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μ-Statistical Relative Uniform Convergence of Sequences of Functions

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Abstract: In this paper we have introduced the notion of convergence in μ-density and μ- statistical relative uniform convergence of sequences of functions defined on a compact subset $\mathcal{D}$ of real numbers, where μ is finitely additive measure. We introduce the concept μ- statistical relative uniform convergence which inherits the basic properties of uniform convergence.

Key Words: Pointwise convergence, uniform convergence, μ-statistical convergence, convergence in μ-density, finitely additive measure, additive property for null sets, μ- statistical relatively uniform convergence.

Contents

1. Introduction

The idea of statistical convergence was introduced by Steinhaus [19] and Fast [12] independently. If $\mathcal{K}$ is a subset of $\mathcal{N}$, the set of natural numbers, then the asymptotic density of $\mathcal{K}$, denoted by $\delta(\mathcal{K})$, is given by

$$\delta(\mathcal{K}) = \lim_{n\to\infty} \frac{1}{n}|\{k \leq n : k \in \mathcal{K}\}|,$$

whenever the limit exists, where $|\mathcal{B}|$ denotes the cardinality of the set $\mathcal{B}$. A sequence $x = (x_k)$ of real or complex numbers is said to be statistically convergent to L if $\delta(\{k : |x_k - L| \geq \varepsilon\}) = 0$ for every $\varepsilon > 0$. In this case we write $st - \lim x_k = L$ or $x_k \to L(stat)$. It is to be noted that convergent sequences are statistically convergent but not necessarily conversely.

Later on the concept of statistical convergence was investigated by many researcher from different aspects. It can be found in a number of recent papers, Connor [3,4], Datta and Tripathy [5], Demirci and Orhan [6], Fridy [13], Saha and Tripathy [16], Salat [17], Rath and Tripathy [20], Tripathy [21], Tripathy and Sen [24], Tripathy and Goswami [22], Tripathy and Nath [23] and others. Connor [3] has extended the notion of statistical convergence, where the asymptotic density is replaced by finitely additive set function. In this paper, μ denotes a finitely additive set function taking values in $[0, 1]$ defined on a field Γ of subsets of $\mathcal{N}$ such that if $|\mathcal{A}| < \infty$ then $\mu(\mathcal{A}) = 0$; if $\mathcal{A} \subseteq \mathcal{B}$ and $\mu(\mathcal{B}) = 0$, then $\mu(\mathcal{A}) = 0$; $\mu(\mathcal{N}) = 1$. Such a set function satisfying the above criteria will be called measure.

We have procured the followings from Connor [3,4].

(i) x is μ-density convergent to L if there is an $\mathcal{A} \in \Gamma$ such that $(x - L)\chi_{\mathcal{A}}$ is a null sequence and $\mu(\mathcal{A}) = 1$ where $\chi_{\mathcal{A}}$ is the characteristic function of $\mathcal{A}$.
(ii) x is μ-statistically convergent to L and write $st_\mu - \lim x = L$, provided
$\mu(\{k \in \mathcal{N} : |x_k - L| \geq \varepsilon\}) = 0$, for every $\varepsilon > 0$.
If $\mathcal{T} = (t_{nk})$ is a non-negative regular summability method, then $\mathcal{T}$ is used to generate a measure as follows:

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For each $n \in \mathcal{N}$, set $\mu_n(\mathcal{A}) = \sum_{k=1}^{\infty} t_{nk}\chi_{\mathcal{A}}(k)$, for each $\mathcal{A} \subseteq \mathcal{N}$. Let $\Gamma = \{\mathcal{A} \subseteq \mathcal{N} : \lim_n \mu_n(\mathcal{A}) = 0 \ \ or \ \ \lim_n \mu_n(\mathcal{A}) = 1\}$.
Define, $\mu_T : \Gamma \to [0,1]$ by

$$\mu_{\mathcal{T}}(\mathcal{A}) = \lim_{n\to\infty} \mu_n(\mathcal{A}) = \lim_{n\to\infty} \sum_{k=1}^{\infty} t_{nk}\chi_{\mathcal{A}}(k),$$

where $\mu_{\mathcal{T}}$ and Γ satisfy the requirements of the preceding definitions. If $\mathcal{T}$ is a Cesàro matrix of order one, the $\mu_{\mathcal{T}}$-statistical convergence is equivalent to statistical convergence.

