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On neutrosophic b-open sets and b-separation axioms

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ABSTRACT: In this article, we begin by presenting key findings related to neutrosophic b-open sets. The concepts of neutrosophic b-closure and neutrosophic b-interior for a neutrosophic set are introduced, and their various properties are explored. Subsequently, we introduce neutrosophic bT_0-space, neutrosophic bT_1-space, and neutrosophic bT_2-space, provide illustrative examples and investigate their distinct properties. We establish that a neutrosophic bT_2-space (respectively, neutrosophic bT_1-space) is a neutrosophic bT_1-space (respectively, neutrosophic bT_0-space), but the converse is not necessarily true. Additionally, we investigate the relationships between neutrosophic separation axioms and neutrosophic b-separation axioms. Furthermore, we show that the property of a space being neutrosophic bT_0-space (respectively, neutrosophic bT_1-space, neutrosophic bT_2-space) is a hereditary property.

Key Words: Neutrosophic bT_0-space, Neutrosophic bT_1-space, Neutrosophic bT_2-space, Neutrosophic b-open function, Neutrosophic b-continuous function, Neutrosophic b^*-continuous function, Neutrosophic b^{**}-continuous function.

Contents

1. Introduction

The idea of a fuzzy set was coined by L.A. Zadeh [19] in the year 1965 and a generalized version of a fuzzy set, acknowledged as the intuitionistic fuzzy set, was uncovered by K. Atanassov [18] in 1986. Afterwards, Florentin Smarandache [7,8,9] developed and studied the concept of a neutrosophic set. It had been shown by Smarandache [9] that a neutrosophic set was a generalization of an intuitionistic fuzzy set. A neutrosophic set is knotted with three membership functions which are the truth-membership function, falsity-membership function, and indeterminacy-membership function and it is remarkable that all these three neutrosophic factors are unbiased to one another. After Smarandache had added the thought of neutrosophy, many researchers [26,1,2,16] across the globe studied and contributed for the upliftment of this theory. There were some innate difficulties in the earlier techniques (classical or fuzzy) because of the deficiency of parametrizing tools, and so, those techniques are inadequate to deal with several real-life problems. These issues can be dealt with in a more general and appropriate way with the help of neutrosophic theory. Different kinds of practical-based works such as decision-making problems [20,29], clinical diagnosis [25,34], image processing [41], and many more had been carried out in a neutrosophic environment.

In the year 2002, Smarandache [8] introduced the notion of neutrosophic topology on the non-standard interval. Lupiáñez [10,11] investigated some properties of neutrosophic topological spaces. Salama & Alblowi [1], in 2012, introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space which was developed by D.Coker [6] in 1997. Later, a range of notions related to neutrosophic topological spaces had been developed by many researchers [17,35,21,27,40,12,13,2,28,3,5,

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14,15,32,30,33,31,23,36,38]. Ebenanjar *et al.* [22] introduced the concept of neutrosophic b-open sets in 2018. In recent years, separation properties had been studied by some researchers [4,24,37,39]. But the separation properties using neutrosophic b-open sets have not been studied so far.

In this article, our primary motive is to study the separation axioms using neutrosophic b-open sets. But, before that, we try to set up a few results on neutrosophic b-open sets. The article is organized by conferring some basic concepts in section 2. In section 3, we establish some results on neutrosophic b-open sets. In section 4, we define neutrosophic bT_0, bT_1, bT_2-spaces and study their various properties. In section 5, we confer a conclusion.

2. Preliminaries

In this section we put forward some basic concepts.

Definition 2.1 *[7] Let X be the universe of discourse. A neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where the functions $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are real standard or non-standard subsets of $]^-0, 1^+[$, i.e., $\mathcal{T}_A : X \to]^-0, 1^+[$, $\mathcal{I}_A : X \to]^-0, 1^+[$, $\mathcal{F}_A : X \to]^-0, 1^+[$ and $^-0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3^+$.*

The neutrosophic set A is characterized by truth-membership function $\mathcal{T}_A$, indeterminacy-membership function $\mathcal{I}_A$, falsity-membership function $\mathcal{F}_A$.

Definition 2.2 *[16] Let X be the universe of discourse. A single-valued neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are functions from X to $[0, 1]$ and $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$.*

The set of all single-valued neutrosophic sets over X is denoted by $\mathcal{N}(X)$.

Throughout this article, a neutrosophic set (NS, for short) will mean a single-valued neutrosophic set.

Definition 2.3 *[27] Let $A, B \in \mathcal{N}(X)$. Then*

(i) (Inclusion): If $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$ then A is said to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.

(ii) (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

(iii) (Intersection): The intersection of A and B, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x)\rangle : x \in X\}$.

(iv) (Union): The union of A and B, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x)\rangle : x \in X\}$.

(v) (Complement): The complement of the NS A, denoted by A^c, is defined as $A^c = \{\langle x, \mathcal{F}_A(x), 1 - \mathcal{I}_A(x), \mathcal{T}_A(x)\rangle : x \in X\}$

(vi) (Universal Set): If $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

(vii) (Empty Set): If $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\tilde{\emptyset}$.

Definition 2.4 *[1] Let $\{A_i : i \in \triangle\} \subseteq \mathcal{N}(X)$, where $\triangle$ is an index set. Then*

(i) $\cup_{i\in\triangle} A_i = \{\langle x, \vee_{i\in\triangle} \mathcal{T}_{A_i}(x), \wedge_{i\in\triangle} \mathcal{I}_{A_i}(x), \wedge_{i\in\triangle} \mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

(ii) $\cap_{i\in\triangle} A_i = \{\langle x, \wedge_{i\in\triangle} \mathcal{T}_{A_i}(x), \vee_{i\in\triangle} \mathcal{I}_{A_i}(x), \vee_{i\in\triangle} \mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

Theorem 2.1 *[3] Let $f : X \to Y$ be a function. Also let $A, A_i \in \mathcal{N}(X), i \in I$ and $B, B_j \in \mathcal{N}(Y), j \in J$. Then the following hold.*

(i) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$, $B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$.

(ii) $A \subseteq f^{-1}(f(A))$ *and if* f *is injective then* $A = f^{-1}(f(A))$.

(iii) $f^{-1}(f(B)) \subseteq B$ *and if* f *is surjective then* $f^{-1}(f(B)) = B$.

(iv) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$ *and* $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$.

(v) $f(\cup A_i) = \cup f(A_i)$, $f(\cap A_i) \subseteq \cap f(A_i)$ *and if* f *is injective then* $f(\cap A_i) = \cap f(A_i)$.

(vi) $f^{-1}(\tilde{\emptyset}_Y) = \tilde{\emptyset}_X$, $f^{-1}(\tilde{Y}) = \tilde{X}$.

(vii) $f(\tilde{\emptyset}_X) = \tilde{\emptyset}_Y$, $f(\tilde{X}) = \tilde{Y}$ *if* f *is surjective.*

Definition 2.5 *[3] Let X and Y be two non-empty sets and $f : X \to Y$ be a function. Also let $A \in \mathcal{N}(X)$ and $B \in \mathcal{N}(Y)$. Then*

(1) Image of A under f is defined by
$f(A) = \{\langle y, f(\mathcal{T}_A)(y), f(\mathcal{I}_A)(y), (1 - f(1 - \mathcal{F}_A))(y)\rangle : y \in Y\}$, *where*

$$f(\mathcal{T}_A)(y) = \begin{cases} \sup\{\mathcal{T}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

$$f(\mathcal{I}_A)(y) = \begin{cases} \inf\{\mathcal{I}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

$$(1 - f(1 - \mathcal{F}_A))(y) = \begin{cases} \inf\{\mathcal{F}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

(2) Pre-image of B under f is defined by
$f^{-1}(B) = \{\langle x, f^{-1}(\mathcal{T}_B)(x), f^{-1}(\mathcal{I}_B)(x), f^{-1}(\mathcal{F}_B)(x)\rangle : x \in X\}$

