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Fixed Point Theorems for α-Admissible Mappings in Metric Like Spaces

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ABSTRACT: In this paper, we shall introduce some α-admissible contractions and prove some fixed point theorems for such contractions in complete metric like spaces. We shall also provide examples to support the main results.

Key Words: Fixed point, α-admissible, metric like spaces.

Contents

1. Introduction

In 1922, Banach [6] gave a contraction principle to obtain the fixed point of a mapping in complete metric space. Since then, the researchers had generalized the principle to obtain new fixed point theorems. Many other researchers had generalized the concept of metric space. In 2000, the concept of dislocated metric space was introduced by Hitzler and Seda [8]. Metric like spaces were discovered by Amini and Harandi [4] in 2012. Some fixed point point results had been proved by various authors (see [1,2], [5], [7], [10,13,9,11,12,14,15,16]) in this direction. In 2013, Akbar *et al.* [3] proved some fixed point results for α-admissible mappings in metric spaces. In this paper, we shall prove fixed theorems for α-admissible mappings in metric like spaces. To prove the main results, we need some pre-requisite from literature as follows:

Definition 1.1 **([4])** *Let X be a non empty set and $d : X \times X \to [0, \infty)$ be a function satisfying the following conditions:*

(1) $d(x, y) = 0 \Rightarrow x = y$;

(2) $d(x, y) = d(y, x)$;

(3) $d(x, z) \le d(x, y) + d(y, z)$,

for all $x, y, z \in X$.
Then d is called metric like (dislocated metric) and (X, d) is called metric like (dislocated metric) space.

Definition 1.2 **([4])** *Let (X, d) be a metric like space.*

(1) *A sequence $\{x_n\}$ in X is a Cauchy sequence if $\lim_{n,m\to\infty} d(x_n, y_m)$ exists and is finite.*

(2) *(X, d) is complete if every Cauchy sequence $\{x_n\}$ in X converges to a point $x \in X$, that is,*

$$\lim_{n\to\infty} d(x, x_n) = d(x, x) = \lim_{n,m\to\infty} d(x_n, x_m).$$

(3) *A mapping $T : (X, d) \to (X, d)$ is continuous if for any sequence x_n in X such that $d(x_n, x) \to d(x, x)$ as $n \to \infty$, we have $d(Tx_n, Tx) \to d(Tx, Tx)$ as $n \to \infty$.*

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Lemma 1.1 ([4]) *Let (X,d) be a metric like space. Let $\{X_n\}$ be a sequence in X such that $X_n \to X$ where $X \in X$ and $d(x,x)=0$. Then for all $y \in X$, we have*

$$\lim_{n\to\infty} d(x_n,y) = d(x,y).$$

Definition 1.3 ([14]) *Let $T: X \to X$ and $\alpha: X \times X \to [0,\infty)$. We say that T is an α-admissible mapping if*

$$\alpha(x,y) \geq 1 \quad \textit{implies} \quad \alpha(Tx,Ty) \geq 1, \quad \textit{for all } x,y \in X.$$

2. Main Results

In 2013, Akbar *et al.* [3] proved some fixed point results for α-admissible mappings in metric spaces. In this section, we shall prove fixed theorems for α-admissible mappings in the mentioned spaces.

Theorem 2.1 *Let (X,d) be a complete metric like space and $T: X \to X$ be an α-admissible mapping. Assume that a function $\beta: [0,\infty) \to [0,1]$, such that, for any bounded sequence $\{t_n\}$ of positive reals, $\beta(t_n) \to 1$ implies $t_n \to 0$ and*

$$(d(Tx,Ty)+l)^{\alpha(x,Tx)\alpha(y,Ty)} \leq \beta(d(x,y))d(x,y)+l. \tag{2.1}$$

for all $x,y \in X$ where $l \geq 1$.
Suppose that T is continuous, then T has a fixed point.

