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Generalized Factor-Type Exponential Estimators of Population mean in Sample Surveys

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ABSTRACT: The present article deals with a generalized class of estimators for estimating the population mean in sample surveys, employing various combinations of auxiliary variables and considering some values of characterizing constant alpha ranging from -1 to +1. The proposed estimator may be considered an efficient extension to the work of Singh and Shukla (Metron, 45(1-2): 273-283, 1987), Bahl and Tuteja (Journal of information and optimization sciences, 12(1), 159-164, 1991) and Kadilar (Journal of Modern Applied Statistical Methods: Vol. 15 : Iss. 2 , Article 15, 2016). The sampling properties of the suggested estimators have been derived up to the first degree of large sample approximations. The suggested estimators are shown to have smaller mean squared errors than the existing exponential estimators considered in this paper. The percent relative efficiencies with respect to the usual mean estimator are calculated. An improvement has been shown over the existing exponential estimators through theoretical conditions as well as a numerical and simulation study based on COVID-19 deaths in India.

Key Words: Bias, Mean Square Error, Auxiliary information, Exponential Estimator, Factor-type Estimator, Percent Relative Efficiency.

Contents

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1. Introduction

The coronavirus disease 2019 (COVID-19), which originated in Wuhan, China, and has spread to every country in the world, has been a significant global catastrophe since January 2020. COVID-19 was classified as a "global epidemic" by the WHO on March 11. Since it originally appeared in India in the first week of January, COVID-19 has grown significantly [1]. However, there may be regional and city-specific variations in the incidence and death rates. This could be the result of varied population densities, healthcare systems, safety precautions, substructures, or climatic conditions in each section (interested unit). The parameter value most frequently used to demonstrate this discrepancy is the population mean.

In sampling theory, one of the important objectives is to estimate the study variable with more precision. To obtain these objective statistics, statisticians have been incorporating auxiliary information for several decades and have recognised that using this information helps us improve precision. If the correlation coefficients between the auxiliary and study variables are positive, then ratio estimators are used, and if the correlation is negative, then product estimators are used for estimating the population mean. Ratio and product estimators make use of supplementary information to produce accurate and efficient results. To improve the efficiency of factor type exponential estimators, information on auxiliary variable characteristics is used.

Later, statisticians focused their efforts on developing modified ratio, product, and exponential estimators. In defining modified estimators based on unknown parameters, a class of estimators was developed that included a number of classical estimators. Cochran [2] , Murthy [3], Hartley and Ross [4], Sisodia and Dwivedi [5], Goodman and Hartley [6], Yan and Tian [7], Williams [8], Tin [9], Srivastava [10], Walsh [11], Srivenkataramana and Tracy [12], Vos [13] and Srivenkataramana [14], Isaki [15], R.V.K. Singh & B. K. Singh [16], Zakari et al. [17], Ahmad et al. [18] and Singh et al [19] did some major, remarkable work in this direction.

In the succession of improvements to classical ratio and product estimators, the suggestion of the exponential methods was first proposed by Bahl and Tuteja [20], when the correlation is not strong between auxiliary and study variables. Further, their work was extended; the survey statisticians have developed and are developing exponential type estimators for different sampling situations. Exponential type estimators were discussed by Sharma and Tailor [21], Monika and Kumar [22], Upadhyaya et al. [23], Solanki et al. [24], Kadilar [25], Irfan et al. ([26], [27]) , Zaman et al. ([28], [29], [30]), Sahzad et al. [31] and Prasad [33] proposed some linear regression-type ratio exponential estimators for estimating the population mean in survey sampling using known values of quartile deviation and deciles of an auxiliary variable. Readers who are fascinated by exploring deeply into the subject in order to gain more information can refer to the extensive studies that was conducted by Prasad and Yadav [34], Ahmad et al. [32], Zaman et al. [35], Singh et al. [36] and Audu et al. [37].

The remaining parts of the manuscript are structured as follows: In Section 2, we offered the various notations that are used in this manuscript. In Section 3, we offered a literature review of existing modified exponential estimators as well as their biases and MSEs. In Section 4, we offered the proposed factor type exponential ratio estimator for various combinations of auxiliary information. In Section 5, we derive an expression for the bias and MSE of suggested estimators. In Section 6, particular cases of the proposed class of estimators are given. In Section 7, we derive the limiting value of our proposed estimators. In Section 8, we derive the theoretical comparison of various existing estimators with respect to proposed estimators. In Sections 9 & 10, we conducted a numerical and simulation study considering COVID-19 death data in India, and in the last Sections, 11, 12 we arrived at the results and conclusions.

2. Notations

Suppose that the $Z = (Z_1, Z_2, ..., Z_N)$ be the finite population of size N and the variables under study and auxiliary are denoted by v and u respectively. Let $\left(\bar{V}, \bar{U}\right)$ be the population means of (v, u) respectively. It is desired to estimate the population mean of $\bar{V}$ using information on population parameters. Let's define some parameters that are used in this manuscript.

N : Population size.

n : Sample size.

$f = \frac{n}{N}$: Sampling fraction.
V : Study Variable.
U : Auxiliary Variable.
$\bar{V}$: Population mean of Study Variable.
$\bar{U}$: Population mean of Auxiliary Variable.
$S_v^2 = (N-1)^{-1}\sum_{i=1}^{N}(v_i - \bar{V})^2$: Population Variance of the Study variable V.
$S_u^2 = (N-1)^{-1}\sum_{i=1}^{N}(u_i - \bar{U})^2$: Population Variance of the Auxiliary variable U.
$C_v = \frac{S_v}{\bar{V}}$: Coefficient of variation of the study variable V.
$C_u = \frac{S_u}{\bar{U}}$: Coefficient of variation of the auxiliary variable U
$S_{uv} = (N-1)^{-1}\sum_{i=1}^{N}(u_i - \bar{U})(v_i - \bar{V})$: Covariance between v and u.
ρ: Correlation coefficient.
$C = \frac{\rho C_v}{C_u}$.
M_d : Population median of u.
Q_i : i^{th} population quartile (i=1,2,3).
T_m : $\frac{Q_1+Q_2+Q_3}{2}$: Tri mean.
$Q_r = (Q_3 - Q_1)$: Inter-quartile range.
$Q_d = \frac{(Q_3-Q_1)}{2}$: Semi-quartile range.
$\beta_1(u)$: Coefficients of skewness of the auxiliary variable u.
$\beta_2(u)$: Coefficients of kurtosis of the auxiliary variable u.
$Q_a = \frac{(Q_3+Q_1)}{2}$: Quartile average.
D_i : i^{th} Population Deciles $(i = 1, 2, ...10)$.
H_l= Median $(\frac{U_j+U_k}{2}, 1 \leq j \leq k \leq N)$ Hodges-Lehmann estimators.
$U_{(1)}$: Lowest order statistics in a population of size N.
$U_{(N)}$: Highest order statistics in a population of size N.
$S_k = \frac{Q_3+Q_1-2Q_2}{Q_2-Q_1}$: Bowley's Coefficient of Skewness.
$M_r = \frac{U_{(1)}+U_{(N)}}{2}$: Mid range.
G : Gini's Mean Difference.
D : Downton's method.
S_{pw} : Probability weighted moments.
$\mu_r(u) = (1/N)\sum_{i=1}^{N}((u_i) - \bar{U})$: r being non-negative integer.

