $t \in \mathcal{M}$ replacing t by $-v$, where $v \in \mathcal{A}$ in last relation implies $\delta(-v)(-\mathrm{u}) \in Z(\mathcal{M})$ and use Lemma 2.1 (a) to get $\delta(v) = 0$ or $-\mathcal{A} \subseteq Z(\mathcal{M})$, which means that $\mathcal{M}$ is a commutative ring or $\delta(v) = 0$ according to Lemma 2.2, it is obvious that the second case leads to $\delta = 0$ (a contradiction).

If δ is a right multiplier of $\mathcal{M}$ and $\delta(-\mathcal{A}) \subseteq \mathcal{Z}(\mathcal{M})$, then $\delta(-tu) = \delta(t(-u) = \mathrm{t}\delta(-u) \in \mathcal{Z}(\mathcal{M})$ for all u in $\mathcal{A}$ and $t \in \mathcal{M}$ use Lemma 2.1 (a) to get $\delta(u) = 0$ for all $u \in \mathcal{A}$ or $\mathcal{M}$ is a commutative ring or $\delta(u) = 0$, it is obvious that the second case leads to $\delta = 0$.
All cases conclude that $\mathcal{M}$ is a commutative ring.

Lemma 2.6 *Let d be a right n-derivation of $\mathcal{M}$ and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ are a nonzero semigroup ideals of $\mathcal{M}$, if $d(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n) = 0$ then $d = 0$.*

Proof: From hypothesis $d(u_1, u_2, \ldots, u_n) = 0$ for all $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_n \in \mathcal{A}_n$ it follows $0 = d(x_1 u_1, u_2, \ldots, u_n) = d(x_1, u_2, \ldots, u_n) u_1$ for all $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}, \mathrm{x}_1 \in \mathcal{M}$, that is $d(x_1, u_2, \ldots, u_n)\mathcal{A}_1 = \{0\}$ for all $\mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}, \mathrm{x}_1 \in \mathcal{M}$, using Lemma 2.3(a), we obtain $d(x_1, u_2, \ldots, u_n) = 0$ for all $\mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}, \mathrm{x}_1 \in \mathcal{M}$, replace u_2 by $x_2 u_2$, in last relation and by the same way as used above we obtain $d(x_1, x_2, \mathfrak{u}_3, \ldots, u_n) = 0$ for all $\mathfrak{u}_3 \in \mathcal{A}_3, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}, \mathrm{x}_1, \mathrm{x}_2 \in \mathcal{M}$ proceeding inductively we conclude that $d = 0$.

3. Results

Theorem 3.1 *Let d be a nonzero right n-derivation of $\mathcal{M}$ and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ are a nonzero semigroup ideals of $\mathcal{M}, \delta$ is a nonzero right multiplier of $\mathcal{M}$, if it is true that any of the following statements:*

(i) $d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, [\mathfrak{u}_\mathrm{j}, \mathfrak{v}_\mathrm{j}]_\delta, \ldots, \mathfrak{u}_n\right) = 0$

(ii) $d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{u}_\mathrm{j} \circ \mathfrak{v}_\mathrm{j})_\delta, \ldots, \mathfrak{u}_n\right) = 0$

hold for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_\mathrm{j}, \mathfrak{v}_\mathrm{j} \in \mathcal{A}_\mathrm{j}, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}$, then $\mathcal{M}$ is a commutative ring.

Proof: (i) Suppose that $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, [\mathfrak{u}_j, \mathfrak{v}_j]_\delta, \ldots, \mathfrak{u}_n) = 0$

$$for\ all\ \mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n \in \mathcal{A}_n. \tag{3.1}$$

Putting $\mathfrak{v}_j\mathfrak{u}_j$ instead of $\mathfrak{u}_j$ in (3.1) and use it implies

$$\begin{aligned} 0 &= d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, [\mathfrak{v}_j\mathfrak{u}_j, \mathfrak{v}_j]_\delta, \ldots, \mathfrak{u}_n) = d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{v}_j [\mathfrak{u}_j, \mathfrak{v}_j]_\delta, \ldots, \mathfrak{u}_n\right) \\ &= d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{v}_j, \ldots, \mathfrak{u}_n) [\mathfrak{u}_j, \mathfrak{v}_j]_\delta \end{aligned}$$

It follows, $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{v}_j, \ldots, \mathfrak{u}_n) \mathfrak{u}_j\delta(\mathfrak{v}_j) = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{v}_j, \ldots, \mathfrak{u}_n) \mathfrak{v}_j\mathfrak{u}_j$, Put $u_j t$, where $t \in \mathcal{M}$ instead of $\mathfrak{u}_j$ in last relation and use it to obtain
$d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{v}_j, \ldots, \mathfrak{u}_n) \mathcal{A}_j [t, \delta(\mathfrak{v}_j)] = \{0\}$ for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n \in \mathcal{A}_u$. According to Lemma 2.3 (b), it follows Either $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{v}_j, \ldots, \mathfrak{u}_n) = 0$ or $\delta(\mathfrak{v}_j) \in \mathcal{Z}(\mathcal{M})$

$$for\ any\ \mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n \in \mathcal{A}_n. \tag{3.2}$$

If there is $\mathfrak{v}_{j0} \in \mathcal{A}_j$ such that $\delta(\mathfrak{v}_{j0}) \in Z(\mathcal{M})$, replacing u_j by $\delta(\mathfrak{v}_{j0})\mathfrak{u}_j$ in (3.1) and using it once more involves $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j-1}, \delta(\mathfrak{v}_{j0}), \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) [\mathfrak{u}_j, \mathfrak{v}_j]_\delta = 0$ for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{j-1} \in \mathcal{A}_{j-1}, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \mathfrak{u}_{j+1} \in \mathcal{A}_{j+1}, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}$.
Afterward, for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{j-1} \in \mathcal{A}_{\mathfrak{n}-1}, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \mathfrak{u}_{j+1} \in \mathcal{A}_{\mathfrak{n}+1}, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}$
$d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j-1}, \delta(\mathfrak{v}_{j0}), \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) \mathfrak{u}_j\delta(\mathfrak{v}_j) = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j-1}, \delta(\mathfrak{v}_{j0}), \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) \mathfrak{v}_j\mathfrak{u}_j$.
Put $u_j t$, where $t \in \mathcal{M}$ instead of $\mathfrak{u}_j$ in last equation and use it to find
$d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j-1}, \delta(\mathfrak{v}_{j0}), \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) \mathcal{A}_j [t, \delta(\mathfrak{v}_j)] = \{0\}$ for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{j-1} \in \mathcal{A}_{j-1}, \mathfrak{v}_j \in \mathcal{A}_j, \mathfrak{u}_{j+1} \in \mathcal{A}_{j+1}, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}, t \in \mathcal{M}$.
According to Lemma 2.3 (b) it follows either $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j-1}, \delta(\mathfrak{v}_{j0}), \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) = 0$ or $\delta(\mathcal{A}_j) \subseteq \mathcal{Z}(\mathcal{M})$ for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{j-1} \in \mathcal{A}_{j-1}, \mathfrak{u}_{j+1} \in \mathcal{A}_{j+1}, \ldots, \mathfrak{u}_n \in \mathcal{A}_n$. Using Lemma 2.5 implies:
Either $\mathcal{M}$ is a commutative ring or $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j-1}, \delta(\mathfrak{v}_{j0}), \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) = 0$

$$for\ any\ \mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{j-1} \in \mathcal{A}_{j-1}, \mathfrak{u}_{j+1} \in \mathcal{A}_{j+1}, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}. \tag{3.3}$$

