Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2025 (43)** : 1–10.
ISSN-0037-8712
doi:10.5269/bspm.66398

On L-fuzzy (K, E)-soft convex space

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ABSTRACT: In this paper, we introduce the concept of L-fuzzy (K, E)-soft convex spaces, and study some of their properties. Also, we study the notion of L-fuzzy soft convexity preserving and, L-fuzzy soft convex-to-convex mappings and an L-fuzzy (K, E)-soft closured convexity space. Also, We study their properties and discuss the relationships between these concepts.
AMS Subject Classification: 54A20, 54A40, 03E72.

Key Words: L-fuzzy soft set, convex space, L-fuzzy convex space, L-fuzzy soft convex space, L-fuzzy soft closure space.

Contents

1. Introduction and Preliminaries

It is well known that the abstract convexity theory deals with set-theoretic structures which satisfies axioms similar to that usual convex sets fulfill and the concept of convex structures can be treated as a special kind of spatial structures and some topology-like properties. The basic concepts of abstract convexity theory can also be found in [15,16]. Some applications of abstract convexity theory can be found in [5,6,14,17]. The concept of a fuzzy convex structure appeared for the first time in [10,11] which is called an I-convex structure. However, similar concepts with slight changes already appeared in [8,9,19,22,23,24]. One of the recent directions is the study of generalized convex structures [12,13,18,20,21] and its applications. In [13], Shi and Xiu studied an (L, M)-fuzzy convex structures as a generalization of L-convex structures and M-fuzzifying convex structures. The main contribution of the present paper is to give some investigations on L-fuzzy (K, E)-soft convex spaces, mainly including L fuzzy soft hull operator with respect to L-fuzzy (K, E)-soft convex spaces where L is completely distributive lattices with order reversing involution " $'$ " where $\perp_L$ and $\top_L$ denote the least and greatest elements in L. An L-fuzzy soft convexity preserving and an L-fuzzy soft convex-to-convex mappings was given. An L-fuzzy (K, E)-soft closured convexity space was introduced. Throughout this paper, let X be a non-empty set, both E and K are the sets of all parameters for X and L be completely distributive lattices with order reversing involution $'$ where $\perp_L$ and $\top_L$ denote the least and the greatest elements in L respectively, and $L_{\perp_L} = L - \{\perp_L\}$.

Definition 1.1 *[1,7] A map f_A is called an L-fuzzy soft set on X, where f_A is a mapping from E into L^X, i.e., $(f_A)_e := f_A(e)$ is an L-fuzzy soft set on X, for each $e \in E$. The set of all L-fuzzy soft set is denoted by $(L^X)^E$. Let $f_A, g_B \in (L^X)^E$.*

(1) f_A is an L-fuzzy soft subset g_B and we write $f_A \sqsubseteq g_B$ if $f_A(e) \le g_B(e)$, for each $e \in E$. f_A and g_B are equal denoted by $f_A \cong g_B$ if $f_A \sqsubseteq g_B$ and $g_B \sqsubseteq f_A$.

(2) The intersection of f_A and g_B is an L-fuzzy soft set $h_C = f_A \sqcap g_B$, where $h_C(e) = f_A(e) \wedge g_B(e)$, for each $e \in E$.

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Submitted December 22, 2022. Published September 24, 2025

(3) The union of f_A and g_B is an L-fuzzy soft set $h_C = f_A \sqcup g_B$, where $h_C(e) = f_A(A) \vee g_B(e)$, for each $e \in E$.

(4) The complement of an L-fuzzy soft sets on X is denoted by f'_A, where $f'_A : E \longrightarrow (L^X)^E$ is a mapping given by $(f'_A)(e) = (f_A(e))'$, for each $e \in E$.

(5) f_A is called a null L-fuzzy soft set and denoted by $\tilde{0}_E$ if $f_A(e)(x) = \perp$, for each $e \in E$, and $x \in X$.

(6) f_A is called absolute L-fuzzy soft set and denoted by $\tilde{1}_E$ if $f_A(e)(x) = \top$, for each $e \in E$, and $x \in X$.

Definition 1.2 *[3,4]. Let $(L^X)^E$ and $(L^Y)^{E^*}$ be classes of L-fuzzy soft sets over X and Y with attributes from E and E^* respectively. Let $\varphi : X \to Y$ and $\psi : E \to E^*$ be mappings. Then a fuzzy soft mapping $\varphi_\psi : (L^X)^E \to (L^Y)^{E^*}$ would be defined as follows:*

(1) For an L-fuzzy soft set f_A in $(L^X)^E, \varphi_\psi^\to(f_A)$ is an L-fuzzy soft set in $(L^Y)^{E^*}$ obtained as follows: for $e^* \in \psi(E) \subseteq E^*$ and $y \in Y$,

$$\varphi_\psi^\to(f_A)(e^*)(y) = \begin{cases} \bigvee_{x\in\varphi^{-1}(y)} \left(\bigvee_{e\in\psi^{-1}} f_A(e)\right)(x), & \text{if } \varphi^{-1}(y) \neq \phi, \\ & \psi^{-1}(e^*) \neq \phi, \\ \perp, & \text{if otherwise.} \end{cases}$$

$\varphi_\psi^\to(f_A)$ is called a fuzzy soft image of an L-fuzzy soft set f_A.

(2) For an L-fuzzy soft set g_B in $(L^Y)^{E^*}, \varphi_\psi^\leftarrow(g_B)$ is an L-fuzzy soft set in $(L^X)^E$ obtained as follows: for $e \in \psi^{-1}(E^*) \subseteq E$ and $x \in X$,

$$\varphi_\psi^\leftarrow(g_B)(e)(x) = g_B(\psi(e))(\varphi(x))$$

$\varphi_\psi^\leftarrow(g_B)$ is called a fuzzy soft inverse image of an L-fuzzy soft set g_B.

(3) A fuzzy soft mapping $\varphi_\psi : (L^X)^E \to (L^Y)^{E^*}$ is called injective (resp. surjective, bijective) if φ and ψ are both injective (resp. surjective, bijective).

