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Exploration of pre-open sets in a fuzzy bitopological space via operation approach

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ABSTRACT: In this article, we propose an operation $(i,j)^*_\gamma$ on the set of all $(i,j)^*$-fuzzy pre-open set in a given fuzzy bitopological space (X,τ_i,τ_j). Using the newly introduced operation, we initiate the $(i,j)^*_\gamma$-fuzzy pre-open set and characterize it upto some extent. Also, we explore $(i,j)^*_\gamma$-fuzzy pre-open set in the light of minimality. Finally, we study locally finiteness of a fuzzy bitopological space via $(i,j)^*_\gamma$-fuzzy pre-open set.

Key Words: $(i,j)^*_\gamma$-operation, $(i,j)^*_\gamma$-fuzzy open set, $(i,j)^*_\gamma$-fuzzy pre-open set, minimal $(i,j)^*_\gamma$-fuzzy pre-open set, $(i,j)^*_\gamma$-fuzzy locally finite pre-open set.

Contents

1. Introduction

Najastad [19] initiated α-open set in topological structure and open sets are extended to the concepts like semiopen, pre-open sets by Levine [16] Mashhour [17] and Andrijevic [1]. The study of operation on topological spaces was first explored by Kasahara [15] and also he defined α-closed graphs of an operation. Furthermore, Andrijevic [2] showed that the class of all pre-open sets generates a topology and he proposed the respective closure and interior. Ogata [20,21] introduced the notion γ-operation, τ_γ in a topology (X,τ). Again, maximal open sets in generalized topology was studied by Roy and Sen [24], whereas minimal open sets in topological spaces was initiated and studied by Nakaoka and Oda [18] . Also, the concept of minimality of open sets in fuzzy environment was studied by Ghour [12]. Carpintero et al. [4] did the same study on the class of all b-open sets, whereas Tahiliani [25] studied the same on the class of all β-open sets in a topological space. Very recently, Hussain [13] introduced operations in generalized closed sets with the same approach and studied some of its applications. In literature, there are several interesting research work on this concept in various environments [3,23,6]. In this current study, we extend the notion of operations in fuzzy sense on pre-open sets in fuzzy bitopological spaces. We define and characterize the concept of $(i,j)^*_\gamma$-fuzzy pre-open set through operation approach in a fuzzy bitopological space extensively and then established the concept of minimal $(i,j)^*_\gamma$-fuzzy pre-open set therein.

We brief a fuzzy bitopological space by FBTS and is denoted by (X,τ_i,τ_j). Before proceeding to the main objective, we require some preparatory concepts which are recalled in this section as follows :

Definition 1.1 *[14] Let τ_1,τ_2 be any two fuzzy topologies defined on a non-empty set X. Then, (X,τ_1,τ_2) is said to be a FBTS.*

Definition 1.2 *[10] In a FBTS (X,τ_i,τ_j), a fuzzy set δ of X is called a (i,j)-fuzzy open set if either $\delta\in\tau_i$ or $\delta\in\tau_j$.*

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In this case, (i,j)-$FO(X)$ and (i,j)-$FC(X)$ denote the set of all (i,j)-fuzzy open sets and (i,j)-fuzzy closed sets respectively.

Definition 1.3 *[9] In a FBTS (X,τ_i,τ_j), a fuzzy set δ of X is called a $(i,j)^*$-fuzzy open set if δ can be expressed as $\delta=\mu_1\vee\mu_2$, where $\mu_1\in\tau_i$ and $\mu_2\in\tau_j$.*
Also here, $(i,j)^*$-$FO(X)$ and $(i,j)^*$-$FC(X)$ denote the set of all $(i,j)^*$-fuzzy open sets and $(i,j)^*$-fuzzy closed sets respectively.

Definition 1.4 *[7] In a FBTS (X,τ_i,τ_j), a fuzzy subset δ is a $(i,j)^*$-fuzzy pre-open if $\delta\le(i,j)^*$-$int((i,j)^*$-$cl(\delta))$.*

Definition 1.5 *[8] A fuzzy subset δ in a FBTS (X,τ_i,τ_j) is said to be $(i,j)^*$-fuzzy γ-open if $\delta\wedge\lambda$ gives a $(i,j)^*$-fuzzy pre-open, for each $(i,j)^*$-fuzzy pre-open set λ in X.*

Definition 1.6 *[11] Let (X,τ_1,τ_2) be a bitopological space. Then an operation γ on $\tau_1\cup\tau_2$ is a function $\gamma:\tau_1\cup\tau_2\longrightarrow P(X)$ s.t. $V\subset V^\gamma$, $\forall\ V\in\tau_1\cup\tau_2$ where the value of γ at V is V^γ.*

Definition 1.7 *[5] Let $P(X)$ be the power set of a crisp set X. A sub class $\tau^*\subset P(X)$ is called an infi-topology on X if*
(i) $\phi,X\in\tau^*$ and
(ii) τ^* is closed under finite intersection.
The members of τ^* are called infi-open sets and $(X,\tau^*)$is called an infi-topological space.

2. $(i,j)^*_\gamma$-Fuzzy Pre-open Set

The recent articulation of the $(i,j)^*_\gamma$-operation within the context of a fuzzy bitopological space [22] represents the latest advancement in research pertaining to the operational approach. We already know the notion of $(i,j)^*$-fuzzy pre-open set. Now, we will define the concept of $(i,j)^*_\gamma$-fuzzy pre-open set using $(i,j)^*$-fuzzy pre-open set and emphasize that the collection of all $(i,j)^*_\gamma$-fuzzy pre-open sets is not closed under the union and intersection. Moreover, we will study some of its basic properties.

Definition 2.1 *In a FBTS (X,τ_i,τ_j), an operation $(i,j)^*_\gamma$ on the FBTS (X,τ_i,τ_j) is a function from $(i,j)^*$-$FPO(X)$ to I^X such that for each $\delta\in(i,j)^*$-$FPO(X)$, $\delta\le\delta^{(i,j)^*_\gamma}$; where $\delta^{(i,j)^*_\gamma}$ is the value of the operation when applied on δ.*
Explicitly,the operation is denoted by $(i,j)^*_\gamma:(i,j)^*$-$FPO(X)\longrightarrow I^X$.

