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On Fuzzy Annihilators and Primitive fuzzy Ideals of a Ring

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ABSTRACT: In this paper our attempt is to study the concept like Noetherian quotient in fuzzy setting. If μ and σ are fuzzy ideals of a ring R, then we define, $(\mu : \sigma) = \vee\{\eta | \eta\sigma \subseteq \mu\}$ and it is proved that $(\mu : \sigma)$ is a fuzzy ideal of R. Using this notion annihilator of fuzzy subset of a ring is defined and we introduce the notion of fuzzy primitive ideal via fuzzy Noetherian quotient. Some results on fuzzy primitive ideal and various properties of fuzzy annihilators have been established.

Key Words: Fuzzy ideals, fuzzy annihilators, primitive fuzzy ideals.

Contents

1. Introduction

The concept of a fuzzy set that was earlier proposed by Zadeh in 1965 [13], which was applied to the elementary theory of groupoids and groups in Rosenfeld [7]. Rosenfeld proposed that μ is called a fuzzy subgroupoid of S if, for all $x, y \in S$, $\mu(xy) \geq min(\mu(x), \mu(y))$. This has opened up a new perception in the field of Mathematical science. From that time only, the study of fuzzy algebraic structure has been followed and applied in many different direction such as groups, rings, semi-rings, near-rings etc. Wang-Jin Liu [12], developed the concept of fuzzy subrings and fuzzy ideals of a ring in their paper "Fuzzy Invariant subgroups and Fuzzy Ideals". Later on, Mukherjee et al. [4], Swamy and Swamy [11] fuzzified some of the concepts and results on rings and ideals. Rosenfeld [7] also studied some of the properties of fuzzy homomorphism where he investigated the characteristics of homomorphic image and homomorphic preimage of a fuzzy subgroupoid. Fuzzy submodules were first proposed by [5] Negoita and Ralescu. Pan [6] studied the fuzzy finitely generated modules and fuzzy quotient modules. After that many researchers have studied fuzzy submodules. Saikia et al. 2009 [8] investigated certain characteristics of annihilator of fuzzy subsets of modules.

In our study, we shall apply the concept of fuzzy sets to the notion of Noetherian quotient that will lead to the theory of fuzzy primitive ideals. And as a result, the concept of fuzzy annihilators will open up a new fuzzy structure namely Goldie fuzzy ring and related concepts. In this paper, we will present some properties of fuzzy primitive ideals and fuzzy annihilators.

2. Basic Definitions

In this paper R denotes commutative ring with unity and $FI(R)$ denotes the set of all fuzzy ideals of R.

Definition 2.1 *[1] Let μ be a fuzzy subset of R then μ is said to be a fuzzy ideal of R if and only if it satisfies,*
(i) $\mu(x - y) \geq \mu(x) \wedge \mu(y)$ and
(ii) $\mu(xy) \geq \mu(x) \vee \mu(y)$, for all $x, y \in R$.

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Definition 2.2 *[1] Let μ and σ be fuzzy ideals of R then their sum is defined as,* $(\mu+\sigma)(x) = \vee\{\mu(y)\wedge\sigma(z)|y,z\in R, x=y+z\}$ *and their product is defined by,* $(\mu\sigma)(x) = \vee\{\mu(y)\wedge\sigma(z)|y,z\in R, x=yz\}$.

Definition 2.3 *[1] Let μ be a fuzzy ideal of R then μ is said to be a fuzzy prime ideal of R if for any fuzzy subsets σ, γ of R such that $\sigma\gamma\subseteq\mu$, then either $\sigma\subseteq\mu$ or $\gamma\subseteq\mu$.*

Definition 2.4 *Let μ be a fuzzy ideal of R then annihilator of μ, denoted by $ann(\mu)$ and is defined as, $ann(\mu) = \vee\{\eta|\eta\in[0,1]^R, \eta\mu\subseteq\chi_0\}$, where χ_0 is the characteristics function on $\{0\}$ and 0 means the additive identity of R.*

Definition 2.5 *[8] Let μ be a fuzzy ideal of R. Then μ is said to be a fuzzy faithful ideal if $ann(\mu)=\chi_0$.*

Definition 2.6 *Let σ be a fuzzy subset of R then fuzzy ideal of the form $ann(\sigma)$ is called a fuzzy annihilator ideal of R. Thus, μ is a fuzzy annihilator ideal if and only if $\mu = ann(\sigma)$, where σ is a fuzzy subset of R such that $\sigma(0)=1$.*