Lemma 1.1 *[4]. Let $\varphi_\psi : (L^X)^E \to (L^Y)^{E^*}$ be a soft mapping. Then we have the following properties. For $f_A, f_{A_i} \in (L^X)^E$ and $g_B, g_{B_i} \in (L^Y)^{E^*}$,*

(1) $\varphi_\psi^\to(\varphi_\psi^\leftarrow(g_B)) \sqsubseteq g_B$ with equality if φ_ψ is surjective.
(2) $\varphi_\psi^\leftarrow(\varphi_\psi^\to(f_A)) \sqsupseteq f_A$ with equality if φ_ψ is injective.
(3) $\varphi_\psi^\leftarrow(g'_B) \cong (\varphi_\psi^\leftarrow(g_B))'$.
(4) $\varphi_\psi^\leftarrow(\sqcup_{i\in\Gamma} g_{B_i}) \cong \sqcup_{i\in\Gamma}\varphi_\psi^\leftarrow(g_{B_i})$.
(5) $\varphi_\psi^\leftarrow(\sqcap_{i\in\Gamma} g_{B_i}) \cong \sqcap_{i\in\Gamma}\varphi_\psi^\leftarrow(g_{B_i})$.
(6) $\varphi_\psi^\to(\sqcup_{i\in\Gamma} f_{A_i}) \cong \sqcup_{i\in\Gamma}\varphi_\psi^\to(f_{A_i})$.
(7) $\varphi_\psi^\to(\sqcap_{i\in\Gamma} f_{A_i}) \sqsubseteq \sqcap_{i\in\Gamma}\varphi_\psi^\to(f_{A_i})$ with equality if φ_ψ is injective.

Definition 1.3 *[2] A map $cl : K \times (L^X)^E \times L_\perp \longleftarrow (L^X)^E$ is called an L-fuzzy (K, E)-soft closure operator if it satisfies the following conditions:*

(1) $cl(k, \tilde{0}_E, r) \cong \tilde{0}_E$.
(2) $f_A \sqsubseteq cl(k, f_A, r)$.
(3) If $f_{A_1} \sqsubseteq f_{A_2}$ then $cl(k, f_{A_1}, r) \sqsubseteq cl(k, f_{A_2}, r)$.
(4) If $r \leq s$ then $cl(k, f_A, r) \sqsubseteq cl(k, f_A, s)$.
(5) $cl(k, f_{A_1} \sqcup f_{A_2}, r \wedge s) \sqsubseteq cl(k, f_{A_1}, r) \sqcup cl(k, f_{A_1}, s)$.

The pair (X, cl) is called an L-fuzzy (K, E)-soft closure space. An L-fuzzy (K, E)-soft closure operator is called topological if

(T) $cl(k, cl(k, f_A, r), r) \sqsubseteq cl(k, f_A, r)$.

Theorem 1.1 *[4] Let $(X, \mathcal{T})$ be an L-fuzzy (K, E)-soft topological space. Define $cl : K \times (L^X)^E \times L_\perp \longleftarrow (L^X)^E$ as*

$$cl(k, f_A, r) \cong \sqcap\{g_B \in (L^X)^E : f_A \sqsubseteq g_B, \mathcal{T}_k(g'_B) \geq r\}.$$

Then cl is a topological L-fuzzy (K, E)-soft closure operator.

2. L-fuzzy (K,E)-soft convex space

Definition 2.1 *A mapping $\mathcal{C}: K \longrightarrow L^{(L^X)^E}$ where ($\mathcal{C}_k := \mathcal{C}(k) : (L^X)^E \longrightarrow L$ is a mapping for each $k \in K$) is called an L-fuzzy (K,E)-soft convexity on X if it satisfies the following conditions for each $k \in K$.*

(1) $\mathcal{C}_k(\tilde{0}_E) = \mathcal{C}_k(\tilde{1}_E) = \top_L$.

(2) If $\{f_{A_i} : i \in \Gamma\} \subseteq (L^X)^E$ is nonempty, then $\mathcal{C}_k(\sqcap_{i\in\Gamma} f_{A_i}) \geq \bigwedge_{i\in\Gamma} \mathcal{C}_k(f_{A_i})$.

(3) If $\{f_{A_i} : i \in \Gamma\} \subseteq (L^X)^E$ is nonempty and totally ordered by inclusion, then $\mathcal{C}_k(\sqcup_{i\in\Gamma} f_{A_i}) \geq \bigwedge_{i\in\Gamma} \mathcal{C}_k(f_{A_i})$.

The pair $(X,\mathcal{C})$ is called an L-fuzzy (K,E)-soft convex space. Let $\mathcal{C}^1, \mathcal{C}^2$ be L-fuzzy (E,K)-soft convexities on X, then $\mathcal{C}^1$ is coarser than $\mathcal{C}^2$ ($\mathcal{C}^2$ is finer than $\mathcal{C}^1$) if $\mathcal{C}^1_k(f_A) \leq \mathcal{C}^2_k(f_A)$ for all $f_A \in (L^X)^E, k \in K$.

Theorem 2.1 *Let $\{\mathcal{C}^i : i \in \Gamma\}$ be a family of L-fuzzy (K,E)-soft convexities on X. Then $\bigwedge_{i\in\Gamma} \mathcal{C}^i$ is an L-fuzzy (K,E)-soft convexity on X, where $\bigwedge_{i\in\Gamma} \mathcal{C}^i : K \longrightarrow L^{(L^X)^E}$ is defined by $(\bigwedge_{i\in\Gamma} \mathcal{C}^i)_k(f_A) = \bigwedge_{i\in\Gamma} \mathcal{C}^i_k(f_A)$ for each $f_A \in (L^X)^E, k \in K$. Obviously, $(\bigwedge_{i\in\Gamma} \mathcal{C}^i)_k$ is coarser than $\mathcal{C}^i_k$ for all $i \in \Gamma, k \in K$.*

Proof: The proof is straightforward. □

Theorem 2.2 *Let $(X,\mathcal{C})$ be an L-fuzzy (K,E)-soft convex space. For each $f_A \in (L^X)^E$ and $r \in L_\perp$ a mapping $CO : K \times (L^X)^E \times L_\perp \longrightarrow (L^X)$ is defined as follows:*

$$CO(k, f_A, r) = \bigwedge\{g_B \in (L^X)^E : f_A \sqsubseteq g_B,\ \mathcal{C}_k(g_B) \geq r\}.$$

For $f_A, f_{A_1} \in (L^X)^E$ and $r, s \in L_\perp$ the operator CO satisfies the following conditions:

(1) $CO(k, \tilde{0}_E, r) \cong \tilde{0}_E$.