Definition 2.6 *[12] Let $\mathcal{N}(X)$ be the set of all neutrosophic sets over X. A NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic point (NP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = \alpha, \mathcal{I}_P(y) = \beta, \mathcal{F}_P(y) = \gamma$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$. An NP $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ will be denoted by $P^x_{\alpha,\beta,\gamma}$ or $P < x, \alpha, \beta, \gamma >$ or simply by $x_{\alpha,\beta,\gamma}$. For the NP $x_{\alpha,\beta,\gamma}$, x will be called its support. The complement of the NP $P^x_{\alpha,\beta,\gamma}$ will be denoted by $(P^x_{\alpha,\beta,\gamma})^c$ or by $x^c_{\alpha,\beta,\gamma}$. A NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic crisp point (NCP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = 1, \mathcal{I}_P(y) = 0, \mathcal{F}_P(y) = 0$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$.*

Definition 2.7 *[13] An NP $x_{\alpha,\beta,\gamma} \in \mathcal{N}(X)$ is said to be quasi-coincident with an NS $A \in \mathcal{N}(X)$, denoted by $x_{\alpha,\beta,\gamma} q A$, iff $\alpha > \mathcal{T}_{A^c}(x)$ or $\beta < \mathcal{I}_{A^c}(x)$ or $\gamma < \mathcal{F}_{A^c}(x)$, i.e., $\alpha > \mathcal{F}_A(x)$ or $\beta < 1 - \mathcal{I}_A(x)$ or $\gamma < \mathcal{T}_A(x)$. An NS A is said to be quasi-coincident with an NS B at $x \in X$, denoted by AqB at x, iff $\mathcal{T}_A(x) > \mathcal{T}_{B^c}(x)$ or $\mathcal{I}_A(x) < \mathcal{I}_{B^c}(x)$ or $\mathcal{F}_A(x) < \mathcal{F}_{B^c}(x)$. If the NP $x_{\alpha,\beta,\gamma}$ (resp. the NS C) is not quasi-coincident with an NS A, we shall denote it by $x_{\alpha,\beta,\gamma} \hat{q} A$ (resp. $C \hat{q} A$).*

Definition 2.8 *[27] Let $\tau \subseteq \mathcal{N}(X)$. Then τ is called a neutrosophic topology on X if*

(i) $\tilde{\emptyset}$ *and* $\tilde{X}$ *belong to* τ.

(ii) Arbitrary union of neutrosophic sets in τ is in τ.

(iii) Intersection of any two neutrosophic sets in τ is in τ.

If τ is a neutrosophic topology on X then the pair (X, τ) is called a neutrosophic topological space (NTS, for short) over X. The members of τ are called neutrosophic open sets (τ-open NSs or open sets, for short) in X. If for an NS A, $A^c \in \tau$ then A is said to be a neutrosophic closed set (τ-closed NS or closed set, for short) in X.

Definition 2.9 *[27] Let (X, τ) be an NTS and $A \in \mathcal{N}(X)$. Then the neutrosophic*

(i) interior of A, denoted by $int(A)$, is defined as $int(A) = \cup\{G : G \in \tau$ and $G \subseteq A\}$.

(ii) closure of A, denoted by $cl(A)$, is defined as $cl(A) = \cap\{G : G^c \in \tau$ and $G \supseteq A\}$.

Theorem 2.2 *[40] In an NTS*

(i) Every neutrosophic open set is a neutrosophic pre-open(NPO) set.

(ii) Every neutrosophic closed set is a neutrosophic pre-closed(NPC) set.

Definition 2.10 *[22] Let (X, τ) be an NTS and G be a NS over X. Then G is called a*

(i) neutrosophic b-open (NBO, for short) set iff $G \subseteq [int(cl(G))] \cup [cl(int(G))]$.

(ii) neutrosophic b-closed (NBC, for short) set iff $G \supseteq [int(cl(G))] \cap [cl(int(G))]$.

If G is an NBO (resp. NBC) set in (X, τ) then we shall also say that G is a τ-NBO (resp. τ-NBC) set.

Theorem 2.3 *[22] Let (X, τ) be an NTS.*

(i) If $G \in \mathcal{N}(X)$ then G is an NBO set iff G^c is an NBC set.

(ii) If $G \in \mathcal{N}(X)$ then G is a NBC set iff G^c is an NBO set.

(iii) In (X, τ), every NPO set is an NBO set.

(iv) In (X, τ), every NPC set is an NBC set.

Definition 2.11 *[22] Let (X, τ) be an NTS and $A \in \mathcal{N}(X)$. Then the neutrosophic*

(i) b-interior of A, denoted by $bint(A)$, is defined as $bint(A) = \cup\{G : G$ is an NBO set in X and $G \subseteq A\}$.

(ii) b-closure of A, denoted by $bcl(A)$, is defined as $bcl(A) = \cap\{G : G$ is an NBC set in X and $G \supseteq A\}$.

Definition 2.12 *[37] Let (X, τ) be an NTS. Let $\emptyset \neq Y \subseteq X$ and $\tau|_Y = \{G|_Y : G \in \tau\}$. Then $(Y, \tau|_Y)$ is an NTS. The topology $\tau|_Y$ is called the neutrosophic relative topology of τ on Y or the neutrosophic subspace topology of Y or the neutrosophic induced topology on Y and the NTS $(Y, \tau|_Y)$ is called a neutrosophic subspace (or a subspace, for short) of the NTS (X, τ). Members of $\tau|_Y$ are called $\tau|_Y$-open sets in Y. A NS $A \in \mathcal{N}(Y)$ such that $A^c \in \tau|_Y$ is called a $\tau|_Y$-closed set in Y.*

Definition 2.13 *[37] A property of an NTS (X, τ) is said to be hereditary if whenever the space X has that property, then so does every subspace of it.*

Definition 2.14 *[37] Let $(Y, \tau|_Y)$ be a neutrosophic subspace of an NTS (X, τ) and $A \in \mathcal{N}(Y)$. Then*

(i) neutrosophic interior of A, denoted by $int_Y(A)$, is defined as $int_Y(A) = \cup\{G : G \in \tau|_Y$ and $G \subseteq A\}$.

(ii) neutrosophic closure of A, denoted by $cl_Y(A)$, is defined as $cl_Y(A) = \cap\{G : G^c \in \tau|_Y$ and $G \supseteq A\}$.

Proposition 2.1 *[37] Let $(Y, \tau|_Y)$ be a neutrosophic subspace of an NTS (X, τ). Then*

(i) $cl_X(G)|_Y = cl_Y(G|_Y)$ for every $G \in \mathcal{N}(X)$, where $cl_Y(G|_Y)$ is the $\tau|_Y$-closure of $G|_Y$ and $cl_X(G)$ is the τ-closure of G.

(ii) $int_X(G)|_Y = int_Y(G|_Y)$ for every $G \in \mathcal{N}(X)$, where $int_Y(G|_Y)$ is the $\tau|_Y$-interior of $G|_Y$ and $int_X(G)$ is the τ-interior of G.

Definition 2.15 *[37] An NTS (X,τ) is called a neutrosophic*

(i) T_0-space or (NT_0-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $U \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ or there exists a $V \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

(ii) T_1-space (NT_1-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $U \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a $V \in \tau$ such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.

(iii) T_2-space or neutrosophic Hausdorff space (NT_2-space or Hausdorff space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exist $U, V \in \tau$ such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \tilde{\emptyset}$.

3. Some results on neutrosophic b-open sets

Here we establish some results related to neutrosophic b-open sets which will be used in the next section.

Proposition 3.1 *In an NTS*

(i) Every neutrosophic open set is an NBO set.

(ii) Every neutrosophic closed set is an NBC set.

Proof: Obvious from the theorems 2.2 and 2.3. □

Proposition 3.2 *Let $(Y, \tau|_Y)$ be a neutrosophic subspace of the NTS (X,τ) and $A \in \mathcal{N}(Y)$. Then A is a $\tau|_Y$-NBC set in Y iff A^c is a $\tau|_Y$-NBO set in Y.*

Proof: A is a $\tau|_Y$-NBC set $\Leftrightarrow [int_Y(cl_Y(G))] \cap [cl_Y(int_Y(G))] \subseteq A \Leftrightarrow A^c \subseteq [[int_Y(cl_Y(G))] \cap [cl_Y(int_Y(G))]]^c = [int_Y(cl_Y(G))]^c \cup [cl_Y(int_Y(G))]^c = [cl_Y(cl_Y(A))^c] \cup [int_Y(int_Y(A))^c] = [cl_Y(int_Y(A^c))] \cup [int_Y(cl_Y(A^c))] \Leftrightarrow A^c \subseteq [cl_Y(int_Y(A^c))] \cup [int_Y(cl_Y(A^c))] \Leftrightarrow A^c$ is a $\tau|_Y$-NBO set. □

Proposition 3.3 *Let $(Y, \tau|_Y)$ be a neutrosophic subspace of the NTS (X,τ). Then*

(i) $G|_Y$ is a $\tau|_Y$-NBO set in Y for every τ-NBO set G in X.