Definition 2.7 *Let σ be a maximal fuzzy right ideal of R. Then $(\mu:\sigma) = \vee\{\eta|\mu\eta\subseteq\sigma\}$ is a right primitive fuzzy ideal of R, where η is a fuzzy subset of R.*

Example 2.1 *We define a fuzzy subset μ of $\mathbf{Z_8}$ by,*

$$\mu(x) = \begin{cases} 0.9, & x\in\{0,2,4,6\} \\ 0, & otherwise \end{cases}$$

Then μ is a fuzzy primitive ideal of $\mathbf{Z_8}$, the ring of integer modulo 8.

3. Preliminary Lemmas

In this section we provide some preliminary lemmas those are derived from [8] are used in our main results.

Lemma 3.1 *Let μ be a fuzzy subset of R. Then the level subset $\mu_t = \{x\in R|\mu(x)\geq t\}$ is ideal of R if and only if μ is a fuzzy ideal of R.*

Lemma 3.2 *Let μ be a fuzzy ideal of R then μ is a fuzzy prime ideal of R if and only if μ_t is prime ideal of R, $t\in Im(\mu)$, the image of μ.*

Lemma 3.3 *If μ_i be a fuzzy ideal of R then $\bigcap_{i\in\Delta}\mu_i$ is also a fuzzy ideal of R, $i\in\Delta$ non empty index set.*

Lemma 3.4 *Let μ, γ be fuzzy ideals of R and σ be any fuzzy subset of R then $\sigma(\mu+\gamma)\subseteq\sigma\mu+\sigma\gamma$.*

Lemma 3.5 *If μ is a fuzzy ideal of R then $\chi_0\subseteq ann(\mu)$.*

Lemma 3.6 *Let μ be a fuzzy ideal of R. Then $ann(\mu)\mu\subseteq\chi_0$. Also, if $\mu(0)=1$, then $ann(\mu)\mu=\chi_0$.*

Proof: Let $x\in R$, since μ is a fuzzy ideal and $ann(\mu)\in[0,1]^R$.
So, $(ann(\mu)\mu)(x) = \bigvee_{x=ry}\{ann(\mu)(r)\wedge\mu(y)|r,y\in R\} = \bigvee_{x=ry}[\{\vee\{\eta(r)|\eta\in[0,1]^R,\eta\mu\subseteq\chi_0\}\}\wedge\mu(y)|r,y\in R] = \bigvee_{x=ry}\{\eta(r)\wedge\mu(y)|\eta\in[0,1]^R,\eta\mu\subseteq\chi_0\} \leq \bigvee_{x=ry}\{(\eta\mu)(ry)|\eta\in[0,1]^R,\eta\mu\subseteq\chi_0\} \leq \vee\{\chi_0(x)|\eta\in[0,1]^R,\eta\mu\subseteq\chi_0\} = \chi_0(x), \forall x\in R$. So, $ann(\mu)\mu\subseteq\chi_0$.
Again by Lemma 3.5, $\chi_0\subseteq ann(\mu) \implies \chi_0(0)\leq ann(\mu)(0) \implies 1\leq ann(\mu)(0) \implies ann(\mu)(0) = 1$. Now, $(ann(\mu)\mu(0) = \vee\{ann(\mu)(r_1)\wedge\mu(r_2)|r_1,r_2\in R, r_1r_2=0\} \geq ann(\mu)(0)\wedge\mu(0) = 1$. So $(ann(\mu)(\mu)(0)) = 1$. □

Lemma 3.7 *Let σ be any fuzzy subset of R and if μ be a fuzzy ideal of R then $\sigma\mu \subseteq \chi_0$ if and only if $\sigma \subseteq ann(\mu)$. Also, if $\sigma(0) = 1$ and $\mu(0) = 1$, then $\sigma\mu = \chi_0$ if and only if $\sigma \subseteq ann(\mu)$.*

Lemma 3.8 *Let μ_i be fuzzy ideals of R, where i is in non-empty index set, say Δ then*

$$ann(\underset{i\in\Delta}{\cup}\mu_i) = \underset{i\in\Delta}{\cap} ann(\mu_i).$$

Lemma 3.9 *Let μ and η be finite valued fuzzy ideals of R. Then $(\mu+\eta)_t = \mu_t + \eta_t, \forall t \in [0,1]$.*