(2) $f_A \sqsubseteq CO(k, f_A, r)$.

(3) If $f_A \sqsubseteq f_{A_1}$, then $CO(k, f_A, r) \sqsubseteq CO(k, f_{A_1}, r)$.

(4) If $r \leq s$, then $CO(k, f_A, r) \sqsubseteq CO(k, f_A, s)$.

(5) $CO(k, CO(k, f_A, r), r) \cong CO(k, f_A, r)$.

(6) For $\{f_{A_i} : i \in \Gamma\} \subseteq (L^X)^E$ is nonempty and totally ordered by inclusion, $CO(k, \sqcup_{i\in\Gamma} f_{A_i}, r) \cong \sqcup_{i\in\Gamma} CO(k, f_{A_i}, r)$.

A mapping CO is called an L-fuzzy soft hull operator.

Proof: (1) For all $r \in L_\perp, k \in K$ we have $\mathcal{C}_k(\tilde{0}_E) \geq r$. So, we obtain $CO(k, \tilde{0}_E, r) \cong \tilde{0}_E$.

(2) and (3) are satisfied from the definition of CO.

(4) Suppose that $r \leq s$. Then by (2) we have

$$CO(k, f_A, r) \sqsubseteq CO(k, CO(k, f_A, s), r).$$

By the definition of CO, we obtain $\mathcal{C}_k(CO(k, f_A, s)) \geq r$. Therefore, $CO(k, CO(k, f_A, s), r) \cong CO(k, f_A, s)$. Hence $CO(k, f_A, r) \sqsubseteq CO(k, f_A, s)$.

(5) For all $f_A \in (L^X)^E, k \in K$ and $r \in L_\perp$. By the definition of $CO(k, f_A, r)$ we have $f_A \sqsubseteq CO(k, f_A, r)$. Hence, $CO(k, CO(k, f_A, r), r) \sqsupseteq CO(k, f_A, r)$. On the other hand

$$\begin{aligned} CO(k, CO(k, f_A, r), r) &\cong CO\big(k, \bigwedge\{g_B \in (L^X)^E : f_A \sqsubseteq g_B,\ \mathcal{C}_k(g_B) \geq r\}, r\big) \\ &\sqsubseteq \bigwedge_{f_A \sqsubseteq g_B,\ \mathcal{C}_k(g_B) \geq r} CO(k, g_B, r) \\ &\cong \bigwedge_{f_A \sqsubseteq g_B,\ \mathcal{C}_k(g_B) \geq r} \ \bigwedge_{g_B \sqsubseteq h_C,\ \mathcal{C}_k(h_C) \geq r} h_C \\ &\cong \bigwedge_{f_A \sqsubseteq h_C,\ \mathcal{C}_k(h_C) \geq r} h_C \\ &\cong CO(k, f_A, r). \end{aligned}$$

Hence, $CO(k, CO(k, f_A, r), r) \cong CO(\mu, r)$.

(6) For $i \in \Gamma$, we have $f_{A_i} \sqsubseteq \sqcup f_{A_i}$. Therefore by (3) we have $CO(k, f_{A_i}, r) \sqsubseteq CO(k, \sqcup f_{A_i}, r)$. Hence,

$$\sqcup CO(k, f_{A_i}, r) \sqsubseteq CO(k, \sqcup f_{A_i}, r). \tag{2.1}$$

On the other hand, by (2), we have $\sqcup f_{A_i} \sqsubseteq \sqcup CO(k, f_{A_i}, r)$. Since $CO(k, f_{A_i}, r)$ are L-fuzzy soft convex sets totally ordered by inclusion, $\sqcup CO(k, f_{A_i}, r)$ is an r-L-fuzzy convex set containing $\sqcup f_{A_i}$. Therefore, $CO(k, \sqcup f_{A_i}, r)$ is the smallest fuzzy convex set containing $\sqcup f_{A_i}$ and hence,

$$\sqcup f_{A_i} \sqsubseteq CO(k, \sqcup f_{A_i}, r) \sqsubseteq \sqcup CO(k, f_{A_i}, r). \tag{2.2}$$

By equations (2.1) and (2.2), we have, $CO(k, \sqcup f_{A_i}, r) \cong \sqcup CO(k, f_{A_i}, r)$. □

Definition 2.2 *Let E be a set of parameters, X be an initial universe, $\emptyset \neq Y \subseteq X$, $\emptyset \neq E^* \subseteq E$ and $f_A \in (L^X)^E$; the restriction of f_A on Y, is denoted by $f_A|Y$ which is defined by: $(f_A|Y)(e^*)(y) = f_A(e^*)(y)$ for all $y \in Y, e^* \in E^*$. Obviously, for $\{f_{A_i} : i \in \Gamma\} \subseteq (L^X)^E$, we have*

(1) $(\sqcup_i f_{A_i})|Y = \sqcup_i (f_{A_i}|Y)$.

(2) $(\sqcap_i f_{A_i})|Y = \sqcap_i (f_{A_i}|Y)$.

(3) $f'_A|Y = (f_A|Y)'$.

For each $f_A \in (L^Y)^{E^*}$ an extension of f_A on X, denoted by $(f_A)_X$, is defined by

$$(f_A)_X(e)(x) = \begin{cases} f_A(e)(x), & \text{if } x \in Y, e \in E^*, \\ \\ \bot, & \text{if } x \in X - Y, e \in E - E^*. \end{cases}$$

Theorem 2.3 *Let $(X, \mathcal{C})$ be an L-fuzzy (K, E)-soft convex space, $\emptyset \neq Y \subseteq X$, $\emptyset \neq E^* \subseteq E$ and $\emptyset \neq K^* \subseteq K$. Define $\mathcal{C}|Y : K^* \longrightarrow L^{(L^Y)^{E^*}}$ where $((\mathcal{C}|Y)_{k^*} := (\mathcal{C}|Y)(k^*) : (L^Y)^{E^*} \longrightarrow L$ is a mapping for each $k^* \in K^*$) as following:*

$$(\mathcal{C}|Y)_k(f_A) = \bigvee \{\mathcal{C}_k(g_B) : g_B \in (L^X)^E, g_B|Y = f_A\}.$$

Then $(Y, \mathcal{C}|Y)$ is an L-fuzzy (K^*, E^*)-soft convex space on Y and we call $(Y, \mathcal{C}|Y)$ an L-fuzzy (K^*, E^*)-soft subspace of $(X, \mathcal{C})$.

