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Preservation Theorems of Weakly $\mu\mathcal{H}$-Countably Compact Spaces

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Abstract: In this paper we study the effect of functions on weakly $\mu\mathcal{H}$-countably compact spaces in generalized topology. The main result is that the $\theta(\mu,\nu)$-continuous image of a weakly $\mu\mathcal{H}$-countably compact (resp. weakly μ-countably compact) space is weakly $\nu f(\mathcal{H})$-countably compact (resp. weakly ν-countably compact).

Key Words: Generalized topology, hereditary class $\mathcal{H}$, weakly μ-countably compact, weakly $\mu\mathcal{H}$-countably compact, $\theta(\mu,\nu)$-continuous.

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1. Introduction and Preliminaries

A generalized topology (breifly GT) [3]) μ on a nonempty set X is a subset of the power set $expX$ such that $\emptyset \in \mu$ and an arbitrary union of elements of μ is belongs to μ. We call the pair (X,μ) a generalized topological space (briefly GTS) on X. The elements of μ are called μ-open sets and their complements are called μ-closed sets. A GTS (X,μ) is called a strong GTS [4] if $X \in \mu$. If A is a subset of a GTS (X,μ), then the μ-closure of A, $c_\mu(A)$, is the intersection of all μ-closed sets containing A and the μ-interior of A, $i_\mu(A)$, is the union of all μ-open sets contained in A (see [3,4]). Observe that i_μ and c_μ are monotonic [6], i.e. if $A \subset B \subset X$, then $c_\mu(A) \subseteq c_\mu(B)$, $i_\mu(A) \subseteq i_\mu(B)$, and idempotent [6], i.e. for any $A \subset X$ then $c_\mu(c_\mu(A)) = c_\mu(A)$ and $i_\mu(i_\mu(A)) = i_\mu(A)$, c_μ is enlarging [6], i.e. if $A \subset X$, then $A \subset c_\mu(A)$, i_μ is restricting [6], i.e. if $A \subset X$, then $i_\mu(A) \subset A$. A subset A of a GTS (X,μ) is μ-open if and only if $A = i_\mu(A)$, and A is μ-closed if and only if $A = c_\mu(A)$, $c_\mu(A)$ is the smallest μ-closed set containing A, $i_\mu(A)$ is the largest μ-open set contained in A. It is also well known form [3,4] that let μ be a GT on X, $A \subseteq X$ and $x \in X$, then $x \in c_\mu(A)$ if and only if $M \cap A \neq \emptyset$ for all $M \in \mu$ and $x \in M$. A strong GTS (X,μ) is a μ-compact space if every μ-open cover of X has a finite subcover [16], more generalizations can be seen in [7,1,13], where some covering spaces are studied in the generalized topology with respect to a hereditary class $\mathcal{H}$. A hereditary class $\mathcal{H}$ is a nonempty subset of the power set $expX$ that satisfies the following property: if $A \in \mathcal{H}$ and $B \subset A$, then $B \in \mathcal{H}$, see [5]. We call $(X,\mu,\mathcal{H})$ a hereditary generalized topological space and briefly we denote it by HGTS. The purpose of this paper is to study the effect of some special types of functions on weakly μ-countably compact and weakly $\mu\mathcal{H}$-countably compact spaces. The main result is that the image of a weakly $\mu\mathcal{H}$-countably compact (resp. weakly μ-countably compact) space under a $\theta(\mu,\nu)$-continuous function is weakly $\nu f(\mathcal{H})$-countably compact (resp. weakly ν-countably compact).

Definition 1.1. *[1] A subset A of a GTS (X,μ) is said to be μ-countably compact if for every countable cover $\{V_\lambda : \lambda \in \Delta\}$ of A by μ-open sets of X, there exists a finite subset subset Δ_0 of Δ such that $A \subseteq \bigcup\{V_\lambda : \lambda \in \Delta_0\}$. If $A = X$, then a strong GTS (X,μ) is called a μ-countably compact space.*

Definition 1.2. *[1] A subset A of a HGTS $(X,\mu,\mathcal{H})$ is said to be $\mu\mathcal{H}$-countably compact if for every countable cover $\{V_\lambda : \lambda \in \Delta\}$ of A by μ-open sets of X, there exists a finite subset Δ_0 of Δ such that $A \setminus \bigcup\{V_\lambda : \lambda \in \Delta_0\} \in \mathcal{H}$. If $A = X$, then a strong HGTS $(X,\mu,\mathcal{H})$ is called a $\mu\mathcal{H}$-countably compact space.*