3. Literature Review of Existing Modified Estimators

The exponential type estimators listed below were taken into consideration in this section:

3.1. Singh and Shukla Estimator

In 1987, Singh and Shukla [39] suggested a traditional factor-type estimator that can be used when the correlations between the study and the auxiliary variables are either negative or positive, as follows

$$t_1 = \bar{v}\left[\frac{(P+R)\bar{U} + fQ\bar{u}}{(P+fQ)\bar{U} + R\bar{u}}\right] \tag{3.1}$$

where; $f = \frac{n}{N}$; $P = (d-1)(d-2)$; $Q = (d-1)(d-4)$; $R = (d-2)(d-3)(d-4)$ and d= 1, 2, 3 and 4. The estimator's bias and MSE are as follows

$$Bias(t_1) = \bar{V}\left(\frac{1}{n} - \frac{1}{N}\right)\left[(b^2 - ab)C_U^2 + (a-b)\rho C_U C_V\right] \tag{3.2}$$

$$MSE(t_1) = \bar{V}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_V^2 + (a^2+b^2)C_U^2 + 2(a-b)\rho C_U C_v - 2abC_U^2\right] \tag{3.3}$$

where $a = \frac{fQ}{P+fQ+R}$ and $b = \frac{R}{P+fQ+R}$

3.2. Bahl and Tuteja Estimators

In 1991, Bahl and Tuteja [20] proposed the modified ratio and product type exponential estimators, which are as follows

$$t_{2Re} = \bar{v} exp \left\{ \frac{\bar{U} - \bar{u}}{\bar{U} + \bar{u}} \right\} \tag{3.4}$$

$$t_{2Pe} = \bar{v} exp \left\{ \frac{\bar{u} - \bar{U}}{\bar{u} + \bar{U}} \right\} \tag{3.5}$$

Bias and MSE of the estimators t_{2Re} and t_{2Pe} are as follows

$$Bias(t_{2Re}) = \left(\frac{1}{n} - \frac{1}{N} \right) \bar{V} \left[\frac{3}{8} C_U^2 - \frac{1}{2} \rho C_U C_V \right] \tag{3.6}$$

$$Bias(t_{2Pe}) = \left(\frac{1}{n} - \frac{1}{N} \right) \bar{V} \left[\frac{3}{8} C_U^2 + \frac{1}{2} \rho C_U C_V \right] \tag{3.7}$$

$$MSE(t_{2Re}) = \left(\frac{1}{n} - \frac{1}{N} \right) \bar{V}^2 \left[C_V^2 + \frac{C_U^2}{4} - \rho C_U C_V \right] \tag{3.8}$$

$$MSE(t_{2Pe}) = \left(\frac{1}{n} - \frac{1}{N} \right) \bar{V}^2 \left[C_V^2 + \frac{C_U^2}{4} + \rho C_U C_V \right] \tag{3.9}$$

3.3. Sharma and Tailor Estimator

In 2010, Sharma and Tailor [21] suggested the exponential dual to ratio estimator in sampling theory is as follows

$$t_3 = \bar{v} exp \left(\frac{\bar{u}^* - \bar{U}}{\bar{u}^* + \bar{U}} \right) \tag{3.10}$$

where $\bar{u}^* = (1+g)\bar{U} - g\bar{u}$
Bias and MSE of t_3 are as

$$Bias(t_3) = \bar{V} \left(\frac{1}{n} - \frac{1}{N} \right) \left[\frac{3}{8} C_U^2 - \frac{1}{2} g \rho C_U C_V \right] \tag{3.11}$$

$$MSE(t_3) = \bar{V}^2 \left(\frac{1}{n} - \frac{1}{N} \right) \left[C_V^2 + \frac{g^2}{4} C_U^2 - g \rho C_U C_V \right] \tag{3.12}$$

where $g = \left(\frac{n}{N-n} \right)$

3.4. Singh et al. Estimators

In 2014, Singh et al. [19] suggested the ratio and the product type exponential estimators as the predictive estimators in sample surveys. The proposed estimators are as

$$t_{4Re} = \left[\frac{n}{N} \bar{V} + \left(\frac{N-n}{N} \right) \bar{V} exp \left(\frac{N(\bar{U} - \bar{u})}{N(\bar{U} - \bar{u}) - 2n\bar{u}} \right) \right] \tag{3.13}$$

$$t_{4Pe} = \left[\frac{n}{N} \bar{V} + \left(\frac{N-n}{N} \right) \bar{V} exp \left(\frac{N(\bar{u} - \bar{U})}{N\bar{U} + (N-2n)\bar{u}} \right) \right] \tag{3.14}$$

The Bias and MSE of t_{4Re} and t_{4Pe} are given as follows:

$$Bias(t_{4Re}) = \bar{V} \left(\frac{1}{n} - \frac{1}{N} \right) \left[\frac{3}{8} C_U^2 - \frac{1}{2} \rho C_U C_V \right] \tag{3.15}$$

$$Bias(t_{4Pe}) = \bar{V} \left(\frac{1}{n} - \frac{1}{N} \right) \left[\frac{3}{8} C_U^2 + \frac{1}{2} \rho C_U C_V \right] \tag{3.16}$$

$$MSE(t_{4Re}) = \bar{V}^2 \left(\frac{1}{n} - \frac{1}{N} \right) \left[C_V^2 + \frac{C_U^2}{4} - \rho C_U C_V \right] \tag{3.17}$$

$$MSE(t_{4Pe}) = \bar{V}^2 \left(\frac{1}{n} - \frac{1}{N} \right) \left[C_V^2 + \frac{C_U^2}{4} + \rho C_U C_V \right] \tag{3.18}$$

3.5. Monika and Kumar Estimator

In 2015, Monika and Kumar [22] suggested an exponential product type estimator utilising auxiliary variables as

$$t_5 = [\bar{v} - k(t-1)] \tag{3.19}$$

$$t_5 = \bar{v} - k\left\{exp\left(\frac{N\bar{U} - n\bar{u}}{N-n} - \bar{U}\right) - 1\right\} \tag{3.20}$$

where

$$t = exp\left(\frac{N\bar{U} - n\bar{u}}{N-n} - \bar{U}\right) \tag{3.21}$$

The MSE of estimator t_5 is given as

$$MSE(t_5) = \bar{V}^2\left(\frac{1}{n} - \frac{1}{N}\right)[C_V^2 + k^2\left(\frac{n}{N-n}\right)^2 C_U^2 + 2\left(\frac{n}{N-n}\right)k\rho C_U C_V] \tag{3.22}$$

where, k = -4, -2, 0, 2, 4.