Proof: (1) Clearly, $(\mathcal{C}|Y)_k(\tilde{0}_{E^*}) = (\mathcal{C}|Y)_k(\tilde{1}_{E^*}) = \top_L$.

(2) For $i \in \Gamma, f_{A_i} \in (L^Y)^{E^*}$ and $k \in K^*$, we have

$$\begin{aligned} \textstyle\bigwedge_i (\mathcal{C}|Y)_k(f_{A_i}) &= \bigwedge_i \bigvee \{\mathcal{C}_k(g_{B_i}) : g_{B_i} \in (L^X)^E, g_{B_i}|Y = f_{A_i}\} \\ &= \bigvee \bigwedge_i \{\mathcal{C}_k(g_{B_i}) : g_{B_i} \in (L^X)^E, g_{B_i}|Y = f_A\} \\ &\leq \bigvee \{\mathcal{C}_k(\sqcap_i g_{B_i}) : \sqcap_i g_{B_i} \in (L^X)^E, (\sqcap_i g_{B_i})|Y = \sqcap_i f_{A_i}\} \\ &= (\mathcal{C}|Y)_k(\sqcap_i f_{A_i}). \end{aligned}$$

(3) Let $i \in \Gamma, \{f_{A_i} : i \in \Gamma\} \subseteq (L^Y)^{E^*}$ is nonempty and totally ordered by inclusion and $k \in K^*$, then

$$\begin{aligned} \textstyle\bigwedge_i (\mathcal{C}|Y)_k(f_{A_i}) &= \bigwedge_i \bigvee \{\mathcal{C}_k(g_{B_i}) : g_{B_i} \in (L^X)^E, g_{B_i}|Y = f_{A_i}\} \\ &= \bigvee \bigwedge_i \{\mathcal{C}_k(g_{B_i}) : g_{B_i} \in (L^X)^E, g_{B_i}|Y = f_A\} \\ &\leq \bigvee \{\mathcal{C}_k(\sqcup_i g_{B_i}) : \sqcup_i g_{B_i} \in (L^X)^E, (\sqcup_i g_{B_i})|Y = \sqcup_i f_{A_i}\} \\ &= (\mathcal{C}|Y)_k(\sqcup_i f_{A_i}). \end{aligned}$$

Hence the proof is complete. □

3. L-fuzzy soft convexity preserving mappings

Definition 3.1 *Let $(X, \mathcal{C})$ be an L-fuzzy (E^1, K^1)-soft convex space and $(Y, \mathcal{D})$ be an L-fuzzy (E^2, K^2)-soft convex space. Let $\varphi : X \longleftarrow Y$, $\psi : E^1 \longleftarrow E^2$ and $\eta : K^1 \longleftarrow K^2$. Then $\varphi_{\psi,\eta}$ from $(X, \mathcal{C}^1)$ into $(Y, \mathcal{C}^2)$ is called:*

(1) L-fuzzy soft convexity preserving if

$$\mathcal{D}_{\eta(k)}(f_A) \leq \mathcal{C}_k(\varphi_\psi^{\leftarrow}(f_A)) \quad \forall f_A \in (L^Y)^{E^2}, k \in K^1.$$

(2) L-fuzzy soft convex-to-convex if

$$\mathcal{C}_k(f_A) \leq \mathcal{D}_{\eta(k)}(\varphi_\psi^{\rightarrow}(f_A)) \quad \forall f_A \in (L^X)^{E^1}, k \in K^1.$$

Theorem 3.1 *Let $(Y, \mathcal{D})$ be an L-fuzzy (K^2, E^2) soft convex space and φ_ψ a surjective mapping. Define a mapping $\varphi_\psi^{\leftarrow}(\mathcal{D}) : K^1 \longrightarrow L^{(L^X)^{E^1}}$ by*

$$\left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(f_A) = \bigvee\{\mathcal{D}_{\eta(k)}(g_B) : \varphi_\psi^{\leftarrow}(g_B) = f_A\} \;\; \forall f_A \in (L^X)^{E^1}, k \in K^1.$$

Then, $(X, \varphi_\psi^{\leftarrow}(\mathcal{D}))$ is an L-fuzzy (K^1, E^1)-soft convex space on X.

Proof: (1) For all $r \in L$ and $k \in K^1$ we obatin

$$\left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(\tilde{0}_{E^1}) = \bigvee\{\mathcal{D}_{\eta(k)}(g_B) : \varphi_\psi^{\leftarrow}(g_B) = \tilde{0}_{E^1}\} = \mathcal{D}_{\eta(k)}(\tilde{0}_{E^2}) = \top_L$$

and

$$\left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(\tilde{1}_{E^1}) = \bigvee\{\mathcal{D}_{\eta(k)}(g_B) : \varphi_\psi^{\leftarrow}(g_B) = \tilde{1}_{E^1}\} = \mathcal{D}_{\eta(k)}(\tilde{1}_{E^2}) = \top_L.$$

(2) Suppose that $r \in L$ and $r \lhd \bigwedge_i \left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(f_{A_i})$. Then $r \lhd \left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(f_{A_i})$ for $k \in K^1, f_{A_i} \in (L^X)^{E^1}$ and $i \in \Gamma$. There exists $r_0^i \in L$ such that

$$\left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(f_{A_i}) = \bigvee\{\mathcal{D}_{\eta(k)}(g_B) : \varphi_\psi^{\leftarrow}(g_B) = f_{A_i}\} \geq r_0^i \text{ and } r \lhd r_0^i$$