Proof: Let $x_0 \in X$ such that $\alpha(x_0,Tx_0) \geq 1$. Construct a sequence $\{x_n\}$ in X such that $x_n = Tx_{n-1}$ for all $n \in \mathbb{N}$. If $x_{m+1} = x_m$ for some $m \in \mathbb{N}$, then $Tx_m = x_m$ and we are done. Hence we suppose that $x_n \neq x_{n+1}$ for any $n \in \mathbb{N}$. Since T is α-admissible mapping so $\alpha(x_0,Tx_0) \geq 1$ implies that $\alpha(x_1,x_2) = \alpha(Tx_0,T^2x_0) \geq 1$.
Continuing this process we get $\alpha(x_n,x_{n+1}) \geq 1$, for all $n \in \mathbb{N}$.
By the inequality (2.1), we have

$$\begin{aligned} d(Tx_{n-1},Tx_n)+l &\leq (d(Tx_{n-1},Tx_n)+l)^{\alpha(x_{n-1},Tx_{n-1})\alpha(x_n,Tx_n)} \\ &\leq \beta(d(x_{n-1},x_n))(d(x_{n-1},x_n))+l, \end{aligned}$$

then

$$d(x_n,x_{n+1}) \leq \beta(d(x_{n-1},x_n))d(x_{n-1},x_n). \tag{2.2}$$

which implies that $d(x_n,x_{n+1}) \leq d(x_{n-1},x_n)$. It follows that the sequence $\{d(x_n,x_{n+1})\}$ is a decreasing sequence of positive reals. So there exists $d \in [0,\infty)$ such that $\lim_{n\to\infty} d(x_n,x_{n+1}) = d$.
We will prove that $d=0$.
From equation (2.2), we get

$$\frac{d(x_n,x_{n+1})}{d(x_{n-1},x_n)} \leq \beta(d(x_{n-1},x_n)) \leq 1,$$

which implies

$$\lim_{n\to\infty} \beta(d(x_{n-1},x_n)) = 1.$$

Using the property of β function, we conclude that

$$\lim_{n\to\infty} d(x_n,x_{n+1}) = 0. \tag{2.3}$$

Next, we will show that $\{x_n\}$ is a Cauchy sequence. Suppose, to the contrary, that $\{x_n\}$ is not a Cauchy sequence. Then there is $\epsilon > 0$ and sequences $m(k)$ and $n(k)$ such that for all positive integer k, we have

$$n(k) > m(k) > k, d(x_{n(k)},x_{m(k)}) \geq \epsilon$$

and

$$d(x_{n(k)}, x_{m(k)-1}) \leq \epsilon.$$

By triangle inequality, we get

$$\begin{aligned}\epsilon &\leq d(x_{n(k)}, x_{m(k)}) \\ &\leq d(x_{n(k)}, x_{m(k)-1}) + d(x_{m(k)-1}, x_{m(k)}) \\ &< \epsilon + d(x_{m(k)-1}, x_{m(k)}),\end{aligned}$$

Taking $k \to \infty$ and using equation (2.3), we get

$$\lim_{n\to\infty} d(x_{n(k)}, x_{m(k)}) = \epsilon\,. \tag{2.4}$$

Again, by triangle inequality we find that

$$d(x_{n(k)}, x_{m(k)}) \leq d(x_{m(k)}, x_{m(k)+1}) + d(x_{m(k)+1}, x_{n(k)+1}) + d(x_{n(k)+1}, x_{n(k)})$$

and

$$d(x_{n(k)+1}, x_{m(k)} + 1) \leq d(x_{m(k)}, x_{m(k)+1}) + d(x_{m(k)}, x_{n(k)}) + d(x_{n(k)+1}, x_{n(k)}).$$

Taking $k \to \infty$ and using equations (2.3) and (2.4), we get

$$\lim_{n\to\infty} d(x_{n(k)+1}, x_{m(k)+1}) = \epsilon. \tag{2.5}$$

From equations (2.1), (2.4) and (2.5), we have

$$\begin{aligned}d(x_{n(k)+1}, x_{m(k)+1}) + l &\leq (d(x_{n(k)+1}, x_{m(k)+1}) + l)^{\alpha(x_{n(k)}, Tx_{n(k)})\alpha(x_{m(k)}, Tx_{m(k)})} \\ &= (d(Tx_{n(k)}, Tx_{m(k)}) + l)^{\alpha(x_{n(k)}, Tx_{n(k)})\alpha(x_{m(k)}, Tx_{m(k)})} \\ &\leq \beta(d(x_{n(k)}, x_{m(k)}))d(x_{n(k)}, x_{m(k)}) + l.\end{aligned}$$