From (3.2) and (3.3), we can say
Either $\mathcal{M}$ is a commutative ring or $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j-1}, \delta(\mathfrak{v}_j), \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) = 0$
for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{j-1} \in \mathcal{A}_{j-1}, \mathfrak{v}_j \in \mathcal{A}_j, \mathfrak{u}_{j+1} \in \mathcal{A}_{j+1}, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}$. If the second case hold, then

$$\begin{aligned} 0 = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j-1}, \delta(x_j\mathfrak{v}_j), \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) &= d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j-1}, x_j\delta(\mathfrak{v}_j), \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) \\ &= d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j-1}, x_j, \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) \delta(\mathfrak{v}_j) \end{aligned}$$

for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{j-1} \in \mathcal{A}_{j-1}, \mathfrak{v}_j \in \mathcal{A}_j, \mathfrak{u}_{j+1} \in \mathcal{A}_{j+1}, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}, x_j \in \mathcal{M}$.
Therefore,
$0 = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j+1}, x_j, \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) \delta(\mathfrak{v}_j) = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j+1}, x_j, \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) t\delta(\mathfrak{v}_j)$ for any $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{j-1} \in \mathcal{A}_{j-1}, \mathfrak{v}_j \in \mathcal{A}_j, \mathfrak{u}_{j+1} \in \mathcal{A}_{j+1}, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n}, x_j \in \mathcal{M}$. Three primeness of $\mathcal{M}$ implies $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{u}_{j+1}, x_j, \mathfrak{u}_{j+1}, \ldots, \mathfrak{u}_n) t = 0$ or $\delta(\mathfrak{v}_j) = 0$, when $\delta(\mathfrak{v}_j) = 0$ for any $\mathfrak{v}_j \in \mathcal{A}_j$, we can easily find that $\delta = 0$ (a contradiction)
Continuing inductively, we obtain either $d(x_1, x_2, \ldots, x_j, \ldots, x_n) = 0$ for any
$x_1, x_2, \ldots, x_j, \ldots, x_n \in \mathcal{M}$ or there is $j \in \{1, 2, \ldots, n\}$ such that $\delta(\mathcal{A}_j) \subseteq Z(\mathcal{M})$, since $d \neq 0$, hence $\mathcal{M}$ is commutative ring according to Lemma 2.5

(ii) Assume that $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{u}_j \circ \mathfrak{v}_j)_\delta, \ldots, \mathfrak{u}_n) = 0$

$$for\ any\ \mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j, \mathfrak{v}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n \in \mathcal{A}_\mathfrak{n} \tag{3.4}$$

Putting $\mathfrak{v}_j\mathfrak{u}_j$ instead of $\mathfrak{u}_j$ in (3.4) and use it implies:

$$\begin{aligned} 0 = d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{v}_j\mathfrak{u}_j \circ \mathfrak{v}_j)_\delta, \ldots, \mathfrak{u}_n) &= d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{v}_j (\mathfrak{u}_j \circ \mathfrak{v}_j)_\delta, \ldots, \mathfrak{u}_n) \\ &= d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, \mathfrak{v}_j, \ldots, \mathfrak{u}_n) (\mathfrak{u}_j \circ \mathfrak{v}_j)_\delta . \end{aligned}$$

It follows, $d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{v}_\mathrm{j},\ldots,\mathfrak{u}_n)\mathfrak{u}_\mathrm{j}\delta(\mathfrak{v}_\mathrm{j}) = -d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{v}_\mathrm{j},\ldots,\mathfrak{u}_n)\mathfrak{v}_\mathrm{j}\mathfrak{u}_\mathrm{j}$. Put $u_\mathrm{j}t$, where $t\in\mathcal{M}$ instead of $\mathfrak{u}_\mathrm{j}$ in last relation and use it to obtain
$d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{v}_\mathrm{j},\ldots,\mathfrak{u}_n)\mathcal{A}_\mathrm{j}[t,\delta(-\mathfrak{v}_\mathrm{j})] = \{0\}$ for any $\mathfrak{u}_1\in\mathcal{A}_1,\mathfrak{u}_2\in\mathcal{A}_2,\ldots,\mathfrak{v}_\mathrm{j}\in\mathcal{A}_\mathrm{j},\ldots,\mathfrak{u}_n\in\mathcal{A}_\mathfrak{n}$. According to Lemma 2.3(b), it follows either $d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{v}_\mathrm{j},\ldots,\mathfrak{u}_n)=0$ or $\delta(-\mathfrak{v}_\mathrm{j})\in\mathcal{Z}(\mathcal{M})$

$$for\ any\ \mathfrak{u}_1\in\mathcal{A}_1,\mathfrak{u}_2\in\mathcal{A}_2,\ldots,\mathfrak{v}_\mathrm{j}\in\mathcal{A}_\mathrm{j},\ldots,\mathfrak{u}_n\in\mathcal{A}_\mathfrak{n}. \tag{3.5}$$

If there is $\mathfrak{v}_{j0}\in\mathcal{A}_\mathrm{j}$ such that $\delta(-\mathfrak{v}_{j0})\in\mathcal{Z}(\mathcal{M})$, replacing u_j by $\delta(-\mathfrak{v}_{j0})\mathfrak{u}_\mathrm{j}$ in (3.4) and using it once more involves

$$d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1},\delta(-\mathfrak{v}_{j0}),\mathfrak{u}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n)(\mathfrak{u}_\mathrm{j}\circ\mathfrak{v}_\mathrm{j})_\delta = 0$$

for any $\mathfrak{u}_1\in\mathcal{A}_1,\mathfrak{u}_2\in\mathcal{A}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1}\in\mathcal{A}_{\mathfrak{n}-1},\mathfrak{u}_\mathrm{j},\mathfrak{v}_\mathrm{j}\in\mathcal{A}_\mathrm{j},\mathfrak{u}_{\mathrm{j}+1}\in\mathcal{A}_{\mathfrak{n}+1},\ldots,\mathfrak{u}_n\in\mathcal{A}_\mathfrak{n}$.
Afterward, for any $\mathfrak{u}_1\in\mathcal{A}_1,\mathfrak{u}_2\in\mathcal{A}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1}\in\mathcal{A}_{\mathrm{j}-1},\mathfrak{u}_\mathrm{j},\mathfrak{v}_\mathrm{j}\in\mathcal{A}_\mathrm{j},\mathfrak{u}_{\mathrm{j}+1}\in\mathcal{A}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n\in\mathcal{A}_\mathfrak{n}$.
$d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1},\delta(-\mathfrak{v}_{j0}),\mathfrak{u}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n)\mathfrak{u}_\mathrm{j}\delta(\mathfrak{v}_\mathrm{j}) = -d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1},\delta(-\mathfrak{v}_{j0}),\mathfrak{u}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n)\mathfrak{v}_j\mathfrak{u}_\mathrm{j}$.
Put $u_\mathrm{j}t$, where $t\in\mathcal{M}$ instead of $\mathfrak{u}_\mathrm{j}$ in last equation and use it to find
$d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1},\delta(-\mathfrak{v}_{j0}),\mathfrak{u}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n)\mathcal{A}_\mathrm{j}[t,\delta(-\mathfrak{v}_\mathrm{j})] = \{0\}$.
According to Lemma 2.3(b) it follows either $d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1},\delta(-\mathfrak{v}_{j0}),\mathfrak{u}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n)=0$ or $\delta(-\mathcal{A}_\mathrm{j})\in\mathcal{Z}(\mathcal{M})$ for any $\mathfrak{u}_1\in\mathcal{A}_1,\mathfrak{u}_2\in\mathcal{A}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1}\in\mathcal{A}_{\mathrm{j}-1},\mathfrak{u}_{\mathrm{j}+1}\in\mathcal{A}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n\in\mathcal{A}_\mathfrak{n}$. Then (3.5) becomes
$\delta(-\mathcal{A}_\mathrm{j})\in Z(\mathcal{M})$ or $d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1},\delta(\mathfrak{v}_\mathrm{j}),\mathfrak{u}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n)=0$ for any $\mathfrak{u}_1\in\mathcal{A}_1,\mathfrak{u}_2\in\mathcal{A}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1}\in\mathcal{A}_{\mathrm{j}-1},\mathfrak{v}_\mathrm{j}\in\mathcal{A}_\mathrm{j},\mathfrak{u}_{\mathrm{j}+1}\in\mathcal{A}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n\in\mathcal{A}_\mathfrak{n}$. If the second case hold, then