3.6. Kadilar Estimator

In 2016, Kadilar [25] proposed the modified exponential type estimator utilising single auxiliary information in sample surveys as

$$t_6 = \bar{v}\left(\frac{\bar{u}}{\bar{U}}\right)^{\alpha} exp\left(\frac{\bar{U} - \bar{u}}{\bar{U} + \bar{u}}\right) \tag{3.23}$$

where α is the constant.
The Bias and MSE of estimator t_6 are as

$$Bias(t_6) = \bar{V}\left(\frac{1}{n} - \frac{1}{N}\right)\left[\frac{3}{8}C_U^2 + \frac{\alpha(\alpha-1)}{2}C_U^2 - \frac{\alpha}{2}C_U^2 + \alpha\rho C_U C_V - \frac{1}{2}\rho C_U C_V\right] \tag{3.24}$$

$$MSE(t_6) = \bar{V}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_V^2 + \frac{C_U^2}{4} + 2\alpha\rho C_V C_U + \rho C_V C_U + \alpha^2 C_U^2 + \alpha C_U^2\right] \tag{3.25}$$

4. Proposed Estimators

Following on from the preceding works, and motivated by the generic nature of exponential and factor-type estimator, a generalized class of factor-type estimators has been proposed as

$$t_{sv}^{FT} = \bar{v}\left[\frac{(P+R)\bar{U} + fQ\bar{u}}{(P+fQ)\bar{U} + R\bar{u}}\right]^{\alpha} exp\left\{\frac{\Psi(\bar{U} - \bar{u})}{\Psi(\bar{U} + \bar{u}) + 2\kappa}\right\} \tag{4.1}$$

where, $P = (d-1)(d-2)$; $Q = (d-1)(d-4)$; $R = (d-2)(d-3)(d-4)$, α and d are suitably choosen constants, such that MSE of the our proposed estimator is minimum, Ψ and κ are real numbers (constants) or functions of population parameters of the known auxiliary variables. For the fixed value of α, we get some generalized exponential estimators for different values of d, which is discuss later on in Section - (6) as a particular cases.

It is to mentioned that:

(i) For $\alpha = 0$, in equation (4.1), our proposed estimator t_{sv}^{FT} reduces to modified exponential estimators for suitable values of Ψ & κ proposed by Singh et al. [38].

$$t_{sv}^{exp} = \bar{v}exp\left\{\frac{\Psi(\bar{U} - \bar{u})}{\Psi(\bar{U} + \bar{u}) + 2\kappa}\right\}$$

(ii) For $\alpha = 1$, in equation (4.1), our proposed estimator t_{sv}^{FT} reduces to factor type ratio exponential estimator for suitable values of Ψ & κ some members of this class of estimators are given in Tables [6-9].

$$t_{sv}^{Y} = \bar{v}\left[\frac{(P+R)\bar{U} + fQ\bar{u}}{(P+fQ)\bar{U} + R\bar{u}}\right] exp\left\{\frac{\Psi(\bar{U} - \bar{u})}{\Psi(\bar{U} + \bar{u}) + 2\kappa}\right\}$$

5. Bias and Mean Square Error

To derive the expression for Bias and MSE, consider the transformations as follows
$\bar{v} = \bar{V}(1+e_0)$ and $\bar{u} = \bar{U}(1+e_1)$ thus

$$E(e_0) = E(e_1) = 0, E(e_0^2) = \left(\frac{1}{n} - \frac{1}{N}\right) C_V^2, E(e_1^2) = \left(\frac{1}{n} - \frac{1}{N}\right) C_U^2, E(e_0 e_1) = \left(\frac{1}{n} - \frac{1}{N}\right) \rho C_U C_V$$

From the above considered transformation expressing estimator t_{sv}^{FT} in terms of $e's$, we have

$$\begin{aligned} t_{sv}^{FT} - \bar{V} = \bar{v}[1 + e_0 + \alpha(a-b)e_1 - \frac{\xi e_1}{2} + \alpha(a-b)e_0 e_1 - \frac{\xi e_0 e_1}{2} - \alpha(ab-b^2)e_1^2 \\ + \frac{\alpha(a-b)}{2}(a-b)^2 e_1^2 - \frac{\alpha\xi(a-b)}{2} e_1^2 + \frac{3}{8}\xi^2 e_1^2] \end{aligned} \tag{5.1}$$

where, $\xi = \frac{\Psi\bar{U}}{\Psi\bar{U}+\kappa}$, $a = \frac{fQ}{P+fQ+R}$ and $b = \frac{R}{P+fQ+R}$
To obtain the bias of the estimator t_{sv}^{FT} in terms of the first degree of large sample approximations, we will take the expectation of equation (5.1) and, then by substituting the value of the considered transformation, we get

$$\begin{aligned} Bias[t_{sv}^{FT}] = \bar{V}\left(\frac{1}{n} - \frac{1}{N}\right)[\frac{3}{8}\xi^2 C_U^2 + \frac{\alpha(\alpha-1)}{2}(a-b)^2 C_U^2 - \frac{\xi\alpha(a-b)}{2} C_U^2 \\ -\alpha(ab-b^2)C_U^2 + \alpha(a-b)\rho C_V C_U - \frac{1}{2}\xi\rho C_V C_U] \end{aligned} \tag{5.2}$$

Squaring both sides of the equation (5.1) and then taking expectation on both sides, the MSE will take the structure as

$$MSE[t_{sv}^{FT}] = \bar{V}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_V^2 + \frac{\xi^2}{4}C_U^2 + \alpha^2\theta^2 C_U^2 - \xi\alpha\theta C_U^2 + 2\alpha\theta\rho C_V C_U - \xi\rho C_V C_U\right] \tag{5.3}$$

where $\theta = a - b$
We can obtain the optimal value of α and d by differentiating equation (5.3) with respect to α and d and equating them to zero, as follows

$$(\alpha)_{optm} = \frac{1}{\theta}\left\{\frac{\xi}{2} - W\right\} \tag{5.4}$$

and

$$(\theta)_{optm} = \frac{1}{\alpha}\left\{\frac{\xi}{2} - W\right\} \tag{5.5}$$

where, $W = \rho\frac{C_V}{C_U}$
Now , we can get the minimum MSE of t_{sv}^{FT} by substituting any of these in equation (5.3).

$$MSE[t_{sv}^{FT}]_{min} = \bar{V}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_V^2 + W^2 C_U^2 - 2W\rho C_V C_U\right] \tag{5.6}$$

Remark 1: If we put the value of $W = \rho\frac{C_V}{C_U}$ in equation (5.6) then we see that this is the MSE of the linear regression estimator. Hence, it is equally efficient as that of the linear regression estimator for the optimum values of α and d.

