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Prime Weakly Standard Ring

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ABSTRACT: In this paper, we prove that a prime weakly standard ring is either a (-1,1) ring or a commutative ring. In general, weakly standard rings are nonassociative rings which are not (-1, 1) rings, but by applying some additional conditions we prove that these are (-1,1) rings.

Key Words: Weakly standard rings, Prime rings, (-1,1) rings, Right alternative rings and Commutative Rings.

Contents

1. Introduction

Weakly standard rings were introduced by Sansoucie [3]. Thedy [4] proved that a simple 2-divisible nonassociative ring satisfying the identity $((x, y, z), w) = 0$. is either commutative or associative. Kleinfeld [2] proved that the identity $((x, y, z)v, w) = 0$ holds in accessible rings under the assumption that the rings are without nilpotent elements in the center. Without this assumption we show that the identity $((x, y, y)v, w) = 0$hold in prime weakly standard rings. Also, we show that all commutators are in the center of a prime weakly standard ring. Using these identities, we prove that a prime weakly standard ring is either a (- 1,1) ring or a commutative ring.

2. Preliminaries

A nonassociative ring R [3] is said to be a weakly standard ring, if it satisfies the following identities:

$$(x, y, x) = 0. \tag{2.1}$$

$$((w, x), y, z) = 0. \tag{2.2}$$

and

$$(w, (x, y), z) = 0. \tag{2.3}$$

Definition : A (-1, 1) ring is a nonassociative ring in which the right alternative law $(x, y, y) = 0$, i.e., $(x, y, z) + (x, z, y) = 0$, and $(x, y, z) + (y, z, x) + (z, x, y) = 0$ hold.

Throughout this paper R denotes a weakly standard ring. The nucleus N of R is the set of all elements n in R such that $(n, R, R) = (R, R, n) = (R, n, R) = 0$. If we define $S(x, y, z) = (x, y, z)+(y, z, x)+(z, x, y)$, we have the following identities in any ring [1].

$$(wx, y, z) - (w, xy, z) + (w, x, yz) = w(x, y, z) + (w, x, y)z \tag{2.4}$$

$$(xy, z) - x(y, z) - (x, z)y = (x, y, z) + (z, x, y) - (x, z, y) \tag{2.5}$$

$$(xy, z) + (yz, x) + (zx, y) = S(x, y, z) \tag{2.6}$$

$$((x, y), z) + ((y, z), x) + ((z, x), y) = S(x, y, z) - S(x, z, y) \tag{2.7}$$

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Substituting $z = x$ in (2.5), we get

$$(xy, x) + x(x, y) = 0 \tag{2.8}$$

Substituting $w = n$ in (2.4) we obtain $(nx, y, z) = n(x, y, z)$.

From (2.2), we get $((n, x), y, z) = 0$

$$(nx, y, z) = n(x, y, z) = (xn, y, z) \tag{2.9}$$

In any ring the associator ideal A is the set of all sums of elements of the form $(R, R, R) + (R, R, R)R$. A ring R is prime, if A and B are ideals of R such that $AB = 0$ then either $A = 0$ or $B = 0$. A linearization of the flexible law yields the identity, $(x, y, z) = -(z, y, x)$.

3. Main Results

To prove the result first we prove the following Lemmas.

Lemma 1: If R is a prime nonassociative weakly standard ring, then all commutators are in the center.

Proof: Forming associators of (2.8) and using (2.4) we obtain
$((xy, x), r, s) + (x(x, y), r, s) = 0$
i.e, $(x(x, y), r, s) = 0$
i.e, $((x, y)x, r, s) = (x, y)(x, r, s) = 0$
Linearizing the above equation with$x = x + a$, we obtain
$(x, y)(a, r, s) + (a, y)(x, r, s) = 0.$
If we substitute a commutator v for a, we see that
$(x, y)(v, r, s) + (v, y)(x, r, s) = 0. i.e,$
$(v, y)(x, r, s) = 0.$
This can be restated as $((R, R), R)(R, R, R) = 0$. But now the ideal generated by the double commutators $((R, R), R)$ annihilates the associator ideal. Since $R$is prime and not associative, we conclude that $((R, R), R) = 0$.
Hence in a prime weakly standard ring all commutators are in the center.