(thus $r \leq r_0^i$). Put $s = \bigwedge_{i \in \Gamma} r_0^i$ then $r \leq s$. Therefore for each $i \in \Gamma$ there exists $g_{B_i} \in (L^Y)^{E^2}$ such that $\varphi_\psi^{\leftarrow}(g_{B_i}) = f_{A_i}$ and $\mathcal{D}_{\eta(k)}(g_{B_i}) \geq s$. Since $\varphi_\psi^{\leftarrow}(\sqcap_i g_{B_i}) = \sqcap_i \varphi_\psi^{\leftarrow}(g_{B_i}) = \sqcap_i f_{A_i}$ and $\mathcal{D}_{\eta(k)}(\sqcap_i g_{B_i}) \geq \bigwedge_i \mathcal{D}_{\eta(k)}(g_{B_i}) \geq s$ we have

$$\begin{aligned}\left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(\sqcap_i f_{A_i}) &= \bigvee\{\mathcal{D}_{\eta(k)}(\sqcap_i g_{B_i}) : \varphi_\psi^{\leftarrow}(\sqcap_i g_{B_i}) = \sqcap_i f_{A_i}\} \\ &\geq \mathcal{D}_{\eta(k)}(\sqcap_i g_{B_i}) \geq s \geq r.\end{aligned}$$

Hence, $\left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(\sqcap_i f_{A_i}) \geq \bigwedge_i \left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(f_{A_i})$.

(3) Let $\{f_{A_i} : i \in \Gamma\} \subseteq (L^X)^{E^1}$ is totally ordered by inclusion, $r \in L$ and $r \lhd \bigwedge_i \left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(f_{A_i})$. Then $r \lhd \left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(f_{A_i})$ for $k \in K^1, f_{A_i} \in (L^X)^{E^1}$ and $i \in \Gamma$. There exists $r_0^i \in L$ such that

$$\left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(f_{A_i}) = \bigvee\{\mathcal{D}_{\eta(k)}(g_B) : \varphi_\psi^{\leftarrow}(g_B) = f_{A_i}\} \geq r_0^i \text{ and } r \lhd r_0^i$$

(thus $r \leq r_0^i$). Put $s = \bigwedge_{i \in \Gamma} r_0^i$ then $r \leq s$. Therefore for each $i \in \Gamma$ there exists $g_{B_i} \in (L^Y)^{E^2}$ such that $\varphi_\psi^{\leftarrow}(g_{B_i}) = f_{A_i}$ and $\mathcal{D}_{\eta(k)}(g_{B_i}) \geq s$. Since φ_ψ is surjective and $\{f_{A_i} : i \in \Gamma\}$ is totally ordered by inclusion we have $\{g_{B_i} : i \in \Gamma\}$ is totally ordered by inclusion. Since $\varphi_\psi^{\leftarrow}(\sqcup_i g_{B_i}) = \sqcup_i \varphi_\psi^{\leftarrow}(g_{B_i}) = \sqcup_i f_{A_i}$ and $\mathcal{D}_{\eta(k)}(\sqcup_i g_{B_i}) \geq \bigwedge_i \mathcal{D}_{\eta(k)}(g_{B_i}) \geq s$ we have

$$\begin{aligned}\left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(\sqcup_i f_{A_i}) &= \bigvee\{\mathcal{D}_{\eta(k)}(\sqcup_i g_{B_i}) : \varphi_\psi^{\leftarrow}(\sqcup_i g_{B_i}) = \sqcup_i f_{A_i}\} \\ &\geq \mathcal{D}_{\eta(k)}(\sqcup_i g_{B_i}) \geq s \geq r.\end{aligned}$$

Hence, $\left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(\sqcup_i f_{A_i}) \geq \bigwedge_i \left(\varphi_\psi^{\leftarrow}(\mathcal{D})\right)_k(f_{A_i})$. □

Theorem 3.2 *Let $(X,\mathcal{C})$ be an L-fuzzy (E^1,K^1)-soft convex space and $(Y,\mathcal{D})$ be an L-fuzzy (E^2,K^2)-soft convex space. A surjective mapping $\varphi_{\psi,\eta}$ from $(X,\mathcal{C})$ into $(Y,\mathcal{D})$ is an L-fuzzy soft convexity preserving if and only if $\left(\varphi_{\psi}^{\leftarrow}(\mathcal{D})\right)_k(f_A)\leq \mathcal{C}_k(f_A)$ for all $k\in K^1, f_A\in (L^X)^{E^1}$.*

Proof: Let $\varphi_{\psi,\eta}$ from $(X,\mathcal{C})$ into $(Y,\mathcal{D})$ is an L-fuzzy soft convexity preserving mapping, then $\mathcal{D}_{\eta(k)}(g_B)\leq \mathcal{C}_k(\varphi_{\psi}^{\leftarrow}(g_B))$ for all $g_B\in (L^Y)^{E^2}, k\in K^1$. Hence for all $f_A\in (L^X)^{E^1}$ we have

$$\begin{aligned}\left(\varphi_{\psi}^{\leftarrow}(\mathcal{D})\right)_k(f_A) &= \bigvee\{\mathcal{D}_{\eta(k)}(g_B):\varphi_{\psi}^{\leftarrow}(g_B)=f_A\}\\ &\leq \bigvee\{\mathcal{C}_k(\varphi_{\psi}^{\leftarrow}(g_B)):\varphi_{\psi}^{\leftarrow}(g_B)=f_A\}\\ &= \mathcal{C}_k(f_A).\end{aligned}$$

Conversely, Let $\left(\varphi_{\psi}^{\leftarrow}(\mathcal{D})\right)_k(f_A)\leq \mathcal{C}_k(f_A)$ for all $k\in K^1, f_A\in (L^X)^{E^1}$. Then for all $k\in K^1, g_B\in (L^Y)^{E^2}$ we have

$$\begin{aligned}\mathcal{D}_{\eta(k)}(g_B) &= \bigvee\{\mathcal{D}_{\eta(k)}(h_C):\varphi_{\psi}^{\leftarrow}(h_C)=\varphi_{\psi}^{\leftarrow}(g_B)\}\\ &= \left(\varphi_{\psi}^{\leftarrow}(\mathcal{D})\right)_k(\varphi_{\psi}^{\leftarrow}(g_B))\\ &\leq \mathcal{C}_k(\varphi_{\psi}^{\leftarrow}(g_B)).\end{aligned}$$

Therefore $\varphi_{\psi,\eta}$ from $(X,\mathcal{C})$ into $(Y,\mathcal{D})$ is an L-fuzzy soft convexity preserving mapping. □