$$MSE[t_{sv}^{FT}]_{min} = \bar{V}^2\left(\frac{1}{n} - \frac{1}{N}\right) C_V^2\left(1 - \rho^2\right)$$

The symbols have their usual meaning and some specific symbols are already defined. Now, for getting the optimum value of d . Writing equation (5.5) in terms of P, Q and R, we have

$$\theta = a - b = \frac{fQ - R}{P + FQ + R} = \frac{1}{\alpha}\left\{\frac{\xi}{2} - W\right\} \tag{5.7}$$

Clearly equation (5.7) can be solved for d^*, The following can be a standard for choosing a good d^*: "Out of all feasible d^*'s, select that d^* as the best option, resulting in the least $|Bias[t_{sv}^{FT}]|$".
On solving the equation (5.7) the cubic equation in the form of d is given by:
$\left\{\alpha - \frac{\xi}{2} + W\right\} d^3 + \left\{(-9\alpha + 4\xi - 8W) + f(-\alpha - \frac{\xi}{2} + W)\right\} d^2 + \left\{(26\alpha - \frac{23}{2}\xi + W) + f(5\alpha + \frac{5}{2} - 5W)\right\} d + \{f(-4\alpha - 2\xi + 4W) + (-24\alpha + 11\xi - 22W)\} = 0$

6. Particular Cases

In this section, we get some generalized exponential estimators for different values of d, Ψ and κ which is shown in Table [1-4]

6.1 Generalized Exponential Ratio-Type Estimator

For $d = 1$, we get the values $P = 0$, $Q = 0$ and $R = -6$. Substituting these values in equation (4.1) we get a generalized exponential ratio type estimator:

$$t_{sv}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\Psi(\bar{U} - \bar{u})}{\Psi(\bar{U} + \bar{u}) + 2\kappa}\right\}$$

After that, Bias and MSE expression are obtained by substituting the values of a , b in equation (5.2) and equation (5.3). We have:

$$Bias[t_{sv}^{Re}] = \bar{V}\left(\frac{1}{n} - \frac{1}{N}\right)\left[\frac{3}{8}\xi^2 C_U^2 + \frac{\alpha(\alpha-1)}{2}C_U^2 + \frac{\xi\alpha}{2}C_U^2 + \alpha C_U^2 - \alpha\rho C_V C_U - \frac{1}{2}\xi\rho C_V C_U\right]$$

$$MSE[t_{sv}^{Re}] = \bar{V}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_V^2 + \frac{\xi^2}{4}C_U^2 + \alpha^2 C_U^2 + \xi\alpha C_U^2 - 2\alpha\rho C_V C_U - \xi\rho C_V C_U\right]$$

6.2 Generalized Exponential Product-Type Estimators

For $d = 2$, we get the values $P = 0$, $Q = -2$ and $R = 0$. Substituting these values in equation (4.1) we get a generalized exponential product type estimator :

$$t_{sv}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\Psi(\bar{U} - \bar{u})}{\Psi(\bar{U} + \bar{u}) + 2\kappa}\right\}$$

After that, Bias and MSE expression are obtained by substituting the values of a and b in equation (5.2) and equation (5.3). We have:

$$Bias[t_{sv}^{Pe}] = \bar{V}\left(\frac{1}{n} - \frac{1}{N}\right)\left[\frac{3}{8}\xi^2 C_U^2 + \frac{\alpha(\alpha-1)}{2}C_U^2 - \frac{\xi\alpha}{2}C_U^2 + \alpha\rho C_V C_U - \frac{1}{2}\xi\rho C_V C_U\right]$$

$$MSE[t_{sv}^{Pe}] = \bar{V}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_V^2 + \frac{\xi^2}{4}C_U^2 + \alpha^2 C_U^2 - \xi\alpha C_U^2 + 2\alpha\rho C_V C_U - \xi\rho C_V C_U\right]$$

6.3 Generalized Exponential Dual to Ratio-Type Estimators

For $d = 3$, we get the values $P = 2$, $Q = -2$ and $R = 0$. Substituting these values in equation (4.1), we get a generalized exponential dual to ratio type estimator:

$$t_{sv}^{DR} = \bar{v}\left[\frac{\bar{U} - \bar{u}f}{\bar{U} - \bar{U}f}\right]^{\alpha} exp\left\{\frac{\Psi(\bar{U} - \bar{u})}{\Psi(\bar{U} + \bar{u}) + 2\kappa}\right\}$$

After that, Bias and MSE expression are obtained by substituting the values of a and b in equation (5.2) and equation (5.3). We have:

$Bias[t_{sv}^{DR}] = \bar{V}\left(\frac{1}{n} - \frac{1}{N}\right)\left[\frac{3}{8}\xi^2 C_U^2 + \frac{\alpha(\alpha-1)}{2}\left(\frac{-n}{N-n}\right)^2 C_U^2 - \frac{\xi\alpha}{2}\left(\frac{n}{N-n}\right)C_U^2 + \left(\frac{n}{N-n}\right)\alpha\rho C_V C_U - \frac{1}{2}\xi\rho C_V C_U\right]$

$MSE[t_{sv}^{DR}] = \bar{V}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_V^2 + \frac{\xi^2}{4}C_U^2 + \alpha^2\left(\frac{-n}{N-n}\right)^2 C_U^2 + \left(\frac{n}{N-n}\right)\xi\alpha C_U^2 - 2\alpha\left(\frac{n}{N-n}\right)\rho C_V C_U - \xi\rho C_V C_U\right]$

6.4 Generalized Exponential Type Estimators

For $d = 4$, we get the values $P = 6$, $Q = 0$ and $R = 0$. Substituting these values in equation (4.1), we get:

$$t_{sv}^{exp} = \bar{v}exp\left\{\frac{\Psi(\bar{U} - \bar{u})}{\Psi(\bar{U} + \bar{u}) + 2\kappa}\right\}$$

After that, Bias and MSE expression are obtained by substituting the values of a and b in equation (5.2) and equation (5.3). We have:

$$Bias[t_{sv}^{exp}] = \bar{V}\left(\frac{1}{n} - \frac{1}{N}\right)\left[\frac{3}{8}\xi^2 C_U^2 - \frac{1}{2}\xi\rho C_V C_U\right]$$

$$MSE[t_{sv}^{exp}] = \bar{V}^2\left(\frac{1}{n} - \frac{1}{N}\right)\left[C_V^2 + \frac{\xi^2}{4}C_U^2 - \xi\rho C_V C_U\right]$$

Note : This estimator is similar to special case for $\alpha = 0$ and is proposed by singh et al. [38].

