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The Use of Latest Kudryashov's Integration Scheme for Two Networking Models *

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ABSTRACT: In this study, we concentrate on the equations which arised in network system and telecommini-cations. The newest Kudryashov method is used to construct traveling wave solutions of nonlinear electrical transmission equation and Lonngren equation. By means of Mathematica 11.3 package program, new types of solutions are obtained. The main goal of this study is to acquire the solitary wave solutions with reduced the number of computations.

Key Words: Latest Kudryashov scheme, solitary wave solutions, nonlinear electrical transmission line equation, Lonngren wave equation.

Contents

1. Introduction

In line with the requirements of the modern world, energy and energy transmission problems have attracted great attention by researchers in recent years. With industrialization, increasing population and production needs arising from globalization, commercial growth and developing technology, the need for energy resources and their use are increasing rapidly. The electrical transmission line model, which we encounter in communication engineering and physics, is a structure specially designed to carry high frequency currents or alternating radio currents. Electrical transmission lines are used for purposes such as distributing telephone signals, routing calls between telephone exchanges, interconnecting radio transmitters and receivers, and computer network connections. In particular, the Lonngren wave equation is also used in the field of telecommunications and network engineering.

Over the past century, soliton waves have been shown to be self-reinforcing wave structures that can be generated in channels. In many studies conducted in recent years, the solutions of real-world problems encountered in all areas of daily life in the traveling wave structure have been examined. For this purpose, methods such as exponential function method [3], modified exponential function method [4,5], exponential rational function method [6,7], (G'/G) expansion method [8,9], Benoulli (G'/G) expansion method [10], sub-equation method [11,12], sine-Gordon expansion method (SGEM) [13], rational SGEM [13], Kudryashov methods [14,15,16], unified method [17] have been developed and various soliton-like solutions have been put forward. Recently, N. R. Kudryashov proposed a new integration scheme with an auxilary logistic function [18,19]. The proposed method is convenient for the solitary wave solutions of nonlinear partial differential equations. Therefore, some researchers have handled the method to find semianalytical solutions for some nonlinear models [20,21,22].

It is important to obtain soliton wave solutions of the nonlinear electrical transmission models, since

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solitons are wave structures that travel without changing shape over long distances, and electrical conduction is also the process of transmitting the generated electricity over long distances. For the last decade, many researchers have concentrated on the the periodic solutions [23,31], singular periodic solitions [24], solitary wave solutions, singular solitary wave solutions [25], complex soliton solutions [26,27,28], soliton solutions [29,32], analytical solutions [33] of the nonlinear electrical line equation. In addition, soliton, invariant and shock solutions of Lonngren wave equation have been obtained in the references [34,35,37,5].

In this paper, we use the latest version of Kudryashov's method to obtain soliton type solutions of two networking models. In Section 2 we give the methodology to figure out the solitary wave solitons. In section 3, we apply the proposed method to the nonlinear electrical transmission line equation and the Lonngren wave equation. In the last section, we give the concluding remarks of the obtained solutions.

2. Mathematical Algorithm

2.1. Outline of the latest Kudryashov's integration scheme

The major items of the method proposed above are indicated as:
In the first item, we handle the general impression of nonlinear partial differential equations in the form:

$$\varphi\left(\Phi, \Phi_x, \Phi_y, ..., \Phi_t, \Phi_{xx}, \Phi_{xy}, ...\right) = 0, \tag{2.1}$$

where φ is polynomial function in Φ and its assorted order partial derivatives in addition to nonlinear terms are included.
Secondly, we assume that the subsequent traveling wave transformation is done to reduce 2.1 to ordinary differential equation.

$$\Phi(x, y, z, w, t) = \Phi(\eta), \quad \eta = k(x + y - ct) \tag{2.2}$$

where k and c are arbitrary constants. After applying the above transformation and the chain rule, we get

$$\mu(\Phi, \Phi', \Phi'', ...) = 0. \tag{2.3}$$

In this step, according to the proposed method, we assume Eq.2.3 has a solution in the form:

$$\Phi(\eta) = \sum_{j=0}^{N} a_j \Psi^j(\eta), \tag{2.4}$$

where

$$\Psi(\eta) = \frac{1}{mexp(\eta) + nexp(-\eta)} \tag{2.5}$$

and $\Psi(\eta)$ adopts the given ordinary differential equation:

$$\Psi_\eta^2 = \Psi^2(1 - \chi\Psi^2), \tag{2.6}$$

where $\chi = 4mn$. The major characteristic of $\Psi(\eta)$ is that while the higher order even derivatives have only Ψ and its powers, higher order odd derivatives holds both Ψ and Ψ_η. Therefore, the termss of the higher order derivatives from second to forth order as follows:

$$\Psi_{\eta\eta} = \Psi - 2\chi\Psi^3, \tag{2.7}$$

$$\Psi_{\eta\eta\eta} = \Psi_\eta - 6\chi\Psi^2\Psi_\eta, \tag{2.8}$$

$$\Psi_{\eta\eta\eta\eta} = \Psi - 20\chi\Psi^3 + 24\chi^2\Psi^5, \tag{2.9}$$

$$\Psi_{\eta\eta\eta\eta\eta} = \Psi_\eta - 60\chi\Psi^2\Psi_\eta + 120\chi^2\Psi^4\Psi_\eta, \tag{2.10}$$

$$\Psi_{\eta\eta\eta\eta\eta\eta} = \Psi - 182\chi\Psi^3 + 840\chi^2\Psi^5 - 720\chi^4\Psi^7, \tag{2.11}$$