Connor [3] has established that (i) implies (ii), but not necessarily conversely. These two definitions are equivalent (Connor [3,4]), if μ has additive property for null sets: if given a collection of null sets $\{\mathcal{A}_j\}_{j\in\mathcal{N}} \subseteq \Gamma$, there exists a collection $\{\mathcal{B}_i\}_{i\in\mathcal{N}}$ with the properties $|\mathcal{A}_i \Delta \mathcal{B}_i| < \infty$, for each $i \in \mathcal{N}$, $\mathcal{B} = \bigcup_{i=1}^{\infty} \mathcal{B}_i \in \Gamma$ and $\mu(\mathcal{B}) = 0$, where Δ denotes the symmetric difference of $\mathcal{A}_i$ and $\mathcal{B}_i$, for each $i \in \mathcal{N}$.

Moore [15] was the first who introduced the notion of relative uniform convergence of sequences of functions. Thereafter, Chittenden [1] studied the notion (which is equivalent to Moore's definition)as follows:

A sequence of functions (g_n), defined on $\mathcal{J} = [a,b]$ converges relatively uniformly to a limit function g if there is a function $\gamma(t)$ called a scale function such that for every $\varepsilon > 0$, there exists an integer $m = m(\varepsilon)$ such that
$|g_n(t) - g(t)| < \varepsilon|\gamma(t)|$, uniformly in t on $\mathcal{J}$, for all $n \geq m$.
.

On the basis of this definition, Demirci and Orhan [6] and Dirik and Sahin [10] introduced the concepts of statistical relatively uniform convergence and statistical relatively equal convergence respectively and used to establish approximation results. This was followed by the works on relative uniform convergence of sequences by Devi and Tripathy [7,8,9].

Duman et al. [11] introduced convergence in μ-density and μ-statistical convergence of sequence of functions defined on a subset $\mathcal{D}$ of $\mathcal{R}$, the set of real numbers. They also introduced μ- statistical uniform convergence and μ-statistical pointwise convergence. Gürdal [14] studied μ-statistical characterization of normed and inner spaces. In this paper we introduce the concept of μ-statistical relative uniform convergence and observe that μ-statistical uniform convergence inherits the basic properties of uniform convergence. The notion of μ- statistical convergence was studied by Sen et al. [18].

Let $\mathcal{D} = [a,1] \subseteq \mathcal{R}$, where $0 < a < 1$ and (f_n) be a sequence of real functions on $\mathcal{D}$.

Now we recall some definitions used in this paper.

Definition 1.1 *The sequence of functions* (f_n) *converges* μ- *density pointwise to* f *if and only if for a given* $\varepsilon > 0$, *for all* $x \in \mathcal{D}$, *there exists* $\mathcal{K}_x \in \Gamma$, $\mu(\mathcal{K}_x) = 1$ *and there exists* $n_0 = n_0(\varepsilon, x) \in \mathcal{K}_x$ *such that for all* $n \geq n_0$ *and* $n \in \mathcal{K}_x$, $|f_n(x) - f(x)| < \varepsilon$.
In this case we write $f_n \to f(\mu - density)$ *on* $\mathcal{D}$.

Definition 1.2 *The sequence of functions* (f_n) *converges* μ-*density uniform to* f *if and only if for a given* $\varepsilon > 0$ *and there exists* $K \in \Gamma$, $\mu(\mathcal{K}) = 1$ *and there exists* $n_0 = n_0(\varepsilon) \in \mathcal{K}$ *such that for all* $n \geq n_0$ *and* $n \in \mathcal{K}$ *and for every* $x \in \mathcal{D}$, $|f_n(x) - f(x)| < \varepsilon$.
In this case we write $f_n \to f(\mu - density)$ *on* $\mathcal{D}$.