Example 2.1 *Let (X,τ_i,τ_j) be a FBTS where $X=\{a,b\}$, $\tau_i=\{0_X,1_X,\{(a,0.5),(b,0.5)\}\}$, and $\tau_j=\{0_X,1_X,\}$.*
We now define a $(i,j)^*_\gamma$ -operation as follows:

$$(i,j)^*_\gamma(\delta)=\begin{cases}\delta, & \text{if } r\in\delta\\ (i,j)^*\text{-}pcl(\delta), & \text{otherwise,}\end{cases}$$

where $r=\{\{(a,\alpha),(b,\beta)\}:\alpha\ge0.8,\beta\ge0.9\}$.
Here for any fuzzy set δ, we have $\delta\le\delta^{(i,j)^*_\gamma}$. Hence the operation defined above is a $(i,j)^*_\gamma$-operation.

Definition 2.2 *Let (X,τ_i,τ_j) be a FBTS and f be an $(i,j)^*_\gamma$-operation on (X,τ_i,τ_j). Then a fuzzy set μ is called $(i,j)^*_\gamma$-fuzzy pre-open set if $\forall$ fuzzy point $x_p\in\mu$, $\exists$ a $(i,j)^*$-fuzzy pre-open set δ containing x_p s.t. $f(\delta)\le\mu$ i.e. $\delta^{(i,j)^*_\gamma}\le\mu$.*
The complement of a $(i,j)^*_\gamma$-fuzzy pre-open set is known as a $(i,j)^*_\gamma$-fuzzy preclosed set.
$(i,j)^*_\gamma$-$FPO(X)$ and $(i,j)^*_\gamma$-$FPC(X)$ denotes the set of all $(i,j)^*_\gamma$-fuzzy pre-open sets and $(i,j)^*_\gamma$-fuzzy preclosed sets respectively.

Theorem 2.1 *The empty set 0_X and the whole set 1_X are always $(i,j)^*_\gamma$-fuzzy pre-open set.*

Proof: Obvious from the definition of the $(i,j)^*_\gamma$-fuzzy pre-open set. □

Theorem 2.2 *Every $(i,j)^*_\gamma$-fuzzy pre-open set is a $(i,j)^*$-fuzzy pre-open set.*

Proof: Let us take μ be a $(i,j)^*_\gamma$-fuzzy pre-open set in (X,τ_i,τ_j) and we consider any fuzzy point $x_p \in \mu$. Then $\exists$ a $(i,j)^*$-fuzzy pre-open set δ containing x_p s.t. $\delta^{(i,j)^*_\gamma} \le \mu$. Consequently, we have $x_p \in \delta \le \mu$. So x_p belongs to $(i,j)^*$-interior of μ. Hence μ is $(i,j)^*$-fuzzy pre-open. □

Remark 2.1 *Every $(i,j)^*$-fuzzy pre-open set may not be a $(i,j)^*_\gamma$-fuzzy pre-open set.*

Example 2.2 *Let $X = \{a,b\}$ and consider two topologies $\tau_i = \{0_X, 1_X, \{(a,0.3),(b,0.5)\}\}$, $\tau_j = \{0_X, 1_X, \{(a,0.7),(b,0.3)\}\}$ defined on X. Then (X,τ_i,τ_j) is a FBTS.*
We define a $(i,j)^*_\gamma$-operation f on $(i,j)^*$-$FPO(X)$ as follows :

$$f(\delta) = (i,j)^*\text{-}pcl(\delta).$$

Here $(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(a,0.3),(b,0.5)\}, \{(a,0.7),(b,0.5)\}, \{(a,0.7),(b,0.3)\}\}$.
So, $(i,j)^*$-$FPO(X) = \{0_X, 1_X, \{\{(a,\alpha),(b,\beta)\} : \forall\alpha,\beta \le 0.5; \forall\alpha,\beta > 0.7; \alpha > 0.3, 0.5 < \beta \le 0.7\}\}$. Clearly, $\{(a,0.7),(b,0.3)\}$ is a $(i,j)^*$-fuzzy pre-open set here but not a $(i,j)^*_\gamma$-fuzzy pre-open set.

Remark 2.2 *The union of two $(i,j)^*_\gamma$-fuzzy pre-open sets may not be a $(i,j)^*_\gamma$-fuzzy pre-open set. Also, the intersection of two $(i,j)^*_\gamma$-fuzzy pre-open set fails to be a $(i,j)^*_\gamma$-fuzzy pre-open set.*

Example 2.3 *Let us consider the FBTS taken in Example 2.2 and also consider the same operation therein. It can be easily observed that the fuzzy sets $\lambda = \{(x,0.3),(y,0.8)\}$ and $\mu = \{(x,0.5),(y,0.6)\}$ are $(i,j)^*_\gamma$-fuzzy pre-open sets but both their union $\lambda \vee \mu = \{(x,0.5),(y,0.8)\}$ and intersection $\lambda \wedge \mu = \{(x,0.3),(y,0.6)\}$ are not $(i,j)^*_\gamma$-fuzzy pre-open set.*

Definition 2.3 *A fuzzy $(i,j)^*_\gamma$-operation on (X,τ_i,τ_j) is called regular if for any two $(i,j)^*$-fuzzy pre-open sets μ_1 and μ_2 containing the fuzzy point x_p, $\exists$ a $(i,j)^*$-fuzzy pre-open set δ containing x_p s.t. $\delta^{(i,j)^*_\gamma} \le \mu_1^{(i,j)^*_\gamma} \wedge \mu_2^{(i,j)^*_\gamma}$.*

Theorem 2.3 *In a regular $(i,j)^*_\gamma$-operation, the intersection of any two $(i,j)^*_\gamma$-fuzzy pre-open set is a $(i,j)^*_\gamma$-fuzzy pre-open set.*