Then, considering $\Phi(\eta) = \sum_{j=0}^{N} a_m \Psi^m(\eta)$ and using homogenous balance principle, we can show the bonds of derivatives $\Phi_\eta, \Phi_{\eta\eta}, \Phi_{\eta\eta\eta}$ and $\Phi_{\eta\eta\eta\eta}$ as follows:

$$\Phi_\eta = \sum_{j=1}^{N} a_j j \Psi^{j-1}\Psi_\eta, \tag{2.12}$$

$$\Phi_{\eta\eta} = \sum_{j=1}^{N} a_j [j^2\Psi^j - j^2\chi\Psi^{j+2} - j\chi\Psi^{j+2}], \tag{2.13}$$

$$\Phi_{\eta\eta\eta} = \sum_{j=1}^{N} a_j [j^3\Psi^{j-1} - \chi j^2(j+2)\Psi^{j+1} - \chi j(j+2)\Psi^{j+1}]\Psi_\eta, \tag{2.14}$$

$$\Phi_{\eta\eta\eta\eta} = \sum_{j=1}^{N} j a_j \Psi^j [j^3 + (j^3\chi^2 + 6j^2\chi^2 + 11j\chi^2 + 6\chi^2)\Psi^4 - (2j^3\chi + 6j^2\chi + 8j\chi + 4\chi)\Psi^2], \tag{2.15}$$

$$\Phi_{\eta\eta\eta\eta\eta} = \sum_{j=1}^{N} j a_j \Psi^{j-1} [j^4 + (j+4)(j^3\chi^2 + 6j^2\chi^2 + 11j\chi^2 + 6\chi^2)\Psi^4 - (j+2)(2j^3\chi + 6j^2\chi + 8j\chi + 4\chi)\Psi^2]\Psi_\eta. \tag{2.16}$$

Then, switching out Eq.(2.4) and (2.12)-(2.16) into Eq.2.3 we can obtain the induced form of the equation as polynomial that comprises of Ψ and Ψ_η. after that, equating each coefficient for different degrees of $\Psi^n \Psi_\eta^h$ $(h = 0, 1)$ we get an algebraic equations system. Then, by calculating this set, we get coefficients of basic polynomial and the parameters of the wave transformation. In the end, solitary wave solutions of Eq.(2.3) can be obtained.

3. Mathematical Applications

3.1. Nonlinear Electrical Transmission Line Equation

The nonlinear electrical transmission line equation has been employed to represent the wave propation on the network system. Transmission lines have served purposes such as telephone transmitters, high speed computer buses, radio transmitters. Tala-Taube et al. have investigated the voltage wave propagation to describe pulse narrowing nonlinear transmission lines [23].

$$\Phi_{tt} - \alpha(\Phi^2)_{tt} + \beta(\Phi^3)_{tt} - \omega^2\Delta^2\Phi_{xx} - \omega^2\frac{\Delta^4}{12}\Phi_{xxxx} - \omega^2\theta^2\Phi_{yy} - \omega^2\frac{\theta^4}{12}\Phi_{yyyy} = 0, \tag{3.1}$$

where $\Phi = \Phi(x, y, t)$ is the voltage in the electrical transmission line, α,β,ω,δ,θ are real nonzero constants, and x and y are interpreted like the promulgation distance.

$$\Phi(x, y, t) = \Phi(\eta), \quad \eta = k(x + y - ct). \tag{3.2}$$

where k and c are also nonzero constants. After using the above transformation, Eq.(3.1) is reduced to ODE which is in the below:

$$c^2\Phi_{\eta\eta} - \alpha c^2(\Phi^2)_{\eta\eta} + \beta c^2(\Phi^3)_{\eta\eta} - \omega^2\Delta^2\Phi_{\eta\eta} - k^2\omega^2\frac{\Delta^4}{12}\Phi_{\eta\eta\eta\eta} - \omega^2\theta^2\Phi_{\eta\eta} - k^2\omega^2\frac{\theta^2}{12}\Phi_{\eta\eta\eta\eta} = 0. \tag{3.3}$$

integrating by twice and taking integration constants zero for simplicity, we obtain:

$$12[c^2 - \omega^2\Delta^2 - \omega^2\theta^2]\Phi + 12\beta c^2\Phi^3 - 12\alpha c^2\Phi^2 - k^2[\omega^2\Delta^4 + \omega^2\theta^4]\Phi^{''} = 0. \tag{3.4}$$

With the help of the homogenous balance principle in Eq.(3.3), we balance the highest degree of nonlinear term and highest order derivative. Then we get the degree of the auxilary polynomial $N = 1$ and we can write the first degree polynomial as follows:

$$\Phi(\eta) = a_0 + a_1\Psi(\eta). \tag{3.5}$$

where

$$\Psi(\eta) = \frac{1}{mexp(\eta) + nexp(-\eta)}$$

and $\Psi(\eta)$ satsifies the differential equation:

$$\Psi_\eta^2 = \Psi^2(1 - \chi\Psi^2),$$

where $\chi = 4mn$.
Taking into account the derivatives of Eq.(3.5) from first order to second order, we obtain the following equations.

$$\begin{aligned} \Phi'(\eta) &= a_1\Psi_\eta, \\ \Phi''(\eta) &= a_1\left(\Psi(\eta) - 2\chi\Psi^3(\eta)\right). \end{aligned}$$