Definition 1.3. *Let (X,μ) and (Y,ν) be two GTSs, then a function $f : (X,\mu) \to (Y,\nu)$ is said to be:*

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1. *(μ,ν)-continuous [3] if $U \in \nu$ implies $f^{-1}(U) \in \mu$;*

2. *almost (μ,ν)-continuous [8] if for each $x \in X$ and each ν-open set V containing $f(x)$, there exists a μ-open set U containing x such that $f(U) \subseteq i_\nu(c_\nu(V))$;*

3. *(μ,ν)-precontinuous [9] if $f^{-1}(V) \subseteq i_\mu(c_\mu(f^{-1}(V)))$ for every ν-open set V in Y;*

4. *$\delta(\mu,\nu)$-continuous [11] (resp. almost $\delta(\mu,\nu)$-continuous) if for each $x \in X$ and each ν-open set V of Y containing $f(x)$, there exists a μ-open set U of X containing x such that $f(i_\mu(c_\mu(U))) \subseteq i_\nu(c_\nu(V))$ (resp. $f(i_\mu(c_\mu(U))) \subseteq c_\nu(V)$);*

5. *$\theta(\mu,\nu)$-continuous [3] (resp. strongly $\theta(\mu,\nu)$-continuous [10]) if for every $x \in X$ and every ν-open subset V of Y containing $f(x)$, there exists a μ-open subset U in X containing x such that $f(c_\mu(U)) \subseteq c_\nu(V)$ (resp. $f(c_\mu(U)) \subseteq V$).*

6. *contra-(μ,ν)-continuous [12] if $f^{-1}(V)$ is μ-closed in X for every ν-open set V in Y.*

2. Weakly $\mu\mathcal{H}$-Countably Compact Spaces

Most of the results in this section are proved with respect to weakly $\mu\mathcal{H}$-countably compact spaces. By taking $\mathcal{H} = \{\emptyset\}$, we get directly the results for weakly μ-countably compact spaces.

Definition 2.1. *[15] A subset A of a GTS (X,μ) is said to be weakly μ-countably compact if for every countable cover $\{V_\lambda : \lambda \in \Delta\}$ of A by μ-open sets of X, there exists a finite subset subset Δ_0 of Δ such that $A \subseteq \bigcup\{c_\mu(V_\lambda) : \lambda \in \Delta_0\}$. If $A = X$, then (X,μ) is called a weakly μ-countably compact space.*

Definition 2.2. *[15] A subset A of a HGTS $(X,\mu,\mathcal{H})$ is said to be weakly $\mu\mathcal{H}$-countably compact if for every countable cover $\{V_\lambda : \lambda \in \Delta\}$ of A by μ-open sets of X, there exists a finite subset subset Δ_0 of Δ such that $A \setminus \bigcup\{c_\mu(V_\lambda) : \lambda \in \Delta_0\} \in \mathcal{H}$. If $A = X$, then $(X,\mu,\mathcal{H})$ is called a weakly $\mu\mathcal{H}$-countably compact space.*

Lemma 2.3. *[7,2] Let $f : X \to Y$ be a function.*

1. *If $\mathcal{H}$ is a hereditary class on X, then $f(\mathcal{H})$ is a hereditary class on Y.*

2. *If $\mathcal{H}$ is a hereditary class on Y, then $f^{-1}(\mathcal{H})$ is a hereditary class on X.*

Lemma 2.4. *Let X be an arbitrary set, (Y,ν) a GTS, and $f : X \to (Y,\nu)$ be a function. Then $f^{-1}(\nu)$ is a GTS on X induced by f and ν.*

Proof. Since $\emptyset \in \nu$, then $\emptyset \in f^{-1}(\nu)$. Let $\{G_\lambda : \lambda \in \Delta\}$ be a collection of subsets of $f^{-1}(\nu)$. Since $f\big(\bigcup_{\lambda\in\Delta} G_\lambda\big) = \bigcup_{\lambda\in\Delta} f(G_\lambda)$ and ν is a GTS on Y, then $\bigcup_{\lambda\in\Delta} f(G_\lambda) \in \nu$. This means that $\bigcup_{\lambda\in\Delta} G_\lambda \in f^{-1}(\nu)$ and this completes the proof. □