Hence,

$$\begin{aligned}\frac{d(x_{n(k)+1}, x_{m(k)+1})}{d(x_{n(k)}, x_{m(k)})} &\leq \beta(d(x_{n(k)}, x_{(m(k)}) \\ &\leq 1.\end{aligned}$$

Letting $k \to \infty$ in the above inequality, we get

$$\lim_{k\to\infty} \beta(d(x_{n(k)}, x_{m(k)})) = 1.$$

That is,

$$\lim_{k\to\infty} d(x_{n(k)}, x_{m(k)}) = 0.$$

a contradiction.
Hence $\{x_n\}$ is a Cauchy sequence. Since X is a complete metric like space, then there exists $u \in X$ such that

$$\begin{aligned}\lim_{n\to\infty} d(x_n, u) &= d(u, u) \\ &= \lim_{n,m\to\infty} d(x_n, x_m) = 0.\end{aligned} \tag{2.6}$$

Since T is continuous, from equation (2.6), we get

$$\begin{aligned}\lim_{n\to\infty} d(x_{n+1}, Tu) &= \lim_{n\to\infty} d(Tx_n, Tu) \\ &= d(Tu, Tu).\end{aligned} \tag{2.7}$$

Using Lemma 1.1 and equation (2.6), we obtain

$$\lim_{n\to\infty} d(x_{n+1}, Tu) = d(u, Tu). \tag{2.8}$$

Comparing equation (2.6) and (2.7), we get

$$d(u, Tu) = d(Tu, Tu).$$

From equation (2.1) and using above equality, we get

$$\begin{aligned} d(u, Tu) + l &= d(Tu, Tu) + l \\ &\leq (d(Tu, Tu) + l)^{\alpha(u,Tu)\alpha(u,Tu)} \\ &\leq \beta(d(u, u))d(u, u) + l\,. \end{aligned}$$

This implies that $d(u, Tu) \leq d(u, u)$.
Using equation (2.6), we get

$$d(u, Tu) = 0,$$

that is, $Tu = u$. □

Theorem 2.2 *Let (X, d) be a complete metric like space and $T : X \to X$ be an α-admissible mapping. Assume that a function $\beta : [0,\infty) \to [0, 1]$, such that, for any bounded sequence $\{t_n\}$ of positive reals, $\beta(t_n) \to 1$ implies $t_n \to 0$ and*

$$(\alpha(x, Tx)\alpha(y, Ty) + 1)^{d(Tx,Ty)} \leq 2^{\beta(d(x,y))d(x,y)}. \tag{2.9}$$

for all $x, y \in X$.
Suppose that T is continuous, then T has a fixed point.

Proof: Let $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Construct a sequence $\{x_n\}$ in X such that $x_n = Tx_{n-1}$ for all $n \in \mathbb{N}$. If $x_{m+1} = x_m$ for some $m \in \mathbb{N}$, then $Tx_m = x_m$ and we are done. Hence we suppose that $x_n \neq x_{n+1}$ for any $n \in \mathbb{N}$.
Since T is α-admissible mapping so $\alpha(x_0, Tx_0) \geq 1$ implies that $\alpha(x_1, x_2) = \alpha(Tx_0, T^2x_0) \geq 1$.
Continuing this process we get $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$.
By the equation (2.9), we have

$$\begin{aligned} 2^{d(Tx_{n-1},Tx_n)} &\leq (\alpha(x_{n-1}, Tx_{n-1})\alpha(x_n, Tx_n) + 1)^{d(Tx_{n-1},Tx_n)} \\ &\leq 2^{\beta(d(x_{n-1},x_n))(d(x_{n-1},x_n))}, \end{aligned}$$

then

$$d(x_n, x_{n+1}) \leq \beta(d(x_{n-1}, x_n))d(x_{n-1}, x_n). \tag{2.10}$$

which implies that $d(x_n, x_{n+1}) \leq d(x_{n-1}, x_n)$. It follows that the sequence $\{d(x_n, x_{n+1})\}$ is a decreasing sequence of positive reals.
So there exists $d \in [0,\infty)$ such that $\lim_{n\to\infty} d(x_n, x_{n+1}) = d$.
We will prove that $d = 0$.
From equation (2.10), we get