7. Limiting Value of Suggested Estimators

Assuming that $d > 0$, it seems sense to consider the limiting case of the suggested estimators as d increases sufficiently. By dividing the numerator and denominator of the RHS of equation (4.1), and choosing the limit to be $d \to \infty$, we obtain

$$\lim_{d\to\infty} t_{sv}^{FT} = t_{sv}^{Re}$$

which is the usual exponential ratio estimator. Similarly, using the expressions of Bias and MSE of t_{sv}^{FT}. It is easy to see that

$$\lim_{d\to\infty} Bias[t_{sv}^{FT}] = Bias[t_{sv}^{Re}]$$

and

$$\lim_{d\to\infty} MSE[t_{sv}^{FT}] = MSE[t_{sv}^{Re}]$$

As a result, the suggested sequence of estimators is convergent and has a limited value that is both finite and clear. Additionally, the Bias and MSE converge asymptotically to those of the estimators of the exponential ratio type.

8. Comparison of Suggested Estimators with Existing Estimators

In this section the MSE expression of proposed estimators t_{sv}^{FT} is campared with the MSE expression of the estimators t_1, t_{2Re}, t_{2Pe}, t_3, t_{4Re}, t_{4Pe} and t_5. Therefore the conditions under which the efficiency of the proposed estimators outproforms the efficiency of existing estimators are obtained by comparing the equations (3.3), (3.8), (3.9), (3.12), (3.17), (3.18) and (3.22). The conditions obtainded as

1. $MSE[t_{sv}^{FT}]_{min} < MSE[t_1]$ if, $\quad (C_V\rho + \theta C_U)^2 > 0$
2. $MSE[t_{sv}^{FT}]_{min} < MSE[t_{2Re}]$ if, $\quad \left(\frac{C_U}{2} - \rho C_V\right)^2 > 0$
3. $MSE[t_{sv}^{FT}]_{min} < MSE[t_{2Pe}]$ if, $\quad \left(\frac{C_U}{2} + \rho C_V\right)^2 > 0$
4. $MSE[t_{sv}^{FT}]_{min} < MSE[t_3]$ if, $\quad \left(\left(\frac{n}{N-n}\right)\frac{C_U}{2} - \rho C_V\right)^2 > 0$
5. $MSE[t_{sv}^{FT}]_{min} < MSE[t_{4Re}]$ if, $\quad \left(\frac{C_U}{2} - \rho C_V\right)^2 > 0$
6. $MSE[t_{sv}^{FT}]_{min} < MSE[t_{4Pe}]$ if, $\quad \left(\frac{C_U}{2} + \rho C_V\right)^2 > 0$
7. $MSE[t_{sv}^{FT}]_{min} < MSE[t_5]$ if, $\quad \left(k\left(\frac{n}{N-n}\right)C_U + \rho C_V\right)^2 > 0$

9. Numerical Illustration

To compare the performance of the suggested estimator to the other traditional estimators, we are considering COVID-19 deaths data in India. COVID-19 data were retrieved from WHO websites (download link: https://covid19.who.int/WHO-COVID-19-global-data.csv). A total of 943 days' data (from the period of 01-February-2020 to 31-August-2022) were taken to examine the impact of mortality in India. When the correlation coefficients between new cases and new deaths in the COVID-19 data are calculated, we can see that these two variables have a strong correlation, with $\rho = 0.7393576$. In other words, there is a strong correlation between the COVID-19 new cases and deaths. Utilizing data on daily new cases, one can forecast the COVID-19 cumulative mortality in India. With this supplementary characteristic, a different family of predictors has been developed to estimate the COVID-19 total mortality mean. With the help of these suggested estimators, the MSE estimate of the COVID-19 deaths in India is calculated.
Following Figure 1 represents the graph of the real data of confirmed cases of COVID-19.

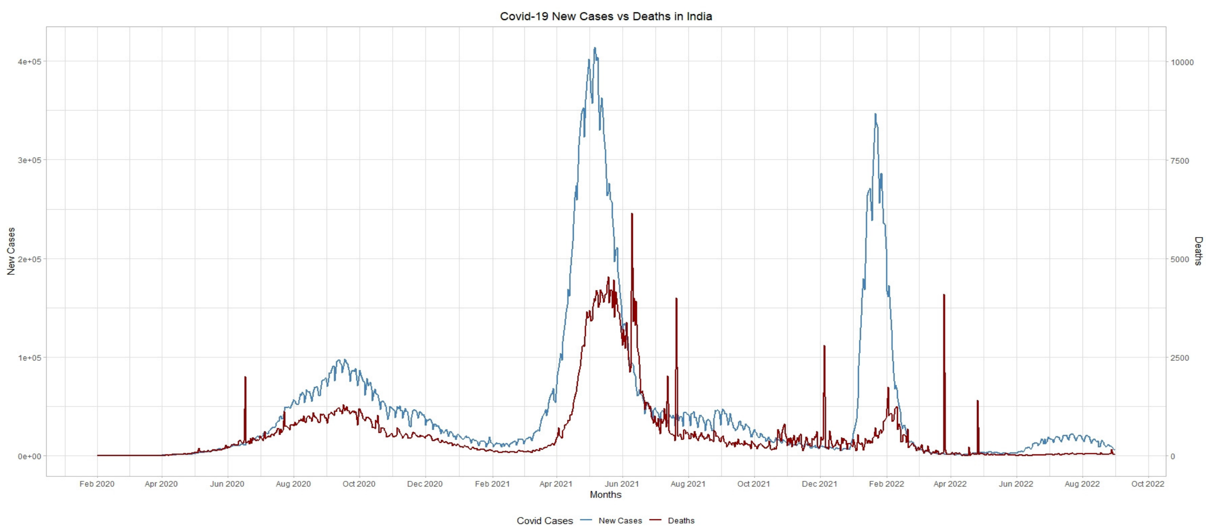


Figure 1: Graph for COVID-19 Data in India.

The formula to calculate percent relative efficiency is

$$PRE = \frac{MSE(Usual\ mean\ Estimators)}{MSE(Proposed\ Estimators)} \times 100 \tag{9.1}$$