Substituting Eq.(3.5) and Eqns.(3.6) into Eq.(3.3), we get an equation that involves the powers of Ψ. Then, banding together the same degreess of Ψ and matching the all coefficients to zero, we obtain the following algebraic equations system:

$$\begin{aligned} 12c^2a_0 - 12\omega^2a_0 - 12\Delta^2a_0 - 12\omega^2\Theta^2a_0 - 12c^2\alpha a_0^2 + 12c^2\beta a_0^3 = 0, \\ 12c^2a_1 - 12\omega^2a_1 - 12\Delta^2a_1 - k^2\omega^2\Delta^4a_1 - 12\omega^2\Theta^2a_1 = 0, \\ -12c^2\alpha a_1^2 + 36c^2\beta a_0a_1^2 = 0, \\ 2k^2\omega^2\Delta^4\chi a_1 + 2k^2\omega\Delta^4\chi a_1 + 12c^2\beta a_1^3 = 0. \end{aligned} \tag{3.6}$$

Finally, solving this system by means of Mathematica 11.3 we obtain seven conditions of traveling wave solutions of the Eq.(3.1) as follows:

$$G(t,s) = \begin{cases} \frac{st^2}{2} - \frac{s^2t}{2} - \frac{s\eta^2}{2} + \frac{s^2\eta}{2}, & s \leq \min\{\eta, t\}; \\ \frac{t^3}{6} - \frac{s\eta^2}{2} + \frac{s^2\eta}{2} - \frac{s^3}{6}, & t \leq s \leq \eta; \\ \frac{st^2}{2} - \frac{s^2t}{2} + \frac{s^3}{6} - \frac{\eta^3}{3}, & \eta \leq s \leq t; \\ \frac{t^3}{6} - \frac{\eta^3}{3}, & \max\{\eta, t\} \leq s. \end{cases} \tag{3.7}$$

Case I:

$$a_0 = \frac{\alpha}{3\beta}, \quad a_1 = -\frac{\alpha\sqrt{2\chi}}{3\beta}, \quad k = -2\alpha\sqrt{\frac{3\omega^2 + \Delta^2 + \omega^2\Theta^2}{(2\alpha^2 - 9\beta)(\omega^2\Delta^4 + \omega\Theta^4)}}, \quad c = -3i\sqrt{\frac{\beta(\omega^2 + \Delta^2 + \omega^2\Theta^2)}{2\alpha^2 - 9\beta}},$$

$$\Phi(\eta) = \frac{\alpha}{3\beta} - \frac{\alpha\sqrt{2\chi}}{3\beta}\left(\frac{4m}{4m^2e^\eta + \chi e^{-\eta}}\right),$$

$$\begin{aligned} \Phi(\eta) &= \frac{\alpha}{3\beta} - \frac{\alpha\sqrt{2\chi}}{3\beta}4m \\ &\times \left(4m^2e^{-2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y+3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right. \\ &+ \left.\chi e^{2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y+3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right)^{-1}. \end{aligned} \tag{3.8}$$

Case II:

$$a_0 = \frac{\alpha}{3\beta}, \quad a_1 = \frac{\alpha\sqrt{2\chi}}{3\beta}, \quad k = -2\alpha\sqrt{\frac{3\omega^2 + \Delta^2 + \omega^2\Theta^2}{(2\alpha^2 - 9\beta)(\omega^2\Delta^4 + \omega\Theta^4)}}, \quad c = -3i\sqrt{\frac{\beta(\omega^2 + \Delta^2 + \omega^2\Theta^2)}{2\alpha^2 - 9\beta}},$$

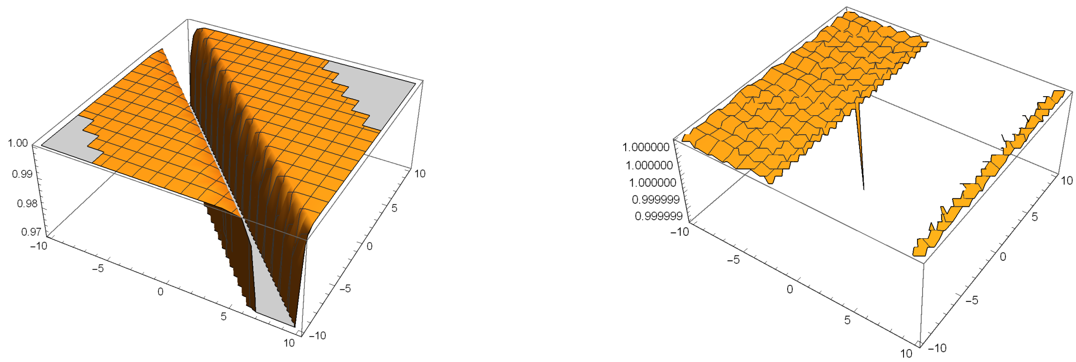

Figure 1: 3D surfaces of real and imaginary parts of the solution (3.8) respectively.

