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Stability of Quadratic and Orthogonally Quadratic Functional Equations

G. Yagachitradevi*, S. Lakshminarayanan and P. Ravindiran

ABSTRACT: In this study, the authors examine the generalized Hyers-Ulam stability of the following quadratic functional equation

$$g(3x+2y+z)+g(3x+2y-z)+g(3x-2y+z)+g(3x-2y-z) = 8[g(x+y)+g(x-y)]+2[g(x+z)+g(x-z)]+16g(x)$$

The preceding equation is changed and its generalized Hyers-Ulam stability for the following quadratic functional equation

$$f(3x+2y+z)+f(3x+2y-z)+f(3x-2y+z)+f(3x-2y-z) = 8[f(x+y)+f(x-y)]+2[f(x+z)+f(x-z)]+16f(x)$$

for any $x, y, z \in X$ with $x \perp y, y \perp z$ and $z \perp x$ is studied in orthogonality space in the sense of Rätz.

Key Words: Quadratic functional equations, generalized Hyers-Ulam stability, orthogonality space.

Contents

1. Introduction

The classic stability issue of functional equation started from a query of Ulam [21] about the stability of group homomorphism in 1940. In 1941, Hyers [12] presented a partial response to the issue of Ulam for Banach spaces. In 1950, Aoki [4] extended the Hyers theorem for additive maps. In 1978, Th.M. Rassias [16] offered a generalization of Hyers' theorem which permits the *Cauchy difference to be boundless.*

The functional equation

$$h(x+y)+h(x-y)=2h(x)+2h(y) \tag{1.1}$$

is termed a *quadratic functional equation.* In specifically, any solution of the quadratic functional equation is considered to be *quadratic mapping.* A Hyers-Ulam-Rassias stability problem for the quadratic functional equation was shown by Skof [20](see also [1], [14]) for mappings $f : X \to Y$, where X is a normed space and Y is a Banach space.

I.S. Chung and H.M. Kim [7] proposed the following quadratic functional equation

$$f(2x+y)+f(2x-y)=f(x+y)+f(x-y)+6f(x) \tag{1.2}$$

and they proved the general solution and the generalized Hyers-Ulam-Rassias stability problem for the functional equation (1.2).

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* Corresponding author.
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K. Balamurugan et. al., [5] presented the following cubic functional equation

$$g(3x+2y+z)+g(3x+2y-z)+g(3x-2y+z)+g(3x-2y-z) \\ =24[g(x+y)+g(x-y)]+6[g(x+z)+g(x-z)]+48g(x). \tag{1.3}$$

Then they examined the generalized Hyers-Ulam stability for Eq.(1.3). The stability problems of many functional equations have been widely addressed by a number of academics and there are several interesting findings touching this problem (see [2], [3], [6], [8], [9], [13], [15], [17], [18]).

In Section 2, authors study the general solution and the generalized Hyers-Ulam stability of the following quadratic functional equation

$$g(3x+2y+z)+g(3x+2y-z)+g(3x-2y+z)+g(3x-2y-z) \\ =8[g(x+y)+g(x-y)]+2[g(x+z)+g(x-z)]+16g(x) \tag{1.4}$$

for any $x, y, z \in X$.

In Section 3, the Hyers-Ulam stability of the following modified orthogonally quadratic functional equation

$$f(3x+2y+z)+f(3x+2y-z)+f(3x-2y+z)+f(3x-2y-z) \\ =8[f(x+y)+f(x-y)]+2[f(x+z)+f(x-z)]+16f(x) \tag{1.5}$$

for all $x, y, z \in X$ with $x \perp y, y \perp z$ and $z \perp x$ in the sense of Rätz [19] is examined. It is simple to see that the mapping $g(x)=ax^2$ is a solution of the functional equation (1.4).

2. General Solution (1.4)

In this part, the authors explored the general solution of the functional equation (1.4) by treating X and Y are real vector spaces.