(ii) $G|_Y$ is a $\tau|_Y$-NBC set in Y for every τ-NBC set G in X.

Proof: (i) G is a τ-NBO set in $X \Rightarrow G \subseteq [int_X(cl_X(G))] \cup [cl_X(int_X(G))] \Rightarrow G|_Y \subseteq [(int_X(cl_X(G))) \cup (cl_X(int_X(G)))]|_Y \Rightarrow G|_Y \subseteq [(int_X(cl_X(G)))|_Y] \cup [(cl_X(int_X(G)))|_Y] \Rightarrow G|_Y \subseteq [int_Y(cl_X(G)|_Y)] \cup [cl_Y(int_X(G)|_Y)]$ [by prop. 2.1] $\Rightarrow G|_Y \subseteq [int_Y(cl_Y(G)|_Y)] \cup [cl_Y(int_Y(G)|_Y)]$ [by prop. 2.1] $\Rightarrow G|_Y$ is a $\tau|_Y$-NBO set in Y.

(ii) G is a τ-NBC set $\Rightarrow G^c$ is a τ-NBO set $\Rightarrow G^c|_Y$ is a $\tau|_Y$-NBO set [by (i)] $\Rightarrow (G|_Y)^c$ is a $\tau|_Y$-NBO set $\Rightarrow G|_Y$ is a $\tau|_Y$-NBC set in Y [by prop. 3.2]. □

Proposition 3.4 *In an NTS, union of an arbitrary collection of NBO sets is an NBO set.*

Proof: Let (X,τ) be an NTS and $\{G_i : i \in \triangle\}$ be an arbitrary collection of NBO sets in X, where $\triangle$ is an index set. Now G_i is an NBO set for each $i \in \triangle \Rightarrow G_i \subseteq [int(cl(G_i))] \cup [cl(int(G_i))]$ for each $i \in \triangle \Rightarrow \cup_{i\in\triangle} G_i \subseteq \cup_{i\in\triangle}[(int(cl(G_i))) \cup (cl(int(G_i)))] \subseteq [int(cl(\cup_{i\in\triangle} G_i))] \cup [cl(int(\cup_{i\in\triangle} G_i))] \Rightarrow \cup_{i\in\triangle} G_i \subseteq [int(cl(\cup_{i\in\triangle} G_i))] \cup [cl(int(\cup_{i\in\triangle} G_i))] \Rightarrow \cup_{i\in\triangle} G_i$ is an NBO set. □

Proposition 3.5 *In an NTS, intersection of an arbitrary collection of NBC sets is an NBC set.*

Proof: Let (X, τ) be an NTS and $\{G_i : i \in \triangle\}$ be an arbitrary collection of NBC sets in X, where $\triangle$ is an index set. Now G_i is an NBC set for each $i \in \triangle \Rightarrow G_i^c$ is an NBO set for each $i \in \triangle \Rightarrow \cup_{i\in\triangle} G_i^c$ is an NBO set [by prop. 3.4] $\Rightarrow (\cap_{i\in\triangle} G_i)^c$ is an NBO set $\Rightarrow \cap_{i\in\triangle} G_i$ is an NBC set. □

Proposition 3.6 *Let (X, τ) be an NTS and $A \in \mathcal{N}(X)$. Then the following hold.*

(i) $bcl(A)$ *is an NBC set.*

(ii) $A \subseteq bcl(A)$.

(iii) A *is an NBC set iff* $A = bcl(A)$.

(iv) $bcl(\tilde{\emptyset}) = \tilde{\emptyset}$

(v) $bcl(\tilde{X}) = \tilde{X}$

(vi) $bcl(bcl(A)) = bcl(A)$

Proof: (i) Since $bcl(A)$ is the intersection of all NBC sets, so by prop. 3.5, $bcl(A)$ is an NBC set.

(ii) Since $bcl(A)$ is the intersection of all NBC sets containing A, so $A \subseteq bcl(A)$.

(iii) Since A is an NBC set and since $A \supseteq A$, so $bcl(A) \subseteq A$. Again by (ii), $A \subseteq bcl(A)$. Therefore, $A = bcl(A)$. Conversely, if $A = bcl(A)$ then by (i), A is an NBC set. Hence proved.

(iv) Since $\tilde{\emptyset}$ is an NBC set [by prop. 3.1(ii)], so by (iii), $bcl(\tilde{\emptyset}) = \tilde{\emptyset}$.

(v) Since $\tilde{X}$ is an NBC set [by prop. 3.1(ii)], so by (iii), $bcl(\tilde{X}) = \tilde{X}$.

(vi) Since by (i), $bcl(A)$ is an NBC set, so by (iii), $bcl(bcl(A)) = bcl(A)$. □

Proposition 3.7 *Let (X, τ) be an NTS and $A, B \in \mathcal{N}(X)$. Then the following hold.*

(i) $A \subseteq B \Rightarrow bcl(A) \subseteq bcl(B)$

(ii) $bcl(A \cup B) \supseteq bcl(A) \cup bcl(B)$.

(iii) $bcl(A \cap B) \subseteq bcl(A) \cap bcl(B)$.

Proof: (i) $A \subseteq B$ and $B \subseteq bcl(B)$, so $A \subseteq bcl(B)$. Since $bcl(B)$ is an NBC set such that $A \subseteq bcl(B)$, so $bcl(A) \subseteq bcl(B)$.

(ii) $A \subseteq A \cup B \Rightarrow bcl(A) \subseteq bcl(A \cup B)$ [by (i)]. Similarly $bcl(B) \subseteq bcl(A \cup B)$. Therefore, $bcl(A \cup B) \supseteq bcl(A) \cup bcl(B)$.

(iii) $A \cap B \subseteq A \Rightarrow bcl(A \cap B) \subseteq bcl(A)$ [by (i)]. Similarly $bcl(A \cap B) \subseteq bcl(B)$. Therefore, $bcl(A \cap B) \subseteq bcl(A) \cap bcl(B)$. □

Proposition 3.8 *Let (X, τ) be an NTS and $A, B \in \mathcal{N}(X)$. Then the following hold.*

(i) $bint(A)$ *is an NBO set.*

(ii) $bint(A) \subseteq A$

(iii) A *is an NBO set iff* $A = bint(A)$.

(iv) $bint(\tilde{\emptyset}) = \tilde{\emptyset}$

(v) $bint(\tilde{X}) = \tilde{X}$

(vi) $bint(bint(A)) = bint(A)$

Proof: (i) Since $bint(A)$ is the union of all NBO sets, so by prop. 3.4, $bint(A)$ is an NBO set.

(ii) Since $bint(A)$ is the union of all NBO sets contained in A, so $bint(A) \subseteq A$.

(iii) Since A is an NBO set and $A \subseteq A$, so $A \subseteq bint(A)$. Again by (ii), $bint(A) \subseteq A$. Therefore, $A = bint(A)$. Conversely, if $A = bint(A)$ then by (i), A is an NBO set. Hence proved.

(iv) Since $\tilde{\emptyset}$ is an NBO set [by prop. 3.1(i)], so by (iii), $bint(\tilde{\emptyset}) = \tilde{\emptyset}$.

(v) Since $\tilde{X}$ is an NBO set [by prop. 3.1(i)], so by (iii), $bint(\tilde{X}) = \tilde{X}$.

(vi) Since by (i), $bint(A)$ is an NBO set, so by (iii), $bint(bint(A)) = bint(A)$. □

Proposition 3.9 *Let (X, τ) be an NTS and $A, B \in \mathcal{N}(X)$. Then the following hold.*

(i) $A \subseteq B \Rightarrow bint(A) \subseteq bint(B)$

(ii) $bint(A \cup B) \supseteq bint(A) \cup bint(B)$.

(iii) $bint(A \cap B) \subseteq bint(A) \cap bint(B)$.