Theorem 3.3 *Let $(X,\mathcal{C})$ be an L-fuzzy (K^1,E^1) soft convex space and φ_{ψ} a surjective mapping. Define a mapping $\mathcal{C}_{/\varphi_{\psi}}:K^2\longrightarrow L^{(L^Y)^{E^2}}$ by*

$$\left(\mathcal{C}_{/\varphi_{\psi}}\right)_{\eta(k)}(f_A)=\mathcal{C}_k(\varphi_{\psi}^{\leftarrow}(f_A))\ \ \forall f_A\in (L^Y)^{E^2}, k\in K^1.$$

Then:

(1) $(Y,\mathcal{C}_{/\varphi_{\psi}})$ is an L-fuzzy (K^2,E^2)-soft convex space on Y and we call $\mathcal{C}_{/\varphi_{\psi}}$ a quotient L-fuzzy soft convexity on Y with respect to $\mathcal{C}$ and φ_{ψ}.

(2) φ_{ψ} is an L-fuzzy soft convexity preserving mapping from $(X,\mathcal{C})$ to $(Y,\mathcal{C}/\varphi_{\psi})$.

Proof: (1)

(1) For all $r\in L$ and $k\in K^1$ we obatin

$$\left(\mathcal{C}_{/\varphi_{\psi}}\right)_{\eta(k)}(\tilde{0}_{E^2})=\mathcal{C}_k(\varphi_{\psi}^{\leftarrow}(\tilde{0}_{E^2}))=\mathcal{C}_k(\tilde{0}_{E^1})=\top_L$$

and

$$\left(\mathcal{C}_{/\varphi_{\psi}}\right)_{\eta(k)}(\tilde{1}_{E^2})=\mathcal{C}_k(\varphi_{\psi}^{\leftarrow}(\tilde{1}_{E^2}))=\mathcal{C}_k(\tilde{1}_{E^1})=\top_L$$

(2) Let $f_{A_i}\in (L^Y)^{E^1}$. Then for $i\in\Gamma$ and $k\in K^1$ we have

$$\begin{aligned}\left(\mathcal{C}_{/\varphi_{\psi}}\right)_{\eta(k)}(\sqcap_i f_{A_i}) &= \mathcal{C}_k(\varphi_{\psi}^{\leftarrow}(\sqcap_i f_{A_i}))=\mathcal{C}_k(\sqcap_i\varphi_{\psi}^{\leftarrow}(f_{A_i}))\\ &\geq \bigwedge_i \mathcal{C}_k(\varphi_{\psi}^{\leftarrow}(f_{A_i}))=\bigwedge_i \left(\mathcal{C}_{/\varphi_{\psi}}\right)_{\eta(k)}(f_{A_i})\end{aligned}$$

(3) Let $\{f_{A_i}:i\in\Gamma\}\subseteq (L^Y)^{E^2}$ is totally ordered by inclusion. Then for $i\in\Gamma$ and $k\in K^1$ we have

$$\begin{aligned}\left(\mathcal{C}_{/\varphi_{\psi}}\right)_{\eta(k)}(\sqcup_i f_{A_i}) &= \mathcal{C}_k(\varphi_{\psi}^{\leftarrow}(\sqcup_i f_{A_i}))=\mathcal{C}_k(\sqcup_i\varphi_{\psi}^{\leftarrow}(f_{A_i}))\\ &\geq \bigwedge_i \mathcal{C}_k(\varphi_{\psi}^{\leftarrow}(f_{A_i}))=\bigwedge_i \left(\mathcal{C}_{/\varphi_{\psi}}\right)_{\eta(k)}(f_{A_i})\end{aligned}$$

(2) Obvious. □

Theorem 3.4 *Let $(X,\mathcal{C})$ be an L-fuzzy (K^1,E^1) soft convex space and φ_ψ a surjective mapping. Then $\mathcal{C}_{/\varphi_\psi}$ is finer than $\mathcal{D}$ such that φ_ψ is an L-fuzzy soft convexity preserving mapping from $(X,\mathcal{C})$ to $(Y,\mathcal{D})$.*

Proof: Let $\mathcal{D}$ is L-fuzzy soft convexity on Y such that φ_ψ is an L-fuzzy soft convexity preserving mapping from $(X,\mathcal{C})$ to $(Y,\mathcal{D})$. Then we have $\mathcal{D}_{\eta(k)}(f_A)\le\mathcal{C}_k(\varphi_\psi^{\leftarrow}(f_A))$ for all $f_A\in(L^Y)^{E^2}$ and $k\in K^1$. Therefore $\mathcal{D}_{\eta(k)}(f_A)\le\mathcal{C}_k(\varphi_\psi^{\leftarrow}(f_A))=(\mathcal{C}_{/\varphi_\psi})_{\eta(k)}(f_A)$. Hence $\mathcal{C}_{/\varphi_\psi}$ is finer than $\mathcal{D}$. □

Theorem 3.5 *Let φ_ψ is a surjective L-fuzzy soft convexity preserving mapping and an L-fuzzy soft convex-to-convex mapping from $(X,\mathcal{C})$ to $(Y,\mathcal{D})$. Then $\mathcal{D}$ is a quotient L-fuzzy soft convexity.*

Proof: Since φ_ψ is a surjective L-fuzzy soft convexity preserving mapping and an L-fuzzy soft convex-to-convex mapping from $(X,\mathcal{C})$ to $(Y,\mathcal{D})$. We obtain

$$\mathcal{D}_{\eta(k)}(f_A)\le\mathcal{C}_k(\varphi_\psi^{\leftarrow}(f_A))\quad\forall f_A\in(L^Y)^{E^2},k\in K^1$$

and

$$\mathcal{C}_k(f_A)\le\mathcal{D}_{\eta(k)}(\varphi_\psi^{\rightarrow}(f_A))\quad\forall f_A\in(L^X)^{E^1},k\in K^1.$$

Since φ_ψ is a surjective we have $\varphi_\psi^{\rightarrow}(\varphi_\psi^{\leftarrow}(f_A))=f_A$. Hence

$$\begin{aligned}\mathcal{D}_{\eta(k)}(f_A)\quad&=\mathcal{D}_{\eta(k)}(\varphi_\psi^{\rightarrow}(\varphi_\psi^{\leftarrow}(f_A)))\\&\ge\mathcal{C}_k(\varphi_\psi^{\leftarrow}(f_A))\ge\mathcal{D}_{\eta(k)}(f_A)\quad\forall f_A\in(L^Y)^{E^2},k\in K^1.\end{aligned}$$