Proposition 2.5. *Let $f : (X,\mu) \to (Y,\nu,\mathcal{H})$ be a surjective function, $\mu = f^{-1}(\nu)$ and $(Y,\nu,\mathcal{H})$ be $\nu\mathcal{H}$-countably compact. Then (X,μ) is $\mu f^{-1}(\mathcal{H})$-countably compact.*

Proof. From Lemma 2.2, we have $\mu = f^{-1}(\nu)$ is a GTS on X induced by f and ν and hence let $\{f^{-1}(V_\lambda) : \lambda \in \Delta\}$ be a countable μ-covering of X. Then $\{V_\lambda : \lambda \in \Delta\}$ is a countable ν-open cover of Y. From assumption, there is a finite subset Δ_0 of Δ such that $Y \setminus \bigcup\{V_\lambda : \lambda \in \Delta_0\} \in \mathcal{H}$ and hence $f^{-1}\big(Y \setminus \bigcup\{V_\lambda : \lambda \in \Delta_0\}\big) = X \setminus \bigcup\{f^{-1}(V_\lambda) : \lambda \in \Delta_0\} \in f^{-1}(\mathcal{H})$. Thus, (X,μ) is $\mu f^{-1}(\mathcal{H})$-countably compact. □

Proposition 2.6. *Let (X,μ) and (Y,ν) be strong GTSs, $f : (X,\mu) \to (Y,\nu)$ be a surjective function, $\mu = f^{-1}(\nu)$ and (Y,ν) be ν-countably compact. Then (X,μ) is μ-countably compact.*

The main result of this paper is stated and proved in the following.

Theorem 2.7. *Let $f : (X,\mu,\mathcal{H}) \to (Y,\nu,f(\mathcal{H}))$ be a $\theta(\mu,\nu)$-continuous function. If A is a weakly $\mu\mathcal{H}$-countably compact subset of X, then $f(A)$ is weakly $\nu f(\mathcal{H})$-countably compact.*

Proof. Let $\mathcal{V} = \{V_\lambda : \lambda \in \Delta\}$ be a countable ν-open cover of $f(A)$. Let $x \in A$ and $V_{\lambda(x)}$ be a ν-open set in Y such that $f(x) \in V_{\lambda(x)}$. Since f is $\theta(\mu,\nu)$-continuous, there exists a μ-open set $U_{\lambda(x)}$ of X containing x such that $f(c_\mu(U_{\lambda(x)})) \subseteq c_\nu(V_{\lambda(x)})$. Since the collection $\{U_{\lambda(x)} : \lambda(x) \in \Delta\}$ is a countable μ-open cover of A and A is weakly $\mu\mathcal{H}$-countably compact, there exists a finite subset Δ_0 of Δ such that $A \setminus \bigcup\{c_\mu(U_{\lambda(x)}) : \lambda(x) \in \Delta_0\} = H_0$, where $H_0 \in \mathcal{H}$. Therefore, we have $f(A) \subseteq f\big(\cup_{\lambda(x)\in\Delta_0} c_\mu(U_{\lambda(x)})\big) \cup f(H_0) = \big[\cup_{\lambda(x)\in\Delta_0} f\big(c_\mu(U_{\lambda(x)})\big)\big] \cup f(H_0)$. Since, $f(c_\mu(U_{\lambda(x)})) \subseteq c_\nu(V_{\lambda(x)})$, then $f(A) \subseteq \big(\cup_{\lambda(x)\in\Delta_0} c_\nu(V_{\lambda(x)})\big) \cup f(H_0)$. Therefore $f(A) \setminus \cup_{\lambda(x)\in\Delta_0} c_\nu(V_{\lambda(x)}) \subseteq f(H_0) \in f(\mathcal{H})$. Hence $f(A)$ is weakly $\nu f(\mathcal{H})$-countably compact. □

Corollary 2.8. *Let $f : (X,\mu,\mathcal{H}) \to (Y,\nu,f(\mathcal{H}))$ be a $\theta(\mu,\nu)$-continuous surjection. If $(X,\mu,\mathcal{H})$ is weakly $\mu\mathcal{H}$-countably compact, then $(Y,\nu,f(\mathcal{H}))$ weakly $\nu f(\mathcal{H})$-countably compact.*