Proof: Let (X,τ_i,τ_j) be a FBTS and f be a regular $(i,j)^*_\gamma$-operation defined on $(i,j)^*$-$FPO(X)$. Let us consider λ_1, λ_2 be any two $(i,j)^*_\gamma$-fuzzy pre-open set and a fuzzy point $x_p \in \lambda_1 \wedge \lambda_2$. Then $x_p \in \lambda_1$ and $x_p \in \lambda_2$. So, $\exists$ two $(i,j)^*$-fuzzy pre-open sets δ_1 and δ_2 such that $f(\delta_1) \le \lambda_1$ and $f(\delta_2) \le \lambda_2$. Since f is a regular $(i,j)^*_\gamma$-operation, $\exists$ a $(i,j)^*$-fuzzy pre-open set μ in X containing x_p s.t. $f(\mu) \le f(\delta_1) \wedge f(\delta_2)$ and so $f(\mu) \le \lambda_1 \wedge \lambda_2$. Hence, $\lambda_1 \wedge \lambda_2$ is a $(i,j)^*_\gamma$-fuzzy pre-open set. □

Combining Theorem 2.1 and Theorem 2.3, we have the following :

Theorem 2.4 *For a regular $(i,j)^*_\gamma$-operation, the set of all $(i,j)^*_\gamma$-fuzzy pre-open sets forms a fuzzy infi topology.*

3. Minimal $(i,j)^*_\gamma$-Fuzzy Pre-open Set

In this particular part of this research work, we initiate the concept of minimal $(i,j)^*_\gamma$-fuzzy pre-open set and further characterize the space based on the conception.

Definition 3.1 *Let (X,τ_i,τ_j) be a FBTS and f be a $(i,j)^*_\gamma$-operation. Then a non-empty $(i,j)^*_\gamma$-fuzzy pre-open set μ is called a minimal $(i,j)^*_\gamma$-fuzzy pre-open set in X if there does not exists any $(i,j)^*_\gamma$-fuzzy pre-open subset of μ other than the fuzzy set 0_X.*

Remark 3.1 *A minimal $(i,j)^*_\gamma$-fuzzy pre-open set can not be a subset of any other minimal $(i,j)^*_\gamma$-fuzzy pre-open set. This claim can be verified from the following example:*

Example 3.1 *Let $X=\{a,b\}$ and consider two topologies $\tau_i=\{0_X,1_X,\{(a,0.4),(b,0.6)\}\}$, $\tau_j=\{0_X,1_X,\{(a,0.6),(b,0.4)\}\}$ defined on X. Then (X,τ_i,τ_j) is a FBTS.*
We define a $(i,j)^*_\gamma$-operation f on $(i,j)^*$-$FPO(X)$ as follows :

$$f(\delta)=(i,j)^*\text{-}cl(\delta).$$

Here $(i,j)^*$-$FO(X)=\{0_X,1_X,\{(a,0.4),(b,0.6)\},\{(a,0.6),(b,0.4)\},\{(a,0.6),(b,0.6)\}\}$.

So, $(i,j)^*$-$FPO(X)=\{0_X,1_X,\{\{(a,\alpha),(b,\beta)\}:\forall\alpha,\beta>0.4;\alpha>0.4,\beta\leq 0.4\}\}$. Calculation for $(i,j)^*_\gamma$-fuzzy pre-open sets gives us $(i,j)^*_\gamma$-$FPO(X)=\{0_X,1_X,\{(a,0.4),(b,0.6)\},\{(a,0.6),(b,0.4)\}\}$. Clearly, here both the fuzzy sets $\{(a,0.4),(b,0.6)\}$ and $\{(a,0.6),(b,0.4)\}$ are minimal $(i,j)^*_\gamma$-fuzzy pre-open set but they are not subset of each other.

Theorem 3.1 *Let $(i,j)^*_\gamma:(i,j)^*$-$FPO(X)\longrightarrow I^X$ be a regular operation in (X,τ_i,τ_j). If μ is any minimal $(i,j)^*_\gamma$-fuzzy pre-open set and δ is a $(i,j)^*_\gamma$-fuzzy pre-open set, then either $\mu\wedge\delta=0_X$ or $\mu\leq\delta$.*

Proof: There is nothing to prove in the case $\mu\wedge\delta=0_X$.
So, we consider the case $\mu\wedge\delta\neq 0_X$. Since μ is a minimal $(i,j)^*_\gamma$-fuzzy pre-open set, hence μ is a $(i,j)^*_\gamma$-fuzzy pre-open set. Then from theorem 2.3, $\mu\wedge\delta$ is also a $(i,j)^*_\gamma$-fuzzy pre-open set. By the minimality condition, we have $\mu\leq\mu\wedge\delta$. Therefore, $\mu\leq\delta$. □

Theorem 3.2 *Let (X,τ_i,τ_j) be a FBTS and $(i,j)^*_\gamma:(i,j)^*$-$FPO(X)\longrightarrow I^X$ be a regular operation in (X,τ_i,τ_j). If μ is a minimal $(i,j)^*_\gamma$-fuzzy pre-open set containing x_p, then $\mu\leq\nu$, for all $(i,j)^*_\gamma$-fuzzy pre-open set ν which contains x_p.*

Proof: Here both μ and ν are $(i,j)^*_\gamma$-fuzzy pre-open sets containing a fuzzy point x_p. Let μ is not a fuzzy subset of ν. Then $\mu\wedge\nu$ is a $(i,j)^*_\gamma$-fuzzy pre-open set since $(i,j)^*_\gamma$ is a regular operation. So, $\mu\wedge\nu$ is not a fuzzy subset of ν and $\nu\wedge\mu\neq\ 0_X$ (as $x_p\in\mu,\nu$). Hence $\mu\wedge\nu$ is a minimal $(i,j)^*_\gamma$-fuzzy pre-open set, which leads to a contradiction. Eventually, we obtain $\mu\leq\nu$. □

From the above theorem, we find the following result:

Corollary 3.1 *If $(i,j)^*_\gamma:(i,j)^*-FPO(X)\longrightarrow I^X$ be a regular operation in a FBTS (X,τ_i,τ_j) and μ be any minimal $(i,j)^*_\gamma$-fuzzy pre-open set in X, then*

$$\mu=\bigwedge\{\nu:\nu\text{ is a }(i,j)^*_\gamma\text{-fuzzy pre-open set containing }x_p\}.$$

for any fuzzy point $x_p\in\mu$.