Definition 1.3 *The sequence of functions* (f_n) *converges* μ-*density relatively uniform to* f *if and only if there exists a function* $\sigma(x)$ *called scale function with* $|\sigma(x)| > 0$ *and for a given* $\varepsilon > 0$ *and* $\mathcal{K} \in \Gamma$, $\mu(\mathcal{K}) = 1$ *there exists* $n_0 = n_0(\varepsilon) \in \mathcal{K}$ *such that for all* $n \geq n_0$ *and* $n \in \mathcal{K}$ *and for each* $x \in \mathcal{D}$,

$$\left|\frac{f_n(x) - f(x)}{\sigma(x)}\right| < \varepsilon.$$

In this case we will write $f_n \rightrightarrows f(\mathcal{D};\sigma)(\mu - density)$

Definition 1.4 *The sequence of functions* (f_n) *converges* μ*-statistically pointwise to* f *if and only if for every* $\varepsilon > 0$ *and for each* $x \in \mathcal{D}$, $\mu(\{n : |f_n(x) - f(x)| \geq \varepsilon) = 0$
It is written as $f_n \rightarrow f(\mu - stat)$ *on* $\mathcal{D}$.

Definition 1.5 *The sequence* (f_n) *of bounded functions on* $\mathcal{D}$ *converges* μ*-statistically uniform to* f *if and only if* $st_\mu - \lim \|f_n - f\|_{\mathcal{B}} = 0$, *where the norm* $\|.\|_{\mathcal{B}}$ *is the usual supremum norm on* $\mathcal{B}(\mathcal{D})$, *the space of bounded functions on* $\mathcal{D}$.
It is written as $f_n \rightrightarrows f(\mu - stat)$ *on* $\mathcal{D}$.

Definition 1.6 *The sequence of functions* (f_n) *converges relatively uniform to* f *if there exists a function* $\sigma(x)$ *such that for every* $\varepsilon > 0$, *there exists an integer* n_ε *and for every* $n > n_\varepsilon$, *the inequality* $|f_n(x) - f(x)| < \varepsilon|\sigma(x)|$ *holds uniformly in* x. *The sequence* (f_n) *is said to converge uniformly relative to the scale function* $\sigma(x)$ *or more simply, relatively uniformly.*

It is observed that uniform convergence is the special case of relatively uniform convergence in which scale function is a nonzero constant.

2. μ-statistical relative uniform convergence of sequences of functions

In this section we introduced the following definition and established the results of this article.

Definition 2.1 *The sequence of functions* (f_n) *converges* μ*-statistically relatively uniform to* f *if and only if there exists a function* $\sigma(x)$ *with* $|\sigma(x)| > 0$ *called scale function* $\sigma(x)$ *such that for every* $\varepsilon > 0$,

$$\mu\{n \in \mathcal{N} : \sup_{x\in\mathcal{D}} |\tfrac{f_n(x)-f(x)}{\sigma(x)}| \geq \varepsilon\} = 0.$$

In this case we will write $f_n \rightrightarrows f(\mathcal{D};\sigma)(\mu - stat)$.

Lemma 2.1 *The sequence of functions* (f_n) *converges* μ*-density uniformly on* $\mathcal{D}$ *implies*

$f_n \rightrightarrows f(\mathcal{D};\sigma)(\mu - density)$ *on* $\mathcal{D}$, *which implies* $f_n \rightrightarrows f(\mathcal{D};\sigma)(\mu - stat)$.

Remark 2.1

The converse of Lemma 2.1 is not necessarily true in general. We discuss the following example.