Theorem 2.9. *Let $f : (X,\mu) \to (Y,\nu)$ be a $\theta(\mu,\nu)$-continuous function. If A is a weakly μ-countably compact subset of X, then $f(A)$ is weakly ν-countably compact.*

Corollary 2.10. *Let $f : (X,\mu) \to (Y,\nu)$ be a $\theta(\mu,\nu)$-continuous surjection. If (X,μ) is weakly μ-countably compact, then (Y,ν) weakly ν-countably compact.*

Lemma 2.11. *[14] If $f : (X,\mu) \to (Y,\nu)$ is almost (μ,ν)-continuous, then f is $\theta(\mu,\nu)$-continuous.*

By Corollary 2.1**A** and Lemma 2.3, we obtain the following corollaries.

Corollary 2.12. *Let $f : (X,\mu,\mathcal{H}) \to (Y,\nu,f(\mathcal{H}))$ be an almost (μ,ν)-continuous surjection. If $(X,\mu,\mathcal{H})$ is weakly $\mu\mathcal{H}$-countably compact, then $(Y,\nu,f(\mathcal{H}))$ is weakly $\nu f(\mathcal{H})$-countably compact.*

Corollary 2.13. *Let $f : (X,\mu) \to (Y,\nu)$ be an almost (μ,ν)-continuous surjection. If (X,μ) is weakly μ-countably compact, then Y is weakly ν-countably compact.*

Every (μ,ν)-continuous function is almost (μ,ν)-continuous and by Corollaries 2.2**B** and 2.2**C**, we obtain the following corollary.

Corollary 2.14. (1) *Weakly $\mu\mathcal{H}$-countably compact property is a GT property.*
(2) *Weakly μ-countably compact property is a GT property.*

Proposition 2.15. *Let $f : (X,\mu,\mathcal{H}) \to (Y,\nu,f(\mathcal{H}))$ be a strongly $\theta(\mu,\nu)$-continuous function. If A is a weakly $\mu\mathcal{H}$-countably compact subset of X, then $f(A)$ is $\nu f(\mathcal{H})$-countably compact.*

Proof. Let $\mathcal{V} = \{V_\lambda : \lambda \in \Delta\}$ be a countable cover of $f(A)$ by ν-open subsets of Y. For each $x \in A$, there exists $\lambda(x) \in \Delta$ such that $f(x) \in V_{\lambda(x)}$. Since f is strongly $\theta(\mu,\nu)$-continuous, there exists a μ-open set $U_{\lambda(x)}$ of X containing x such that $f(c_\mu(U_{\lambda(x)})) \subseteq V_{\lambda(x)}$. Since $\{U_{\lambda(x)} : \lambda(x) \in \Delta\}$ is a countable μ-open cover of A and A is weakly $\mu\mathcal{H}$-countably compact, there exists a finite subset Δ_0 of Δ such that $A \setminus \bigcup\{c_\mu(U_{\lambda(x)}) : \lambda(x) \in \Delta_0\} = H_0$, where $H_0 \in \mathcal{H}$. Therefore, we have $f(A) \subseteq f\big(\cup_{\lambda(x)\in\Delta_0} c_\mu(U_{\lambda(x)})\big) \cup f(H_0) = \big[\cup_{\lambda(x)\in\Delta_0} f\big(c_\mu(U_{\lambda(x)})\big)\big] \cup f(H_0)$. Since $f(c_\mu(U_{\lambda(x)})) \subseteq V_{\lambda(x)}$, then $f(A) \subseteq \big(\cup_{\lambda(x)\in\Delta_0} V_{\lambda(x)}\big) \cup f(H_0)$ and hence $f(A) \setminus \cup_{\lambda(x)\in\Delta_0} V_{\lambda(x)} \subseteq f(H_0) \in f(\mathcal{H})$. That means $f(A)$ is $\nu f(\mathcal{H})$-countably compact. □

Corollary 2.16. *Let $f : (X,\mu,\mathcal{H}) \to (Y,\nu,f(\mathcal{H}))$ be a strongly $\theta(\mu,\nu)$-continuous surjection. If $(X,\mu,\mathcal{H})$ is weakly $\mu\mathcal{H}$-countably compact, then $(Y,\nu,f(\mathcal{H}))$ is $\nu f(\mathcal{H})$-countably compact.*