$$\begin{aligned}0 &= d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1},\delta(x_j\mathfrak{v}_\mathrm{j}),\mathfrak{u}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n)\\ &= d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1},x_j\delta(\mathfrak{v}_\mathrm{j}),\mathfrak{u}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n)\\ &= d(\mathfrak{u}_1,\mathfrak{u}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1},x_j,\mathfrak{u}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n)\delta(\mathfrak{v}_\mathrm{j})\end{aligned}$$

for any $\mathfrak{u}_1\in\mathcal{A}_1,\mathfrak{u}_2\in\mathcal{A}_2,\ldots,\mathfrak{u}_{\mathrm{j}-1}\in\mathcal{A}_{\mathrm{j}-1},\mathfrak{v}_\mathrm{j}\in\mathcal{A}_\mathrm{j},\mathfrak{u}_{\mathrm{j}+1}\in\mathcal{A}_{\mathrm{j}+1},\ldots,\mathfrak{u}_n\in\mathcal{A}_\mathfrak{n},x_j\in\mathcal{M}$.
Therefore, for any $\mathrm{u}_1\in\mathcal{A}_1,\mathrm{u}_2\in\mathcal{A}_2,\ldots,\mathrm{u}_{\mathrm{j}-1}\in\mathcal{A}_{\mathrm{j}-1},\mathrm{v}_\mathrm{j}\in\mathcal{A}_\mathrm{j},\mathrm{u}_{\mathrm{j}+1}\in\mathcal{A}_{\mathrm{j}+1},\ldots,\mathrm{u}_n\in\mathcal{A}_\mathrm{u},x_j\in\mathcal{M}$,
$0 = d(u_1,u_2,\ldots,u_{j-1},x_j,u_{j+1},\ldots,u_n)\delta(tx_j) = d(u_1,u_2,\ldots,u_{j+1},x_j,u_{j+1},\ldots,u_n)t\delta(\mathfrak{v}_j)$.
3-primeness of $\mathcal{M}$ implies

$$d(u_1,u_2,\ldots,u_{j-1},x_j,u_{j+1},\ldots,u_n)=0\ or\ \delta(\mathfrak{v}_\mathrm{j})=0$$

for any $\mathrm{u}_1\in\mathcal{A}_1,\mathrm{u}_2\in\mathcal{A}_2,\ldots,\mathrm{u}_{\mathrm{j}-1}\in\mathcal{A}_{\mathrm{j}-1},\mathfrak{v}_\mathrm{j}\in\mathcal{A}_\mathrm{j},\mathrm{u}_{\mathrm{j}+1}\in\mathcal{A}_{\mathrm{j}+1},\ldots,\mathrm{u}_n\in\mathcal{A}_\mathrm{u},x_j\in\mathcal{M}$.
When $\delta(\mathfrak{v}_\mathrm{j})=0$ for any $\mathrm{u}_\mathrm{j}\in\mathcal{A}_\mathrm{j}$, we can easily find that $\delta=0$ (a contradiction).
Continuing inductively, we obtain either $d(x_1,x_2,\ldots,x_j,\ldots,x_n)=0$ for any
$x_1,x_2,\ldots,x_j,\ldots,x_n\in\mathcal{M}$ or there is $j\in\{1,2,\ldots,n\}$ such that $\delta(-\mathcal{A}_\mathrm{j})\subseteq Z(\mathcal{M})$, since $d\neq 0$ hence $\mathcal{M}$ is commutative ring according to Lemma 2.5.

Corollary 3.1 *Let d be a nonzero right n-derivation of $\mathcal{M}$ and $\mathcal{A}_1,\mathcal{A}_2,\ldots,\mathcal{A}_n$ are nonzero semigroup ideals of $\mathcal{M}$, if one of the following assertions:*

(i) $d(u_1,u_2,\ldots,[u_i,v_j],\ldots,u_n)=0$

(ii) $d(u_1,u_2,\ldots,(u_j\circ v_j),\ldots,u_n)=0$

hold for any $\mathrm{u}_1\in\mathcal{A}_1,\mathrm{u}_2\in\mathcal{A}_2,\ldots,\mathrm{u}_\mathrm{j},\mathrm{v}_\mathrm{j}\in\mathcal{A}_\mathrm{j},\ldots,\mathrm{u}_n\in\mathcal{A}_\mathrm{u}$*, then $\mathcal{M}$ is a commutative ring.*

Corollary 3.2 *Let d be a nonzero right derivation of $\mathcal{M}$ and $\mathcal{A}$ is a nonzero semigroup ideal of $\mathcal{M},\delta$ is a nonzero right multiplier of $\mathcal{M}$, if it is true that any of the following statements:*

(i) $d([u,v]_\delta)=0$

(ii) $d((u\circ v)_\delta)=0$

hold for any $\mathrm{u},\mathrm{v}\in\mathcal{A}$*, then $\mathcal{M}$ is a commutative ring.*

Corollary 3.3 *Let d be a nonzero right n-derivation of $\mathcal{M}$ and δ is a nonzero right multiplier of $\mathcal{M}$, if it is true that any of the following statements:*

(i) $d\left(x_1, x_2, \ldots, [x_{\mathrm{j}}, y_{\mathrm{j}}]_\delta, \ldots, x_n\right) = 0$

(ii) $d\left(x_1, x_2, \ldots, (x_{\mathrm{j}} \circ y_{\mathrm{j}})_\delta, \ldots, x_n\right) = 0$

hold for any $x_1, x_2, \ldots, x_{\mathrm{j}}, y_{\mathrm{j}}, \ldots, x_n \in \mathcal{M}$, then $\mathcal{M}$ is a commutative ring.

Corollary 3.4 *Let d be anonzero right* n*-derivation of $\mathcal{M}$, if it is true that any of the following statements:*

(i) $d\left(x_1, x_2, \ldots, [x_{\mathrm{j}}, y_{\mathrm{j}}], \ldots, x_n\right) = 0$

(ii) $d\left(x_1, x_2, \ldots, (x_{\mathrm{j}} \circ y_{\mathrm{j}}), \ldots, x_n\right) = 0$

hold for any $x_1, x_2, \ldots, x_{\mathrm{j}}, y_{\mathrm{j}}, \ldots, x_n \in \mathcal{M}$, then $\mathcal{M}$ is a commutative ring.