Lemma 2.1 *If a mapping $g : X \to Y$ satisfies the functional equation (1.4) for every $x, y, z \in X$, then $g : X \to Y$ is quadratic.*

Proof: Let $g : X \to Y$ satisfies the functional equation (1.4) for any $x, y, z \in X$. Putting (x, y, z) by $(0, 0, 0)$ in (1.4), we get $g(0) = 0$. Again putting (x, y, z) by $(0, 0, x)$ in (1.4), we achieve $g(-x) = g(x)$ for every $x \in X$. Therefore g is an even function. Substituting (x, y, z) with $(x, 0, x)$ in (1.4) and utilizing evenness of f, we derive

$$g(2x) = 4g(x) \tag{2.1}$$

for each $x \in X$. Setting (x, y, z) by $(x, 0, 0)$ in (1.4), we obtain

$$g(3x) = 9g(x) \tag{2.2}$$

for any $x, \in X$. In general, for each positive integer a, we get, $g(ax) = a^2 g(x)$ for all $x \in X$. Hence g is quadratic. □

Theorem 2.1 *If the mapping $g : X \to Y$ satisfies the functional equation (1.4) for every $x, y, z \in X$, then $g : X \to Y$ satisfies the functional equation (1.2) for every $x, y \in X$.*

Proof: Let $g : X \to Y$ satisfy the functional equation (1.4) for any $x, y, z \in X$. Replacing (x, y, z) by (x, y, x) in (1.4), we obtain

$$g(4x+2y)+g(4x-2y)+g(2x+2y)+g(2x-2y) \\ =8[g(x+y)+g(x-y)]+2g(2x)+16g(x)$$

for every $x, y \in X$. With the aid of Lemma 2.1, we reach

$$g(2x+y)+g(2x-y)=g(x+y)+g(x-y)+6g(x) \qquad \forall x, y \in X.$$

□

Theorem 2.2 *If a function $g : X \to Y$ satisfies the functional equation (1.4) for any $x, y, z \in X$, then $g : X \to Y$ satisfies the functional equation (1.1) for every $x, y \in X$*

Proof: According to Theorem 2.1, if $g : X \to Y$ satisfies the functional equation (1.4) for any $x, y, z \in X$, then $g : X \to Y$ must satisfy the functional equation (1.2) for every $x, y \in X$. Due to Theorem 2.1 of [7], we sought our result. □

3. Generalized Hyers-Ulam Stability of (1.4)

Let X be a normed space and Y be a Banach space in this section. Create a difference operator $Dg : X \times X \times X \to Y$ by

$$\begin{aligned} Dg(x,y,z) = g(3x+2y+z) + g(3x+2y-z) + g(3x-2y+z) + g(3x-2y-z) \\ -8[g(x+y)+g(x-y)] - 2[g(x+z)+g(x-z)] - 16g(x) \end{aligned}$$

for any $x, y, z \in X$ and analyze its generalized Hyers - Ulam stability of the functional equation (1.4).

Theorem 3.1 *Let $j = \pm 1$ and $\psi : X^3 \to [0, \infty)$ be a function such that*

$$\lim_{n\to\infty} \frac{\psi\left(3^{nj}x, 3^{nj}y, 3^{nj}z\right)}{9^{nj}} = 0 \tag{3.1}$$

for every $x, y, z \in X$. Let $g : X \to Y$ be a function fulfilling the inequality

$$\|Dg(x,y,z)\| \le \psi\left(x,y,z\right) \qquad \forall x, y, z \in X. \tag{3.2}$$

Then there exists a unique quadratic mapping $Q : X \to Y$ which satisfies (1.4) and

$$\|g(x) - Q(x)\| \le \frac{1}{36} \sum_{k=\frac{1-j}{2}}^{\infty} \frac{\psi\left(3^{kj}x, 0, 0\right)}{9^{kj}} \tag{3.3}$$

where $Q(x)$ is given by

$$Q(x) = \lim_{n\to\infty} \frac{g(3^{nj}x)}{9^{nj}} \qquad \forall x \in X. \tag{3.4}$$