Proposition 2.17. *Let $f : (X,\mu) \to (Y,\nu)$ be a strongly $\theta(\mu,\nu)$-continuous function. If A is a weakly μ-countably compact subset of X, then $f(A)$ is ν-countably compact.*

Corollary 2.18. *Let (Y,ν) be a strong GTS and $f : (X,\mu) \to (Y,\nu)$ be a strongly $\theta(\mu,\nu)$-continuous surjection. If (X,μ) is weakly μ-countably compact, then (Y,ν) is ν-countably compact.*

Theorem 2.19. *Let $f : (X,\mu,\mathcal{H}) \to (Y,\nu,f(\mathcal{H}))$ be an almost $\delta(\mu,\nu)$-continuous function. If for every countable μ-open cover $\{U_\lambda : \lambda \in \Delta\}$ of $A \subseteq X$, there is a finite subset Δ_0 of Δ such that $A \setminus \bigcup\{i_\mu\big(c_\mu(U_{\lambda(x)})\big) : \lambda(x) \in \Delta_0\} \in \mathcal{H}$, then $f(A)$ is weakly $\nu f(\mathcal{H})$-countably compact.*

Proof. Let $\mathcal{V} = \{V_\lambda : \lambda \in \Delta\}$ be a countable cover of $f(A)$ by ν-open subsets of Y. For each $x \in A$, there exists $\lambda(x) \in \Delta$ such that $f(x) \in V_{\lambda(x)}$. Since f is almost $\delta(\mu,\nu)$-continuous, there exists a μ-open set $U_{\lambda(x)}$ of X containing x such $f\big(i_\mu(c(f(U_{\lambda(x)})))\big) \subseteq c_\mu(V_{\lambda(x)})$. So $\big\{U_{\lambda(x)} : \lambda(x) \in \Delta\big\}$ is a countable μ-open cover of A. By assumption, there is a finite subset Δ_0 of Δ such that $A \setminus \bigcup\{i_\mu\big(c_\mu(U_{\lambda(x)})\big) : \lambda(x) \in \Delta_0\} = H_0$, where $H_0 \in \mathcal{H}$ and hence $f(A) \subseteq f\big(\cup_{\lambda(x)\in\Delta_0} i_\mu(c_\mu(U_{\lambda(x)}))\big) \cup f(H_0) = \big[\cup_{\lambda(x)\in\Delta_0} f\big(i_\mu(c_\mu(U_{\lambda(x)}))\big)\big] \cup f(H_0)$. Since $f\big(i_\mu(c(f(U_{\lambda(x)})))\big) \subseteq c_\mu(V_{\lambda(x)})$, then $f(A) \subseteq \big(\cup_{\lambda(x)\in\Delta_0} c_\mu(V_{\lambda(x)})\big) \cup f(H_0)$. Therefore, $f(A) \setminus \cup_{\lambda(x)\in\Delta_0} c_\mu(V_{\lambda(x)}) \subseteq f(H_0) \in f(\mathcal{H})$. This shows that $f(A)$ is weakly $\nu f(\mathcal{H})$-countably compact. □

Corollary 2.20. *Let $f : (X,\mu,\mathcal{H}) \to (Y,\nu,f(\mathcal{H}))$ be an almost $\delta(\mu,\nu)$-continuous surjection. If for every countable μ-open cover $\{U_\lambda : \lambda \in \Delta\}$ of X, there is a finite subset Δ_0 of Δ such that $X \setminus \bigcup\{i_\mu\big(c_\mu(U_{\lambda(x)})\big) : \lambda(x) \in \Delta_0\} \in \mathcal{H}$, then $(Y,\nu,f(\mathcal{H}))$ is weakly $\nu f(\mathcal{H})$-countably compact.*

Theorem 2.21. *Let $f : (X,\mu) \to (Y,\nu)$ be an almost $\delta(\mu,\nu)$-continuous function. If for every countable μ-open cover $\{U_\lambda : \lambda \in \Delta\}$ of $A \subseteq X$, there is a finite subset Δ_0 of Δ such that $A \subseteq \bigcup\{i_\mu\big(c_\mu(U_{\lambda(x)})\big) : \lambda(x) \in \Delta_0\}$, then $f(A)$ is weakly ν-countably compact.*