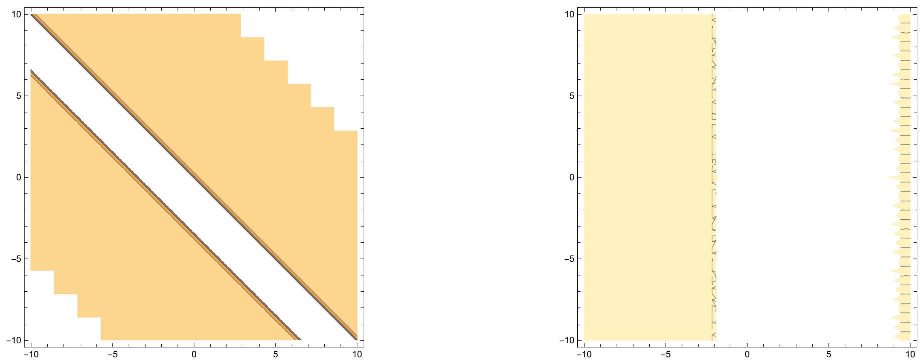

Figure 2: Contour surfaces of real and imaginary parts of the solution (3.8) respectively.

$$\Phi(\eta) = \frac{\alpha}{3\beta} + \frac{\alpha\sqrt{2\chi}}{3\beta}\left(\frac{4m}{4m^2e^{\eta} + \chi e^{-\eta}}\right),$$

$$\begin{aligned}\Phi(x,y,t) &= \frac{\alpha}{3\beta} + \frac{\alpha\sqrt{2\chi}}{3\beta}4m \\ &\times \left(4m^2 e^{-2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y+3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right. \\ &+ \left.\chi e^{2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y+3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right)^{-1}. \qquad (3.9)\end{aligned}$$

Case III:

$$a_0 = \frac{\alpha}{3\beta}, \quad a_1 = -\frac{\alpha\sqrt{2\chi}}{3\beta}, \quad k = -2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}, \quad c = 3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}},$$

$$\Phi(\eta) = \frac{\alpha}{3\beta} - \frac{\alpha\sqrt{2\chi}}{3\beta}\left(\frac{4m}{4m^2e^{\eta} + \chi e^{-\eta}}\right),$$

$$\begin{aligned}\Phi(x,y,t) &= \frac{\alpha}{3\beta}-\frac{\alpha\sqrt{2\chi}}{3\beta}4m\\ &\times \left(4m^2 e^{-2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y-3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right.\\ &+ \left.\chi e^{2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y-3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right)^{-1}. \qquad (3.10)\end{aligned}$$

Case IV:

$$a_0=\frac{\alpha}{3\beta},\quad a_1=\frac{\alpha\sqrt{2\chi}}{3\beta},\quad k=-2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}},\quad c=3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}},$$

$$\Phi(\eta)=\frac{\alpha}{3\beta}+\frac{\alpha\sqrt{2\chi}}{3\beta}\left(\frac{4m}{4m^2e^{\eta}+\chi e^{-\eta}}\right),$$

$$\begin{aligned}\Phi(x,y,t) &= \frac{\alpha}{3\beta}+\frac{\alpha\sqrt{2\chi}}{3\beta}4m\\ &\times \left(4m^2 e^{-2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y-3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right.\\ &+ \left.\chi e^{2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y-3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right)^{-1}. \qquad (3.11)\end{aligned}$$

Case V:

$$a_0=\frac{\alpha}{3\beta},\quad a_1=-\frac{\alpha\sqrt{2\chi}}{3\beta},\quad k=2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}},\quad c=-3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}},$$

$$\Phi(\eta)=\frac{\alpha}{3\beta}-\frac{\alpha\sqrt{2\chi}}{3\beta}\left(\frac{4m}{4m^2e^{\eta}+\chi e^{-\eta}}\right),$$

$$\begin{aligned}\Phi(x,y,t) &= \frac{\alpha}{3\beta}-\frac{\alpha\sqrt{2\chi}}{3\beta}4m\\ &\times \left(4m^2 e^{2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y+3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right.\\ &+ \left.\chi e^{-2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y+3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right)^{-1}. \qquad (3.12)\end{aligned}$$

Case VI:

$$a_0=\frac{\alpha}{3\beta},\quad a_1=\frac{\alpha\sqrt{2\chi}}{3\beta},\quad k=2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}},\quad c=-3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}},$$

$$\Phi(\eta)=\frac{\alpha}{3\beta}+\frac{\alpha\sqrt{2\chi}}{3\beta}\left(\frac{4m}{4m^2e^{\eta}+\chi e^{-\eta}}\right),$$

$$\begin{aligned}\Phi(x,y,t) &= \frac{\alpha}{3\beta}+\frac{\alpha\sqrt{2\chi}}{3\beta}4m\\ &\times \left(4m^2 e^{2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y+3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right.\\ &+ \left.\chi e^{-2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y+3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right)^{-1}. \qquad (3.13)\end{aligned}$$

Case VII:

$$a_0=\frac{\alpha}{3\beta},\quad a_1=\frac{\alpha\sqrt{2\chi}}{3\beta},\quad k=2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}},\quad c=3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}},$$

$$\Phi(\eta)=\frac{\alpha}{3\beta}+\frac{\alpha\sqrt{2\chi}}{3\beta}\left(\frac{4m}{4m^2e^{\eta}+\chi e^{-\eta}}\right),$$

$$\begin{aligned}\Phi(x,y,t) &= \frac{\alpha}{3\beta}+\frac{\alpha\sqrt{2\chi}}{3\beta}4m\\ &\times \left(4m^2 e^{2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y-3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right.\\ &+ \left.\chi e^{-2\alpha\sqrt{\frac{3\omega^2+\Delta^2+\omega^2\Theta^2}{(2\alpha^2-9\beta)(\omega^2\Delta^4+\omega\Theta^4)}}\left(x+y-3i\sqrt{\frac{\beta(\omega^2+\Delta^2+\omega^2\Theta^2)}{2\alpha^2-9\beta}}t\right)}\right)^{-1}. \qquad (3.14)\end{aligned}$$