Proof: Let $x \in (\mu+\eta)_t \implies t \leq (\mu+\eta)(x) = \underset{x=y+z}{\vee}(\mu(y)\wedge\eta(z)) = \mu(r_1)\wedge\eta(r_2)$, for some $r_1, r_2 \in R$, such that $x = r_1 + r_2$.
$\implies t \leq \mu(r_1),\ t \leq \eta(r_2) \implies r_1 \in \mu_t,\ r_2 \in \eta_t \implies x = r_1 + r_2 \in \mu_t + \eta_t$.
Therefore, $(\mu+\eta)_t \subseteq \mu_t + \eta_t$
Conversely let, $u \in \mu_t + \eta_t$. Then there exists $a, b \in R$ such that $a \in \mu_t$, $b \in \eta_t$ and $u = a + b$.
So, $\mu(a) \geq t$, $\eta(b) \geq t \implies \mu(a)\wedge\eta(b) \geq t$, where $u = a + b$.
$\implies \underset{x=p+q}{\vee}(\mu(p)\wedge\eta(q)) \geq t \implies (\mu+\eta)(u) \geq t \implies u \in (\mu+\eta)_t$
Hence, $(\mu+\eta)_t = \mu_t + \eta_t, \forall t \in [0,1]$.

□

4. Main Results

In this section we provide the main results.

Theorem 4.1 *Let μ and σ be fuzzy ideals of R then $(\mu:\sigma) = \vee\{\eta|\eta\sigma \subseteq \mu\}$ is a fuzzy ideal of R, η is a fuzzy subset of R.*

Proof: Let $x, y \in R$ such that $(\mu:\sigma)(x) = t_1$ and $(\mu:\sigma)(y) = t_2$ with $t_1 \neq 0$, $t_2 \neq 0$, $t_1 < t_2$.
If $t_1 < t_2$ then $(\mu:\sigma)_{t_2} \subseteq (\mu:\sigma)_{t_1}$.
Now, $(\mu:\sigma)(x) = t_2 \implies x \in (\mu:\sigma)_{t_2}$ and $(\mu:\sigma)(y) = t_1 \implies y \in (\mu:\sigma)_{t_1}$.
Thus we get, $x, y \in (\mu:\sigma)_{t_1}$. Therefore, $x-y \in (\mu:\sigma)_{t_1} \implies (\mu:\eta)(x-y) \geq t_1 > (\mu:\eta)(x)\wedge(\mu:\eta)(y)$.
If $(\mu:\eta)(x) = 0$ and $(\mu:\eta)(y) = t$ then $(\mu:\eta)(x-y) \geq 0 = (\mu:\eta)(x)\wedge(\mu:\eta)(y)$.
Again, let $x, y \in R$ such that $(\mu:\sigma)(x) = t_1$ and $(\mu:\sigma)(y) = t_2$, where $0 < t_1 < t_2$.
So, $(\mu:\sigma)_{t_2} \subseteq (\mu:\sigma)_{t_1}$. Thus we get, $x, y \in (\mu:\sigma)_{t_1} \implies xy \in (\mu:\sigma)_{t_1}$
$\implies (\mu:\eta)(xy) \geq t_1 = (\mu:\eta)(x)\vee(\mu:\eta)(y)$.
If $(\mu:\eta)(x) = 0$ and $(\mu:\eta)(y) = t$ then $xy \in (\mu:\eta)_t$. $\implies (\mu:\eta)(xy) \geq t = (\mu:\eta)(x)\vee(\mu:\eta)(y)$. If $(\mu:\eta)(x) = 0$ and $(\mu:\eta)(y) = 0$ then $(\mu:\eta)(xy) \geq 0 = (\mu:\eta)(x)\vee(\mu:\eta)(y)$. Hence $(\mu:\eta)$ is a fuzzy ideal of R.