Corollary 3.5 *Let d be a nonzero right derivation of $\mathcal{M}$, if one of the following assertions:*

(i) $d([x, y]) = 0$

(ii) $d(x \circ y) = 0$

hold for any $x, y \in \mathcal{M}$, then $\mathcal{M}$ is a commutative ring. The following corollary is direct result of Corollary 3.4(i).

Corollary 3.6 *Let $\mathcal{M}$ be a near ring admitting anonzero n-derivation d were $(\mathcal{M}, +)$ is abeliain then $\mathcal{M}$ is a commutative ring.*

Proof: Since $(\mathcal{M}, +)$ is abelian, then $d\left(x_1, x_2, \ldots, [x_{\mathrm{j}}, y_{\mathrm{j}}], \ldots, x_n\right) = 0$ for any $x_1, x_2, \ldots, x_{\mathrm{j}}, y_{\mathrm{j}}, \ldots, x_n \in \mathcal{M}$, it follows that then $\mathcal{M}$ is a commutative ring according to Corollary 3.4(i).

Theorem 3.2 *Let d be a nonzero right n-derivation of $\mathcal{M}$ and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ are nonzero semigroup ideals of $\mathcal{M}$, if $d\left(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n\right) \subseteq \mathcal{Z}(\mathcal{M})$, then $\mathcal{M}$ is a commutative ring.*

Proof: If $d\left(\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n\right) = \{0\}$ then by Lemma 2.6, we obtain $d = 0$, which contradicts our assumption, then there is $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_n \in \mathcal{A}_n$, all being nonzero such that $d\left(u_1, u_2, \ldots, u_n\right) \in Z(\mathcal{M})/\{0\}$ and $d\left(u_1 + u_1, u_2, \ldots, u_n\right) = d\left(u_1, u_2, \ldots, u_n\right) + d\left(u_1, u_2, \ldots, u_n\right) \in \mathcal{Z}(\mathcal{M})$, therefore $(\mathcal{M}, +)$ is abelian according to Lemma 2.1(b), by Corollary 3.6, we conclude that $\mathcal{M}$ is a commutative ring.

Theorem 3.3 *Let d be a nonzero right n-derivation of $\mathcal{M}$ and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A},$ are a nonzero semigroup ideals of $\mathcal{M}, \delta$ is a nonzero right multiplier of $\mathcal{M}$, if one of the following assertions:*

(i) $d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, [\mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}}]_\delta, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) = 0$

(ii) $d\left((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, (\mathfrak{u}_i \circ \mathfrak{v}_{\mathrm{j}})_\delta, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta\right) = 0$

hold for any $\mathfrak{u}_1, v_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathfrak{n}}$, then $\mathcal{M}$ is a commutative ring.

Proof:(i) Suppose that $d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, [\mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}}]_\delta, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) = 0$

$$\text{for all } \mathfrak{u}_1, v_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathrm{j}}. \tag{3.6}$$

Putting $\mathfrak{v}_{\mathrm{j}}\mathfrak{u}_{\mathrm{j}}$ instead of $\mathfrak{u}_{\mathrm{j}}$ in (3.6) and use it implies

$$\begin{aligned}
0 &= d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, [\mathfrak{v}_{\mathrm{j}}\mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}}]_\delta, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \\
&= d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}\,[\mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}}]_\delta, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \\
&= d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right)[\mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}}]_\delta
\end{aligned}$$

It follows, for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathrm{u}}$, we have

$$d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \mathfrak{u}_{\mathrm{j}} \delta\left(\mathfrak{v}_{\mathrm{j}}\right) = d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \mathfrak{v}_{\mathrm{j}} \mathfrak{u}_{\mathrm{j}}$$

. Put $u_{\mathrm{j}} t$ where $t \in \mathcal{M}$ instead of $\mathfrak{u}_{\mathrm{j}}$ in last relation and use it to obtain

$$d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \mathcal{A}_{\mathrm{j}} \left[t, \delta\left(\mathfrak{v}_{\mathrm{j}}\right)\right] = \{0\}$$

for any $t \in \mathcal{M}$, $\mathrm{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathrm{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathrm{n}}$

According to Lemma 2.3(b), it follows
Either $d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) = 0$ or $\delta\left(\mathfrak{v}_{\mathrm{j}}\right) \in Z(\mathcal{M})$

$$\text{for any } \mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathfrak{n}}. \tag{3.7}$$

If there is $\mathfrak{v}_{j0} \in \mathcal{A}_{\mathrm{j}}$ such that $\delta\left(\mathfrak{v}_{j0}\right) \in \mathcal{Z}(\mathcal{M})$, replacing u_{j} by $\delta\left(\mathfrak{v}_{j0}\right) \mathfrak{u}_{\mathrm{j}}$ in (3.6) and using it once more involves

$$d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \delta\left(\mathfrak{v}_{j0}\right), \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) [\mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}}]_\delta = 0$$

for any $\mathfrak{u}_1, v_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathfrak{n}}$.

Afterward,$d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \delta\left(\mathfrak{v}_{j0}\right), \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \mathfrak{u}_j \delta\left(\mathfrak{v}_{\mathrm{j}}\right) =$
$d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \delta\left(\mathfrak{v}_{j0}\right), \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \mathfrak{v}_j \mathfrak{u}_{\mathrm{j}}$
Put $u_{\mathrm{j}} t$, where $t \in \mathcal{M}$ instead of $\mathfrak{u}_{\mathrm{j}}$ in last equation and use it to find

$$d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \delta\left(\mathfrak{v}_{j0}\right), \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \mathcal{A}_{\mathrm{j}} \left[t, \delta\left(\mathfrak{v}_{\mathrm{j}}\right)\right] = \{0\}$$

for any $\mathfrak{u}_1, v_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathfrak{n}}$

According to Lemma 2.3(b) it follows
Either $d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \delta\left(\mathfrak{v}_{j0}\right), \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) = 0$ or $\delta\left(\mathcal{A}_{\mathrm{j}}\right) \subseteq Z(\mathcal{M})$

for any $\mathfrak{u}_1, v_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathfrak{n}}$

Therefore (3.7) becomes
Either $\delta\left(\mathcal{A}_{\mathrm{j}}\right) \subseteq \mathcal{Z}(\mathcal{M})$ or $d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \delta\left(\mathfrak{v}_{\mathrm{j}}\right), \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) = 0$

for any $\mathrm{u}_1, v_1 \in \mathcal{A}_1, \mathrm{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathrm{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathrm{n}}$.