Proof: (i) Since $A \subseteq B$ and $bint(A) \subseteq A$, so $bint(A) \subseteq B$. Since $bint(A)$ is an NBO set such that $bint(A) \subseteq B$ and since $bint(B)$ is the largest NBO set contained in B, so $bint(A) \subseteq bint(B)$.

(ii) $A \subseteq A \cup B \Rightarrow bint(A) \subseteq bint(A \cup B)$ [by (i)]. Similarly $bint(B) \subseteq bint(A \cup B)$. Therefore, $bint(A \cup B) \supseteq bint(A) \cup bint(B)$.

(iii) $A \cap B \subseteq A \Rightarrow bint(A \cap B) \subseteq bint(A)$ [by (i)]. Similarly $bint(A \cap B) \subseteq bint(B)$. Therefore, $bint(A \cap B) \subseteq bint(A) \cap bint(B)$. □

4. Neutrosophic b-separation axioms

Here we define neutrosophic bT_0, neutrosophic bT_1, and neutrosophic bT_2 spaces with examples and investigate various properties. But, before that, we put forward a few definitions.

Definition 4.1 *Let (X, τ) and (X, τ^*) be two NTSs. If every τ-NBO set in X is a τ^*-NBO set in X then τ is said to be b-coarser than τ^* or τ^* is said to be b-finer than τ.*

Example 4.1 *Let $X = \{a, b\}$, $\tau^* = \{\tilde{\emptyset}, \tilde{X}\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A\}$, where $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$. Obviously, both (X, τ) and (X, τ^*) are NTSs. It is also clear that every τ-NBO set is a τ^*-NBO set. Therefore, by the definition 4.1, τ is b-coarser than τ^*, i.e., τ^* is b-finer than τ.*

Definition 4.2 *Let f be a neutrosophic function from an NTS (X, τ) to another NTS (Y, σ). If $f(G)$ is a σ-NBO set in Y for every τ-open NS G in X then f is called a neutrosophic b-open function.*

Example 4.2 *Let $X = \{a, b\}, Y = \{x, y\}, \tau = \{\tilde{\emptyset}, \tilde{X}, A\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{X}, B\}$, where $A = \{\langle a, 0.6, 0.7, 0.2\rangle, \langle b, 0.8, 0.4, 0.3\rangle\} \in \mathcal{N}(X)$ and $B = \{\langle x, 0.8, 0.4, 0.3\rangle, \langle y, 0.6, 0.7, 0.2\rangle\} \in \mathcal{N}(Y)$. Obviously, both (X, τ) and (Y, σ) are NTSs. It is also clear that $f(G)$ is a σ-NBO set for every τ-open NS G in X. Therefore, by def. 4.2, f is a neutrosophic b-open function.*

Definition 4.3 *A neutrosophic function f from an NTS (X, τ) to another NTS (Y, σ) is called a neutrosophic*

(i) b-continuous function if $f^{-1}(G)$ is a τ-NBO set in X for every σ-open NS G in Y.

(ii) b^*-continuous function if $f^{-1}(G)$ is a τ-NBO set in X for every σ-NBO set G in Y.

(iii) b^{**}-continuous function if $f^{-1}(G)$ is a τ-open NS in X for every σ-NBO set G in Y.

Definition 4.4 *An NTS (X, τ) is called a neutrosophic bT_0 space (NBT_0-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists an NBO set U in X such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ or there exists an NBO set V in X such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.*

Example 4.3 *Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$ and $B = \{\langle a, 0, 1, 1\rangle, \langle b, 1, 0, 0\rangle\}$. Clearly the NTS (X, τ) is an NBT_0-space.*

Proposition 4.1 *Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is b-finer than τ. If (X, τ) is an NBT_0-space then (X, τ^*) is also an NBT_0-space.*

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X, τ) is an NBT_0-space, so there exists a τ-NBO set G in X such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-NBO set H in X such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since τ^* is b-finer than τ, so every τ-NBO set in X is a τ^*-NBO set in X. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exists a τ^*-NBO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ^*-NBO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X, τ^*) is an NBT_0-space. □

Proposition 4.2 *Let (X, τ) be an NTS. If X is NT_0-space then X is an NBT_0-space.*

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since X is an NT_0-space, so there exists a τ-open NS G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-open NS H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since every neutrosophic open set is an NBO set [by prop. 3.1(i)], so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a τ-NBO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-NBO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X, τ) is an NBT_0-space. □

Remark 4.1 *Converse of the proposition 4.2 is not true in general. We establish it by the following counter example.*

Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}\}$ Clearly (X, τ) is not an NT_0-space.

We now show that (X, τ) is an NBT_0-space. Let $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$ be any two NPs in X such thar $a \neq b$. Also let $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$. Obviously, $A \in \mathcal{N}(X)$, $a_{\alpha,\beta,\gamma} \in A$ but $b_{\alpha',\beta',\gamma'} \notin A$. Now $int(cl(A)) \cup cl(int(A)) \supseteq int(cl(A)) = int(\tilde{X}) = \tilde{X} \supseteq A$. Therefore, A is an NBO set in X. Thus for any two NPs $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$, $a \neq b$, there exists an NBO set A in X such that $a_{\alpha,\beta,\gamma} \in A$ but $b_{\alpha',\beta',\gamma'} \notin A$. Therefore, (X, τ) is an NBT_0-space.

Hence the NTS (X, τ) is an NBT_0-space but not an NT_0-space.

Proposition 4.3 *Let (X, τ) be an NBT_0-space. Then every neutrosophic subspace of X is an NBT_0-space and hence the property is hereditary.*

Proof: Let $(Y, \tau\mid_Y)$ be a neutrosophic subspace of (X, τ). Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X, τ) is an NBT_0-space, so there exists a τ-NBO set U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ or there exists a τ-NBO set V such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Then $(x_{\alpha,\beta,\gamma} \in U\mid_Y,\ y_{\alpha',\beta',\gamma'} \notin U\mid_Y)$ or $(x_{\alpha,\beta,\gamma} \notin V\mid_Y,\ y_{\alpha',\beta',\gamma'} \in V\mid_Y)$. Also by the prop. 3.3(i), $U\mid_Y, V\mid_Y$ are $\tau\mid_Y$-NBO sets in Y as U, V are τ-NBO sets in X. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $\tau\mid_Y$-NBO set $U\mid_Y$ such that $x_{\alpha,\beta,\gamma} \in U\mid_Y$, $y_{\alpha',\beta',\gamma'} \notin U\mid_Y$ or there exists a $\tau\mid_Y$-NBO set $V\mid_Y$ such that $x_{\alpha,\beta,\gamma} \notin V\mid_Y$, $y_{\alpha',\beta',\gamma'} \in V\mid_Y$ Therefore, $(Y, \tau\mid_Y)$ is an NBT_0-space and hence the property is hereditary. □

Proposition 4.4 *Let (X, τ) be an NTS. Then X is NBT_0 iff for any two neutrosophic crisp points $x_{1,0,0}$ and $y_{1,0,0}$ in X, $(x_{1,0,0})\hat{q}[bcl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[bcl(x_{1,0,0})]$.*

Proof: Necessary part: Suppose that the statement $(x_{1,0,0})\hat{q}[bcl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[bcl(x_{1,0,0})]$ is false. Then both $(x_{1,0,0})q[bcl(y_{1,0,0})]$ and $(y_{1,0,0})q[bcl(x_{1,0,0})]$ are true. Now $(x_{1,0,0})q[bcl(y_{1,0,0})] \Rightarrow x_{1,0,0} \notin [bcl(y_{1,0,0})]^c \Rightarrow x_{1,0,0} \notin [\cap\{G : G \text{ is an NBC set and } y_{1,0,0} \in G\}]^c \Rightarrow x_{1,0,0} \notin \cup\{G^c : G^c \text{ is an NBO set and } y_{1,0,0} \notin G^c\} \Rightarrow x_{1,0,0} \notin G^c$ for all NBO sets G^c in X such that $y_{1,0,0} \notin G^c$. This implies that every NBO set containing $y_{1,0,0}$ must contain $x_{1,0,0}$. Similarly $(y_{1,0,0})q[bcl(x_{1,0,0})]$ will imply that every NBO set containing $x_{1,0,0}$ must contain $y_{1,0,0}$. Thus every NBO set containing one of $x_{1,0,0}$ and $y_{1,0,0}$ must contain the other. But this is a contradiction to the assumption that X is an NBT_0-space. Therefore, $(x_{1,0,0})\hat{q}[bcl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[bcl(x_{1,0,0})]$.