Proof: In (3.2), if $(x, 0, 0)$ is substituted for (x, y, z), we obtain

$$\|g(3x) - 9g(x)\| \le \frac{\psi(x,0,0)}{4} \tag{3.5}$$

for every $x \in X$. Dividing the previous inequality by 9, we get

$$\left\|\frac{g(3x)}{9} - g(x)\right\| \le \frac{\psi(x,0,0)}{36} \tag{3.6}$$

for every $x \in X$. Now replacing x by $3x$ and dividing by 9 in (3.6)), we obtain

$$\left\|\frac{g(3^2x)}{9^2} - \frac{g(3x)}{9}\right\| \le \frac{\psi(3x,0,0)}{36 \cdot 9} \tag{3.7}$$

for every $x \in X$. From (3.6) and (3.7), we get

$$\begin{aligned} \left\|\frac{g(3^2x)}{9^2} - g(x)\right\| &\le \left\|\frac{g(3x)}{9} - g(x)\right\| + \left\|\frac{g(3^2x)}{9^2} - \frac{g(3x)}{9}\right\| \\ &\le \frac{1}{36}\left[\psi(x,0,0) + \frac{\psi(3x,0,0)}{9}\right] \end{aligned} \tag{3.8}$$

for every $x \in X$. Proceeding further and using induction on a positive integer n, we obtain

$$\left\| \frac{g(3^n x)}{9^n} - g(x) \right\| \leq \frac{1}{36} \sum_{k=0}^{n-1} \frac{\psi(3^k x)}{9^k} \leq \frac{1}{36} \sum_{k=0}^{\infty} \frac{\psi(3^k x)}{9^k} \tag{3.9}$$

for every $x \in X$. In order to show the convergence of the sequence $\left\{ \frac{g(3^n x)}{9^n} \right\}$, replace x with $3^m x$ and dividing it by 9^m in (3.9), for each $m, n > 0$, we determine

$$\begin{aligned} \left\| \frac{g(3^{n+m} x)}{9^{(n+m)}} - \frac{g(3^m x)}{9^m} \right\| &= \frac{1}{9^m} \left\| \frac{g(3^n \cdot 3^m x)}{9^n} - g(3^m x) \right\| \\ &\leq \frac{1}{36} \sum_{k=0}^{n-1} \frac{\psi(3^{k+m} x, 0, 0)}{9^{(k+m)}} \\ &\leq \frac{1}{36} \sum_{k=0}^{\infty} \frac{\psi(3^{k+m} x, 0, 0)}{9^{(k+m)}} \\ &\to 0 \quad as \quad m \to \infty \end{aligned}$$

for every $x \in X$. Hence the sequence $\left\{ \frac{g(3^n x)}{9^n} \right\}$ is a Cauchy sequence. Since Y is complete, there exists a mapping $C : X \to Y$ such that

$$C(x) = \lim_{n \to \infty} \frac{g(3^n x)}{9^n}, \quad \forall \ x \in X.$$

Letting $n \to \infty$ in (3.9), we find that (3.3) holds for any $x \in X$. To establish that C meets (1.4), substituting (x, y, z) by $(3^n x, 3^n y, 3^n z)$ and dividing by 9^n in (3.2)), we get

$$\begin{aligned} \frac{1}{9^n} \Big\| & g(3^n(3x + 2y + z)) + g(3^n(3x + 2y - z)) + g(3^n(3x - 2y + z)) + g(3^n(3x - 2y - z)) \\ & - 8[g(3^n(x + y)) + g(3^n(x - y))] - 2[g(3^n(x + z)) + g(3^n(x - z))] - 16g(3^n x) \Big\| \\ & \leq \frac{1}{9^n} \psi(3^n x, 3^n y, 3^n z) \end{aligned}$$

for every $x, y, z \in X$. Letting $n \to \infty$ in the previous inequality and using the definition of $Q(x)$, we see that

$$\begin{aligned} Q(3x + 2y{+}z) + Q(3x + 2y - z) + Q(3x - 2y + z) + Q(3x - 2y - z) \\ = 8[Q(x + y) + Q(x - y)] + 2[Q(x + z) + Q(x - z)] + 16Q(x). \end{aligned}$$