Theorem 3.3 *For a regular operation $(i,j)^*_\gamma:(i,j)^*$-$FPO(X)\longrightarrow I^X$ in a FBTS (X,τ_i,τ_j), there exists exactly one non-empty minimal $(i,j)^*_\gamma$-fuzzy pre-open set .*

Proof: Let us assume that, there are two different minimal $(i,j)^*_\gamma$-fuzzy pre-open sets λ and δ in a FBTS (X,τ_i,τ_j). Then both λ and δ are $(i,j)^*_\gamma$-fuzzy open sets and so $\lambda\wedge\delta$ is also $(i,j)^*_\gamma$-fuzzy pre-open set. Considering the fact that λ is a minimal $(i,j)^*_\gamma$-fuzzy pre-open set, we have $\lambda\leq\delta$ from the Theorem 3.4 and we have $\delta\leq\lambda$, considering δ as a minimal $(i,j)^*_\gamma$-fuzzy pre-open set. Combining both of them, we have $\lambda=\delta$. □

Theorem 3.4 *Let $(i,j)^*_\gamma:(i,j)^*$-$FPO(X)\longrightarrow I^X$ be a regular operation in (X,τ_i,τ_j). Then μ is a minimal $(i,j)^*_\gamma$-fuzzy pre-open set iff $\mu\leq(i,j)^*_\gamma$-$pcl(\nu)$ and $(i,j)^*_\gamma$-$pcl(\mu)=(i,j)^*_\gamma$-$pcl(\nu)$, for any non-empty fuzzy subset ν of μ.*

Proof: Let us consider a fuzzy point $x_p\in\mu$ and δ be a $(i,j)^*_\gamma$-fuzzy pre-open set containing x_p. Then $\mu\leq\delta$ and $\nu=\mu\wedge\nu\leq\delta\wedge\nu$. Hence $\mu\wedge\nu\neq 0_X$ and so $x_p\in(i,j)^*_\gamma$-$pcl(\nu)$. Then $\mu\leq(i,j)^*_\gamma$-$pcl(\nu)$. This implies $(i,j)^*_\gamma-pcl(\mu)\leq(i,j)^*_\gamma$-$pcl(\nu)$. Also, for any non-empty fuzzy subset ν of μ, we have $(i,j)^*_\gamma$-$pcl(\mu)\leq(i,j)^*_\gamma$-$pcl(\mu)$. Thus, $(i,j)^*_\gamma$-$pcl(\mu)=(i,j)^*_\gamma$-$pcl(\nu)$.

Conversely, let μ is not a minimal $(i,j)^*_\gamma$-fuzzy pre-open set. So, $\exists$ a non-empty $(i,j)^*_\gamma$-fuzzy pre-open set ν which is not a fuzzy pre-open set of μ in X. Thus there is a fuzzy point $x_p \in \mu$ such that $x_p \notin \nu$ and hence $(i,j)^*_\gamma$-$pcl(x_p) \subseteq 1_X - \nu$, that is $(i,j)^*_\gamma$-$pcl(x_p) \neq (i,j)^*_\gamma$-$pcl(\mu)$, which leads to a contradiction. Consequently, μ is a minimal $(i,j)^*_\gamma$-fuzzy pre-open set. □

Theorem 3.5 *Let $(i,j)^*_\gamma$ be a regular operation in (X,τ_i,τ_j). If μ is a minimal $(i,j)^*_\gamma$-fuzzy pre-open set with $x_p \in 1_X - \mu$ and $\chi_x = \bigwedge\{\delta \in (i,j)^*_\gamma\text{-}FPO(X) : x_p \in \delta\}$. Then either $\chi_x \wedge \mu = 0_x$ or $\lambda \leq \chi_x$.*

Proof: The following two cases may arise :
Case I : If $\mu \leq \delta$ with $x_p \in \delta$, then $\mu \leq \bigwedge\{\delta \in (i,j)^*\text{-}FO(X) : x_p \in \delta\}$. That is, $\mu \leq \chi_x$.
Case II : If $\delta < \mu$ with $x_p \in \delta$, then $\exists$ a $(i,j)^*_\gamma$-fuzzy pre-open set δ containing x_p such that $\mu \wedge \delta = 0_x$ and hence $\chi_x \wedge \mu = 0_x$. □

Theorem 3.6 *If $(i,j)^*_\gamma$ be an operation defined in a FBTS (X,τ_i,τ_j) and μ be a proper $(i,j)^*_\gamma$-fuzzy pre-open set, then $\exists$ a minimal $(i,j)^*_\gamma$-fuzzy pre-open set δ s.t. $\delta \leq \mu$.*

Proof: Let μ be a proper $(i,j)^*_\gamma$-fuzzy pre-open set in (X,τ_i,τ_j). Then the following cases arise :
Case I : If μ is a minimal $(i,j)^*_\gamma$-fuzzy pre-open set, then by assuming $\mu = \delta$, the proof is straightforward.
Case II : If μ is not a minimal $(i,j)^*_\gamma$-fuzzy pre-open set, then $\exists$ a proper $(i,j)^*_\gamma$-fuzzy pre-open set ν of μ. If ν is a minimal $(i,j)^*_\gamma$-fuzzy pre-open set, then by setting $\delta = \nu$ we have, $\delta \leq \mu$. If ν is not a minimal $(i,j)^*_\gamma$-fuzzy pre-open set, then we continue the process until we get a minimal $(i,j)^*_\gamma$-fuzzy pre-open set. Since the process will stop after some steps (say, n), we will get a minimal $(i,j)^*_\gamma$-fuzzy pre-open set $\delta = \mu_n$ such that $\delta \leq \mu$. Hence the proof. □