Converse part: $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ be two NPs in X such that $x \neq y$. By hypothesis, $(x_{1,0,0})\hat{q}[bcl(y_{1,0,0})]$ or $(y_{1,0,0})\hat{q}[bcl(x_{1,0,0})]$. Now, if $(x_{1,0,0})\hat{q}[bcl(y_{1,0,0})]$ then $x_{1,0,0} \in [bcl(y_{1,0,0})]^c$, which implies $x_{\alpha,\beta,\gamma} \in [bcl(y_{1,0,0})]^c$. Obviously $y_{p,q,r} \notin [bcl(y_{1,0,0})]^c$. Since $bcl(y_{1,0,0})$ is an NBC set, so $[bcl(y_{1,0,0})]^c$ is an NBO set. Thus there exists an NBO set $[bcl(y_{1,0,0})]^c$ in X such that $x_{\alpha,\beta,\gamma} \in [bcl(y_{1,0,0})]^c$ but $y_{p,q,r} \notin [bcl(y_{1,0,0})]^c$. Similarly if $(y_{1,0,0})\hat{q}[bcl(x_{1,0,0})]$ then there exists an NBO set $[bcl(x_{1,0,0})]^c$ in X such that $x_{\alpha,\beta,\gamma} \notin [bcl(x_{1,0,0})]^c$ but $y_{p,q,r} \in [bcl(x_{1,0,0})]^c$. Therefore, (X, τ) is an NBT_0-space. □

Proposition 4.5 *Let f be a bijective neutrosophic b-open function from an NTS (X, τ) to another NTS (Y, σ). If (X, τ) is NT_0 then (Y, σ) is an NBT_0-space.*

Proof: Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ be two NPs in Y such that $y^1 \neq y^2$. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_0, so there exists a τ-open NS G such that $x^1_{\alpha,\beta,\gamma} \in G$ and $x^2_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-open NS H such that $x^1_{\alpha,\beta,\gamma} \notin H$ and $x^2_{\alpha',\beta',\gamma'} \in H$. Suppose G exists. Since f is a neutrosophic b-open function, so $f(G)$ is a σ-NBO set [by def. 4.2] such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly if H exists then $f(H)$ is a σ-NBO set such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \notin f(H)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y there exists a σ-NBO set $f(G)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \notin f(G)$ or there exists a σ-NBO set $f(H)$ such that $y^1_{p,q,r} \notin f(H)$, $y^2_{p',q',r'} \in f(H)$. Therefore, (Y, σ) is an NBT_0-space. Hence proved. □

Proposition 4.6 *Let f be a one-one neutrosophic b-continuous function from an NTS (X, τ) to another NTS (Y, σ). If (Y, σ) is NT_0 then (X, τ) is an NBT_0-space.*

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NT_0, so there exists a σ-open NS G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ or there exists a σ-open NS H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Suppose G exists. Since f is a neutrosophic b-continuous function, so $f^{-1}(G)$ is a τ-NBO set [by def. 4.3(i)] in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly if H exists then $f^{-1}(H)$ is a τ-NBO set in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NBO set $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-NBO set $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore, (X, τ) is an NBT_0-space. Hence proved. □

Proposition 4.7 *Let f be a one-one neutrosophic b^*-continuous function from an NTS (X, τ) to another NTS (Y, σ). If (Y, σ) is NBT_0 then (X, τ) is an NBT_0-space.*

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NBT_0, so there exists a σ-NBO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ or there exists a σ-NBO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Suppose G exists. Since f is a neutrosophic b^*-continuous function, so $f^{-1}(G)$ is a τ-NBO set [by def. 4.3(ii)] in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly if H exists then $f^{-1}(H)$ is a τ-NBO set in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NBO set $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-NBO set $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore, (X, τ) is an NBT_0-space. Hence proved. □

Proposition 4.8 *Let f be a one-one neutrosophic b^{**}-continuous function from an NTS (X,τ) to another NTS (Y,σ). If (Y,σ) is NBT_0 then (X,τ) is an NT_0-space.*

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NBT_0, so there exists a σ-NBO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ or there exists a σ-NBO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Suppose G exists. Since f is a neutrosophic b^{**}-continuous function, so $f^{-1}(G)$ is a τ-open NS [by def. 4.3(iii)] in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly if H exists then $f^{-1}(H)$ is a τ-open NS in X such that $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$, $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-open NS $f^{-1}(G)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin G$ or there exists a τ-open NS $f^{-1}(H)$ in X such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore, (X,τ) is an NT_0-space. Hence proved. □

Definition 4.5 *An NTS (X,τ) is called a neutrosophic bT_1 space (NBT_1-space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists an NBO set U in X such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists an NBO set V in X such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$.*

Example 4.4 *Let $X = \{a,b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$ and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly, the NTS (X,τ) is an NBT_1-space.*

Proposition 4.9 *Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is b-finer than τ. If (X,τ) is an NBT_1-space then (X,τ^*) is also an NBT_1-space.*

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is an NBT_1-space, so there exists a τ-NBO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-NBO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since τ^* is b-finer than τ, so G and H are τ^*-NBO sets. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a τ^*-NBO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ^*-NBO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X,τ^*) is an NBT_1-space. □

Proposition 4.10 *Let (X,τ) be an NTS. If X is NT_1-space then X is an NBT_1-space.*

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since X is an NT_1-space, so there exists a τ-open NS G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-open NS H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Since every neutrosophic open set in X is an NBO set in X, so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a τ-NBO set G such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-NBO set H such that $x_{\alpha,\beta,\gamma} \notin H$, $y_{\alpha',\beta',\gamma'} \in H$. Hence (X,τ) is an NBT_1-space. □

Remark 4.2 *Converse of the proposition 4.10 is not true in general. We establish it by the following counter example.*

Let $X = \{a,b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}\}$ Clearly (X,τ) is not an NT_1-space.

We now show that (X,τ) is an NBT_1-space. Let $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $a \neq b$. Also let $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$ and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly A and B are two NBO sets in X. Thus for any two NPs $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$, $a \neq b$, there exists an NBO set A in X such that $a_{\alpha,\beta,\gamma} \in A$, $b_{\alpha',\beta',\gamma'} \notin A$ and there exists an NBO set B in X such that $a_{\alpha,\beta,\gamma} \notin B$, $b_{\alpha',\beta',\gamma'} \in B$. Therefore, (X,τ) is an NBT_1-space.

Hence the NTS (X,τ) is an NBT_1-space but not an NT_1-space.

Proposition 4.11 *Let (X,τ) be an NBT_1-space. Then every neutrosophic subspace of X is an NBT_1-space and hence the property is hereditary.*

Proof: Let $(Y, \tau|_Y)$ be a neutrosophic subspace of (X, τ). Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X, τ) is an NBT_1-space, so there exists a τ-NBO set U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists a τ-NBO set V such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Then $(x_{\alpha,\beta,\gamma} \in U|_Y,\ y_{\alpha',\beta',\gamma'} \notin U|_Y)$ and $(x_{\alpha,\beta,\gamma} \notin V|_Y,\ y_{\alpha',\beta',\gamma'} \in V|_Y)$. Also by the prop. 3.3(i), $U|_Y, V|_Y$ are $\tau|_Y$-NBO sets in Y. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exists a $\tau|_Y$-NBO set $U|_Y$ such that $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \notin U|_Y$ and there exists a $\tau|_Y$-NBO set $V|_Y$ such that $x_{\alpha,\beta,\gamma} \notin V|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ Therefore, $(Y, \tau|_Y)$ is an NBT_1-space and hence the property is hereditary. □

Proposition 4.12 *Let (X, τ) be an NTS. Then every neutrosophic crisp point in X is an NBC set iff X is an NBT_1-space.*

Proof: Necessary part: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in X such that $x \neq y$. By hypothesis, neutrosophic crisp points $x_{1,0,0}$ and $y_{1,0,0}$ are NBC sets in X. Then $(x_{1,0,0})^c$ and $(y_{1,0,0})^c$ are NBO sets in X such that $x_{\alpha,\beta,\gamma} \in (y_{1,0,0})^c, y_{\alpha',\beta',\gamma'} \notin (y_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma} \notin (x_{1,0,0})^c, y_{\alpha',\beta',\gamma'} \in (x_{1,0,0})^c$. Therefore, (X, τ) is an NBT_1-space.