$$\frac{d(x_n, x_{n+1})}{d(x_{n-1}, x_n)} \leq \beta(d(x_{n-1}, x_n)) \leq 1,$$

which implies

$$\lim_{n\to\infty} \beta(d(x_{n-1}, x_n)) = 1.$$

Using the property of β function, we conclude that

$$\lim_{n\to\infty} d(x_n, x_{n+1}) = 0. \tag{2.11}$$

Next, we will show that $\{x_n\}$ is a Cauchy sequence. Suppose, to the contrary, that $\{x_n\}$ is not a Cauchy sequence. Then there is $\epsilon > 0$ and sequences $m(k)$ and $n(k)$ such that for all positive integer k, we have

$$n(k) > m(k) > k, d(x_{n(k)}, x_{m(k)}) \geq \epsilon$$

and

$$d(x_{n(k)}, x_{m(k)-1}) \leq \epsilon.$$

By triangle inequality, we get

$$\begin{aligned}\epsilon &\leq d(x_{n(k)}, x_{m(k)})\\ &\leq d(x_{n(k)}, x_{m(k)-1}) + d(x_{m(k)-1}, x_{m(k)})\\ &< \epsilon + d(x_{m(k)-1}, x_{m(k)}).\end{aligned}$$

Taking $k \to \infty$ and using equation (2.11), we get

$$\lim_{n\to\infty} d(x_{n(k)}, x_{m(k)}) = \epsilon. \tag{2.12}$$

Again, by triangle inequality we find that

$$d(x_{n(k)}, x_{m(k)}) \leq d(x_{m(k)}, x_{m(k)+1}) + d(x_{m(k)+1}, x_{n(k)+1}) + d(x_{n(k)+1}, x_{n(k)})$$

and

$$d(x_{n(k)+1}, x_{m(k)} + 1) \leq d(x_{m(k)}, x_{m(k)+1}) + d(x_{m(k)}, x_{n(k)}) + d(x_{n(k)+1}, x_{n(k)}).$$

Taking $k \to \infty$ and using equations (2.11) and (2.12), we get

$$\lim_{n\to\infty} d(x_{n(k)+1}, x_{m(k)+1}) = \epsilon. \tag{2.13}$$

From equations (2.9), (2.12) and (2.13), we have

$$\begin{aligned}2^{d(x_{n(k)+1}, x_{m(k)+1})} &\leq 2^{(d(x_{n(k)+1}, x_{m(k)+1}))\alpha(x_{n(k)}, Tx_{n(k)})\alpha(x_{m(k)}, Tx_{m(k)})}\\ &= 2^{(d(Tx_{n(k)}, Tx_{m(k)}))\alpha(x_{n(k)}, Tx_{n(k)})\alpha(x_{m(k)}, Tx_{m(k)})}\\ &\leq 2^{\beta(d(x_{n(k)}, x_{m(k)}))d(x_{n(k)}, x_{m(k)})}.\end{aligned}$$

Hence,

$$\frac{d(x_{n(k)+1}, x_{m(k)+1})}{d(x_{n(k)}, x_{m(k)})} \leq \beta(d(x_{n(k)}, x_{(m(k)}) \leq 1.$$

Letting $k \to \infty$ in the above inequality, we get

$$\lim_{k\to\infty} \beta(d(x_{n(k)}, x_{m(k)})) = 1.$$

That is,

$$\lim_{k\to\infty} d(x_{n(k)}, x_{m(k)}) = 0.$$

a contradiction.
Hence $\{x_n\}$ is a Cauchy sequence. Since X is a complete metric like space, then there exists $u \in X$ such that

$$\lim_{n\to\infty} d(x_n, u) = d(u, u) = \lim_{n,m\to\infty} d(x_n, x_m) = 0. \tag{2.14}$$

Since T is continuous, from equation (2.14), we get

$$\lim_{n\to\infty} d(x_{n+1}, Tu) = \lim_{n\to\infty} d(Tx_n, Tu) = d(Tu, Tu). \tag{2.15}$$

Using Lemma 1.1 and equation (2.14), we obtain

$$\lim_{n\to\infty} d(x_{n+1}, Tu) = d(u, Tu). \tag{2.16}$$

Comparing equation (2.15) and (2.16), we get

$$d(u, Tu) = d(Tu, Tu).$$

From equation (2.9) and using above equality, we get

$$\begin{aligned} 2^{d(u,Tu)} &= 2^{d(Tu,Tu)} \\ &\leq (\alpha(u, Tu)\alpha(u, Tu) + 1)^{d(Tu,Tu)} \\ &\leq 2^{\beta(d(u,u))d(u,u)}. \end{aligned}$$