So, $\mathcal{C}_k(\varphi_\psi^{\leftarrow}(f_A))=\mathcal{D}_{\eta(k)}(f_A)$ $\forall f_A\in(L^Y)^{E^2},k\in K^1$ and hence $\mathcal{D}$ is a quotient L-fuzzy soft convexity. □

4. L-fuzzy (K,E)-soft closure L-fuzzy (K,E)-soft convexity spaces

Definition 4.1 *A triple $(X,\mathcal{C},cl)$ consisting of a set X, an L-fuzzy (K,E)-soft convexity on X and an L-fuzzy (K,E)-soft closure cl is called an L-fuzzy (K,E)-soft closure L-fuzzy (K,E)-soft convexity spaces.*

Theorem 4.1 *Let (X,cl) be an L-fuzzy (K,E)-soft closure space, $\emptyset\ne Y\subseteq X$, $\emptyset\ne E^*\subseteq E$ and $\emptyset\ne K^*\subseteq K$. Then an L-fuzzy (K^*,E^*)-soft closure cl_Y on Y is defined as*

$$cl_Y(k^*,g_B,r)=cl(k,f_A,r)\mid Y \text{ for all } f_A\in(L^X)^E,k\in K \text{ such that } g_B=f_A\mid Y.$$

Proof: (1) For $r\in L_\perp$ and $k\in K$ we have

$$cl_Y(k^*,\tilde{0}_{E^*},r)\cong cl(k,\tilde{0}_E,r)\mid Y\cong\tilde{0}_E\mid Y\cong\tilde{0}_{E^*}.$$

(2) Since $f_A\sqsubseteq cl(k,f_A,r)$ we obtain $f_A\mid Y\sqsubseteq cl(k,f_A,r)\mid Y$. Put $g_B=f_A\mid Y$ then $g_B\sqsubseteq cl_Y(k,g_B,r)$.
(3) Let $f_{A_1}\sqsubseteq f_{A_2}$. Then by Definition 2.2 we obtain $f_{A_1}\mid Y\sqsubseteq f_{A_2}\mid Y$. Therefore by Definition 1.3 (3) we have $cl(k,f_{A_1},r)\mid Y\sqsubseteq cl(k,f_{A_2},r)\mid Y$. Hence $cl_Y(k,g_{B_1},r)\sqsubseteq cl_Y(k,g_{B_2},r)$.
(4) Let $r\le s$. Then we have from Definition 1.3 (4) $cl(k,f_A,r)\mid Y\sqsubseteq cl(k,f_A,s)\mid Y$. Hence $cl_Y(k,g_B,r)\sqsubseteq cl_Y(k,g_B,s)$.
(5) By Definition 1.3 (5) we have

$$\begin{aligned}cl_Y(k,g_{B_1}\sqcup g_{B_2},r\wedge s)\quad&\cong cl(k,f_{A_1}\sqcup f_{A_2},r\wedge s)\mid Y\\&\sqsubseteq\big(cl(k,f_{A_1},r)\sqcup cl(k,f_{A_2},s)\big)\mid Y\\&\cong\big(cl(k,f_{A_1},r)\mid Y\big)\sqcup\big(cl(k,f_{A_2},s)\mid Y\big)\\&\cong cl_Y(k,g_{B_1},r)\sqcup cl_Y(k,g_{B_2},s).\end{aligned}$$

(T) Form Definition 1.3 (T) we have

$$\begin{aligned}cl_Y(k,cl_Y(k,g_B,r),r)\quad&\cong cl(k,cl(k,f_A,r)\mid Y,r)\mid Y\\&\sqsubseteq cl(k,f_A,r)\mid Y\\&\cong cl_Y(k,g_B,r).\end{aligned}$$

Hence the proof is complete. □

Definition 4.2 *Let $(X, \mathcal{C}, cl)$ be an L-fuzzy (K, E)-soft closure L-fuzzy (K, E)-soft convexity spaces, $\emptyset \neq Y \subseteq X$, $\emptyset \neq E^* \subseteq E$ and $\emptyset \neq K^* \subseteq K$. Then, the corresponding triple $(Y, \mathcal{C} \mid Y, cl_Y)$ is an L-fuzzy (K^*, E^*)-soft subspace of $(X, \mathcal{C}, cl)$.*

Definition 4.3 *Let $\mathcal{C}, cl$ be an L-fuzzy (K, E)-soft convexity and an L-fuzzy (K, E)-soft closure operator respectively. Then cl is said to be compatible with $\mathcal{C}$ if*

$$cl(k, CO(k, f_A, r), r) = CO(k, f_A, r) \text{ for each } f_A \in (L^X)^E, k \in K$$

and the triple $(X, \mathcal{C}, cl)$ is called an L-fuzzy (K, E)-soft closured convexity space.

Remark 4.1 *It is obvious that an L-fuzzy (K, E)-soft closured convexity space is always an L-fuzzy (K, E)-soft closure L-fuzzy (K, E)-soft convexity space and the converse is not true.*

Example 4.1 Let $L = [0, 1]$, $X = \{a, b, c\}$ and $E = \{e_1, e_2\}$. Let $f_{A_i} \in ([0, 1]^X)^E$ where $i = \{1, 2\}$ defined as follows:

$$\begin{array}{ll}
(f_{A_1})_{e_1} = (0.5, 0.4, 0.4), & (f_{A_1})_{e_2} = (0.2, 0.6, 0.7), \\
(f_{A_2})_{e_1} = (0.6, 0.6, 0.6), & (f_{A_2})_{e_2} = (0.8, 0.8, 0.8), \\
(f_{A_3})_{e_1} = (0.2, 0.3, 0.4), & (f_{A_3})_{e_2} = (1.0, 1.0, 0.3), \\
(f_{A_4})_{e_1} = (0.7, 0.8, 0.6), & (f_{A_4})_{e_2} = (0.8, 0.7, 0.6), \\
(f_{A_5})_{e_1} = (0.2, 0.9, 0.6), & (f_{A_5})_{e_2} = (0.4, 0.4, 0.7).
\end{array}$$