Definition 3.2 *Let $(i,j)^*_\gamma : (i,j)^*\text{-}FPO(X) \longrightarrow I^X$ be an operation defined on a FBTS (X,τ_i,τ_j). Then a fuzzy set μ is called a $(i,j)^*_\gamma$-fuzzy γ-open set if the intersection of μ with every $(i,j)^*_\gamma$-fuzzy pre-open set gives a $(i,j)^*_\gamma$-fuzzy pre-open set.*

From the definition it is evident:

Theorem 3.7 *Every $(i,j)^*_\gamma$-fuzzy γ-open set is a $(i,j)^*_\gamma$-fuzzy pre-open set.*

Remark 3.2 *A $(i,j)^*_\gamma$-fuzzy pre-open set is not always a $(i,j)^*_\gamma$-fuzzy γ-open set.*

Example 3.2 *Let $X = \{a,b\}$ and consider two topologies defined on X which are*

$$\tau_i = \{0_X, 1_X, \{(a,0.6),(b,0.2)\}\} \text{ and } \tau_j = \{0_X, 1_X, \{(a,0.4),(b,0.5)\}\}.$$

Then (X,τ_i,τ_j) is a FBTS.
We define a $(i,j)^*_\gamma$-operation f on $(i,j)^*\text{-}FPO(X)$ as follows :

$$f(\lambda) = \lambda$$

Here $(i,j)^*\text{-}FO(X) = \{0_X, 1_X, \{(a,0.6),(b,0.2)\}, \{(a,0.4),(b,0.5)\}, \{(a,0.6),(b,0.5)\}\}$.
So, $(i,j)^*\text{-}FPO(X) = \{0_X, 1_X, \{\{(a,\alpha),(b,\beta)\} : \forall\alpha,\beta < 0.5; \alpha \geq 0.4, \beta \geq 0.5; \alpha < 0.4, \beta > 0.8\}\}$.
Calculation for $(i,j)^*_\gamma$-fuzzy pre-open sets gives us
$(i,j)^*_\gamma\text{-}FPO(X) = \{0_X, 1_X, \{\{(a,\alpha),(b,\beta)\} : \forall\alpha,\beta < 0.5; \alpha \geq 0.4, \beta \geq 0.5; \alpha < 0.4, \beta > 0.8\}\}$
and $(i,j)^*_\gamma$-fuzzy γ-open sets are 0_X,1_X,$\{(a,\alpha),(b,\beta) : \forall\alpha,\beta < 0.5\}$.
Clearly, $\{(a,0.5),(b,0.5)\}$ is not a $(i,j)^*_\gamma$-fuzzy γ-open set though it is a $(i,j)^*_\gamma$-fuzzy pre-open set.

Theorem 3.8 *Every $(i,j)^*_\gamma$-fuzzy pre-open set is a $(i,j)^*_\gamma$-fuzzy γ-open set in a singleton FBTS (X,τ_i,τ_j).*

Proof: Assume that, δ and λ are two $(i,j)^*_\gamma$-fuzzy pre-open sets in a FBTS (X,τ_i,τ_j). Since X is singleton, then either $\delta \leq \lambda$ or $\lambda \leq \delta$. This implies that $\delta \wedge \lambda = \delta$ or λ. So, $\delta \wedge \lambda$ is always a $(i,j)^*_\gamma$-fuzzy pre-open set. Therefore, δ is a $(i,j)^*_\gamma$-fuzzy γ-open set in (X,τ_i,τ_j). □

Definition 3.3 *Let (X, τ_i, τ_j) be a FBTS and $(i,j)^*_\gamma : (i,j)^*$-$FPO(X) \longrightarrow I^X$ be an operation defined on a FBTS (X, τ_i, τ_j). Then X is called a $(i,j)^*_\gamma$-fuzzy locally finite space if for every fuzzy point $x_p \in X$, $\exists$ a $(i,j)^*_\gamma$-fuzzy pre-open set $\lambda \neq 1_X$ in X s.t. $x_p \leq \lambda$.*

Theorem 3.9 *Let (X, τ_i, τ_j) be a FBTS which is $(i,j)^*_\gamma$-fuzzy locally finite and $(i,j)^*_\gamma$ be a regular operation defined on $(i,j)^*$-$FPO(X)$. If μ be any non-empty $(i,j)^*_\gamma$-fuzzy open set then $\exists$ at least one minimal $(i,j)^*_\gamma$-fuzzy pre-open set λ s.t. $\lambda \leq \mu$.*

Proof: Let us consider a fuzzy point $x_p \in \mu$. X being a $(i,j)^*_\gamma$-fuzzy locally finite space, $\exists$ a $(i,j)^*_\gamma$-fuzzy pre-open set δ s.t. $x_p \leq \delta$. As $(i,j)^*_\gamma$ is a regular operation, $\delta \wedge \mu \neq 0_X$ is also a $(i,j)^*_\gamma$-fuzzy pre-open set. So, there is a minimal $(i,j)^*_\gamma$-fuzzy pre-open set λ s.t. $\lambda \leq \delta \wedge \mu$. Hence we have $\lambda \leq \mu$. □

4. Local Finiteness in Fuzzy Bitopological Space

In this particular section, we introduce α-locally finite set in a given FBTS via operation approach and characterize this notion up to some extent.

Definition 4.1 *A fuzzy set μ in a FBTS (X, τ_i, τ_j) is called an empty fuzzy set of order α if $\mu(x) \leq \alpha$ for each $x \in X$ and $\alpha \in [0, 1)$.*

A fuzzy set μ is said to be non-empty fuzzy set of order α if $\exists$ $x_0 \in X$ s.t. $\mu(x_0) > \alpha$ where $\alpha \in [0, 1)$.