This implies that $d(u, Tu) \leq d(u, u)$.
Using equation (2.14), we get
$d(u, Tu) = 0$, that is, $Tu = u$. □

Theorem 2.3 *Let (X, d) be a complete metric like space and $T : X \to X$ be an α-admissible mapping. Assume that a function $\beta : [0, \infty) \to [0, 1]$, such that, for any bounded sequence $\{t_n\}$ of positive reals, $\beta(t_n) \to 1$ implies $t_n \to 0$ and*

$$\alpha(x, Tx)\alpha(y, Ty)d(Tx, Ty) \leq \beta(d(x, y))d(x, y). \tag{2.17}$$

for all $x, y \in X$.
Suppose that T is continuous, then T has a fixed point.

Proof: Let $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$. Construct a sequence $\{x_n\}$ in X such that $x_n = Tx_{n-1}$ for all $n \in \mathbb{N}$. If $x_{m+1} = x_m$ for some $m \in \mathbb{N}$, then $Tx_m = x_m$ and we are done. Hence we suppose that $x_n \neq x_{n+1}$ for any $n \in \mathbb{N}$.
Since T is α-admissible mapping so $\alpha(x_0, Tx_0) \geq 1$ implies that

$$\alpha(x_1, x_2) = \alpha(Tx_0, T^2x_0) \geq 1.$$

Continuing this process we get $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N}$.
By the equation (2.17), we have

$$\begin{aligned} d(Tx_{n-1}, Tx_n) &\leq (\alpha(x_{n-1}, Tx_{n-1})\alpha(x_n, Tx_n) + 1)d(Tx_{n-1}, Tx_n), \\ &\leq \beta(d(x_{n-1}, x_n))(d(x_{n-1}, x_n)), \end{aligned}$$

then

$$d(x_n, x_{n+1}) \leq \beta(d(x_{n-1}, x_n))d(x_{n-1}, x_n), \tag{2.18}$$

which implies that $d(x_n, x_{n+1}) \leq d(x_{n-1}, x_n)$. It follows that the sequence $\{d(x_n, x_{n+1})\}$ is a decreasing sequence of positive reals. So there exists $d \in [0, \infty)$ such that $\lim_{n\to\infty} d(x_n, x_{n+1}) = d$. We will prove that $d = 0$.
From equation (2.18), we get

$$\frac{d(x_n, x_{n+1})}{d(x_{n-1}, x_n)} \leq \beta(d(x_{n-1}, x_n)) \leq 1,$$

which implies $\lim_{n\to\infty}\beta(d(x_{n-1},x_n))=1$.
Using the property of β function, we conclude that

$$\lim_{n\to\infty} d(x_n,x_{n+1})=0. \tag{2.19}$$

Next, we will show that $\{x_n\}$ is a Cauchy sequence. Suppose, to the contrary, that $\{x_n\}$ is not a Cauchy sequence. Then there is $\epsilon>0$ and sequences $m(k)$ and $n(k)$ such that for all positive integer k, we have

$$n(k)>m(k)>k, d(x_{n(k)},x_{m(k)})\geq\epsilon$$

and

$$d(x_{n(k)},x_{m(k)-1})\leq\epsilon.$$

By triangle inequality, we get

$$\begin{aligned}\epsilon &\leq d(x_{n(k)},x_{m(k)})\\ &\leq d(x_{n(k)},x_{m(k)-1})+d(x_{m(k)-1},x_{m(k)})\\ &<\epsilon+d(x_{m(k)-1},x_{m(k)}).\end{aligned}$$

Taking limit as $k\to\infty$ in the above inequality and using equation (2.19), we get

$$\lim_{n\to\infty} d(x_{n(k)},x_{m(k)})=\epsilon. \tag{2.20}$$

Again, by triangle inequality we find that

$$d(x_{n(k)},x_{m(k)})\leq d(x_{m(k)},x_{m(k)+1})+d(x_{m(k)+1},x_{n(k)+1})+d(x_{n(k)+1},x_{n(k)})$$

and

$$d(x_{n(k)+1},x_{m(k)}+1)\leq d(x_{m(k)},x_{m(k)+1})+d(x_{m(k)},x_{n(k)})+d(x_{n(k)+1},x_{n(k)}).$$