3.2. Lonngren Wave Equation

The Lonngren equation, which was first introduced to the literature by Lonngren and named after the person who made it, plays an important role in telecommunications [34]. Specifically describing electrical signals on telegraph lines known as a model of the tunnel diode,

$$\left(\Phi_{xx}-\alpha\Phi+\beta\Phi^2\right)_{tt}+\Phi_{xx}=0, \qquad (3.15)$$

where α and β arbitrary constants $\Phi=\Phi(x,t)$ is used to explain the tightness throughout the electrical line, and x and t are interpreted like the promulgation distance. At first, we apply traveling wave transformation,

$$\Phi(x,t)=\Phi(\eta),\quad \eta=k(x-ct). \qquad (3.16)$$

where k and c are nonzero constants. By means of the wave transformation, Eq.(3.15) is reduced to the following ordinary differential equation:

$$c^2k^2\Phi''+(1-c^2\alpha)\Phi+c^2\beta\Phi^2=0. \qquad (3.17)$$

In that step, we apply the homogenous balance principle to Eq.(3.17) and we balance the highest degree nonlinear term and highest order derivative. Then we get the balance degree $N=1$. Hence, the first degree basic polynomial can be written as:

$$\Phi(\eta)=a_0+a_1\Psi(\eta)+a_2\Psi^2(\eta). \qquad (3.18)$$

where

$$\Psi(\eta)=\frac{1}{mexp(\eta)+nexp(-\eta)}$$

and $\Psi(\eta)$ adopts the differential equation:

$$\Psi_\eta^2=\Psi^2(1-\chi\Psi^2),$$

where $\chi = 4mn$.
Taking account the derivatives of Eq.(3.18) from first order to second order, the following expressions are gained.

$$\begin{aligned} \Phi'(\eta) &= \Psi_\eta[a_1 + 2a_2\Psi(\eta)], \\ \Phi''(\eta) &= a_1\left(\Psi(\eta) - 2\chi\Psi^3(\eta)\right). \end{aligned}$$

Substituting Eq.(3.18) and Eqns.(3.19) into Eq.(3.17), we obtain an equation containing the degrees of Ψ. In addition, banding together all the same degrees of Ψ and matching the coefficients to zero, we obtain the following algebraic equations system:

$$\begin{aligned} 12c^2a_0 - 12\omega^2a_0 - 12\Delta^2a_0 - 12\omega^2\Theta^2a_0 - 12c^2\alpha a_0^2 + 12c^2\beta a_0^3 = 0, \\ 12c^2a_1 - 12\omega^2a_1 - 12\Delta^2a_1 - k^2\omega^2\Delta^4a_1 - 12\omega^2\Theta^2a_1 = 0, \\ -12c^2\alpha a_1^2 + 36c^2\beta a_0a_1^2 = 0, \\ 2k^2\omega^2\Delta^4\chi a_1 + 2k^2\omega\Delta^4\chi a_1 + 12c^2\beta a_1^3 = 0. \end{aligned} \tag{3.19}$$

Finally, solving this system by means of Mathematica 11.3 we obtain three conditions of traveling wave solutions of the Eq.(3.15)as follows:

Case I:

$$a_0 = \frac{c^2\alpha - 1}{c^2\beta}, \quad a_1 = 0, \quad a_2 = -\frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}, \quad k = -\frac{\sqrt{1-c^2\alpha}}{2\sqrt{c}}$$

$$\Phi(\eta) = \frac{c^2\alpha - 1}{c^2\beta} - \frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}\left(\frac{4m}{4m^2e^{\eta} + \chi e^{-\eta}}\right)^2,$$

$$\Phi(x,t) = \frac{c^2\alpha - 1}{c^2\beta} - \frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}\left(\frac{4m}{4m^2e^{-\frac{\sqrt{1-c^2\alpha}}{2\sqrt{c}}(x-ct)} + \chi e^{\frac{\sqrt{1-c^2\alpha}}{2\sqrt{c}}(x-ct)}}\right)^2. \tag{3.20}$$

Case II:

$$a_0 = \frac{c^2\alpha - 1}{c^2\beta}, \quad a_1 = 0, \quad a_2 = -\frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}, \quad k = \frac{\sqrt{1-c^2\alpha}}{2\sqrt{c}}$$

$$\Phi(\eta) = \frac{c^2\alpha - 1}{c^2\beta} - \frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}\left(\frac{4m}{4m^2e^{\eta} + \chi e^{-\eta}}\right)^2,$$

$$\Phi(x,t) = \frac{c^2\alpha - 1}{c^2\beta} - \frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}\left(\frac{4m}{4m^2e^{\frac{\sqrt{1-c^2\alpha}}{2\sqrt{c}}(x-ct)} + \chi e^{-\frac{\sqrt{1-c^2\alpha}}{2\sqrt{c}}(x-ct)}}\right)^2. \tag{3.21}$$

Case III:

$$a_0 = 0, \quad a_1 = 0, \quad a_2 = \frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}, \quad k = -\frac{\sqrt{c^2\alpha - 1}}{2\sqrt{c}}$$

$$\Phi(\eta) = \frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}\left(\frac{4m}{4m^2e^{\eta} + \chi e^{-\eta}}\right)^2,$$

$$\Phi(x,t) = \frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}\left(\frac{4m}{4m^2e^{-\frac{\sqrt{c^2\alpha-1}}{2\sqrt{c}}(x-ct)} + \chi e^{\frac{\sqrt{c^2\alpha-1}}{2\sqrt{c}}(x-ct)}}\right)^2. \tag{3.22}$$