Definition 4.2 *Let (X, τ_i, τ_j) be a FBTS and $\alpha \in [0, 1)$. A family $\{\lambda_n : n \in \Lambda\}$ of fuzzy sets in (X, τ_i, τ_j) is called α-point finite if $\forall$ $x \in X$, $\lambda_n(x) > \alpha$ for atmost finitely many $n \in \Lambda$.*

Definition 4.3 *Let (X, τ_i, τ_j) be a FBTS and $\alpha \in [0, 1)$. A family $\{\lambda_n : n \in \Lambda\}$ of fuzzy sets in (X, τ_i, τ_j) is called $(i,j)^*_\gamma$-fuzzy α-locally finite set in X if $\forall$ $x \in X$ $\exists$ an $(i,j)^*_\gamma$-fuzzy pre-open set δ s.t. $\delta(x) = 1_X$ and $\delta \wedge \lambda_n$ is non-empty of order α for atmost finitely many $n \in \Lambda$.*

Example 4.1 *Let $X = \{a, b\}$ and $\tau_i = \{0_X, 1_X, \{(a, 0.5), (b, 0.5)\}\}$, $\tau_j = \{0_X, 1_X, \{(a, 0.7), (b, 0.2)\}\}$ be two fuzzy topologies defined on X. Then (X, τ_i, τ_j) is a FBTS.*
We define a $(i,j)^*_\gamma$-operation f on $(i,j)^*$-$FPO(X)$ as follows :

$$f(\lambda) = (i,j)^*\text{-}cl(\lambda).$$

Here $(i,j)^*$-$FO(X) = \{0_X, 1_X, \{(a, 0.5), (b, 0.5)\}, \{(a, 0.7), (b, 0.2)\}, \{(a, 0.7), (b, 0.5)\}\}$.

So, $(i,j)^*$-$FPO(X) = \{0_X, 1_X, \{\{(a, \alpha), (b, \beta)\} : \forall\alpha, \beta > 0.8; \alpha > 0.3, \beta \leq 0.8\}\}$. By the calculation, $(i,j)^*_\gamma$-$FPO(X) = \{0_X, 1_X, \{(a, 0.5), (b, 0.5)\}\}$.

Let $\lambda_n = \{\{(x, 1/n), (y, 1/n)\} : n \in \Lambda\}$ be a family of fuzzy sets.
Let us consider, $\alpha = 0.15$ then $\exists$ $(i,j)^*_\gamma$-fuzzy pre-open set δ s.t. $\delta(x) = 1_X$ and $\delta \wedge \lambda_n$ is non-empty of order α for λ_1, λ_2, λ_3, λ_4, λ_5,λ_6 . Hence, $\{\lambda_n : n \in \Lambda\}$ is $(i,j)^*_\gamma$-fuzzy α-locally finite set.

Theorem 4.1 *In (X, τ_i, τ_j), every $(i,j)^*_\gamma$-fuzzy α-locally finite is α-point finite.*

Proof: Let $\{\lambda_n : n \in \Lambda\}$ be any family of fuzzy sets which is $(i,j)^*_\gamma$-fuzzy α-locally finite in X. Then $\forall$ $x \in X$ $\exists$ an $(i,j)^*_\gamma$-fuzzy pre-open set δ in X s.t. $\delta(x) = 1_X$ and $\delta \wedge \lambda_n$ is non-empty of order α for atmost finitely many $n \in \Lambda$. That means, for each $x \in X$, $(\delta \wedge \lambda_n)(x) > \alpha$ for atmost finitely many $n \in \Lambda$. Hence for each $x \in X$, $\lambda_n \geq (\delta \wedge \lambda_n)(x) > \alpha$ for atmost finitely many $n \in \Lambda$. Therefore, $\{\lambda_n : n \in \Lambda\}$ is α-point finite. □

Theorem 4.2 *Let (X, τ_i, τ_j) be a FBTS and $\alpha \in [0, 1)$. If $\{\lambda_n : n \in \Gamma\}$ and $\{\mu_m : m \in \Lambda\}$ are any two $(i,j)^*_\gamma$-fuzzy α-locally finite families of fuzzy sets in X, then $\{\lambda_n \wedge \mu_m : (m, n) \in \Gamma \times \Lambda\}$ is also $(i,j)^*_\gamma$-fuzzy α-locally finite.*

Proof: Let $\{\lambda_n : n \in \Gamma\}$ and $\{\mu_m : m \in \Lambda\}$ are any two $(i,j)^*_\gamma$-fuzzy α-locally finite families of fuzzy sets in X then for each $x \in X$, $\exists$ two $(i,j)^*_\gamma$-fuzzy pre-open sets δ and β s.t. $\delta(x) = 1_X$, $\beta(x) = 1_X$ and $\delta \wedge \lambda_n$, $\beta \wedge \mu_m$ are non-empty of order α for atmost finitely many $n \in \Gamma$,$m \in \Lambda$.
Let us consider for each $y \in X$, $[(\delta \wedge \beta) \wedge (\lambda_n \wedge \lambda_m)](y) > \alpha$ is true for infinitely many $(n,m) \in \Gamma \times \Lambda$. It implies that $[(\delta \wedge \lambda_n) \wedge (\beta \wedge \mu_m)] > \alpha$ for infinitely many $(n,m) \in \Gamma \times \Lambda$ which contradicts that $\delta \wedge \lambda_n$ and $\beta \wedge \mu_m$ are non-empty of order α for atmost finitely many $n \in \Gamma$,$m \in \Lambda$. Hence, $\{\lambda_n \wedge \mu_m : (m,n) \in \Gamma \times \Lambda\}$ is $(i,j)^*_\gamma$-fuzzy α-locally finite. □

Definition 4.4 *In a FBTS (X, τ_i, τ_j), a family $\{\lambda_n : n \in \Lambda\}$ of fuzzy sets is called $(i,j)^*_\gamma$-fuzzy α-discrete if $\forall$ $x \in X$ $\exists$ an $(i,j)^*_\gamma$-fuzzy pre-open set δ in X s.t. $\delta(x) = 1_X$ and $\delta \wedge \lambda_n$ is non-empty of order α for atmost one member $n \in \Lambda$ and $\alpha \in [0,1)$.*

Theorem 4.3 *In a FBTS (X, τ_i, τ_j), every $(i,j)^*_\gamma$-fuzzy α-discrete family is $(i,j)^*_\gamma$-fuzzy α-locally finite.*