Hence Q satisfies (1.4) for any $x, y, z \in X$. To establish that Q is unique, let $R(x)$ be another quadratic mapping satisfies (1.4) and (3.3), then

$$\begin{aligned} \|Q(x) - R(x)\| &= \frac{1}{9^n} \|Q(3^n x) - R(3^n x)\| \\ &\leq \frac{1}{9^n} \{\|Q(3^n x) - g(3^n x)\| + \|g(3^n x) - R(3^n x)\|\} \\ &\leq \frac{2}{36} \sum_{k=0}^{\infty} \frac{\psi(3^{k+n} x, 0, 0)}{9^{(k+n)}} \\ &\to 0 \quad as \quad n \to \infty \end{aligned}$$

for every $x \in X$. Thus Q is unique. Hence for $j = 1$ the theorem holds. Now, replace x by $\frac{x}{3}$ in (3.5)), we reach

$$\left\| g(x) - 9g(\frac{x}{3}) \right\| \leq \frac{\psi(\frac{x}{3}, 0, 0)}{4} \tag{3.10}$$

for every $x \in X$. The remainder of the proof is the same as for $j = 1$. As a result, the theorem holds for $j = -1$. This concludes the theorem's proof. □

Corollary 3.1 *Let $\rho, s \geq 0$. Let $g : X \to Y$ be a mapping satisfying the inequality*

$$\|Dg(x,y,z)\| \leq \begin{cases} \rho, & \\ \rho\left\{||x||^s + ||y||^s + ||z||^s\right\}, & s \neq 2; \\ \rho\left\{||x||^s||y||^s||z||^s + \left\{||x||^{3s} + ||y||^{3s} + ||z||^{3s}\right\}\right\}, & 3s \neq 2; \end{cases} \tag{3.11}$$

for any $x, y, z \in X$. Then there exists a unique quadratic function $Q : X \to Y$ such that

$$\|g(x) - C(x)\| \leq \begin{cases} \dfrac{\rho}{4|3^2 - 1|}, \\ \dfrac{\rho||x||^s}{4|3^2 - 3^s|}, \\ \dfrac{\rho||x||^{3s}}{4|3^2 - 3^{3s}|} \end{cases} \tag{3.12}$$

for any $x \in X$.

4. Introduction and Preliminaries

Now, we present the fundamental ideas of orthogonality and orthogonality normed spaces.

Definition 4.1 *[11] A vector space X is considered an orthogonality vector space if there is a relation $x \perp y$ on X such that*
(i) $x \perp 0$, $0 \perp x$ for every $x \in X$;
(ii) if $x \perp y$ and $x, y \neq 0$, then x, y are linearly independent;
(iii) $x \perp y$, $ax \perp by$ for every $a, b \in \mathbb{R}$;
(iv) if P is a two-dimensional subspace of X ; then
(a) for every $x \in P$ there exists $0 \neq y \in P$ such that $x \perp y$;
(b) there are vectors $x, y \neq 0$ such that $x \perp y$ and $x + y \perp x - y$.
Any vector space may be turned into an orthogonality vector space if we define $x \perp 0, 0 \perp x$ for all x and for non zero vector x, y define $x \perp y$ iff x, y are linearly independent. The relation $\perp$ is termed symmetric if $x \perp y$ implies that $y \perp x$ for any x, $y \in X$. The pair $(x, \perp)$ is termed an orthogonality space. When the orthogonality space is supplied with a norm, this becomes orthogonality normed space

S. Gudder and D. Strawther [11] were first to investigate the orthogonal Cauchy functional equation

$$f(x + y) = f(x) + f(y), x \perp y \tag{4.1}$$

where $\perp$ is an abstract orthogonal. In [10], R. Ger and J. Sikorska investigated the orthogonal stability of the equation (4.1) .