Case IV:

$$a_0 = 0, \quad a_1 = 0, \quad a_2 = \frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}, \quad k = \frac{\sqrt{c^2\alpha - 1}}{2\sqrt{c}}$$

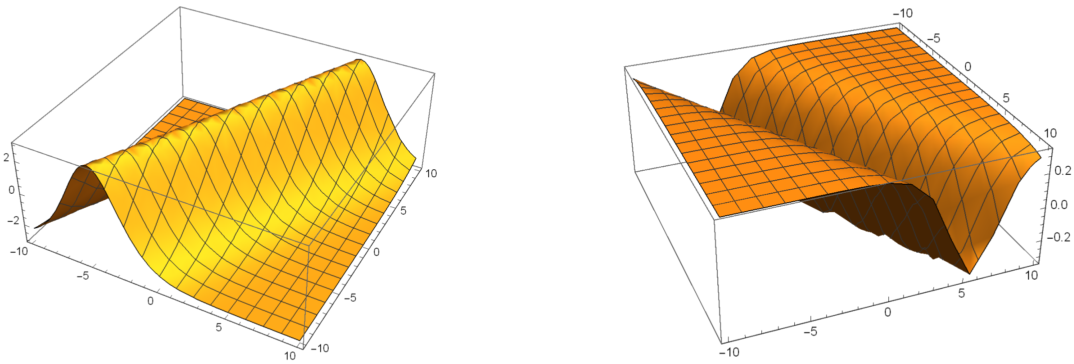

Figure 3: 3D Plot of the solution (3.21) for $c = 0.5$, $\alpha = 1$, $\beta = 1$, $m = 1$, $n = 1$ and $c = 1.73$, $\alpha = 0.5$, $\beta = 0.5$, $m = 1$, $n = 1$ respectively.

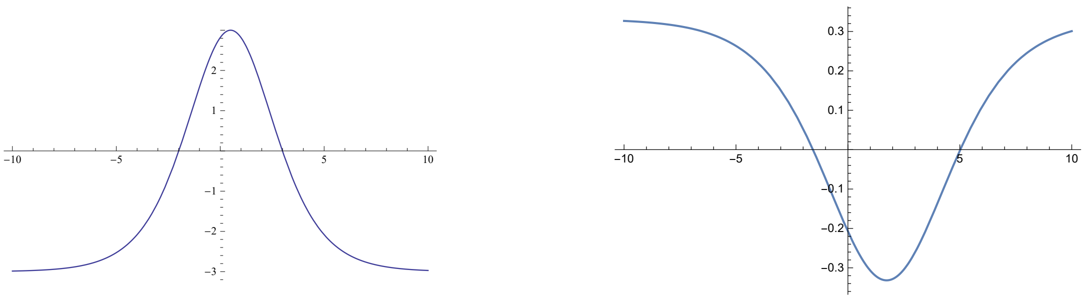

Figure 4: Plot of the solution (3.21) for $c = 0.5$, $\alpha = 1$, $\beta = 1$, $m = 1$, $n = 1$ and $c = 1.73$, $\alpha = 0.5$, $\beta = 0.5$, $m = 1$, $n = 1$ respectively.

$$\Phi(\eta) = \frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}\left(\frac{4m}{4m^2 e^{\eta} + \chi e^{-\eta}}\right)^2,$$

$$\Phi(x,t) = \frac{2(-\chi + c^2\alpha\chi)}{c^2\beta}\left(\frac{4m}{4m^2 e^{\frac{\sqrt{c^2\alpha-1}}{2\sqrt{c}}(x-ct)} + \chi e^{\frac{\sqrt{c^2\alpha-1}}{2\sqrt{c}}(x-ct)}}\right)^2. \tag{3.23}$$

4. Concluding Remarks

We have carried out the latest version of Kudryashov's method to the nonlinear electrical transmission line equation and the Lonngren equation which appears in communication engineering and mathematical physics. The applied wave transforms and the Ψ function used in the algorithm made it easy to obtain solutions. For both models, the obtained results have been indicated as polynomials of the logistic function which in turn are solutions of the Riccati equation. For the nonlinear electrical transmission line equation, we have obtained soliton solutions in seven cases which are the special forms of the solutions in [33]. But also in some cases we have obtained complex constants c which determines the time variable. Therefore, some disjuctions have been occured in the wave that can be seen in graphical representation. On the other hand, for Lonngren equation, soliton solutions are also obtained and constant k which effects the distance directly depends on the constant c. So, in graphical representation when we take c as a real number the obtained waves will be smooth.

Finally, one can see that the new integration scheme may be applied many nonlinear models. When we compare this method with other methods, we see that it is more easily applicable for the solution of nonlinear differential equations containing even-order derivatives. Therefore, one can draw a conclusion that the algorithm can be applied to many integer order evolution equations.