Corollary 2.22. *Let (X,μ) and (Y,ν) be strong GTSs and $f : (X,\mu) \to (Y,\nu)$ be an almost $\delta(\mu,\nu)$-continuous surjection. If for every countable μ-open cover $\{U_\lambda : \lambda \in \Delta\}$ of X, there is a finite subset Δ_0 of Δ such that $X = \bigcup\{i_\mu\big(c_\mu(U_{\lambda(x)})\big) : \lambda(x) \in \Delta_0\}$, then (Y,ν) is weakly ν-countably compact.*

Theorem 2.23. *Let $f : (X,\mu,\mathcal{H}) \to (Y,\nu,f(\mathcal{H}))$ be a contra (μ,ν)-continuous and (μ,ν)-precontinuous function. If A is a weakly $\mu\mathcal{H}$-countably compact subset of X, then $f(A)$ is $\nu f(\mathcal{H})$-countably compact.*

Proof. Let $\mathcal{V} = \{V_\lambda : \lambda \in \Delta\}$ be a countable cover of $f(A)$ by ν-open sets of Y. For each $x \in A$, there exists $\lambda(x) \in \Delta$ such that $f(x) \in V_{\lambda(x)}$. Since f is contra (μ,ν)-continuous and (μ,ν)-precontinuous, $f^{-1}(V_{\lambda(x)})$ is μ-closed in X and $f^{-1}(V_{\lambda(x)}) \subseteq i_\mu(c_\mu(f^{-1}(V_{\lambda(x)}))) = i_\mu(f^{-1}(V_{\lambda(x)}))$. So $f^{-1}(V_{\lambda(x)}) = i_\mu(f^{-1}(V_{\lambda(x)}))$ which means that $f^{-1}(V_{\lambda(x)})$ is μ-clopen. Since the family $\{f^{-1}(V_{\lambda(x)}) : \lambda(x) \in \Delta\}$ is a countable μ-clopen cover of A and A is weakly $\mu\mathcal{H}$-countably compact, there is a finite subset Δ_0 of Δ such that $A \setminus \bigcup\{c_\mu\big(f^{-1}(V_{\lambda(x)})\big) : \lambda(x) \in \Delta_0\} = A \setminus \bigcup\{f^{-1}(V_{\lambda(x)}) : \lambda(x) \in \Delta_0\} = H_0$, where $H_0 \in \mathcal{H}$. Therefore, we have $f(A) \subseteq f\big(\cup_{\lambda(x)\in\Delta_0} f^{-1}(V_{\lambda(x)})\big) \cup f(H_0) = \big[\cup_{\lambda(x)\in\Delta_0} f\big(f^{-1}(V_{\lambda(x)})\big)\big] \cup f(H_0) \subseteq \big(\cup_{\lambda(x)\in\Delta_0} V_{\lambda(x)})\big) \cup f(H_0)$ and hence $f(A) \setminus \cup_{\lambda(x)\in\Delta_0} V_{\lambda(x)} \subseteq f(H_0) \in f(\mathcal{H})$. Thus $f(A)$ is $\nu f(\mathcal{H})$-countably compact. □

Corollary 2.24. *Let $f : (X,\mu,\mathcal{H}) \to (Y,\nu,f(\mathcal{H}))$ be a contra (μ,ν)-continuous and (μ,ν)-precontinuous surjection. If $(X,\mu,\mathcal{H})$ is weakly $\mu\mathcal{H}$-countably compact, then $(Y,\nu,f(\mathcal{H}))$ is $\nu f(\mathcal{H})$-countably compact.*

Theorem 2.25. *Let $f : (X,\mu) \to (Y,\nu)$ be a contra (μ,ν)-continuous and (μ,ν)-precontinuous function. If A is a weakly μ-countably compact subset of X, then $f(A)$ is ν-countably compact.*

Corollary 2.26. *Let (Y,ν) be a strong GTS and $f : (X,\mu) \to (Y,\nu)$ be a contra (μ,ν)-continuous and (μ,ν)-precontinuous surjection. If (X,μ) is is weakly μ-countably compact, then (Y,ν) is ν-countably compact.*

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