Definition 4.2 *Assume that X is an orthogonality space and Y is a real Banach space. For any $x, y \in X$ with $x \perp y$, a function $f : X \to Y$ is orthogonally quadratic if it obeys the following Euler-Lagrange quadratic functional equation*

$$f(x + y) + f(x - y) = 2f(x) + 2f(y). \tag{4.2}$$

F. Vajzovic [22] was the first to investigate the orthogonality Hilbert space for the orthogonally quadratic functional equation (4.2).

Here after, let $(A, \perp)$ be an orthogonality normed space with norm $\|\cdot\|_A$ and $(B, \|\cdot\|_B)$ is a Banach space. We construct a difference operator $Df : X^3 \to Y$ by

$$\begin{aligned} Df(x,y,z) = f(3x{+}2y+z) + f(3x+2y-z) + f(3x-2y+z) + f(3x-2y-z) \\ - 8[f(x+y) + f(x-y)] - 2[f(x+z) + f(x-z)] - 16f(x) \end{aligned}$$

for any $x, y, z \in X$ with $x \perp y, y \perp z$ and $z \perp x$ in the sense of Rätz.

5. Generalized Hyers-Ulam Stability of Modified Orthogonally Quadratic Functional Equation (1.5)

We describe the generalized Hyers-Ulam stability of the functional equation (1.5) in this section.

Theorem 5.1 *Assume that μ and $s(s < 2)$ are nonnegative real numbers. If $f : A \to B$ is a mapping satisfying*

$$\|Df(x,y,z)\|_B \leq \mu \left\{ \|x\|_A^s + \|y\|_A^s + \|z\|_A^s \right\} \tag{5.1}$$

for any $x, y, z \in A$ with $x \perp y, y \perp z$ and $z \perp x$, then there is a unique orthogonally quadratic mapping $Q : A \to B$ such that

$$\|f(x) - C(x)\|_B \leq \frac{\mu}{4 \cdot (9 - 3^s)} \|x\|_A^s \tag{5.2}$$

for every $x \in A$. The function $Q(x)$ is given by

$$C(x) = \lim_{n\to\infty} \frac{f(3^n x)}{9^n} \qquad \forall x \in A. \tag{5.3}$$

Proof: Replacing (x, y, z) with $(0, 0, 0)$ in (5.1), we obtain $f(0) = 0$. Setting (x, y, z) by $(x, 0, 0)$ in (5.1), we get

$$\|f(3x) - 9f(x)\|_B \leq \frac{\mu}{4} \|x\|_A^s \tag{5.4}$$

for all $x \in A$. Since $x \perp 0$, we have

$$\left\| \frac{f(3x)}{9} - f(x) \right\|_B \leq \frac{\mu}{36} \|x\|_A^s \tag{5.5}$$

for every $x \in A$. Now changing x by $3x$ and dividing by 9 in (5.5) and adding yielding inequality with (5.5)), we reach

$$\left\| \frac{f(3^2x)}{9^2} - f(x) \right\|_B \leq \frac{\mu}{36} \left\{ 1 + \frac{3^s}{9} \right\} \|x\|_A^s \tag{5.6}$$

for every $x \in A$. In general, using induction on a positive integer n, we derive that

$$\begin{aligned} \left\| \frac{f(3^n x)}{9^n} - f(x) \right\|_B &\leq \frac{\mu}{36} \sum_{k=0}^{n-1} \left(\frac{3^s}{9} \right)^k \|x\|_A^s \\ &\leq \frac{\mu}{36} \sum_{k=0}^{\infty} \left(\frac{3^s}{9} \right)^k \|x\|_A^s \end{aligned} \tag{5.7}$$