Example 2.1

$$f_n(x) = \begin{cases} 0, & x = 0; \\ \frac{2n^2x}{1+n^3x^2}, & x \neq 0. \end{cases}$$

Define

$$\sigma(x) = \begin{cases} 1, & x = 0; \\ \frac{1}{x}, & x \in (0,1]. \end{cases}$$

Then, $f_n \rightarrow \theta(\mathcal{D};\sigma)(\mu - density)$. Hence, $f_n \rightarrow \theta(\mathcal{D};\sigma)(\mu - stat)$. But (f_n) is not μ-statistical uniformly convergent to $f = \theta$ in $[0,1]$, where θ is the null function.

Theorem 2.1 *Let the sequence of functions* (f_n) *be each continuous on* $\mathcal{D}$, *a compact subset of* $\mathcal{R}$ *and let* μ *be a measure with additive property for null sets. If*
$f_n \rightrightarrows \theta(\mathcal{D};\sigma)(\mu - stat)$ *on* $\mathcal{D}$ *and* $\sigma(x)$ *is continuous, then* f *is continuous on* $\mathcal{D}$.

Proof: By hypothesis, $\mathcal{D}$ is a compact subset of $\mathcal{R}$ and for each $n \in \mathcal{N}$, f_n is a continuous function. Thus it is clear that for each $n \in \mathcal{N}$, f_n is bounded on $\mathcal{D}$. Hence, there exists $M > 0$, where $M = \sup_{x \in \mathcal{D}}\{f_n(x)\}$ such that $|f_n(x)| \leq M$.

Also, $\sigma(x)$ continuous implies it is bounded. So there exists $G > 0$ such that $|\sigma(x)| \leq G$. Let, $\mathcal{L} = \max(\mathcal{M}, G)$ and $f_n \rightrightarrows \theta(\mathcal{D};\sigma)(\mu - stat)$. Then for every $\varepsilon > 0$,

$$\mu\left\{n \in \mathcal{N} : \sup_{x \in \mathcal{D}}\left|\frac{f_n(x) - f(x)}{\sigma(x)}\right| \geq \frac{\varepsilon}{3L}\right\} = 0.$$

Let $x_0 \in \mathcal{D}$. Since (f_n) is continuous for each $n \in \mathcal{N}$ at $x_0 \in \mathcal{D}$, there exists a $\delta > 0$ such that $|x - x_0| < \delta$ implies $|f_n(x) - f(x)| < \frac{\varepsilon}{3}$, for each $x \in \mathcal{D}$. Thus for all $x \in \mathcal{D}$, for which $|x - x_0| < \delta$ we have

$$|f(x) - f(x_0)| \leq |f(x) - f_n(x)| + |f_n(x) - f_n(x_0)| + |f_n(x_0) - f(x_0)| < \frac{\varepsilon}{3} + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = \varepsilon.$$

Since, $x_0 \in \mathcal{D}$ is arbitrary, so f is continuous on $\mathcal{D}$. □

In view of the above theorem we state the following result.

Corollary 2.1 *Let the terms of the sequence of functions (f_n) be continuous on a compact subset $\mathcal{D}$ of $\mathcal{R}$. If $f_n \to f(\mathcal{D};\sigma)(\mu - density)$ on $\mathcal{D}$, then f is continuous on $\mathcal{D}$.*

Theorem 2.2 *A necessary and sufficient condition for a real valued continuous function f on $\mathcal{D}$ be the μ-stat relative uniform limit of a sequence of real valued continuous functions f_n is that there exists a sequence $(\mathcal{D}_r)$ of subsets of $\mathcal{D}$ such that $\mathcal{D} = \bigcup_{j=1}^{\infty} \mathcal{D}_j$ and the restricted functions $f_{j|\mathcal{D}_j|}$ converges $\mu-stat$ relative uniformly to f.*

Proof: Let, f_n be convergent μ-statistically relative uniform to f. To prove that $f_{j|\mathcal{D}_j|}$ converges μ-statistically relative uniformly to f.