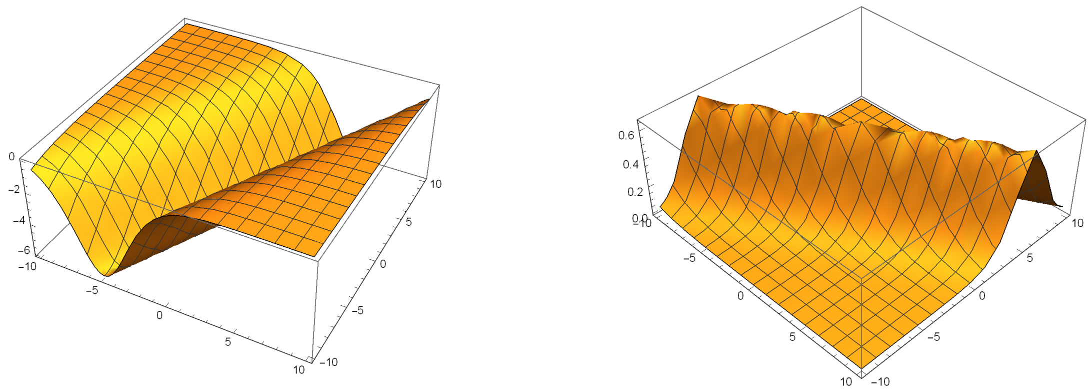

Figure 5: 3D Plot of the solution (3.22) for $c = 0.5$, $\alpha = 1$, $\beta = 1$, $m = 1$, $n = 1$ and $c = 1.73$, $\alpha = 0.5$, $\beta = 0.5$, $m = 1$, $n = 1$ respectively.

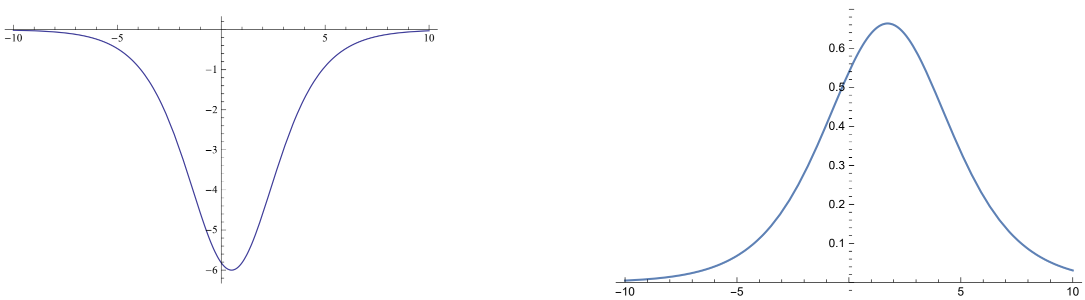

Figure 6: Plot of the solution (3.22) for $c = 0.5$, $\alpha = 1$, $\beta = 1$, $m = 1$, $n = 1$ and $c = 1.73$, $\alpha = 0.5$, $\beta = 0.5$, $m = 1$, $n = 1$ respectively.

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References

1. Lions, J. L., *Exact Controllability, Stabilizability, and Perturbations for Distributed Systems*, Siam Rev. 30, 1-68, (1988).

2. C. M. Dafermos, C. M., *An abstract Volterra equation with application to linear viscoelasticity.* J. Differential Equations 7, 554-589, (1970).

3. E. Misirli, Y. Gurefe, *Exp-Function Method for Solving Nonlinear Evolution Equations.* Mathematical and Computational Applications, 16 (1) (2011), 258-266.

4. T. Akturk, G. Yel, *Modified exponential function method for the KP-BBM equation*, Math. Nat. Sci, 6 (2020), 1-7.

5. O. Kirci, T. Akturk, H. Bulut, *Simulation of Wave Solutions of a Mathematical Model Representing Communication Signals*, Journal of the Institute of Science and Technology , 11 (4) (2021), 3086-3097.

6. M. Ekici, M. Ünal, *Application of the Exponential Rational Function Method to Some Fractional Soliton Equations*, Emerging Applications of Differential Equations and Game Theory, (2020), 13-32.

7. B. Ghanbari, M. S. Osman, D. Baleanu, *Generalized exponential rational function method for extended Zakharov–Kuzetsov equation with conformable derivative*, Modern Physics Letters A, 34 (20)(2019), 1-16.

8. A. M. Shahoot, K. A. E. Alurrfi, M. O. M. Elmrid, A. M. Almsiri, A. M. H. Arwiniya, *The (G'/G)-expansion method for solving a nonlinear PDE describing the nonlinear low-pass electrical lines*, Journal of Taibah University for Science, 13 (1) (2019), 63-70.

9. S. Akcagil, T. Aydemir, *Comparison between the $(G'/G)-$ expansion method and the modified extended tanh method.* Open Phys., 14 (2016) 88-94.

10. N. Aminakbari, Y. Gu, W. Yuan , *Bernoulli (G'/G)-expansion method for nonlinear Schrödinger equation under effect of constant potential*, Optical and Quantum Electronics, 53 (331) (2021), 1-11.

11. L. Akinyemi, M. Şenol, O. S. Iyiola , *Exact solutions of the generalized multidimensional mathematical physics models via sub-equation method*, Mathematics and Computers in Simulation, 182 (2021), 211-233.

12. H. Durur, A. Kurt, O. Tasbozan, *New Travelling Wave Solutions for KdV6 Equation Using Sub Equation Method*, Applied Mathematics and Nonlinear Sciences, 5 (1) (2020), 455-460.

13. L. Yan, H. M. Baskonus, C. Cattani, W. Gao *Extraction of the gravitational potential and high-frequencywave perturbation properties of nonlinear(3+1)-dimensional Vakhnenko–Parkes equation via novel approach*, Mathematical Methods in the Applied Sciences, 5 (1) (2022), 1-10.