□

Theorem 4.2 *Let μ and σ be fuzzy ideal of R. If $\mu \subseteq \sigma$ then $ann(\sigma) \subseteq ann(\mu)$.*

Proof: Let, γ be a fuzzy subset of R. So for $r, x \in R$ we have, $\gamma(r)\wedge\mu(x) \leq \gamma(r)\wedge\sigma(x)$.
$\implies \vee\{\gamma(r)\wedge\mu(x)|r, x \in R, t = rx\} \leq \vee\{\gamma(r)\wedge\sigma(x)|r, x \in R, t = rx\}$.
So, $(\gamma\mu)(t) \leq (\gamma\sigma)(t)$, for all $t \in R$. Hence, $\gamma\mu \subseteq \gamma\sigma$. Therefore, $\gamma\sigma \subseteq \chi_0 \implies \gamma\mu \subseteq \chi_0$.
Hence, $\{\gamma|\gamma\sigma \subseteq \chi_0\} \subseteq \{\gamma|\gamma\mu \subseteq \chi_0\} \implies ann(\sigma) \subseteq ann(\mu)$.

□

Theorem 4.3 *If μ and σ be fuzzy ideals of R, then $ann(\mu+\sigma) = ann(\mu)\cap ann(\sigma)$.*

Proof: Let μ and σ be fuzzy ideals of R then $\mu+\sigma$ also be a fuzzy ideal of R.
So, $\mu \subseteq \mu+\sigma$ and $\sigma \subseteq \mu+\sigma$. So by Theorem 4.2, we get $ann(\mu+\sigma) \subseteq ann(\mu)$ and $ann(\mu+\sigma) \subseteq ann(\sigma) \implies ann(\mu+\sigma) \subseteq ann(\mu)\cap ann(\sigma)$. ...(1)

Also, $ann(\mu) \cap ann(\sigma) = (\cup\{\gamma_1 | \gamma_1 \in FI(R), \gamma_1\mu \subseteq \chi_0\}) \cap (\cup\{\gamma_2 | \gamma_2 \in FI(R), \gamma_2\sigma \subseteq \chi_0\})$
$= \cup\{\gamma_1 \cap \gamma_2 | \gamma_1, \gamma_2 \in FI(R), \gamma_1\mu \subseteq \chi_0, \gamma_2\sigma \subseteq \chi_0\} \subseteq \cup\{\gamma | \gamma = \gamma_1 \cap \gamma_2 \in FI(R), \gamma\mu \subseteq \chi_0, \gamma\sigma \subseteq \chi_0\}$
$\subseteq \cup\{\gamma | \gamma \in FI(R), \gamma(\mu + \sigma) \subseteq \chi_0\} = ann(\mu + \sigma)$
Therefore $ann(\mu) \cap ann(\sigma) \subseteq ann(\mu + \sigma)$. ...(2). Hence, $ann(\mu + \sigma) = ann(\mu) \cap ann(\sigma)$.

□

Theorem 4.4 *The following two conditions are identical:*
Let μ and γ be fuzzy ideals of R ,
(i) $ann(\gamma) = ann(\mu)$ for all $\gamma \subseteq \mu$, $\gamma \neq \chi_0$.
(ii) $\gamma\sigma \subseteq \chi_0 \implies \sigma\mu \subseteq \chi_0$, for all $\gamma \subseteq \mu$, $\gamma \neq \chi_0$, σ is a fuzzy subset of R.

Proof: $(i) \implies (ii)$. Let, $\gamma\sigma \subseteq \chi_0$. Then using Lemma 3.7, we have, $\sigma \subseteq ann(\gamma) = ann(\mu)$.
So, $\sigma \subseteq ann(\mu) \implies \sigma\mu \subseteq \chi_0$, by Lemma 3.7.
$(ii) \implies (i)$. Since, γ is a fuzzy ideal of R, so by Lemma 3.6, we have $ann(\gamma)\gamma \subseteq \chi_0$.
So, (ii) implies that $ann(\gamma)\mu \subseteq \chi_0$, where $\gamma \subseteq \mu$, $\gamma \neq \chi_0$. Again by Lemma 3.7, which implies that $ann(\gamma) \subseteq ann(\mu)$. Also, $\gamma \subseteq \mu \implies ann(\mu) \subseteq ann(\gamma)$, by Theorem 4.2. So. $ann(\gamma) = ann(\mu)$.

□

Corollary 4.1 *If $\gamma(0) = 1$ and $\mu(0) = 1$, then the following conditions are identical:*
(i) $ann(\gamma) = ann(\mu)$ for all $\gamma \subseteq \mu$, $\gamma \neq \chi_0$.
(i) $\gamma\sigma = \chi_0 \implies \sigma\mu = \chi_0$, for all $\gamma \subseteq \mu$, $\gamma \neq \chi_0$, σ is a fuzzy subset of R.