Lemma 2: In a 2-divisible weakly standard ring $R, S(x, y, z) = 0$. Proof: If we put $y = x$ in (2.1), then $(x, x, x) = 0$.
If we linearize$(x, x, x) = 0$, we get
$S(x, y, z) + S(x, z, y) = 0.$
By applying Lemma 1 on (2.7), we get
$S(x, y, z) - S(x, z, y) = 0.$
So that $2S(x, y, z) = 0$, since R is 2 divisible then $S(x, y, z) = 0$.
Now we develop further identities which hold in weakly standard rings.
The elementsu, v, w, x, y, z are arbitrary elements of weakly standard rings.
From (2.5) and using $S(x, y, z) = 0$ we get

$$(xy, z) = x(y, z) + (x, z)y \tag{3.1}$$

Through repeated use of (3.1), we break up$((w, x, y), z)$ as
$((w, x, y), z) = (wx.y – w.xy, z)$
$= wx.(y, z) + w(x, z).y + (w, z)x.y – (w, z).xy – w.x(y, z) – w.(x, z)y$
$= (w, x, (y, z)) + (w, (x, z), y) + ((w, z), x, y).$
Since (2.2) implies that every commutator is in nucleus, we obtain$((w, x, y), z) = 0$

$$((w, x, y), z) = 0 \tag{3.2}$$

Hence every associator commutes with every element of R. Because of (2.9) and the fact that every commutator is in the nucleus, we get
$(v, x)(x, y, z) = ((v, x)x, y, z)$

It follows from (3.1) that $(v, x)x = (vx, x)$. Consequently $((v, x)x, y, z) = ((vx, x), y, z) = 0$.
Thus

$$(v, x)(x, y, z) = 0 \tag{3.3}$$

Lemma 3: In a weakly standard ring $R, ((x, y, y)v, w) = 0$.

Proof: Linearization of (3.3) becomes

$$(v, w)(x, y, z) = -(v, x)(w, y, z). \tag{3.4}$$

By using flexible law (2.1), (3.4), $S(x, y, z) = 0$ and (3.3) we obtain
$(v, w)(x, y, y) = -(v, w)(y, y, x)$
$= (v, y)(w, y, x)$
$= (v, y)(-(y, x, w)-(x, w, y))$
$= -(v, y)(y, x, w)-(v, y)(x, w, y)$
$= -(v, y)(y, x, w) + (v, y)(y, w, x)$
$= 0.$

$$i.e, (v, w)(x, y, y) = 0. \tag{3.5}$$

Now from (3.1), (3.4) and (3.2),we get
$((x, y, y)v, w) = (x, y, y)(v, w) + ((x, y, y), w)v = 0.$

Lemma 4: Let R be a weakly standard ring, then

$$S = s \in R/(s, R) = 0 = (sR, R)$$

is an ideal of R.

Proof: If we put $w = s$ in (11), then $((s, x, y), z) = 0$.
From this it follows that
$(sx.y, z)-(s.xy, z) = 0$.
From definition of $S, (sx.y, z) = 0$.
Thus $sx \in S$. So S is a right ideal of R.
Since $(s, R) = 0, (s, x) = 0$. That is, $sx = xs \in S$. So S is a left ideal of R. Hence S is an ideal of R.

Lemma 5: If B is the set of all finite sums of elements of the form $(R, R) + R(R, R)$, then B is an ideal of R.
Proof :- Let $B = (R, R) + R(R, R)$.

$$\forall x, y, z \in R, x(y, z) \in B.$$

Since every commutator is in the nucleus, $(x, (y, z), r) = 0$.
$x(y, z).r = x.(y, z)r$
$= x.(yr, z)-x.y(r, z)$
$= x.(yr, z)-xy.(r, z) \in B$.
Thus, B is a right ideal of R.
Now again using every commutator is in the nucleus we have,
$(r, x, (y, z)) = 0,$
$r.x(y, z) = rx.(y, z) \in B$.
That is $r.x(y, z) \in B$, so that B is left ideal of R.
Hence B is an ideal of R.

Theorem: If R is a prime 2-divisible weakly standard ring then R is either a (-1,1) ring or a commutative ring.

Proof: Substituting $x = s$ in (10), we obtain $(sy, z) = s(y, z) + (s, z)y$.

From definition of S, $(sy, z) = 0and(s, z)y = 0$. Thus $s(y, z) = 0$.

Hence $SB = 0$. From (3.2) and Lemma(3), (x, y, y) is in S.

Since R is prime, if S and B are ideals of R such that $SB = 0$, then either $S = 0orB = 0$. If $S = 0$, then R is right alternative, that is $(x, y, y) = 0$.

Linearization of this yields $(x, y, z) + (x, z, y) = 0$.

From this and $S(x, y, z) = 0$, it follows that R is a (-1,1) ring.

If $B = 0$ Then R is Commutative.

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