14. M. Kaplan, A. Bekir, A. Akbulut , *A generalized Kudryashov method to some nonlinear evolution equations in mathematical physics*, Nonlinear Dynamics, 85 (2016), 2843-2850.

15. Y. Gurefe ,*The generalized Kudryashov method for the nonlinear fractional partial differential equations with the beta-derivative*, Revista mexicana de física, 66 (6) (2020), 771-781.

16. S. M. Ege ,*Traveling Wave Solutions For Two Physical Models via Extended Modified Kudryashov Method*,Journal of the Institute of Science and Technology, 11 (1) (2021), 625-634.

17. S. Akcagil and T. Aydemir, *A new application of the unified method*, NTMSCI, 6 (2018) 185-199.

18. A.N. Kudryashov, *Method for finding highly dispersive optical solitons of nonlinear differential equations*. Opt. Int. J. Light Electr. Opt., 206 (2020) 1-9.

19. N.A. Kudryashov, *Solitary waves of the non-local Schrodinger ¨ equation with arbitrary refractive index*, Optik, 231 (2021) 1-5.

20. Y. Yildirim, A. Biswas, A. H. Kara, P. Guggilla, S. Khan, A. K. Alzahrani and M.R. Belic, *Highly dispersive optical solitons and conservation laws with Kudryashov's sextic power-law of nonlinear refractive index*, Optik, 240 (2021) 1.

21. E.M.E. Zayed,R.M.A. Shohib, M.E.M Alngar, A. Biswas,S. Khan, Y. Yildirim, H. Triki, A.K. Alzahrani and M.R. Belic,*Cubic–quartic optical solitons with Kudryashov's arbitrary form of nonlinear refractive index*, Optik, 238 (2021) 1

22. S. M. Ege, *Solitary Wave Solutions for Some Fractional Evolution Equations via New Kudryashov Approach*, Revista mexicana de física, 68 (1) (2022), 1-11.

23. E. Tala-Tebue, D.C. Tsobgni-Fozap, A. Kenfack-Jiotsa, T.C. Kofane, *Envelope periodic solutions for a discrete network with the Jacobi elliptic functions and the alternative (G'/G)-expansion method including the generalized Riccati equation*, Eur. Phys. J. Plus , 129 (36) (2014), 1-10.

24. M. M. El-Borai, H. M. El-Owaidy, H. M. Ahmed, A. H. Arnous, *Exact and soliton solutions to nonlinear transmission line model*, Nonlinear Dyn, 87 (2017) , 767-773.

25. E. Fendzi-Donfack, J. P. Nguenang, L. Nana, *Fractional analysis for nonlinear electrical transmission line and nonlinear Schroedinger equations with incomplete sub-equation*, Eur. Phys. J. Plus, 133(32)(2018), 1-11.

26. D. Kumar, A. R. Seadawy, R. Chowdhury, *On new complex soliton structures of the nonlinear partial differential equation describing the pulse narrowing nonlinear transmission lines*, Opt Quant Electron , 50 (108) (2018), 1-14.

27. M. T. Gulluoglu, *New Complex Solutions to the Nonlinear Electrical Transmission Line Model*, Open Phys, 17 (2019), 823-830.

28. W. Gao, M. Senel, G. Yel, H. M. Baskonus, B. Senel, *New complex wave patterns to the electrical transmission line model arising in network system*, AIMS Mathematics , 5(3)(2020), 1881-1892.

29. M. A. Kayuma, S. Arab, H. K. Barmana, M. A. Akbar, *Soliton solutions to voltage analysis in nonlinear electrical transmission lines and electric signals in telegraph lines, Journal of the Institute of Science and Technology* , 18 (2020), 1-10.

30. O. Ozer, H. M. Baskonus, H. Bulut, I. Amirali, G. Yel, *A new survey to the nonlinear electrical transmission line model*, International Journal of Cognitive Computing in Engineering, 2 (2021), 208-214.

31. K. J. Wang, G. D. Wang, *Periodic solution of the (2+1)- dimensional nonlinear electrical transmission line equation via variational method* , Results in Physics , 20 (2021), 1-2.

32. A. Zulfiqar,J. Ahmad, A. Rani, Q. M. Hassan, *Wave Propagations in Nonlinear Low-Pass Electrical Transmission Lines through Optical Fiber Medium*, Mathematical Problems in Engineering , 2022 (2022), 1-16.

33. K. K. Ali , M. S. Mehanna, *On some new analytical solutions to the (2+1)-dimensional nonlinear electrical transmission line model*, Eur. Phys. J. Plus , 137 (280) (2022), 1-12.

34. K. E. Lonngren, H. C. S. Hsuan and W. F. Ames, *On the soliton, invariant, and shock solutions of a fourth-order nonlinear equation*.J. Math. Anal. Appl., **52** (1975) 538-545.

35. H. M. Baskonus, H. Bulut, T. A. Sulaiman, *New Complex Hyperbolic Structures to the Lonngren Wave Equation by Using Sine Gordon Expansion Method*, Applied Mathematics and Nonlinear Sciences , 4(1)(2019), 129-138.

36. S. Duran, *Travelling wave solutions and simulation of the Lonngren wave equation for tunnel diode*, Optical and Quantum Electronic , 53(458) (2021), 1-9.

37. H. K. Barman, A. R. Seadawy, R. Roy, M. A. Akbar, M.H. Raddad, *Rational closed form soliton solutions to certain nonlinear evolution equations ascend in mathematical physics*,Results in Physics, 27 (2021), 1-12.

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