Proof: Let us consider a family of fuzzy sets $\{\lambda_n : n \in \Lambda\}$ which is $(i,j)^*_\gamma$-fuzzy α-discrete. Then $\forall$ $x \in X$, $\exists$ an $(i,j)^*_\gamma$-fuzzy pre-open set δ in X s.t. $\delta(x) = 1_X$ and $\delta \wedge \lambda_n$ is non-empty of order α for atmost one member $n \in \Lambda$. Hence for each $x \in X$, $\delta(x) = 1_X$ and $\delta \wedge \lambda_n$ is non-empty of order α for atmost finitely many $n \in \Lambda$. □

Theorem 4.4 *Let (X, τ_i, τ_j) be a FBTS. If $\{\lambda_n : n \in \Lambda\}$ be $(i,j)^*_\gamma$-fuzzy α-locally finite family in X then $\{(i,j)^*_\gamma\text{-}pcl(\lambda_n) : n \in \Lambda\}$ is also $(i,j)^*_\gamma$-fuzzy α-locally finite family.*

Proof: Let $\{\lambda_n : n \in \Lambda\}$ be $(i,j)^*_\gamma$-fuzzy α-locally finite family then for each $x \in X$, $\exists$ an $(i,j)^*_\gamma$-fuzzy pre-open set $\delta \in X$ s.t. $\delta(x) = 1_X$ and $\delta \wedge \lambda_n$ is non-empty of order α for atmost finitely many $n \in \Lambda$. Hence there exists $x_0 \in X$ such that $(\delta \wedge \lambda_n)(x_0) > \alpha$ for atmost finitely many $n \in \Lambda$. This implies that $(i,j)^*_\gamma\text{-}pcl(\lambda_n)(x_0) \geq \lambda_n(x_0) \geq (\delta \wedge \lambda_n)(x_0) > \alpha$ for atmost finitely many $n \in \Lambda$. Consequently, $\{(i,j)^*_\gamma\text{-}pcl(\lambda_n) : n \in \Lambda\}$ is $(i,j)^*_\gamma$-fuzzy α-locally finite family. □

Theorem 4.5 *Let $\{\lambda_n : n \in \Lambda\}$ be a $(i,j)^*_\gamma$-fuzzy α-locally finite family in (X, τ_i, τ_j) then $\bigvee\{(i,j)^*_\gamma\text{-}pcl(\lambda_n) : n \in \Lambda'\}$ is a $(i,j)^*_\gamma$-fuzzy preclosed set for each subset Λ' of Λ.*

Proof: Let $\Lambda' \subset \Lambda$ and $\beta = \bigvee\{(i,j)^*_\gamma\text{-}cl(\lambda_n) : n \in \Lambda'\}$. We now prove $1_X - \beta$ is an $(i,j)^*_\gamma$-fuzzy pre-open set in (X, τ_i, τ_j). Let $x \in X$ such that $(1_X - \beta)(x) > 0$.
Then

$$\begin{aligned}\left(1_X - \bigvee_{n \in \Lambda'} (i,j)^*_\gamma - cl(\lambda_n)\right)(x) &= \bigwedge_{n \in \Lambda'} \left(1_X - ((i,j)^*_\gamma - pcl(\lambda_n))\right)(x) \\ &= inf_{n \in \Lambda'}\{1_X - ((i,j)^*_\gamma - pcl(\lambda_n))(x)\} \\ &> 0.\end{aligned}$$

Hence $1_X - ((i,j)^*_\gamma\text{-}pcl(\lambda_n))(x) > 0$ for each $n \in \Lambda'$.

From the above theorem, we have $\delta \wedge (i,j)^*_\gamma\text{-}pcl(\lambda_n)$ is non-empty of order α for atmost finitely many $n \in \Lambda'$, say n_1, n_2,...,$n_p \in \Lambda'$.

So, there exists $x_0 \in X$ such that $(\delta \wedge (i,j)^*_\gamma\text{-}pcl(\lambda_n))(x_0) > \alpha$ for n_1, n_2,...,n_p and

$$(\delta \wedge (i,j)^*_\gamma\text{-}pcl(\lambda_n))(x_0) \leq \alpha \text{ for } n \neq n_1, n_2, ..., n_p. \qquad \textbf{(I)}$$

Let $\mu = \delta \wedge \left(\bigwedge_{i=1}^{p}(1_X - (i,j)^*_\gamma - pcl(\lambda_i))\right)$.
Then μ is an $(i,j)^*_\gamma$-fuzzy pre-open set and

$$\mu(x) > (1_x - \beta)(x). \qquad \textbf{(II)}$$

i.e. $[\delta \wedge \left(\bigwedge_{i=1}^{p}(1_X - (i,j)^*_\gamma - pcl(\lambda_i))\right)](x) > \bigwedge_{n \in \Lambda'} \left(1_X - ((i,j)^*_\gamma - pcl(\lambda_n))\right)(x)$.

Let us assume that

$$\bigwedge_{n\in\Lambda'}\left(1_X-((i,j)^*_\gamma-pcl(\lambda_n))\right)(x)=inf_{n\in\Lambda'}\{1_X-((i,j)^*_\gamma-pcl(\lambda_n))(x)\}$$
$$=\left(1_X-((i,j)^*_\gamma-pcl(\lambda_{n_0}))\right)(x)\text{ for some }n_0\in\Lambda^{'}.$$