By hypothesis, there exists a scale function $\sigma(x)$ such that $|\sigma(x)| > 0$ and for every $\varepsilon > 0$ we have,

$$\mu\left\{n \in \mathcal{N} : \sup_{x \in \mathcal{D}}\left|\frac{f_n(x) - f(x)}{\sigma(x)}\right| \geq \varepsilon\right\} = 0.$$

By definition of restriction function, we have

$$f_{n|\mathcal{D}_n|}(x) = f_n(x), \qquad \text{for all } x \in \mathcal{D}_n.$$

Also, $\mathcal{D}_n \subseteq \mathcal{D}$ shows that

$$\mu\left\{n \in \mathcal{N} : \sup_{x \in \mathcal{D}_n}\left|\frac{f_n(x) - f(x)}{\sigma(x)}\right| \geq \varepsilon\right\} \leq \mu\left\{n \in \mathcal{N} : \sup_{x \in \mathcal{D}}\left|\frac{f_n(x) - f(x)}{\sigma(x)}\right| \geq \varepsilon\right\}$$
$$\implies \mu\left\{n \in \mathcal{N} : \sup_{x \in \mathcal{D}_n}\left|\frac{f_n(x) - f(x)}{\sigma(x)}\right| \geq \varepsilon\right\} = 0.$$

Conversely, let each $f_{j|\mathcal{D}_j|}$ converges $\mu - stat$ relative uniformly to f. We established that (f_n) converges μ- statistical relative uniform to f.

Here, $\mathcal{D} = \bigcup_{j=1}^{\infty} \mathcal{D}_j$. Therefore,

$$\mu\left\{n \in \mathbb{N} : \sup_{x \in \mathcal{D}}\left|\frac{f_n(x) - f(x)}{\sigma(x)}\right| \geq \varepsilon\right\} \leq \sum_{n=1}^{\infty} \mu\left\{n \in \mathbb{N} : \sup_{x \in \mathcal{D}_n}\left|\frac{f_n(x) - f(x)}{\sigma(x)}\right| \geq \varepsilon\right\}.$$

Hence, we have
$\mu\left\{n\in\mathbb{N}:\sup_{x\in\mathcal{D}}\left|\frac{f_n(x)-f(x)}{\sigma(x)}\right|\geq\varepsilon\right\}=0,$
since $\mu\left\{n\in\mathbb{N}:\sup_{x\in\mathcal{D}_n}\left|\frac{f_n(x)-f(x)}{\sigma(x)}\right|\geq\varepsilon\right\}=0.$
This completes the proof. □

Throughout $\mathcal{C}(\mu,ru)$ will be used to denote the space of all continuous μ-statistical relative uniform convergent sequences of real functions defined on $\mathcal{D}$ and the scale function is continuous. Now we are in a position to define a norm on this class of sequences as follows:
Let, $(f_n)\in\mathcal{C}(\mu,ru)$, define

$$\|(f_n)\|=\sup_{n\geq1}\sup_{x\in\mathcal{D}}\frac{|f_n(x)|}{|\sigma(x)|},\ \text{with } \|x\|=1. \tag{2.1}$$

Now we prove the following result.

Theorem 2.3 *The class of sequences $\mathcal{C}(\mu,ru)$ is a Banach space with respect to the norm defined by* (2.1).

Proof: First we establish that $\mathcal{C}(\mu,ru)$ is a normed linear space. Let $(f_n),(g_n)\in\mathcal{C}(\mu,ru)$

$$\text{(N1) } \|(f_n)\|\geq0 \text{ and } \|(f_n)\|=0 \text{ if and only if } f_n=\theta,\ \text{for each } n\in\mathcal{N}.$$

$$\begin{aligned}\text{(N2) } \|(f_n)+(g_n)\|&=\|(f_n+g_n)\|\\&=\sup_{n\geq1}\sup_{x\in\mathcal{D}}\frac{|(f_n+g_n)(x)|}{|\sigma(x)|}\\&\leq\sup_{n\geq1}\sup_{x\in\mathcal{D}}\frac{|f_n(x)|}{|\sigma(x)|}+\sup_{n\geq1}\sup_{x\in\mathcal{D}}\frac{|g_n(x)|}{|\sigma(x)|}\\&=\|(f_n)\|+\|(g_n)\|.\end{aligned}$$