Sufficient part: Let $x_{1,0,0}$ be an NCP in X. Also let $y_{p,q,r} \in (x_{1,0,0})^c$ be any NP. Then obviously $x \neq y$. Let us consider an NP $x_{\alpha,\beta,\gamma}$ with support x. Since X is a NBT_1-space, so for $y_{p,q,r}$ and $x_{\alpha,\beta,\gamma}$, there exists a τ-NBO set G such that $y_{p,q,r} \in G$ and $x_{\alpha,\beta,\gamma} \notin G$. Since for all α, β, γ with $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$, one such G exists, therefore we must have a τ-NBO set H such that $y_{p,q,r} \in H$ and $x_{1,0,0} \cap H = \tilde{\emptyset}$. Therefore $y_{p,q,r} \in H \subseteq (x_{1,0,0})^c$. So, $(x_{1,0,0})^c$ is a τ-NBO set and consequently, $x_{1,0,0}$ is a τ-NBC set.

Hence proved. □

Proposition 4.13 *Let (X, τ) be an NTS. If (X, τ) is an NBT_1-space then it is an NBT_0-space.*

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since X is NBT_1-space, so there exists a τ-NBO set U such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \notin U$ and there exists τ-NBO set V such that $x_{\alpha,\beta,\gamma} \notin V$, $y_{\alpha',\beta',\gamma'} \in V$. Hence (X, τ) is an NBT_0-space. □

Remark 4.3 *Converse of the proposition 4.13 is not true in general. We establish it by the following counter example.*

Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A\}$, where $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$. Clearly the NTS (X, τ) is an NBT_0-space.

We now show that (X, τ) is not an NBT_1-space.

We first establish that the NCP $a_{1,0,0}$ is a not an NBC set. We have $int(cl(a_{1,0,0})) \cap cl(int(a_{1,0,0})) = int(\tilde{X}) \cap cl(A) = \tilde{X} \cap \tilde{X} = \tilde{X}$. Therefore $int(cl(a_{1,0,0})) \cap cl(int(a_{1,0,0})) \nsubseteq a_{1,0,0}$, i.e., the NCP $a_{1,0,0}$ is not an NBC set. Therefore, by the prop. 4.12 (X, τ) is not an NBT_1-space.

Thus NTS (X, τ) is an NBT_0-space but not an NBT_1-space.

Proposition 4.14 *Let f be a bijective neutrosophic b-open function from an NTS (X, τ) to another NTS (Y, σ). If (X, τ) is NT_1 then (Y, σ) is an NBT_1-space.*

Proof: Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, be two NPs in Y. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_1, so there exists a τ-open NS G such that $x^1_{\alpha,\beta,\gamma} \in G$, $x^2_{\alpha',\beta',\gamma'} \notin G$ and there exists a τ-open NS H such that $x^1_{\alpha,\beta,\gamma} \notin H$, $x^2_{\alpha',\beta',\gamma'} \in H$. Since f is a neutrosophic b-open function, so $f(G)$ is a σ-NBO set such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \notin f(G)$. Similarly $f(H)$ is a σ-NBO set such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \notin f(H)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1 \neq y^2$, there exists a σ-NBO set $f(G)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \notin f(G)$ and there exists a σ-NBO set $f(H)$ such that $y^1_{p,q,r} \notin f(H)$, $y^2_{p',q',r'} \in f(H)$. Therefore, (Y, σ) is an NBT_1-space. Hence proved. □

Proposition 4.15 *Let f be a one-one neutrosophic b-continuous function from an NTS (X,τ) to another NTS (Y,σ). If (Y,σ) is NT_1 then (X,τ) is an NBT_1-space.*

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NT_1, so there exists a σ-open NS G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ and there exists a σ-open NS H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic b-continuous function, so $f^{-1}(G)$ and $f^{-1}(H)$ are τ-NBO sets in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$ and $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NBO set $f^{-1}(G)$ such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ and there exists a τ-NBO set $f^{-1}(H)$ such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore, (X,τ) is an NBT_1-space. Hence proved. □

Proposition 4.16 *Let f be a one-one neutrosophic b^*-continuous function from an NTS (X,τ) to another NTS (Y,σ). If (Y,σ) is NBT_1 then (X,τ) is an NBT_1-space.*

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NBT_1, so there exists a σ-NBO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ and there exists a σ-NBO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic b^*-continuous function, so $f^{-1}(G)$ and $f^{-1}(H)$ are τ-NBO sets in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$ and $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-NBO set $f^{-1}(G)$ such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ and there exists a τ-NBO set $f^{-1}(H)$ such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore, (X,τ) is an NBT_1-space. Hence proved. □

Proposition 4.17 *Let f be a one-one neutrosophic b^{**}-continuous function from an NTS (X,τ) to another NTS (Y,σ). If (Y,σ) is NBT_1 then (X,τ) is an NT_1-space.*

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since Y is NBT_1, so there exists a σ-NBO set G such that $y^1_{p,q,r} \in G$, $y^2_{p',q',r'} \notin G$ and there exists a σ-NBO set H such that $y^1_{p,q,r} \notin H$, $y^2_{p',q',r'} \in H$. Since f is a neutrosophic b^{**}-continuous function, so $f^{-1}(G)$ and $f^{-1}(H)$ are τ-open NSs in X. Also $y^1_{p,q,r} \in G \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(G) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(G)$ and $y^2_{p',q',r'} \notin G \Rightarrow f^{-1}(y^2_{p',q',r'}) \notin f^{-1}(G) \Rightarrow x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$ and $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exists a τ-open NS $f^{-1}(G)$ such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(G), x^2_{\alpha',\beta',\gamma'} \notin f^{-1}(G)$ and there exists a τ-open NS $f^{-1}(H)$ such that $x^1_{\alpha,\beta,\gamma} \notin f^{-1}(H)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H)$. Therefore, (X,τ) is an NT_1-space. Hence proved. □

Definition 4.6 *An NTS (X,τ) is called a neutrosophic bT_2 space or neutrosophic b-Hausdorff space (NBT_2-space or NB-Hausdorff space, for short) iff for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exist two neutrosophic b-open sets U, V in X such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \tilde{\emptyset}$*

Example 4.5 *Let $X = \{a,b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}, A, B\}$, where $A = \{\langle a,1,0,0\rangle, \langle b,0,1,1\rangle\}$ and $B = \{\langle a,0,1,1\rangle, \langle b,1,0,0\rangle\}$. Clearly, the NTS (X,τ) is a neutrosophic NBT_2-space.*

Proposition 4.18 *Let τ and τ^* be two neutrosophic topologies on a set X such that τ^* is b-finer than τ. If (X,τ) is an NBT_2-space then (X,τ^*) is also an NBT_2-space.*

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is an NBT_2-space, so there exist τ-NBO sets G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$. Since τ^* is b-finer than τ, so G and H are τ^*-NBO sets. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exist τ^*-NBO sets G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$. Hence (X,τ^*) is an NBT_2-space. □

Proposition 4.19 *Let (X,τ) be an NTS. If X is NT_2-space then X is an NBT_2-space.*

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, be two NPs in X. Since (X,τ) is an NT_2-space, so there exist τ-open NSs G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$. Since every neutrosophic open set is an NBO set, so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$, $x \neq y$, there exist τ-NBO sets G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$.. Hence (X,τ) is an NBT_2-space. □

Remark 4.4 *Converse of the proposition 4.19 is not true in general. We establish it by the following counter example.*

Let $X = \{a, b\}$ and $\tau = \{\tilde{\emptyset}, \tilde{X}\}$ Clearly (X,τ) is not an NT_2-space.