and MSE/variance of the Usual mean estimator $\bar{V}$ is given by

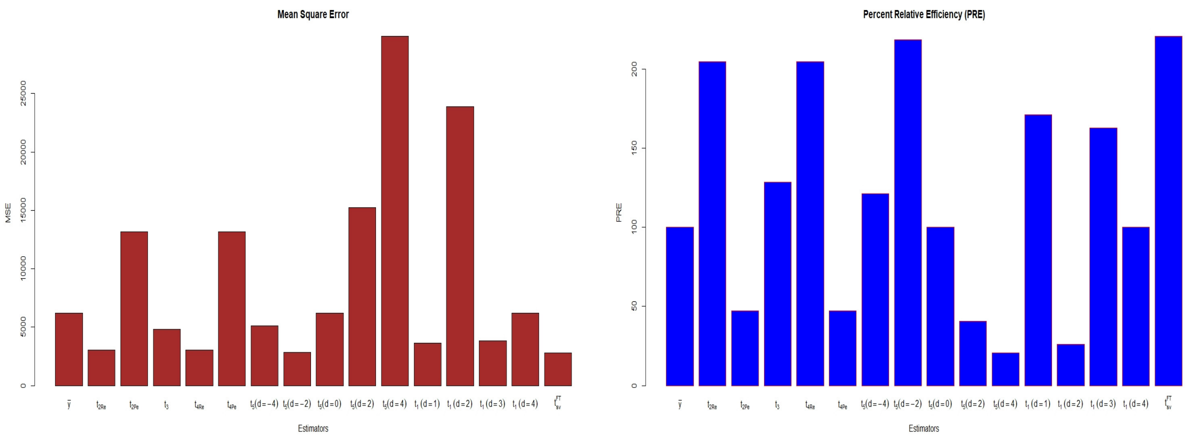
$$MSE(\bar{V}) = \bar{V}^2 \left(\frac{1}{n} - \frac{1}{N}\right) C_V^2 \tag{9.2}$$

In this article, total 56 combinations of the auxiliary variables are given in Tables [1-4] and many more combinations can also be made by choosing various other auxiliary variables. The results of MSE and RE (%) of the last combinations shown in Tables i.e., S_k and Q_r are used here and are compared to existing estimators that is given in Tables [10-13]. In Figure -[2], graphs of MSE and percent relative efficiency are also provided for comparative analysis and the graph given in Figure -[1] is for COVID-19 new cases and deaths in India for the period 01/02/2020 to 31/08/2022 (943 days). It has two Y-axis, one for the new cases and the other for the deaths. So the way to interpret the graph is by the blue line, which is for the new cases, is read with the left y axis ticks and the red line, which is for deaths, needs to be read from the right y axis ticks. As a result, the number of deaths in India is very low when compared to COVID-19 cases. The highest death peak is 6148 on June 10, 2021, which is the average daily death rate over the previous months.

Remark 2 : Some members of our proposed class of estimators are similar to some existing estimators.

1. Our proposed estimator $t_{sv_2}^{Pe}$ is similar to Kadilar [25] estimator.
2. Our proposed estimator $t_{sv_4}^{exp}$ is similar to Bahl and Tuteja [20] estimator.
3. Estimators $t_{sv_8}^{Re}$, $t_{sv_{12}}^{Re}$, $t_{sv_{16}}^{Re}$, $t_{sv_{20}}^{Re}$, $t_{sv_{24}}^{Re}$, $t_{sv_{28}}^{Re}$, $t_{sv_{32}}^{Re}$, $t_{sv_{36}}^{Re}$ and $t_{sv_{40}}^{Re}$ are similar to Singh et al. [38] estimators.

The Figure- [2a] and Figure- [2b] represents the MSE and PRE of different competing and the suggested estimators for the real data on COVID-19 disease.



(a) Mean Squared Error (b) Percent Relative Efficiency

Figure 2: Bar Plot for COVID-19 Data in India.

10. Simulation Study

To achieve RE(%)of the suggested estimators, we perform simulation study that is carried out by using the parameters of the above "COVID-19" data set. For performing the simulation study, we conduct the steps below, which were coded in R-program [40] , and outline the simulation processes considered to determine the MSEs of the suggested estimators .

Step 1 : Select the 5000 sample of the different size n (where n = 150, n = 200 ,n = 300 and n=350) using the "COVID-19" data set using the SRSWOR techniques .

Step 2 : After that we will considered the data from 5000 sample to find the value of the $\hat{\bar{V}}$.

Now, we have the 5000 values of $\hat{\bar{V}}$ from the 5000 samples for each sample n.

Step 3 : The mean squared error of $\hat{\bar{V}}$ is computed for each n by

$$MSE(\hat{\bar{V}}) = \frac{1}{5000}\sum_{i=1}^{5000}(\hat{\bar{V}}_i - \bar{V})^2 \tag{10.1}$$

where $\bar{V} = \frac{\sum_i^{5000}(\hat{\bar{V}}_i)}{5000}$.

The MSEs ratio of the considered estimators to the usual mean estimators for each n is then used to calculate percent relative efficiency. For all sample sizes, all of the proposed estimators are more dominant than existing estimators, therefore we can infer that they are all more efficient than existing estimators. The simulation results back up the theoretical results. Tables [12-13] summaries the simulation results of numerical iterations for different sample sizes n (where n = 150, n = 200 ,n = 300 and n=350).

Table 1: Some members of the proposed class of estimators t_{sv}^{FT}.

S.No.	Ψ	κ	d	Estimators	S.No.	Ψ	κ	d	Estimators
1.	1	0	1	$t_{sv_1}^{Re}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})}\right\}$	17.	C_u	$\beta_2(u)$	1	$t_{sv_{17}}^{Re}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$
2.*			2	$t_{sv_2}^{Pe}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})}\right\}$	18.			2	$t_{sv_{18}}^{Pe}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$
3.			3	$t_{sv_3}^{DR}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})}\right\}$	19.			3	$t_{sv_{19}}^{DR}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$
4.*			4	$t_{sv_4}^{exp}=\bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})}\right\}$	20.*			4	$t_{sv_{20}}^{exp}=\bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$
5.	1	C_u	1	$t_{sv_5}^{Re}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2C_u}\right\}$	21.	1	ρ	1	$t_{sv_{21}}^{Re}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\rho}\right\}$
6.			2	$t_{sv_6}^{Pe}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2C_u}\right\}$	22.			2	$t_{sv_{22}}^{Pe}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\rho}\right\}$
7.			3	$t_{sv_7}^{DR}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2C_u}\right\}$	23.			3	$t_{sv_{23}}^{DR}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\rho}\right\}$
8.*			4	$t_{sv_8}^{exp}=\bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2C_u}\right\}$	24.*			4	$t_{sv_{24}}^{exp}=\bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\rho}\right\}$
9.	1	$\beta_2(u)$	1	$t_{sv_9}^{Re}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	25.	C_u	ρ	1	$t_{sv_{25}}^{Re}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\rho}\right\}$
10.			2	$t_{sv_{10}}^{Pe}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	26.			2	$t_{sv_{26}}^{Pe}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\rho}\right\}$
11.			3	$t_{sv_{11}}^{DR}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	27.			3	$t_{sv_{27}}^{DR}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\rho}\right\}$
12.*			4	$t_{sv_{12}}^{exp}=\bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	28.*			4	$t_{sv_{28}}^{exp}=\bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\rho}\right\}$
13.	$\beta_2(u)$	C_u	1	$t_{sv_{13}}^{Re}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2C_u}\right\}$	29.	ρ	C_u	1	$t_{sv_{29}}^{Re}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2C_u}\right\}$
14.			2	$t_{sv_{14}}^{Pe}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2C_u}\right\}$	30.			2	$t_{sv_{30}}^{Pe}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2C_u}\right\}$
15.			3	$t_{sv_{15}}^{DR}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2C_u}\right\}$	31.			3	$t_{sv_{31}}^{DR}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2C_u}\right\}$
16.*			4	$t_{sv_{16}}^{exp}=\bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2C_u}\right\}$	32.*			4	$t_{sv_{32}}^{exp}=\bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2C_u}\right\}$

Table 2: Some members of the proposed class of estimators t_{sv}^{FT} continued...