Taking $k\to\infty$ and using equations (2.19) and (2.20), we get

$$\lim_{n\to\infty} d(x_{n(k)+1},x_{m(k)+1})=\epsilon. \tag{2.21}$$

From equations (2.18), (2.20) and (2.21), we have

$$\begin{aligned}d(x_{n(k)+1},x_{m(k)+1}) &\leq (d(x_{n(k)+1},x_{m(k)+1}))\alpha(x_{n(k)},Tx_{n(k)})\alpha(x_{m(k)},Tx_{m(k)})\\ &=(d(Tx_{n(k)},Tx_{m(k)}))\alpha(x_{n(k)},Tx_{n(k)})\alpha(x_{m(k)},Tx_{m(k)})\\ &\leq\beta(d(x_{n(k)},x_{m(k)}))d(x_{n(k)},x_{m(k)}).\end{aligned}$$

Hence,

$$\frac{d(x_{n(k)+1},x_{m(k)+1})}{d(x_{n(k)},x_{m(k)})}\leq\beta(d(x_{n(k)},x_{(m(k)})\leq 1.$$

Letting $k\to\infty$ in the above inequality, we get

$$\lim_{k\to\infty}\beta(d(x_{n(k)},x_{m(k)}))=1.$$

That is,

$$\lim_{k\to\infty} d(x_{n(k)},x_{m(k)})=0$$

a contradiction.
Hence $\{x_n\}$ is a Cauchy sequence. Since X is a complete metric like space, then there exists $u\in X$ such that

$$\lim_{n\to\infty} d(x_n,u)=d(u,u)=\lim_{n,m\to\infty} d(x_n,x_m)=0. \tag{2.22}$$

Since T is continuous, from equation (2.22), we get

$$\lim_{n\to\infty} d(x_{n+1}, Tu) = \lim_{n\to\infty} d(Tx_n, Tu) = d(Tu, Tu). \tag{2.23}$$

Using Lemma 1.1 and equation (2.22), we obtain

$$\lim_{n\to\infty} d(x_{n+1}, Tu) = d(u, Tu). \tag{2.24}$$

Comparing equation (2.23) and (2.24), we get

$$d(u, Tu) = d(Tu, Tu).$$

From equation (2.17) and using above equality, we get

$$\begin{aligned} d(u, Tu) &= d(Tu, Tu) \\ &\leq (\alpha(u, Tu)\alpha(u, Tu) + 1)d(Tu, Tu) \\ &\leq \beta(d(u, u))d(u, u). \end{aligned}$$

This implies that $d(u, Tu) \leq d(u, u)$.
Using equation (2.22), we get $d(u, Tu) = 0$, that is, $Tu = u$.
Now we omit continuity for Theorems 2.1, 2.2 and 2.3 by the following hypothesis:

(H) If $\{x_n\}$ is a sequence in X such that $x_n \to x$, $\alpha(x_n, x_{n+1}) \geq 1$ for all n, then $\alpha(x, Tx) \geq 1$. If there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$, then T has a fixed point. □

Theorem 2.4 *If in Theorems 2.1, 2.2 and 2.3, we replace continuity by the above hypothesis, then still T has a fixed point.*

Proof: Following the proof of Theorem 2.1, we get $\{x_n\}$ is convergent sequence, so there exists $u \in X$ such that $x_n \to u$. Then by hypothesis (H) $\alpha(u, Tu) \geq 1$.
So, by using equations (2.1) and (2.6), we obtain

$$\begin{aligned} d(x_{n+1}, Tu) + l &\leq (d(Tx_n, Tu) + l)^{\alpha(x_n, Tx_n)\alpha(x, Tx)} \\ &\leq \beta(d(x_n, u))d(x_n, u) + l. \end{aligned}$$

This implies that

$$d(x_{n+1}, Tu) \leq d(x_n, u).$$

Taking limits $n \to \infty$, we get

$$d(u, Tu) = 0.$$

So $Tu = u$.
Similarly, one can prove easily for Theorems 2.2 and 2.3. □

Theorem 2.5 *Assume that all the conditions of Theorems 2.1, 2.2, 2.3 and 2.4 hold. Adding the following condition:*

(c) *if $x = Tx$ then $\alpha(x, Tx) \geq 1$.*

We obtain the uniqueness of fixed point.