If the second case hold, then

$$\begin{aligned} 0 &= d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, \delta\left(x_j \mathfrak{v}_{\mathrm{j}}\right), \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \\ &= d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, x_j \delta\left(\mathfrak{v}_{\mathrm{j}}\right), \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \\ &= d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, x_j, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \delta\left(\mathfrak{v}_{\mathrm{j}}\right) \end{aligned}$$

for any $\mathrm{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathrm{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathrm{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathrm{j}}, x_j \in \mathcal{M}$.
Therefore,

$$\begin{aligned} 0 &= d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, x_j, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) \delta\left(y \mathfrak{v}_{\mathrm{j}}\right) \\ &= d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, x_j, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) y \delta\left(\mathfrak{v}_{\mathrm{j}}\right) \end{aligned}$$

for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathrm{u}}, x_j, y \in \mathcal{M}$. Three primeness of $\mathcal{M}$ ensures that either $d\left([\mathfrak{u}_1, \mathfrak{v}_1]_\delta, [\mathfrak{u}_2, \mathfrak{v}_2]_\delta, \ldots, x_j, \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]_\delta\right) = 0$ or $\delta\left(\mathfrak{v}_{\mathrm{j}}\right) = 0$

for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_n, x_j \in \mathcal{M}$.
The second case: $\delta(\mathfrak{v}_{\mathrm{j}}) = 0$ for any $\mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$ leads to $\delta = 0$, which is a contradiction.
Continuing inductively, we obtain either $d(x_1, x_2, \ldots, x_j, \ldots, x_n) = 0$ for any $x_1, x_2, \ldots, x_j, \ldots, x_n \in \mathcal{M}$ or there is $j \in \{1, 2, \ldots, n\}$ such that $\delta(\mathcal{A}_{\mathrm{j}}) \subseteq Z(\mathcal{M})$, since $d \neq 0$, we conclude $\mathcal{M}$ is commutative ring according to Lemma 2.5.
(ii) Suppose that
$d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, (\mathfrak{u}_{\mathrm{j}} \circ \mathfrak{v}_{\mathrm{j}})_\delta, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta) = 0$

$$\text{for any } \mathfrak{u}_1, v_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_i, \mathfrak{v}_j \in \mathcal{A}_i, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathrm{n}}. \tag{3.8}$$

Then,

$$\begin{aligned} 0 &= d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, (\mathfrak{v}_{\mathrm{j}}\mathfrak{u}_{\mathrm{j}} \circ \mathfrak{v}_{\mathrm{j}})_\delta, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta) \\ &= d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}(\mathfrak{u}_{\mathrm{j}} \circ \mathfrak{v}_{\mathrm{j}})_\delta, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta) \\ &= d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta)(\mathfrak{u}_{\mathrm{j}} \circ \mathfrak{v}_{\mathrm{j}})_\delta \end{aligned}$$

for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathrm{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathrm{n}}$.
Therefore,

$$\begin{aligned} &d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta)\mathfrak{u}_{\mathrm{j}}\delta(\mathfrak{v}_{\mathrm{j}}) = \\ &-d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta)\mathfrak{v}_{\mathrm{j}}\mathfrak{u}_{\mathrm{j}}. \end{aligned}$$

Put $u_{\mathrm{j}}t$, where $t \in \mathcal{M}$ instead of $\mathfrak{u}_{\mathrm{j}}$ and use it to obtain

$$d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta)\mathcal{A}_{\mathrm{j}}[t, \delta(-\mathfrak{v}_{\mathrm{j}})] = \{0\}$$

for any $\mathfrak{u}_1, v_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathfrak{n}}$.

According to Lemma 2.3(b), it follows
Either $d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \mathfrak{v}_{\mathrm{j}}, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta) = 0$ or $\delta(-\mathfrak{v}_{\mathrm{j}}) \in Z(\mathcal{M})$

$$\text{for any } \mathfrak{u}_1, v_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathrm{n}} \tag{3.9}$$

If there is $\mathfrak{v}_{j0} \in \mathcal{A}_{\mathrm{j}}$ such that $\delta(-\mathfrak{v}_{j0}) \in Z(\mathcal{M})$, replace u_{j} by $\delta(-\mathfrak{v}_{j0})\mathfrak{u}_{\mathrm{j}}$ in (3.8) implies

$$d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \delta(-\mathfrak{v}_{j0}), \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta)(\mathfrak{u}_{\mathrm{j}} \circ \mathfrak{v}_{\mathrm{j}})_\delta = 0$$

for any $\mathfrak{u}_1, \mathfrak{v}_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathfrak{n}}$
Therefore,

$$\begin{aligned} &d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \delta(-\mathfrak{v}_{j0}), \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta)\mathfrak{u}_j\delta(\mathfrak{v}_{\mathrm{j}}) = \\ &-d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \delta(-\mathfrak{v}_{j0}), \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta)\mathfrak{v}_i\mathfrak{u}_{\mathrm{j}}. \end{aligned}$$

Put $u_{\mathrm{j}}t$, where $t \in \mathcal{M}$ instead of $\mathfrak{u}_{\mathrm{j}}$ in last equation and use it to conclude

$$d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \delta(-\mathfrak{v}_{j0}), \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta)\mathcal{A}_{\mathrm{j}}[t, \delta(-\mathfrak{v}_{\mathrm{j}})] = \{0\}$$

for any $\mathrm{u}_1, v_1 \in \mathcal{A}_1, \mathrm{u}_2, \mathrm{v}_2 \in \mathcal{A}_2, \ldots, \mathrm{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathrm{u}_n, \mathrm{v}_n \in \mathcal{A}_{\mathrm{n}}$,

According to Lemma 2.3(b) it follows
either $d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \delta(\mathfrak{v}_{j0}), \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta) = 0$ or $\delta(-\mathcal{A}_j) \in \mathcal{Z}(\mathcal{M})$ Therefore, (3.9) becomes: either $\delta(-\mathcal{A}_{\mathrm{j}}) \in \mathcal{Z}(\mathcal{M})$ or $d((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \delta(\mathfrak{v}_j), \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta) = 0$

$$\text{for any } \mathrm{u}_1, v_1 \in \mathcal{A}_1, \mathrm{u}_2, \mathrm{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathrm{u}_n, \mathrm{v}_n \in \mathcal{A}_{\mathrm{n}}$$

If the second case hold, then
for any $\mathfrak{u}_1, v_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathfrak{n}}, x_j \in \mathcal{M}$, we have

$$\begin{aligned} 0 &= d\left((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, \delta(x_j \mathfrak{v}_j), \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta\right) \\ &= d\left((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, x_j \delta(\mathfrak{v}_j), \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta\right) \\ &= d\left((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, x_j, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta\right) \delta(\mathfrak{p}_j) \end{aligned}$$

It follows, for any $\mathrm{u}_1, v_1 \in \mathcal{A}_1, \mathrm{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathrm{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathrm{n}}, x_j, y \in \mathcal{M}$:

$$\begin{aligned} 0 &= d\left((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, x_j, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta\right) \delta(y \mathfrak{v}_j) \\ &= d\left((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, x_j, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta\right) y \delta(\mathfrak{v}_j) \end{aligned}$$

By three primeness of $\mathcal{M}$, we obtain, for any $\mathfrak{u}_1, v_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_j, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_n$, $x_j \in \mathcal{M}$

$$d\left((\mathfrak{u}_1 \circ \mathfrak{v}_1)_\delta, (\mathfrak{u}_2 \circ \mathfrak{v}_2)_\delta, \ldots, x_j, \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)_\delta\right) = 0 \text{ or } \delta(\mathfrak{v}_{\mathrm{j}}) = 0$$

The second case leads to $\delta = 0$ (contradiction). Continuing inductively, we obtain either $d(x_1, x_2, \ldots, x_j, \ldots, x_n) = 0$ for any $x_1, x_2, \ldots, x_j, \ldots, x_n \in \mathcal{M}$ or there is $j \epsilon \{1, 2, \ldots, n\}$ such that $\delta(-\mathcal{A}_{\mathrm{j}}) \subseteq Z(\mathcal{M})$, since $d \neq 0$, we conclude $\mathcal{M}$ is commutative ring according to Lemma 2.5.