We now show that (X,τ) is an NBT_2-space. Let $a_{\alpha,\beta,\gamma}$ and $b_{\alpha',\beta',\gamma'}$ be any two NPs in X. Also let $A = \{\langle a, 1, 0, 0\rangle, \langle b, 0, 1, 1\rangle\}$ and $B = \{\langle a, 0, 1, 1\rangle, \langle b, 1, 0, 0\rangle\}$. Clearly, A and B are two NBO sets in X such that $A \cap B = \tilde{\emptyset}$. Thus there exist NBO sets A and B such that $a_{\alpha,\beta,\gamma} \in A$, $b_{\alpha',\beta',\gamma'} \in B$ and $A \cap B = \tilde{\emptyset}$. Therefore, (X,τ) is a NBT_2-space.

Thus the NTS (X,τ) is an NBT_2-space but not an NT_2-space.

Proposition 4.20 *Let (X,τ) be an NBT_2-space. Then every neutrosophic subspace of X is an NBT_2-space and hence the property is hereditary.*

Proof: Let $(Y, \tau|_Y)$ be a neutrosophic subspace of (X,τ). Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in Y such that $x \neq y$. Then $x_{\alpha,\beta,\gamma}, y_{\alpha',\beta',\gamma'} \in X$, $x \neq y$. Since (X,τ) is an NBT_2-space, so there exist τ-NBO sets U, V such that $x_{\alpha,\beta,\gamma} \in U$, $y_{\alpha',\beta',\gamma'} \in V$ and $U \cap V = \tilde{\emptyset}$. Then $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ and $(U|_Y) \cap (V|_Y) = (U \cap V)|_Y = \tilde{\emptyset}|_Y = \tilde{\emptyset}$. Also by the prop. 3.3(i), $U|_Y, V|_Y$ are $\tau|_Y$-NBO sets in Y. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in Y such that $x \neq y$, there exist $\tau|_Y$-NBO sets $U|_Y, V|_Y$ such that $x_{\alpha,\beta,\gamma} \in U|_Y$, $y_{\alpha',\beta',\gamma'} \in V|_Y$ and $(U|_Y) \cap (V|_Y) = \tilde{\emptyset}$. Therefore, $(Y, \tau|_Y)$ is an NBT_2-space and hence the property is hereditary. □

Lemma 4.1 *In a neutrosophic co-countable topological space, every countable NS is an NBC set.*

Proof: Let (X,τ) be a neutrosophic co-countable topological space and let U be a countable NS in X. Then $U = (U^c)^c$ is a countable NS and so, U^c is a τ-open NS [as (X,τ) is a neutrosophic co-countable topological space], i.e., U is a τ-closed NS. Since every closed NS is an NBC set, therefore U is an NBC set. □

Remark 4.5 *From lemma 4.1, it is clear that in a neutrosophic co-countable topological space, an NBO set is that NS whose complement is a countable NS. Therefore, in a neutrosophic co-countable topological space, every NBO set is a neutrosophic open set.*

Lemma 4.2 *Let X be an uncountable set. Then the neutrosophic co-countable topological space (X,τ) is not an NBT_2-space.*

Proof: Suppose that (X,τ) is an NBT_2-space. Then for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x \neq y$, there exist τ-NBO sets G, H such that $x_{\alpha,\beta,\gamma} \in G$, $y_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$. Since (X,τ) is a neutrosophic co-countable topological space, so G, H are τ-open NSs [by remark 4.5] and therefore, their complements, i.e., G^c, H^c are countable NSs. Now $G \cap H = \tilde{\emptyset} \Rightarrow (G \cap H)^c = (\tilde{\emptyset})^c \Rightarrow G^c \cup H^c = \tilde{X}$, which is not possible as $\tilde{X}$ is an uncountable neutrosophic set and $G^c \cup H^c$ is a countable neutrosophic set. Therefore, (X,τ) is not an NBT_2-space. □

Proposition 4.21 *Let (X,τ) be an NTS. If (X,τ) is an NBT_2-space then it is an NBT_1-space.*

Proof: Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x\neq y$. Since (X,τ) is a NBT_2-space, so there exist τ-NBO sets H and K such that $x_{\alpha,\beta,\gamma}\in H$, $y_{\alpha',\beta',\gamma'}\in K$ and $H\cap K=\tilde{\emptyset}$. Since $x_{\alpha,\beta,\gamma}\in H$ and $H\cap K=\tilde{\emptyset}$, so $x_{\alpha,\beta,\gamma}\notin K$. Similarly $y_{\alpha',\beta',\gamma'}\notin H$. Thus for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ in X such that $x\neq y$ there exists a τ-NBO set H such that $x_{\alpha,\beta,\gamma}\in H$, $y_{\alpha',\beta',\gamma'}\notin H$ and there exists a τ-NBO set K such that $x_{\alpha,\beta,\gamma}\notin K$, $y_{\alpha',\beta',\gamma'}\in K$. Hence (X,τ) is an NBT_1-space. □

Remark 4.6 *Converse of the proposition 4.21 is not true in general. We establish it by the following counter example.*

Let $\mathbb{R}$ be the set of all real numbers and $\mathcal{N}(\mathbb{R})$ be the set of all neutrosophic sets over $\mathbb{R}$. Also let $\tilde{\mathbb{R}}=\{\langle x,1,0,0\rangle : x\in\mathbb{R}\}$ and $\tilde{\emptyset}=\{\langle x,0,1,1\rangle : x\in\mathbb{R}\}$. Let τ be the set containing $\tilde{\emptyset}$ and all those neutrosophic sets over $\mathbb{R}$ whose complements are countable. Then $(\mathbb{R},\tau)$ is a neutrosophic co-countable topological space.

Since $(\mathbb{R},\tau)$ is a neutrosophic co-countable topological space, so by the lemma 4.2, $(\mathbb{R},\tau)$ is not an NBT_2-space.

We show that $(\mathbb{R},\tau)$ is NBT_1-space.

Let $x_{\alpha,\beta,\gamma}$ and $y_{\alpha',\beta',\gamma'}$ be two NPs in $\mathbb{R}$ such that $x\neq y$. Since $[(x_{1,0,0})^c]^c=x_{1,0,0}$, a countable NS, so $(x_{1,0,0})^c$ is a τ-open NS and therefore, a τ-NBO set. Clearly $y_{\alpha',\beta',\gamma'}\in(x_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma}\notin(x_{1,0,0})^c$. Therefore, $(x_{1,0,0})^c$ is a τ-NBO set such that $y_{\alpha',\beta',\gamma'}\in(x_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma}\notin(x_{1,0,0})^c$. Similarly $(y_{1,0,0})^c$ is a τ-NBO set such that $y_{\alpha',\beta',\gamma'}\notin(y_{1,0,0})^c$ and $x_{\alpha,\beta,\gamma}\in(y_{1,0,0})^c$. Therefore, $(\mathbb{R},\tau)$ is an NBT_1-space.

Thus $(\mathbb{R},\tau)$ is an NBT_1-space but not an NBT_2-space.

Proposition 4.22 *Let (X,τ) be an NBT_2-space. Then for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ such that $x\neq y$, there exists an NBO set G such that $x_{\alpha,\beta,\gamma}\in G$, $y_{p,q,r}\in G^c$ and $y_{p,q,r}\in[bcl(G)]^c$.*

Proof: Since X is NBT_2-space, so for any two NPs $x_{\alpha,\beta,\gamma}$ and $y_{p,q,r}$ such that $x\neq y$, there exist two NBO sets G and H in X such that $x_{\alpha,\beta,\gamma}\in G$, $y_{p,q,r}\in H$ and $G\cap H=\tilde{\emptyset}$. Now $G\cap H=\tilde{\emptyset}\Rightarrow H\subseteq G^c\Rightarrow y_{p,q,r}\in G^c$. Since H^c is an NBC set and $G\subseteq H^c$, so $bcl(G)\subseteq H^c\Rightarrow H\subseteq[bcl(G)]^c\Rightarrow y_{p,q,r}\in[bcl(G)]^c$. Hence proved. □

Proposition 4.23 *Let (X,τ) be an NBT_2-space. Then for every NP $x_{\alpha,\beta,\gamma}$ in X, $x_{\alpha,\beta,\gamma}=\cap\{bcl(G) : x_{\alpha,\beta,\gamma}\in G$ and G is an NBC set $\}$.*