Then,

$$f_{A_1} \sqcap f_{A_2} = f_{A_1}, \qquad f_{A_1} \sqcup f_{A_2} = f_{A_2}.$$

For $K = \{k_1, k_2\}$ we define a $[0, 1]$-fuzzy (K, E)-soft convexity $\mathcal{C} : K \longrightarrow [0, 1]^{([0.1]^X)^E}$ as follows:

$$\mathcal{C}_{k_1}(f_A) = \begin{cases} 1, & \text{if } f_A \in \{\tilde{0}_E, \tilde{1}_E\}, \\ \frac{1}{4}, & \text{if } f_A = f_{A_1}, \\ \frac{1}{5}, & \text{if } f_A = f_{A_2}, \\ 0, & \text{otherwise,} \end{cases} \qquad \mathcal{C}_{k_2}(f_A) = \begin{cases} 1, & \text{if } f_A \in \{\tilde{0}_E, \tilde{1}_E\}, \\ \frac{1}{3}, & \text{if } f_A = f_{A_3}, \\ 0, & \text{otherwise.} \end{cases}$$

Also for $K = \{k_1, k_2\}$ we define a $[0, 1]$-fuzzy (K, E)-soft closure operators $cl_1, cl_2 : K \times ([0, 1]^X)^E \times (0.1] \longrightarrow ([0, 1]^X)^E$ as follows:

$$cl_1(k_1, f_A, r) = \begin{cases} \tilde{1}_E, & \text{if } f_A \cong \tilde{1}_E, r \in (0, 1], \\ f_{A_1}, & \text{if } f_A \sqsubseteq f_{A_1}, r \leq \frac{1}{4}, \\ f_{A_2}, & \text{if } f_A \cong f_{A_2}, r \leq \frac{1}{5}, \\ \tilde{0}_E, & \text{otherwise,} \end{cases}$$

$$cl_1(k_2, f_A, r) = \begin{cases} \tilde{1}_E, & \text{if } f_A \cong \tilde{1}_E, r \in (0, 1], \\ f_{A_3}, & \text{if } f_A \sqsubseteq f_{A_3}, r \leq \frac{1}{3}, \\ \tilde{0}_E, & \text{otherwise,} \end{cases}$$

$$cl_2(k_1, f_A, r) = \begin{cases} \tilde{1}_E, & \text{if } f_A \cong \tilde{1}_E, r \in (0, 1], \\ f_{A_4}, & \text{if } f_A \sqsubseteq f_{A_4}, r \leq \frac{1}{4}, \\ \tilde{0}_E, & \text{otherwise,} \end{cases}$$

$$cl_2(k_2, f_A, r) = \begin{cases} \tilde{1}_E, & \text{if } f_A \cong \tilde{1}_E, r \in (0, 1], \\ f_{A_5}, & \text{if } f_A \sqsubseteq f_{A_5}, r \leq \frac{1}{3}, \\ \tilde{0}_E, & \text{otherwise.} \end{cases}$$

Then $(X,\mathcal{C},cl_1)$ is an L-fuzzy (K,E)-soft closured convexity space. On the other hand, $(X,\mathcal{C},cl_2)$ is an L-fuzzy (K,E)-soft closure L-fuzzy (K,E)-soft convexity space but it is not L-fuzzy (K,E)-soft closured convexity space because $CO(k_1,f_{A_1},\frac{1}{4})=f_{A_1}$ and

$$\begin{aligned}\tilde{0}_E\cong cl_2(k_1,f_{A_1},\tfrac{1}{4}) &\cong cl_2(k_1,CO(k_1,f_{A_1},\frac{1}{4}),\frac{1}{4})\\ &\not\cong CO(k_1,f_{A_1},\frac{1}{4})\cong f_{A_1}.\end{aligned}$$

Also $CO(k_2,f_{A_3},\frac{1}{4})=f_{A_3}$ and

$$\begin{aligned}\tilde{0}_E\cong cl_2(k_1,f_{A_1},\tfrac{1}{4}) &\cong cl_2(k_2,CO(k_2,f_{A_2},\frac{1}{3}),\frac{1}{3})\\ &\not\cong CO(k_2,f_{A_3},\frac{1}{3})\cong f_{A_3}.\end{aligned}$$

Proposition 4.1 *An L-fuzzy (K,E)-soft subspace of L-fuzzy (K,E)-soft closured convexity space is an L-fuzzy (K,E)-soft closured convexity space.*

Proof: Let $(X,\mathcal{C},cl)$ be an L-fuzzy (K,E)-soft closured convexity space and $(Y,\mathcal{C}\mid Y,cl_Y)$ be an L-fuzzy (K,E)-soft subspace of $(X,\mathcal{C},cl)$. Then by Theorem 2.3 and Theorem 4.1, $(Y,\mathcal{C}\mid Y,cl_Y)$ is an L-fuzzy (K,E)-soft closure L-fuzzy (K,E)soft convexity space. Let $g_B\cong CO_{(\mathcal{C}|Y)}(k,f_A,r)$ for $g_B,f_A\in(L^Y)^{E^*}$. Then $(\mathcal{C}\mid Y)_{k^*}(g_B)\geq r$, $g_B\cong h_C\mid Y$ and $\mathcal{C}_k(h_C)\geq r$ such that $h_C\cong CO_{\mathcal{C}}(k,h_{C_1},r)$ for each $h_C,h_{C_1}\in(L^X)^E, k\in K$. Since $(X,\mathcal{C},cl)$ be an L-fuzzy (K,E)-soft closured convexity space,

$$cl(k,CO_{\mathcal{C}}(k,h_C,r),r)\cong CO_{\mathcal{C}}(k,h_C,r).$$

and hence

$$\begin{aligned}cl_Y(k,g_B,r) &\cong cl_Y(k,CO_{(\mathcal{C}|Y)}(k,f_A,r),r)\\ &\cong CO_{(\mathcal{C}|Y)}(k,f_A,r)\cong g_B.\end{aligned}$$

Therefore $(Y,\mathcal{C}\mid Y,cl_Y)$ be an L-fuzzy (K^*,E^*)-soft closured convexity space. □

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