Corollary 3.7 *Let d be a right n-derivation of $\mathcal{M}$ and $\mathcal{A}_1$, $\mathcal{A}_2$, $\ldots, \mathcal{A}_n$ are nonzero semigroup ideals of $\mathcal{M}$, if one of the following assertions:*

(iii) $d([\mathfrak{u}_1, \mathfrak{v}_1], [\mathfrak{u}_2, \mathfrak{v}_2], \ldots, [\mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}}], \ldots, [\mathfrak{u}_n, \mathfrak{v}_n]) = 0$

(iv) $d((\mathfrak{u}_1 \circ \mathfrak{v}_1), (\mathfrak{u}_2 \circ \mathfrak{v}_2), \ldots, (\mathfrak{u}_{\mathrm{j}} \circ \mathfrak{v}_{\mathrm{j}}), \ldots, (\mathfrak{u}_n \circ \mathfrak{v}_n)) = 0$

hold for any $\mathfrak{u}_1, v_1 \in \mathcal{A}_1, \mathfrak{u}_2, \mathfrak{v}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_n, \mathfrak{v}_n \in \mathcal{A}_{\mathfrak{n}}$, then $\mathcal{M}$ is a commutative ring.

Corollary 3.8 *Let d be a nonzero right* n*-derivation of $\mathcal{M}$ and δ is a nonzero right multiplier of $\mathcal{M}$, if one of the following assertions:*

(i) $d([x_1, y_1]_\delta, [x_2, y_2]_\delta, \ldots, [x_{\mathrm{j}}, y_{\mathrm{j}}]_\delta, \ldots, [x_n, y_n]_\delta) = 0$

(ii) $d((x_1 \circ y_1)_\delta, (x_2 \circ y_2)_\delta, \ldots, (x_{\mathrm{j}} \circ y_{\mathrm{j}})_\delta, \ldots, (x_n \circ y_n)_\delta) = 0$

hold for any $\mathrm{x}_1, y_1, \mathrm{x}_2, y_2, \ldots, x_{\mathrm{j}}, y_{\mathrm{j}}, \ldots, x_n, y_n \in \mathcal{M}$, then $\mathcal{M}$ is a commutative ring.

Corollary 3.9 *Let d be a nonzero right* n*−derivation of $\mathcal{M}$, if it is true that any of the following statements:*

(i) $d([x_1, y_1], [x_2, y_2], \ldots, [x_{\mathrm{j}}, y_{\mathrm{j}}], \ldots, [x_n, y_n]) = 0.$

(ii) $d((x_1 \circ y_1), (x_2 \circ y_2), \ldots, (x_{\mathrm{j}} \circ y_{\mathrm{j}}), \ldots, (x_n \circ y_n)) = 0.$

hold for any $\mathrm{x}_1, y_1, \mathrm{x}_2, y_2, \ldots, x_{\mathrm{j}}, y_{\mathrm{j}}, \ldots, x_n, y_n \in \mathcal{M}$, then $\mathcal{M}$ is a commutative ring.

Theorem 3.4 *Let d be nonzero right n-derivation of $\mathcal{M}, \delta$ is a nonzero a right multiplier of $\mathcal{M}$ and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ are nonzero semigroup ideals of $\mathcal{M}$, if any of the following assertions hold:*

(i) $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, [\mathfrak{u}_{\mathrm{j}}, x]_\delta, \ldots, \mathfrak{u}_{\mathfrak{n}}) = [\mathfrak{u}_{\mathrm{j}}, x]_\delta$

(ii) $d(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{u}_{\mathrm{j}} \circ x)_\delta, \ldots, \mathfrak{u}_{\mathfrak{n}}) = (\mathfrak{u}_{\mathrm{j}} \circ x)_\delta$

for any $x \in \mathcal{M}, \mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathrm{u}_{\mathrm{n}} \in \mathcal{A}_{\mathfrak{n}}$, then $\mathcal{M}$ is a commutative ring

Proof:(i) Suppose that: for any $x \in \mathcal{M}, \mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_j \in \mathcal{A}_j, \ldots, \mathfrak{u}_n \in \mathcal{A}_n$

$$d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, [\mathfrak{u}_{\mathrm{j}}, x]_\delta, \ldots, \mathfrak{u}_{\mathfrak{r}}\right) = [\mathfrak{u}_{\mathrm{j}}, x]_\delta \tag{3.10}$$

Replace $\mathfrak{u}_{\mathrm{j}}$ by $x\mathfrak{u}_{\mathrm{j}}$ in (3.10) to get $d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, x[\mathfrak{u}_{\mathrm{j}}, x]_\delta, \ldots, \mathfrak{u}_{\mathfrak{n}}\right) = x[\mathfrak{u}_{\mathrm{j}}, x]_\delta$ for any $x \in \mathcal{M}$, $\mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathfrak{u}_{\mathrm{n}} \in \mathcal{A}_{\mathrm{n}}$. It follows,

$$d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, x, \ldots, u_n\right)[\mathfrak{u}_{\mathrm{j}}, x]_\delta + [\mathfrak{u}_{\mathrm{j}}, x]_\delta\, x = x[\mathfrak{u}_{\mathrm{j}}, x]_\delta \tag{3.11}$$

Put $[v_i, x]_\delta$ instead of x in (3.11) to get

$$d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, [v_{\mathrm{j}}, x]_\delta, \ldots, \mathfrak{u}_{\mathfrak{n}}\right)[\mathfrak{u}_{\mathrm{j}}, [v_{\mathrm{j}}, x]_\delta]_\delta + [\mathfrak{u}_{\mathrm{j}}, [v_{\mathrm{j}}, x]_\delta]_\delta [v_{\mathrm{j}}, x]_\delta = [v_{\mathrm{j}}, x]_\delta [\mathfrak{u}_{\mathrm{j}}, [v_{\mathrm{j}}, x]_\delta]_\delta,$$

Use hypothesis in previous relation to get

$$[\mathfrak{u}_{\mathrm{j}}, [v_{\mathrm{j}}, x]_\delta]_\delta [v_{\mathrm{j}}, x]_\delta = 0 \text{ for any } x \in \mathcal{M}, \mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}} \tag{3.12}$$