$$\text{(N3) } \|\alpha(f_n)\|=\sup_{n\geq1}\sup_{x\in\mathcal{D}}\frac{|\alpha f_n(x)|}{|\sigma(x)|}=\sup_{n\geq1}\sup_{x\in\mathcal{D}}\frac{|\alpha||f_n(x)|}{|\sigma(x)|}=|\alpha|\|(f_n)\|.$$

In order to show that $\mathcal{C}(\mu,ru)$ is complete with respect to the above norm, we start with a Cauchy sequence $(f^{(n)})$, where $f^{(n)}=(f_{n1},f_{n2},f_{n3},...)$. Then by definition of Cauchy sequence, for a given $\varepsilon>0$, there exists a positive integer $n_0\in\mathcal{N}$ such that for all $m,n\geq n_0$,

$$\|(f^{(n)})-(f^{(m)})\|<\varepsilon,\ \text{for all } n,m\geq n_0$$

$$\begin{aligned}&\Longrightarrow \sup_{i\geq1}\sup_{x\in\mathcal{D}}\frac{|(f_{ni}-f_{mi}(x)|}{|\sigma(x)|}<\varepsilon\\&\Longrightarrow \frac{|(f_{ni}-f_{mi}(x)|}{|\sigma(x)|}<\varepsilon,\ \text{with } \|x\|\leq1\\&\Longrightarrow |(f_{ni}-f_{mi})(x)|<\varepsilon|\sigma(x)|.\end{aligned} \tag{2.2}$$

Thus, $(f_{ni}(x))$ is a Cauchy sequence of reals, hence it is convergent.
Let, $\lim_{n\to\infty}f_{ni}(x)=f_i(x)$, for all $x\in\mathcal{D}$. Keeping n fixed and taking $m\to\infty$ in (2.2) we have,

$$|(f_{ni}-f_i)(x)|<\varepsilon|\sigma(x)|<\varepsilon,\ \text{for all } n\geq n_0. \tag{2.3}$$

$$\implies \sup_{x\in\mathcal{D}} \frac{|(f_{ni}-f_i)(x)|}{|\sigma(x)|} < \varepsilon$$
$$\implies \lim_{n\to\infty} \sup_{x\in\mathcal{D}} \frac{|(f_{ni}-f_i)(x)|}{|\sigma(x)|} = 0.$$

Thus, $f_{ni} \to f_i$ uniformly on $\{x \in \mathcal{D} : \|x\| = 1\}$, implying f_{ni} converges relatively uniformly to f_i. Hence, we have

$$f_{ni} \rightrightarrows f_i(\mathcal{D};\sigma)(\mu - stat).$$

Thus, there exists a scale function $\sigma(x)$ such that $|\sigma(x)| > 0$ and for every $\varepsilon > 0$, we have

$$\mu\left\{n \in \mathcal{N} \colon \sup_{x\in\mathcal{D}} |\frac{f_{ni}(x) - f_i(x)}{\sigma(x)}| \geq \varepsilon\right\} = 0.$$

Let, $f = (f_r)$. Then from (2.3) we have,

$$\sup_{i\geq 1}\sup_{x\in\mathcal{D}} \frac{|(f_{ni}-f_i)(x)|}{|\sigma(x)|} < \varepsilon.$$

$\implies \|(f^{(n)}) - (f)\| < \varepsilon$, for all $n \geq n_0$.

Now $(f) = \left\{(f) - (f^{(n)})\right\} + (f^{(n)}) \in \mathcal{C}(\mu, ru)$.

Hence, $(f^{(n)}) \to (f)$ in $\left(\mathcal{C}(\mu, ru), \|.\|\right)$. This completes the proof. □

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