Proof: Let $x_{\alpha,\beta,\gamma}$ be an NP in X. Let $y_{1,0,0}$ be an NCP in X. Since X is NBT_2-space, so there exist two NBO sets G and H in X such that $x_{\alpha,\beta,\gamma}\in G$, $y_{1,0,0}\in H$ and $G\cap H=\tilde{\emptyset}$. Now $G\cap H=\tilde{\emptyset}\Rightarrow H\subseteq G^c\Rightarrow y_{1,0,0}\in G^c$. Since H^c is an NBC set and $G\subseteq H^c$, so $bcl(G)\subseteq H^c\Rightarrow H\subseteq[bcl(G)]^c\Rightarrow y_{1,0,0}\in[bcl(G)]^c\Rightarrow y_{0,1,1}\in bcl(G)$, i.e., $y_{p,q,r}\cap bcl(G)=\tilde{\emptyset}$ for every NP with support y. Thus for every NBO set G such that $x_{\alpha,\beta,\gamma}\in G$, we have $y_{p,q,r}\cap bcl(G)=\tilde{\emptyset}$ for all NP such that $x\neq y$. Obviously $x_{\alpha,\beta,\gamma}\in bcl(G)$ for every G with $x_{\alpha,\beta,\gamma}\in G$. Therefore, $x_{\alpha,\beta,\gamma}=\cap\{bcl(G) : x_{\alpha,\beta,\gamma}\in G$ and G is an NBC set $\}$. Hence proved. □

Proposition 4.24 *In an NBT_2-space, every NP is a NBC set.*

Proof: Immediate from the prop. 4.23. □

Proposition 4.25 *Let f be a bijective neutrosophic b-open function from an NTS (X,τ) to another NTS (Y,σ). If (X,τ) is NT_2 then (Y,σ) is an NBT_2-space.*

Proof: Let $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, be two NPs in Y. Since f is bijective, so there exist two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$, $x^1 \neq x^2$, in X such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$. Since X is NT_2, so there exist τ-open NSs G, H such that $x^1_{\alpha,\beta,\gamma} \in G$, $x^2_{\alpha',\beta',\gamma'} \in H$ and $G \cap H = \tilde{\emptyset}$. Since f is a neutrosophic b-open function, so $f(G), f(H)$ are σ-NBO sets such that $y^1_{p,q,r} = f(x^1_{\alpha,\beta,\gamma}) \in f(G)$ and $y^2_{p',q',r'} = f(x^2_{\alpha',\beta',\gamma'}) \in f(H)$. Again since f is bijective, so $f(G) \cap f(H) = f(G \cap H) = f(\tilde{\emptyset}) = \tilde{\emptyset}$. Thus for any two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$ in Y such that $y^1 \neq y^2$, there exist σ-NBO sets $f(G), f(H)$ such that $y^1_{p,q,r} \in f(G)$, $y^2_{p',q',r'} \in f(H)$ and $f(G) \cap f(H) = \tilde{\emptyset}$. Therefore, (Y, σ) is an NBT_2-space. Hence proved. □

Proposition 4.26 *Let f be a one-one neutrosophic b-continuous function from an NTS (X, τ) to another NTS (Y, σ). If (Y, σ) is NT_2 then (X, τ) is an NBT_2-space.*

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since (Y, σ) is NT_2, so there exist σ-open NSs H_1, H_2 such that $y^1_{p,q,r} \in H_1, y^2_{p',q',r'} \in H_2$ and $H_1 \cap H_2 = \tilde{\emptyset}$. Since f is a neutrosophic b-continuous function, so $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are τ-NBO sets in X. Now $f^{-1}(H_1) \cap f^{-1}(H_2) = f^{-1}(H_1 \cap H_2) = f^{-1}(\tilde{\emptyset}) = \tilde{\emptyset}$. Also $y^1_{p,q,r} \in H_1 \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(H_1) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exist τ-NBO sets $f^{-1}(H_1)$ and $f^{-1}(H_2)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$ and $f^{-1}(H_1) \cap f^{-1}(H_2) = \tilde{\emptyset}$. Therefore, (X, τ) is an NBT_2-space. Hence proved. □

Proposition 4.27 *Let f be a one-one neutrosophic b^*-continuous function from an NTS (X, τ) to another NTS (Y, σ). If (Y, σ) is NBT_2 then (X, τ) is an NBT_2-space.*

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since (Y, σ) is NBT_2, so there exist σ-NBO sets H_1, H_2 such that $y^1_{p,q,r} \in H_1, y^2_{p',q',r'} \in H_2$ and $H_1 \cap H_2 = \tilde{\emptyset}$. Since f is a neutrosophic b^*-continuous function, so $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are τ-NBO sets in X. Now $f^{-1}(H_1) \cap f^{-1}(H_2) = f^{-1}(H_1 \cap H_2) = f^{-1}(\tilde{\emptyset}) = \tilde{\emptyset}$. Also $y^1_{p,q,r} \in H_1 \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(H_1) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exist τ-NBO sets $f^{-1}(H_1)$ and $f^{-1}(H_2)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$ and $f^{-1}(H_1) \cap f^{-1}(H_2) = \tilde{\emptyset}$. Therefore, (X, τ) is an NBT_2-space. Hence proved. □

Proposition 4.28 *Let f be a one-one neutrosophic b^{**}-continuous function from an NTS (X, τ) to another NTS (Y, σ). If (Y, σ) is NBT_2 then (X, τ) is an NT_2-space.*

Proof: Let $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ be any two NPs in X such that $x^1 \neq x^2$. Since f is one-one, so there exist two NPs $y^1_{p,q,r}$ and $y^2_{p',q',r'}$, $y^1 \neq y^2$, in Y such that $f(x^1_{\alpha,\beta,\gamma}) = y^1_{p,q,r}$ and $f(x^2_{\alpha',\beta',\gamma'}) = y^2_{p',q',r'}$, i.e., $x^1_{\alpha,\beta,\gamma} = f^{-1}(y^1_{p,q,r})$ and $x^2_{\alpha',\beta',\gamma'} = f^{-1}(y^2_{p',q',r'})$. Since (Y, σ) is NBT_2, so there exist σ-NBO sets H_1, H_2 such that $y^1_{p,q,r} \in H_1, y^2_{p',q',r'} \in H_2$ and $H_1 \cap H_2 = \tilde{\emptyset}$. Since f is a neutrosophic b^{**}-continuous function, so $f^{-1}(H_1)$ and $f^{-1}(H_2)$ are τ-open NSs in X. Now $f^{-1}(H_1) \cap f^{-1}(H_2) = f^{-1}(H_1 \cap H_2) = f^{-1}(\tilde{\emptyset}) = \tilde{\emptyset}$. Also $y^1_{p,q,r} \in H_1 \Rightarrow f^{-1}(y^1_{p,q,r}) \in f^{-1}(H_1) \Rightarrow x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$. Similarly $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$. Thus for any two NPs $x^1_{\alpha,\beta,\gamma}$ and $x^2_{\alpha',\beta',\gamma'}$ in X such that $x^1 \neq x^2$, there exist τ-open NSs $f^{-1}(H_1)$ and $f^{-1}(H_2)$ in X such that $x^1_{\alpha,\beta,\gamma} \in f^{-1}(H_1)$, $x^2_{\alpha',\beta',\gamma'} \in f^{-1}(H_2)$ and $f^{-1}(H_1) \cap f^{-1}(H_2) = \tilde{\emptyset}$. Therefore, (X, τ) is an NT_2-space. Hence proved. □

5. Conclusion

In this article, we have initially presented key findings grounded in neutrosophic b-open sets in section 3. Moving forward, Section 4 introduces and investigates neutrosophic bT_0, neutrosophic bT_1, and neutrosophic bT_2 spaces, delving into their respective properties. Our exploration reveals that an NBT_2-space (resp. NBT_1-space) is indeed an NBT_1-space (resp. NBT_0-space); however, the converse does not hold true. Additionally, we demonstrate that an NT_0-space (resp. NT_1-space, NT_2-space) is an NBT_0-space (resp. NBT_1-space, NBT_2-space), but the reverse is not necessarily the case. A notable outcome of our study is that in an NTS (X, τ), every neutrosophic crisp point is an NBC set if and only if X is an NBT_1-space. Furthermore, we have established the hereditary nature of the property of being an NBT_0-space (and similarly, an NBT_1-space, NBT_2-space).

In the coming future, we shall try to study some other separation properties in neutrosophic topological space. Hope that the findings of this article will assist the research fraternity to move forward with the development of different aspects of neutrosophic topology.

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