Proof: It is obvious by using Lemma 3.6 and Lemma 3.7. □

Theorem 4.5 *Let μ be a fuzzy ideal of R. Then $ann(\mu) = \vee\{r_\alpha | r \in R, \alpha \in [0,1], r_\alpha\mu \subseteq \chi_0\}$, where r_α is a fuzzy subset of R,*

$$r_\alpha(x) = \begin{cases} \alpha, & if x = r \\ 0, & otherwise \end{cases}$$

.

Proof: Since, $\{r_\alpha | r \in R, \alpha \in [0,1]\} \subseteq [0,1]^R$.
So, $\{r_\alpha | r \in R, \alpha \in [0,1], r_\alpha\mu \subseteq \chi_0\} \subseteq \{\eta | \eta \in [0,1]^R, \eta\mu \subseteq \chi_0\}$.
$\implies \vee\{r_\alpha | r \in R, \alpha \in [0,1], r_\alpha\mu \subseteq \chi_0\} \subseteq \vee\{\eta | \eta \in [0,1]^R, \eta\mu \subseteq \chi_0\} = ann(\mu)$.
Therefore, $\vee\{r_\alpha | r \in R, \alpha \in [0,1], r_\alpha\mu \subseteq \chi_0\} \subseteq ann(\mu)$.
Again let, $\eta \in [0,1]^R$ such that $\eta\mu \subseteq \chi_0$ and $\eta(r) = \alpha$, $r \in R$.
Now, $(r_\alpha\mu)(x) = \vee\{r_\alpha(s) \wedge \mu(y) | s, y \in R, sy = x\} = \vee\{\sigma(r) \wedge \mu(y) | y \in R, ry = x\} \leq \vee\{\sigma(s) \wedge \mu(y) | s, y \in R, sy = x\} = (\sigma\mu)(x) \leq \chi_0(x), \forall x \in R$. Thus, $r_\alpha\mu \subseteq \chi_0$. So, $ann(\mu) \subseteq \vee\{r_\alpha | r \in R, \alpha \in [0,1], r_\alpha\mu \subseteq \chi_0\}$.
Hence $ann(\mu) = \vee\{r_\alpha | r \in R, \alpha \in [0,1], r_\alpha\mu \subseteq \chi_0\}$.

□

Theorem 4.6 *Let μ be a fuzzy ideal of R. If μ is faithful and R is non zero then $\mu \neq \chi_0$.*

Proof: Since, μ is faithful. So, $ann(\mu) = \chi_0$. If $\mu = \chi_0$, then $ann(\mu) = ann(\chi_0) = \chi_R$, which implies that $\chi_0 = \chi_R$. Hence, $R = \{0\}$, a contradiction. Thus, $\mu \neq \chi_0$.

□

Theorem 4.7 *Let μ be a fuzzy ideal of R with $\mu(0) = 1$. Then $\mu \subseteq ann(ann(\mu))$ and $ann(ann(ann(\mu))) = ann(\mu)$.*

Proof: If μ is a fuzzy ideal of R then by Lemma 3.6 we get, $ann(\mu)\mu \subseteq \chi_0$. Again by Lemma 3.7, $\mu \subseteq ann(ann(\mu))$, by using Theorem 4.2, $ann(ann(ann(\mu))) \subseteq ann(\mu)$.
Also, $ann(\mu) \subseteq ann(ann(ann(\mu)))$. Thus, $ann(ann(ann(\mu))) = ann(\mu)$.

□

As a consequence of Theorem 4.5, we get corollary 4.7A.

Corollary 4.2 *If μ is a fuzzy annihilator ideal of R then $ann(ann(\mu)) = \mu$.*

Theorem 4.8 *If μ be a fuzzy prime ideal of R then $ann(\mu)$ is also a fuzzy prime ideal of R.*

Proof: Let μ be a fuzzy prime ideal. To show $ann(\mu)$ is a fuzzy prime ideal, so let $\sigma\gamma \subseteq ann(\mu)$, where σ, γ are two fuzzy subsets of R such that $\gamma \nsubseteq ann(\mu)$. Then $\chi_0 \neq \gamma\mu \subseteq \mu$.
Now, $\sigma\gamma \subseteq ann(\mu) \implies (\sigma\gamma)\mu \subseteq ann(\mu)\mu = \chi_0$, by Lemma 3.6. So, $\sigma \subseteq ann(\gamma\mu) = ann(\mu)$, since μ is fuzzy prime ideal. So. $ann(\mu)$ is also a fuzzy prime ideal.