Proof: For Theorem 2.1, let us suppose u and v are two distinct fixed point of T. Then $\alpha(x, Tx) \geq 1$ and $\alpha(y, Ty) \geq 1$. Using equation (2.1), we get

$$\begin{aligned} d(Tu, Tv) + l &\leq (d(Tu, Tv) + l)^{\alpha(u, Tu)\alpha(v, Tv)} \\ &\leq \beta(d(u, v))d(u, v) + l. \end{aligned}$$

This implies that $\beta(d(u, v)) = 1$.
So $d(u, v) = 0$ implies $u = v$.
Similarly, one can prove for Theorems 2.2, 2.3 and 2.4. □

Example 2.1 *Let*$X = [0, \infty)$ *and* $d : [0, \infty) \times [0, \infty) \to [0, \infty)$.
Define $d(x, y) = \max\{x, y\}$, *and* $T : [0, \infty) \to [0, \infty)$ *as* $Tx = \frac{x}{2}$.
Clearly, (X, d) *is complete metric like space and* T *is continuous function.*
Define $\alpha(x, y) = 1$ *for all* $x, y \in [0, \infty)$ *and* $\beta(t) = \frac{3}{4}$.
Without loss of generality assume that $x \le y$.
Then left hand side of equation (2.1)

$$(d(Tx, Ty) + l)^{\alpha(x,Tx)\alpha(y,Ty)} = \frac{y}{2} + l. \tag{2.25}$$

Similarly, right hand side of equation (2.1)

$$\beta(d(x, y))d(x, y) + l = \frac{3y}{4} + l. \tag{2.26}$$

From equations (2.25) *and* (2.26)*, we obtain that all the conditions of Theorems 2.1 and 2.5.*
So, T *has a unique fixed point.*
Clearly, 0 *is the unique fixed point of* T.

Example 2.2 *Let* $X = [0, \infty)$ *and* $d : [0, \infty) \times [0, \infty) \to [0, \infty)$.
Define $d(x, y) = \max\{x, y\}$, *and* $T : [0, \infty) \to [0, \infty)$ *as* $Tx = \frac{x}{3}$.
Clearly (X, d) *is complete metric like space and* T *is continuous function.*
Define $\alpha(x, y) = 1$ *for all* $x, y \in [0, \infty)$ *and* $\beta(t) = \frac{1}{2}$.
Without loss of generality assume that $x \le y$.
Then left hand side of equation (2.10)

$$(\alpha(x, Tx)\alpha(y, Ty) + 1)^{d(Tx,Ty)} = 2^{\frac{y}{3}}. \tag{2.27}$$

Similarly, right hand side of equation (2.10)

$$2^{\beta(d(x,y))d(x,y)} = 2^{\frac{y}{2}}. \tag{2.28}$$

From equations (2.27) *and* (2.28)*, we obtain that all the conditions of Theorems 2.2 and2.5.*
So, T *has a unique fixed point.*
Clearly, 0 *is the unique fixed point of* T.

Example 2.3 *Let* $X = [0, \infty)$ *and* $d : [0, \infty) \times [0, \infty) \to [0, \infty)$.
Define $d(x, y) = \max\{x, y\}$, *and* $T : [0, \infty) \to [0, \infty)$ *as* $Tx = \frac{x}{5}$.
Clearly (X, d) *is complete metric like space and* T *is continuous function.*
Define $\alpha(x, y) = 1$ *for all* $x, y \in [0, \infty)$ *and* $\beta(t) = \frac{1}{3}$.
Without loss of generality assume that $x \le y$.
Then left hand side of equation (2.17)

$$\alpha(x, Tx)\alpha(y, Ty)d(Tx, Ty) = \frac{y}{5}. \tag{2.29}$$

Similarly, right hand side of equation (2.17)

$$\beta(d(x, y))d(x, y) = \frac{y}{3}. \tag{2.30}$$

From equations (2.29) *and* (2.30)*, we obtain that all the conditions of Theorems 2.3 and 2.5.*
So T *has a unique fixed point.*
Clearly, 0 *is the unique fixed point of* T.

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