for every $x \in A$. In order to verify the convergence of the sequence $\{f(3^n x)/9^n\}$, substitute x with $3^m x$ and divide by 9^m in (5.7), for any $n, m > 0$, we get

$$\begin{aligned} \left\| \frac{f(3^n 3^m x)}{9^{(n+m)}} - \frac{f(3^m x)}{9^m} \right\|_B &= \frac{1}{9^m} \left\| \frac{f(3^n 3^m x)}{9^n} - f(3^m x) \right\|_B \\ &\leq \left(\frac{1}{9^m} \right) \frac{\mu}{36} \sum_{k=0}^{n-1} \left(\frac{3^s}{9} \right)^k \|3^m x\|_A^s \\ &\leq \frac{\mu}{36} \sum_{k=0}^{\infty} \left(\frac{3^s}{9} \right)^{k+m} \|x\|_A^s . \end{aligned} \tag{5.8}$$

The right hand side of (5.8) goes to 0 as $m \to \infty$ for e $x \in A$ as $s < 3$. Thus $\{f(3^n x)/9^n\}$ is a Cauchy sequence. Because B is complete, there is a mapping $Q : A \to B$ such that

$$Q(x) = \lim_{n\to\infty} \frac{f(3^n x)}{9^n} \quad \forall \quad x \in A.$$

The argument is identical to that of Theorem 3.1 in order to establish that Q is unique and that it satisfies the functional equation(1.5). The theorem's proof is now complete. □

Theorem 5.2 *Assume that μ and $s(s > 2)$ are non-negative real numbers. Let $f : A \to B$ be a function satisfying (5.1)) for any $x, y, z \in A$, with $x \perp y, y \perp z$ and $z \perp x$. Then there is a unique orthogonally quadratic mapping $Q : A \to B$ such that*

$$\|f(x) - C(x)\|_B \le \frac{\mu}{4 \cdot (3^s - 9)} \|x\|_A^s \tag{5.9}$$

for every $x \in A$. The function $Q(x)$ is given by

$$Q(x) = \lim_{n\to\infty} 9^n f\left(\frac{x}{3^n}\right) \qquad \forall x \in A. \tag{5.10}$$

Proof: After replacing x with $\frac{x}{3}$ in (5.4), the remainder of the proof is same as in Theorem 5.1. □

Theorem 5.3 *Let $f : A \to B$ be a mapping that satisfies the inequality*

$$\|D\ f(x, y, z)\|_B \le \mu \left(\|x\|_A^s \|y\|_A^s \|z\|_A^s + \left\{\|x\|_A^{3s} + \|y\|_A^{3s} + \|z\|_A^{3s}\right\}\right) \tag{5.11}$$

for every $x, y, z \in A$, with $x \perp y, y \perp z$ and $z \perp x$ where μ and s are constants with, $\mu, s > 0$ and $3s < 2$. Then the limit

$$Q(x) = \lim_{n\to\infty} \frac{f(3^n x)}{9^n} \tag{5.12}$$

exists for every $x \in A$ and $Q : A \to B$ is the unique orthogonally quadratic mapping such that

$$\|f(x) - Q(x)\|_B \le \frac{\mu}{4 \cdot (9 - 3^{3s})} \|x\|_A^{3s} \qquad \forall x \in A \tag{5.13}$$