Therefore

$$\begin{aligned} 0 &= d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, [\mathfrak{u}_{\mathrm{j}}, [v_{\mathrm{j}}, x]_\delta]_\delta [v_{\mathrm{j}}, x]_\delta, \ldots, \mathfrak{u}_{\mathrm{n}}\right) \\ &= d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, [\mathfrak{u}_{\mathrm{j}}, [v_{\mathrm{j}}, x]_\delta]_\delta, \ldots, \mathfrak{u}_r\right)[v_{\mathrm{j}}, x]_\delta \\ &\quad + d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, [v_{\mathrm{j}}, x]_\delta, \ldots, \mathfrak{u}_{\mathrm{r}}\right)[\mathfrak{u}_{\mathrm{j}}, [v_{\mathrm{j}}, x]_\delta]_\delta \\ &= [\mathfrak{u}_{\mathrm{j}}, [v_{\mathrm{j}}, x]_\delta]_\delta [v_{\mathrm{j}}, x]_\delta + [v_{\mathrm{j}}, x]_\delta [\mathfrak{u}_{\mathrm{j}}, [v_{\mathrm{j}}, x]_\delta]_\delta \\ &= [v_{\mathrm{j}}, x]_\delta [\mathfrak{u}_{\mathrm{j}}, [v_{\mathrm{j}}, x]_\delta]_\delta \end{aligned} \tag{3.13}$$

for any $x \in \mathcal{M}, \mathrm{u}_{\mathrm{j}}, \mathrm{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$
Which means that $[v_{\mathrm{j}}, x]_\delta \mathfrak{u}_{\mathrm{j}} \delta\left([\mathfrak{v}_{\mathrm{j}}, x]_\delta\right) = [v_{\mathrm{j}}, x]_\delta [v_{\mathrm{j}}, x]_\delta \mathfrak{u}_{\mathrm{j}}$ for any $x \in \mathcal{M}, \mathfrak{u}_{\mathrm{j}}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$, taking $\mathfrak{u}_{\mathrm{j}} n$ in place of $\mathfrak{u}_{\mathrm{j}}$ in last equation and using it implies that
$[\mathfrak{v}_{\mathrm{j}}, x]_\delta \mathcal{A}_{\mathrm{j}}\left[\delta\left([\mathfrak{v}_{\mathrm{j}}, x]_\delta\right), n\right] = \{0\}$ for any $x, n \in \mathcal{M}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$, and Lemma 2.3 (b) ensures that either $[\mathfrak{v}_{\mathrm{j}}, x]_\delta = 0$ or $\delta\left([\mathfrak{v}_{\mathrm{j}}, x]_\delta\right) \in \mathcal{Z}(\mathcal{M})$ for any $x \in \mathcal{M}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$, which can be reduce to

$$\delta\left([v_{\mathrm{j}}, x]_\delta\right) \in Z(\mathcal{M}) \quad \text{for any} \quad x \in \mathcal{M}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}} \tag{3.14}$$

Replace v_{j} by xv_{j} in (3.14), we obtain $x\delta\left([v_{\mathrm{j}}, x]_\delta\right) \in Z(\mathcal{M})$ for any $x \in \mathcal{M}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$. Using Lemma 2.1 (a) lastly implies

$$x \in Z(\mathcal{M}) \text{ or } \delta\left([v_{\mathrm{j}}, x]_\delta\right) = 0 \quad \text{for any} \quad x \in \mathcal{M}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}. \tag{3.15}$$

If there is $x_0 \in \mathcal{M}$ such that $x_0 \in Z(\mathcal{M})$, from (3.14), we obtain $\delta\left([v_i, x_0]_\delta\right) = \delta\left(v_{\mathrm{j}}\delta(x_0) - x_0 v_j\right) = v_{\mathrm{j}}\delta\left(\delta(x_0) - x_0\right) \in Z(\mathcal{M})$ for any $\mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$. It follows $tv_{\mathrm{j}}\delta\left(\delta(x_0) - x_0\right) \in Z(\mathcal{M})$ for any $t \in \mathcal{M}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$, using Lemma 2.1 (a) another time ensures that $t \in \mathcal{Z}(\mathcal{M})$ for any $t \in \mathcal{M}$ or $v_{\mathrm{j}}\delta\left(\delta(x_0) - x_0\right) = 0$ for any $\mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$, first case leads to the required result and the second case implies $0 = v_{\mathrm{j}}\delta\left(\delta(x_0) - x_0\right) = \delta\left(v_{\mathrm{j}}\delta(x_0) - x_0 v_{\mathrm{j}}\right) = \delta\left([v_{\mathrm{j}}, x_0]_\delta\right)$ for any $\mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$.

Therefore (3.15) becomes either $\mathcal{M}$ is commutative or $\delta\left([v_{\mathrm{j}}, x]_\delta\right) = 0$ for any $x \in \mathcal{M}, \mathfrak{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$. Last result with (3.12) implies $[v_{\mathrm{j}}, x]_\delta \mathfrak{u}_{\mathrm{j}} [v_{\mathrm{j}}, x]_\delta = 0$ for any $x \in \mathcal{M}, \mathfrak{u}_{\mathrm{j}}, v_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$, using Lemma 2.3 (b) to conclude $[v_{\mathrm{j}}, x]_\delta = 0$ for any $x \in \mathcal{M}, \mathrm{v}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}$. Then $\mathcal{M}$ is commutative according to Lemma 2.4 (a) and Lemma 2.5.
(ii) By assumption, we get
$d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{u}_j \circ x)_\delta, \ldots, \mathfrak{u}_{\mathfrak{n}}\right) = (\mathfrak{u}_{\mathrm{j}} \circ x)_\delta$

for any $x \in \mathcal{M}, u_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathrm{u}_{\mathrm{j}} \in \mathcal{A}_{\mathrm{j}}, \ldots, \mathrm{u}_{\mathfrak{n}} \in \mathcal{A}_{\mathfrak{n}}$.

It follows, $d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (x\mathfrak{u}_j \circ x)_\delta, \ldots, \mathfrak{u}_n\right) = (x\mathfrak{u}_j \circ x)_\delta$, that is $d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, x(\mathfrak{u}_j \circ \; x)_\delta, \ldots, \mathfrak{u}_n\right) = x(\mathfrak{u}_{\mathrm{j}} \circ x)_\delta$. Which implies $d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, x, \ldots, \mathfrak{u}_n\right)(\mathfrak{u}_{\mathrm{j}} \circ x)_\delta + (\mathfrak{u}_{\mathrm{j}} \circ x)_\delta\, x = x(\mathfrak{u}_{\mathrm{j}} \circ x)_\delta$, Put $(v_{\mathrm{j}} \circ x)_\delta$ instead of x in last relation to get $d\left(u_1, \mathfrak{u}_2, \ldots, (v_{\mathrm{j}} \circ x)_\delta, \ldots, \mathrm{u}_{\mathrm{r}}\right)\left(\mathrm{u}_{\mathrm{j}} \circ (v_{\mathrm{j}} \circ x)_\delta\right)_\delta + \left(\mathfrak{u}_{\mathrm{j}} \circ (v_{\mathrm{j}} \circ x)_\delta\right)_\delta (v_{\mathrm{j}} \circ x)_\delta =$

$(v_j \circ x)_\delta (u_j \circ (v_j \circ x)_\delta)_\delta$, Use hypothesis to get
$(\mathfrak{u}_j \circ (v_j \circ x)_\delta)_\delta (v_j \circ x)_\delta = 0$ for any $x \in \mathcal{M}, \mathfrak{u}_j, v_j \in \mathcal{A}_j$. thus

$$(\mathfrak{u}_j \circ (v_j \circ x)_\delta)_\delta (v_j \circ x)_\delta = 0 \text{ for any } x \in \mathcal{M}, \mathfrak{u}_j, v_j \in \mathcal{A}_j \tag{3.16}$$