S.No.	Ψ	κ	d	Estimators	S.No.	Ψ	κ	d	Estimators
33.	$\beta_2(u)$	ρ	1	$t_{sv_{33}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2\rho}\right\}$	65.	1	D_i	1	$t_{sv_{65}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D_i}\right\}$
34.			2	$t_{sv_{34}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2\rho}\right\}$	66.			2	$t_{sv_{66}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D_i}\right\}$
35.			3	$t_{sv_{35}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2\rho}\right\}$	67.			3	$t_{sv_{67}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D_i}\right\}$
36.*			4	$t_{sv_{36}}^{exp} = \bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2\rho}\right\}$	68.			4	$t_{sv_{68}}^{exp} = \bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D_i}\right\}$
37.	ρ	$\beta_2(u)$	1	$t_{sv_{37}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	69.	$\beta_1(u)$	Q_d	1	$t_{sv_{69}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
38.			2	$t_{sv_{38}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	70.			2	$t_{sv_{70}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
39.			3	$t_{sv_{39}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	71.			3	$t_{sv_{71}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
40.*			4	$t_{sv_{40}}^{exp} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	72.			4	$t_{sv_{72}}^{exp} = \bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
41.	1	$\beta_1(u)$	1	$t_{sv_{41}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_1(u)}\right\}$	73.	ρ	M_d	1	$t_{sv_{73}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_d}\right\}$
42.			2	$t_{sv_{42}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_1(u)}\right\}$	74.			2	$t_{sv_{74}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_d}\right\}$
43.			3	$t_{sv_{43}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_1(u)}\right\}$	75.			3	$t_{sv_{75}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_d}\right\}$
44.			4	$t_{sv_{44}}^{exp} = \bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_1(u)}\right\}$	76.			4	$t_{sv_{76}}^{exp} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_d}\right\}$
45.	$\beta_1(u)$	$\beta_2(u)$	1	$t_{sv_{45}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	77.	$\beta_2(u)$	Q_1	1	$t_{sv_{77}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_1}\right\}$
46.			2	$t_{sv_{46}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	78.			2	$t_{sv_{78}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_1}\right\}$
47.			3	$t_{sv_{47}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	79.			3	$t_{sv_{79}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_1}\right\}$
48.			4	$t_{sv_{48}}^{exp} = \bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	80.			4	$t_{sv_{80}}^{exp} = \bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_1}\right\}$
49.	1	M_d	1	$t_{sv_{49}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_d}\right\}$	81.	$\beta_2(u)$	Q_3	1	$t_{sv_{81}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_3}\right\}$
50.			2	$t_{sv_{50}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_d}\right\}$	82.			2	$t_{sv_{82}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_3}\right\}$
51.			3	$t_{sv_{51}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_d}\right\}$	83.			3	$t_{sv_{83}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_3}\right\}$
52.			4	$t_{sv_{52}}^{exp} = \bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_d}\right\}$	84.			4	$t_{sv_{84}}^{exp} = \bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_3}\right\}$
53.	C_u	M_d	1	$t_{sv_{53}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_d}\right\}$	85.	$\beta_2(u)$	Q_r	1	$t_{sv_{85}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_r}\right\}$
54.			2	$t_{sv_{54}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_d}\right\}$	86.			2	$t_{sv_{86}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_r}\right\}$
55.			3	$t_{sv_{55}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_d}\right\}$	87.			3	$t_{sv_{87}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_r}\right\}$
56.			4	$t_{sv_{56}}^{exp} = \bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_d}\right\}$	88.			4	$t_{sv_{88}}^{exp} = \bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_r}\right\}$
57.	$\beta_1(u)$	M_d	1	$t_{sv_{57}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2M_d}\right\}$	89.	$\beta_2(u)$	Q_d	1	$t_{sv_{89}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
58.			2	$t_{sv_{58}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2M_d}\right\}$	90.			2	$t_{sv_{90}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
59.			3	$t_{sv_{59}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2M_d}\right\}$	91.			3	$t_{sv_{91}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
60.			4	$t_{sv_{60}}^{exp} = \bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2M_d}\right\}$	92.			4	$t_{sv_{92}}^{exp} = \bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
61.	$\beta_2(u)$	M_d	1	$t_{sv_{61}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2M_d}\right\}$	93.	$\beta_2(u)$	Q_a	1	$t_{sv_{93}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_a}\right\}$
62.			2	$t_{sv_{62}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2M_d}\right\}$	94.			2	$t_{sv_{94}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_a}\right\}$
63.			3	$t_{sv_{63}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2M_d}\right\}$	95.			3	$t_{sv_{95}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_a}\right\}$
64.			4	$t_{sv_{64}}^{exp} = \bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2M_d}\right\}$	96.			4	$t_{sv_{96}}^{exp} = \bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_a}\right\}$

Table 3: Some members of the proposed class of estimators t_{sv}^{FT} continued...