Proof: When we replace (x, y, z) in (5.11) with $(0, 0, 0)$, we obtain $f(0) = 0$. By replacing (x, y, z) with $(x, 0, 0)$ in (5.11)), we get

$$\left\|\frac{f(3x)}{9} - f(x)\right\|_B \le \frac{\mu}{36} \|x\|_A^{3s} \tag{5.14}$$

for every $x \in A$. Now by changing x with $3x$ and dividing by 9 in (5.14), then adding the resultant inequality with (5.14)), we get

$$\left\|\frac{f(3^2 x)}{9^2} - f(x)\right\|_B \le \frac{\mu}{36}\left\{1 + \frac{3^{3s}}{9}\right\} \|x\|_A^{3s} \tag{5.15}$$

for each $x \in A$. We derive via induction on a positive integer n that

$$\begin{aligned}\left\|\frac{f(3^n x)}{9^n} - f(x)\right\|_B &\le \frac{\mu}{36} \sum_{k=0}^{n-1} \left(\frac{3^{3s}}{9}\right)^k \|x\|_A^{3s} \\ &\le \frac{\mu}{36} \sum_{k=0}^{\infty} \left(\frac{3^{3s}}{9}\right)^k \|x\|_A^{3s}\end{aligned} \tag{5.16}$$

for every $x \in A$. To show the convergence of the sequence $\{f(3^n x)/9^n\}$, substitute x with $3^m x$ and divide by 9^m in (5.16)), for every $n, m > 0$, we get

$$\begin{aligned} \left\| \frac{f\left(3^n 3^m x\right)}{9^{(n+m)}} - \frac{f(3^m x)}{9^m} \right\|_B &= \frac{1}{9^m} \left\| \frac{f\left(3^n 3^m x\right)}{9^n} - f\left(3^m x\right) \right\|_B \\ &\leq \left(\frac{1}{9^m}\right) \frac{\mu}{36} \sum_{k=0}^{n-1} \left(\frac{3^{3s}}{9}\right)^k \left\|3^m x\right\|_A^{3s} \\ &\leq \frac{\mu}{36} \sum_{k=0}^{\infty} \left(\frac{3^{3s}}{9}\right)^{k+m} \|x\|_A^{3s}. \end{aligned} \tag{5.17}$$

As $3s < 2$, for every $x \in A$, the right hand side of (5.17)) goes to 0 as $m \to \infty$. As a result, $\{f(3^n x)/9^n\}$ is a Cauchy sequence. Because B is complete, there is a function $Q : A \to B$ such that

$$Q(x) = \lim_{n\to\infty} \frac{f(3^n x)}{9^n} \quad \forall \;\; x \in A.$$

Letting $n \to \infty$ in (5.16), we get the formula (5.13) for any $x \in A$. To prove that Q is unique and it satisfies (1.5), the remainder of the proof is identical to that of Theorem 3.1. □

Theorem 5.4 *For any $x, y, z \in A$, let $f : A \to B$ be a mapping that satisfies the inequality* (5.11) *, with $x \perp y, y \perp z$ and $z \perp x$, where μ and s are constants with $\mu, s > 0$ and $s > 1$. Then the limit*

$$Q(x) = \lim_{n\to\infty} 9^n \; f\left(\frac{x}{3^n}\right) \tag{5.18}$$

exists for all values of $x \in A$ and $Q : A \to B$ is the only quadratic mapping such that

$$\|f(x) - Q(x)\|_B \leq \frac{\mu}{4 \cdot (3^{3s} - 9)} \|x\|_A^{3s} \qquad \forall x \in A. \tag{5.19}$$

Proof: Replacing x with $\frac{x}{3}$ in (5.15), the proof is identical to that in Theorem 5.3. □

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G. Yagachitradevi,
Department of Mathematics,
Siga College of Management and Computer Science,
Villupuram - 605 601, India.
E-mail address: yagachitradevi@gmail.com

and

S. Lakshminarayanan,
Department of Mathematics,
Arignar Anna Government Arts College,
Villupuram - 605 602, India.
E-mail address: slnvpm29@gmail.com

and

P. Ravindiran,
Department of Mathematics,
Arignar Anna Government Arts College,
Villupuram - 605 602, India.
E-mail address: p.ravindiran@gmail.com