Therefore,

$$\begin{aligned} 0 &= d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{u}_j \circ (v_j \circ x)_\delta)_\delta (v_j \circ x)_\delta, \ldots, \mathfrak{u}_n\right) \\ &= d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{u}_j \circ (v_j \circ x)_\delta)_\delta, \ldots, \mathfrak{u}_n\right)(v_j \circ x)_\delta \\ &+ d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (v_j \circ x)_\delta, \ldots, \mathfrak{u}_n\right)(\mathfrak{u}_j \circ (v_j \circ x)_\delta)_\delta \\ &= (\mathfrak{u}_j \circ (v_j \circ x)_\delta)_\delta (v_j \circ x)_\delta + (v_j \circ x)_\delta (\mathfrak{u}_j \circ (v_j \circ x)_\delta)_\delta \\ &= (v_j \circ x)_\delta (\mathfrak{u}_j \circ (v_j \circ x)_\delta)_\delta \text{ for any } x \in \mathcal{M}, \mathfrak{u}_j, v_j \in \mathcal{A}_j. \end{aligned} \tag{3.17}$$

Thus, $(v_j \circ x)_\delta u_j \delta((v_j \circ x)_\delta) = -(v_j \circ x)_\delta (v_j \circ x)_\delta u_j$ for any $x \in \mathcal{M}, u_j, v_j \in \mathcal{A}_j$, taking $u_j n$ in place of $\mathfrak{u}_j$ in last equation and using it implies that
$(v_j \circ x)_\delta \mathcal{A}_j \left[\delta\left(-(v_j \circ x)_\delta\right), n\right] = \{0\}$ for any $x, n \in \mathcal{M}, v_j \in \mathcal{A}_j$, and Lemma 2.3(b) ensures that $(v_j \circ x)_\delta = 0$ or $\delta\left(-(v_j \circ x)_\delta\right) \in \mathcal{Z}(\mathcal{M})$ for any $x \in \mathcal{M}, v_j \in \mathcal{A}_j$. Thus

$$\delta\left(-(v_j \circ x)_\delta\right) \in Z(\mathcal{M}) \text{ for any } x \in \mathcal{M}, \mathfrak{v}_j \in \mathcal{A}_j \tag{3.18}$$

Replace v_j by xv_j in (3.18), we obtain $x\delta\left(-(v_j \circ x)_\delta\right) \in \mathcal{Z}(\mathcal{M})$ for any $x \in \mathcal{M}, v_j \in \mathcal{A}_j$.
Using Lemma 2.1 (a) lastly implies

$$x \in Z(\mathcal{M}) \text{ or } \delta\left((v_j \circ x)_\delta\right) = 0 \text{ for any } x \in \mathcal{M}, \mathfrak{v}_j \in \mathcal{A}_j \tag{3.19}$$

If there is $x_0 \in \mathcal{M}$ such that $x_0 \in Z(\mathcal{M})$, from (3.18), we obtain $-\delta\left((\mathfrak{v}_j \circ x_0)_\delta\right) = -\delta\left(\mathfrak{v}_j \delta(x_0) + x_0 \mathfrak{v}_j\right) = \mathfrak{v}_j \delta\left(-\left(\delta(x_0) + x_0\right)\right) \in Z(\mathcal{M})$ for any $\mathfrak{p}_j \in \mathcal{A}_j$ It follows $t\mathfrak{v}_j \delta\left(-\left(\delta(x_0) + x_0\right)\right) \in Z(\mathcal{M})$ for any $t \in \mathcal{M}, \mathfrak{v}_j \in \mathcal{A}_j$, using Lemma 2.1 (a) another time ensures that $t \in \mathcal{Z}(\mathcal{M})$ for any $t \in \mathcal{M}$ or $v_j \delta\left(-\left(\delta(x_0) + x_0\right)\right) = 0$ for any $t \in \mathcal{M}, v_j \in \mathcal{A}_j$, first case leads to the required result and the second case $0 = v_j \delta\left(-\left(\delta(x_0) + x_0\right)\right)$, that is $\delta\left(-\left(v_j \delta(x_0) + x_0 v_j\right)\right) = \delta\left((v_j \circ x_0)_\delta\right) = 0$ for any $\mathfrak{v}_j \in \mathcal{A}_j$. Therefore (3.19) becomes either $\mathcal{M}$ is commutative or $\delta\left((v_j \circ x)_\delta\right) = 0$ for any $x \in \mathcal{M}, \mathfrak{v}_j \in \mathcal{A}_j$. Last result with (3.16) implies $(v_j \circ x)_\delta u_j (v_j \circ x)_\delta = 0$ for any $x \in \mathcal{M}, \mathfrak{u}_j, v_j \in \mathcal{A}_j$, using Lemma 2.3 (b) to conclude $(v_j \circ x)_\delta = 0$ for any $x \in \mathcal{M}, v_j \in \mathcal{A}_j$. Then $\mathcal{M}$ is commutative according to Lemma 2.4 (b)and Lemma 2.5.

Corollary 3.10 *Let d be a nonzero right n-derivation of $\mathcal{M}$ and $\mathcal{A}_1, \mathcal{A}_2, \ldots, \mathcal{A}_n$ are nonzero semigroup ideals of $\mathcal{M}$, if any of the following assertion hold:*

(iii) $d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, [\mathfrak{u}_j, x], \ldots, \mathfrak{u}_n\right) = [\mathfrak{u}_j, x]$

(iv) $d\left(\mathfrak{u}_1, \mathfrak{u}_2, \ldots, (\mathfrak{u}_j \circ x), \ldots, \mathfrak{u}_n\right) = (\mathfrak{u}_j \circ x)$

for any $x \in \mathcal{M}, \mathfrak{u}_1 \in \mathcal{A}_1, \mathfrak{u}_2 \in \mathcal{A}_2, \ldots, \mathfrak{u}_3 \in \mathcal{A}_n$ then $\mathcal{M}$ is a commutative ring

Corollary 3.11 *Let d be a nonzero right derivation of $\mathcal{M}$ and $\mathcal{A}$ is a nonzero semigroup ideal of $\mathcal{M}, \delta$ is a nonzero right multiplier of $\mathcal{M}$, if one of the following assertions:*

(i) $d([u, x]_\delta) = [u, x]_\delta$

(ii) $d((u \circ x)_\delta) = (u \circ x)_\delta$

hold for any $u \in \mathcal{A}$, xin$\mathcal{M}$ then $\mathcal{M}$ is a commutative ring.

Corollary 3.12 *Let d be a nonzero right n-derivation of $\mathcal{M}, \delta$ is a nonzero a right multiplier of $\mathcal{M}$, if any of the following assertions hold:*

(i) $d\left(x_1, x_2, \ldots, [x_j, x]_\delta, \ldots, x_n\right) = [x_j, x]_\delta$

(ii) $d\left(x_1, x_2, \ldots, (x_\mathrm{j} \circ x)_\delta, \ldots, x_\mathrm{n}\right) = (x_\mathrm{j} \circ x)_\delta$

for any $x_1, x_2, \ldots, x_1, \ldots, x_n, x \in \mathcal{M}$*, then* $\mathcal{M}$ *is a commutative ring*

Corollary 3.13 *Let* d *be a nonzero right* n*-derivation of* $\mathcal{M}$*, if any of the following assertions hold:*

(i) $d\left(x_1, x_2, \ldots, [x_\mathrm{j}, x], \ldots, x_\mathrm{n}\right) = [x_\mathrm{j}, x]$,

(ii) $d\left(x_1, x_2, \ldots, (x_i \circ x), \ldots, x_n\right) = (x_i \circ x)$

for any $x_1, x_2, \ldots, x_j, \ldots, x_n, x \in \mathcal{M}$*, then* $\mathcal{M}$ *is a commutative ring*

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