Then $\delta(x)\wedge\left(\bigwedge_{i=1}^p(1_X-(i,j)^*_\gamma-pcl(\lambda_i))\right)(x)>\left(1_X-((i,j)^*_\gamma-pcl(\lambda_{n_0}))\right)(x)$
and $\left(\bigwedge_{i=1}^p(1_X-(i,j)^*_\gamma-pcl(\lambda_{n_i}))\right)(x)>\left(1_X-((i,j)^*_\gamma-pcl(\lambda_{n_0}))\right)(x)$. **(III)**
If $n_0=n_i$ for some i = 1,2,3,...,p, then from (**III**) we have,
$\left(1_X-(i,j)^*_\gamma-pcl(\lambda_{n_i}))\right)(x)>\left(1_X-((i,j)^*_\gamma-pcl(\lambda_{n_0}))\right)(x)$ for each i = 1,2,3,...,p.
This gives $\left(1_X-((i,j)^*_\gamma-pcl(\lambda_{n_0}))\right)(x)>\left(1_X-((i,j)^*_\gamma-pcl(\lambda_{n_0}))\right)(x)$, which is a contradiction.
Now if $n_0\neq n_i$ for i = 1,2,3,...,p, then we have,
$\left(\bigwedge_{i=1}^p(1_X-(i,j)^*_\gamma-pcl(\lambda_{n_i}))\right)(x)\geq\left(1_X-((i,j)^*_\gamma-pcl(\lambda_{n_0}))\right)(x)$ for $n_0\neq n_1$,n_2,...n_p. **(IV)**
From (**I**) we obtain, $\min\{\delta(x),(i,j)^*_\gamma-pcl(\lambda_n)\}\leq\alpha$ for each $x\in X$ and for $n_0\neq n_1$,n_2,...n_p.
This implies that $(i,j)^*_\gamma-pcl(\lambda_{n_0})(x)\leq\alpha$ for each $x\in X$ and for $n_0\neq n_1$,n_2,...n_p.
From (**IV**), we have

$$\bigwedge_{i=1}^p\left(1_X-((i,j)^*_\gamma-pcl(\lambda_{n_i}))\right)(x)\geq 1_X-\alpha$$

and

$$\left(1_X-((i,j)^*_\gamma-pcl(\lambda_{n_i}))\right)(x)\geq 1_X-\alpha\text{ for }i=1,2,3,...,p.$$

This impies that, $(i,j)^*_\gamma$-$pcl(\lambda_n)(x)\leq\alpha$ for $n=n_1$,n_2,...n_p. **(V)**
Again, (**I**) gives $(\delta\wedge(i,j)^*_\gamma$-$pcl(\lambda_n))(x)>\alpha$ for n_1, n_2,...,n_p.
Thus $(i,j)^*_\gamma$-$pcl(\lambda_n))(x)>\alpha$ for n_1, n_2,...,n_p, which is a contradiction.
Therefore, $\mu(x)=(1_X-\beta)(x)$. So, β is a $(i,j)^*_\gamma$-fuzzy preclosed set. Hence the proof is completed. □

Theorem 4.6 *In (X,τ_i,τ_j), a $(i,j)^*_\gamma$-fuzzy α-locally finite family $\{\lambda_n:n\in\Lambda\}$ of fuzzy sets is closure preserving.*

Proof: Let $\{\lambda_n:n\in\Lambda\}$ be a $(i,j)^*_\gamma$-fuzzy α-locally finite family of fuzzy sets in a FBTS (X,τ_i,τ_j). Then $\lambda_n\leq\bigvee_{n\in\Lambda}\lambda_n$ which gives $(i,j)^*_\gamma$-$pcl(\lambda_n)\leq(i,j)^*_\gamma$-$pcl(\bigvee_{n\in\Lambda}\lambda_n)$ for each $n\in\Lambda$.

Then $\bigvee_{n\in\Lambda}\left((i,j)^*_\gamma-pcl(\lambda_n)\right)\leq(i,j)^*_\gamma$-$pcl(\bigvee_{n\in\Lambda}\lambda_n)$.

Also, we know that, $\lambda_n\leq(i,j)^*_\gamma$-$pcl(\lambda_n)$ for each $n\in\Lambda$, which implies that

$$\bigvee_{n\in\Lambda}\lambda_n\leq\bigvee_{n\in\Lambda}\left((i,j)^*_\gamma-pcl(\lambda_n)\right).$$

Thus by the above theorem, we obtain that $\bigvee_{n\in\Lambda}\left((i,j)^*_\gamma-pcl(\lambda_n)\right)$ is a $(i,j)^*_\gamma$-fuzzy preclosed set containing $\bigvee_{n\in\Lambda}\lambda_n$. So, $(i,j)^*_\gamma-pcl(\bigvee_{n\in\Lambda}\lambda_n)\leq\bigvee_{n\in\Lambda}\left((i,j)^*_\gamma-pcl(\lambda_n)\right)$.

Then,$(i,j)^*_\gamma$-$pcl(\bigvee_{n\in\Lambda}\lambda_n)=\bigvee_{n\in\Lambda}\left((i,j)^*_\gamma-pcl(\lambda_n)\right)$. Eventually, $\{\lambda_n:n\in\Lambda\}$ is closure preserving. □

Theorem 4.7 *Let (X,τ_i,τ_j) be a FBTS. If $\{\lambda_n:n\in\Lambda\}$ be a $(i,j)^*_\gamma$-fuzzy α-locally finite family of $(i,j)^*_\gamma$ fuzzy preclosed sets, then $\bigvee_{n\in\Lambda}\lambda_n$ is a $(i,j)^*_\gamma$ fuzzy preclosed set.*

Proof: Let $\{\lambda_n:n\in\Lambda\}$ be a $(i,j)^*_\gamma$-fuzzy α-locally finite family of $(i,j)^*_\gamma$ fuzzy preclosed sets, then $\lambda_n=(i,j)^*_\gamma-pcl(\lambda_n)$ for each $n\in\Lambda$. From the above theorem, we have $\{\lambda_n:n\in\Lambda\}$ is closure preserving. So, $\bigvee_{n\in\Lambda}\left((i,j)^*_\gamma-pcl(\lambda_n)\right)=(i,j)^*_\gamma$-$pcl(\bigvee_{n\in\Lambda}\lambda_n)$. This gives $\bigvee_{n\in\Lambda}\lambda_n=(i,j)^*_\gamma$-$pcl(\bigvee_{n\in\Lambda}\lambda_n)$.

Therefore, $\bigvee_{n\in\Lambda}\lambda_n$ is a $(i,j)^*_\gamma$ fuzzy preclosed set. □

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