□

Theorem 4.9 *Let μ be a fuzzy ideal of R then $[ann(\mu)]_t \subseteq ann(\mu_t)$, $\forall t \in (0,1]$.*

Proof: Let $x \in [ann(\mu)]_t \implies ann(\mu)(x) \geq t > 0 \implies \vee\{\eta(x) | \eta \in [0,1]^R, \eta\mu \subseteq \chi_0\} \geq t$.
$\implies \eta(x) \geq t$ for some $\eta \in [0,1]^R$, with $\eta\mu \subseteq \chi_0$.
Suppose $x \notin ann(\mu_t)$ then there exists $y \in \mu_t$ such that $xy \neq 0$.
Now, $(\eta\mu)(xy) \geq \eta(x) \wedge \mu(y) \geq t > 0$, a contradiction to $\eta\mu \subseteq \chi_0$. So, $x \in ann(\mu_t)$. Hence, $[ann(\mu)]_t \subseteq ann(\mu_t), \forall t \in (0,1]$.

□

Theorem 4.10 *If μ is a primitive fuzzy ideal of R then the support μ^* is a primitive ideal of R and the level set μ_t for $t \in [0,1]$ is also a primitive ideal of R.*

Proof: Let μ be a fuzzy primitive ideal of R and $x \in \mu^*$ then $\mu(x) > 0$. Since, μ is a fuzzy primitive ideal, so μ is the largest fuzzy ideal contained in some maximal fuzzy ideal σ. So, $\sigma(x) > 0 \implies x \in \sigma^*$. Therefore,$\mu^* \subseteq \sigma^*$. If σ is a maximal fuzzy ideal so, σ^* is a maximal ideal and thus, μ^* is the largest ideal contained in σ^*. So, μ^* is a primitive ideal of R.

In a similar manner it can be shown that, the level set μ_t, for $t \in [0,1]$ is a primitive ideal if μ is a fuzzy primitive ideal of R.

□

Theorem 4.11 *Let μ be a fuzzy ideal and σ be a maximal fuzzy right ideal of R.*
Then $(\mu : \sigma) = \vee\{\eta | \mu\eta \subseteq \sigma\}$ is a right primitive ideal of R, where η is a fuzzy subset of R .

Proof: Let $P = (\mu : \sigma)$ then by Theorem 4.1, P is a fuzzy ideal of R. Now let, $x \in P$ then $\mu x \subseteq \sigma$.

$$\implies x = \chi_R x(x) = \begin{cases} 1, & x \in R \\ 0, & otherwise \end{cases}$$

This implies, $x \in \sigma$. Hence, $P \subseteq \sigma$.
Now, to show P is the largest ideal contained in σ. Let, J be any fuzzy ideal such that $J \subseteq \sigma$.
Let, $j \in J$ then $\mu_j \subseteq J \subseteq \sigma \implies \mu_j \subseteq \sigma \implies j \in P \implies J \subseteq P$. So, P is the largest ideal contained in σ. Hence, P is a right primitive fuzzy ideal of R.

□

Theorem 4.12 *Every primitive fuzzy ideal is a fuzzy prime ideal of R.*

Proof: Let $P = (\mu : \sigma)$ be fuzzy primitive ideal of R, where σ is a maximal fuzzy right ideal of R. Let, θ_1 and θ_2 are two any fuzzy ideal of R such that $\theta_1\theta_2 \subseteq P \subseteq \sigma$. Since, $\theta_1\theta_2 \subseteq P \subseteq \sigma \iff \sigma \subseteq \sigma : \theta_2 \subseteq R$. So, $\sigma : \theta_2 = \sigma$ or $\sigma : \theta_2 = R$. In the first case, $\theta_1 \subseteq \sigma : \theta_2 = \sigma \implies \theta_1 \subseteq P$.
Secondly, $\theta_2 \subseteq (\sigma : \theta_2)\theta_2 \subseteq \sigma \implies \theta_2 \subseteq P$. So, P is a fuzzy prime ideal.

□

Conclusion:

In this paper we have studied the fuzzy Noetherian quotient, fuzzy annihilator as well as primitive fuzzy ideal of a ring. The concept of fuzzy annihilator will open up a new fuzzy structure namely Goldie fuzzy ring and associated concepts.

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