S.No.	Ψ	κ	**d**	**Estimators**	**S.No.**	Ψ	κ	**d**	**Estimators**
97.	$\beta_1(u)$	Q_1	1	$t_{sv_{97}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_1}\right\}$	129.	ρ	Q_a	1	$t_{sv_{129}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_a}\right\}$
98.			2	$t_{sv_{98}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_1}\right\}$	130.			2	$t_{sv_{130}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_a}\right\}$
99.			3	$t_{sv_{99}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_1}\right\}$	131.			3	$t_{sv_{131}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_a}\right\}$
100.			4	$t_{sv_{100}}^{exp} = \bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_1}\right\}$	132.			4	$t_{sv_{132}}^{exp} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_a}\right\}$
101.	$\beta_1(u)$	Q_3	1	$t_{sv_{101}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_3}\right\}$	133.	1	T_m	1	$t_{sv_{133}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2T_m}\right\}$
102.			2	$t_{sv_{102}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_3}\right\}$	134.			2	$t_{sv_{134}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2T_m}\right\}$
103.			3	$t_{sv_{103}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_3}\right\}$	135.			3	$t_{sv_{135}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2T_m}\right\}$
104.			4	$t_{sv_{104}}^{exp} = \bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_3}\right\}$	136.			4	$t_{sv_{136}}^{exp} = \bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2T_m}\right\}$
105.	$\beta_1(u)$	Q_r	1	$t_{sv_{105}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_r}\right\}$	137.	C_u	T_m	1	$t_{sv_{137}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2T_m}\right\}$
106.			2	$t_{sv_{106}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_r}\right\}$	138.			2	$t_{sv_{138}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2T_m}\right\}$
107.			3	$t_{sv_{107}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_r}\right\}$	139.			3	$t_{sv_{139}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2T_m}\right\}$
108.			4	$t_{sv_{108}}^{exp} = \bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_r}\right\}$	140.			4	$t_{sv_{140}}^{exp} = \bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2T_m}\right\}$
109.	$\beta_1(u)$	Q_a	1	$t_{sv_{109}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_a}\right\}$	141.	ρ	T_m	1	$t_{sv_{141}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2T_m}\right\}$
110.			2	$t_{sv_{110}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_a}\right\}$	142.			2	$t_{sv_{142}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2T_m}\right\}$
111.			3	$t_{sv_{111}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_a}\right\}$	143.			3	$t_{sv_{143}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2T_m}\right\}$
112.			4	$t_{sv_{112}}^{exp} = \bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_a}\right\}$	144.			4	$t_{sv_{144}}^{exp} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2T_m}\right\}$
113.	ρ	Q_1	1	$t_{sv_{113}}^{Re}\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_1}\right\}$	145.	1	M_r	1	$t_{sv_{145}}^{Re}\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_r}\right\}$
114.			2	$t_{sv_{114}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_1}\right\}$	146.			2	$t_{sv_{146}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_r}\right\}$
115.			3	$t_{sv_{115}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_1}\right\}$	147.			3	$t_{sv_{147}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_r}\right\}$
116.			4	$t_{sv_{116}}^{exp} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_1}\right\}$	148.			4	$t_{sv_{148}}^{exp} = \bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_r}\right\}$
117.	ρ	Q_3	1	$t_{sv_{117}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_3}\right\}$	149.	C_u	M_r	1	$t_{sv_{149}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_r}\right\}$
118.			2	$t_{sv_{118}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_3}\right\}$	150.			2	$t_{sv_{150}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_r}\right\}$
119.			3	$t_{sv_{119}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_3}\right\}$	151.			3	$t_{sv_{151}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_r}\right\}$
120.			4	$t_{sv_{120}}^{exp} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_3}\right\}$	152.			4	$t_{sv_{152}}^{exp} = \bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_r}\right\}$
121.	ρ	Q_r	1	$t_{sv_{121}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_r}\right\}$	153.	ρ	M_r	1	$t_{sv_{153}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_r}\right\}$
122.			2	$t_{sv_{122}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_r}\right\}$	154.			2	$t_{sv_{154}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_r}\right\}$
123.			3	$t_{sv_{123}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_r}\right\}$	155.			3	$t_{sv_{155}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_r}\right\}$
124.			4	$t_{sv_{124}}^{exp} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_r}\right\}$	156.			4	$t_{sv_{156}}^{exp} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_r}\right\}$
125.	ρ	Q_d	1	$t_{sv_{125}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_d}\right\}$	157.	ρ	D_i	1	$t_{sv_{157}}^{Re} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D_i}\right\}$
126.			2	$t_{sv_{126}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_d}\right\}$	158.			2	$t_{sv_{158}}^{Pe} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D_i}\right\}$
127.			3	$t_{sv_{127}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_d}\right\}$	159.			3	$t_{sv_{159}}^{DR} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D_i}\right\}$
128.			4	$t_{sv_{128}}^{exp} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_d}\right\}$	160.			4	$t_{sv_{160}}^{exp} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D_i}\right\}$

Table 4: Some members of the proposed class of estimators t_{sv}^{FT} continued...

S.No.	Ψ	κ	d	Estimators	S.No.	Ψ	κ	d	Estimators
161.	C_u	D_i	1	$t^{Re}_{sv_{161}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D_i}\right\}$	193.	ρ	D	1	$t^{Re}_{sv_{193}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D}\right\}$
162.			2	$t^{Pe}_{sv_{162}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D_i}\right\}$	194.			2	$t^{Pe}_{sv_{194}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D}\right\}$
163.			3	$t^{DR}_{sv_{163}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D_i}\right\}$	195.			3	$t^{DR}_{sv_{195}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D}\right\}$
164.			4	$t^{exp}_{sv_{164}} = \bar{v} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D_i}\right\}$	196.			4	$t^{exp}_{sv_{196}} = \bar{v} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D}\right\}$
165.	1	H_1	1	$t^{Re}_{sv_{165}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2H_1}\right\}$	197.	C_u	D	1	$t^{Re}_{sv_{197}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D}\right\}$
166.			2	$t^{Pe}_{sv_{166}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2H_1}\right\}$	198.			2	$t^{Pe}_{sv_{198}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D}\right\}$
167.			3	$t^{DR}_{sv_{167}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2H_1}\right\}$	199.			3	$t^{DR}_{sv_{199}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D}\right\}$
168.			4	$t^{exp}_{sv_{168}} = \bar{v} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2H_1}\right\}$	200.			4	$t^{exp}_{sv_{200}} = \bar{v} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D}\right\}$
169.	C_u	H_1	1	$t^{Re}_{sv_{169}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2H_1}\right\}$	201.	1	S_{pw}	1	$t^{Re}_{sv_{201}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2S_{pw}}\right\}$
170.			2	$t^{Pe}_{sv_{170}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2H_1}\right\}$	202.			2	$t^{Pe}_{sv_{202}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2S_{pw}}\right\}$
171.			3	$t^{DR}_{sv_{171}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2H_1}\right\}$	203.			3	$t^{DR}_{sv_{203}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2S_{pw}}\right\}$
172.			4	$t^{exp}_{sv_{172}} = \bar{v} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2H_1}\right\}$	204.			4	$t^{exp}_{sv_{204}} = \bar{v} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2S_{pw}}\right\}$
173.	ρ	H_1	1	$t^{Re}_{sv_{173}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2H_1}\right\}$	205.	ρ	S_{pw}	1	$t^{Re}_{sv_{205}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2S_{pw}}\right\}$
174.			2	$t^{Pe}_{sv_{174}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2H_1}\right\}$	206.			2	$t^{Pe}_{sv_{206}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2S_{pw}}\right\}$
175.			3	$t^{DR}_{sv_{175}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2H_1}\right\}$	207.			3	$t^{DR}_{sv_{207}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2S_{pw}}\right\}$
176.			4	$t^{exp}_{sv_{176}} = \bar{v} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2H_1}\right\}$	208.			4	$t^{exp}_{sv_{208}} = \bar{v} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2S_{pw}}\right\}$
177.	1	G	1	$t^{Re}_{sv_{177}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2G}\right\}$	209.	C_u	S_{pw}	1	$t^{Re}_{sv_{209}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2S_{pw}}\right\}$
178.			2	$t^{Pe}_{sv_{178}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2G}\right\}$	210.			2	$t^{Pe}_{sv_{210}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2S_{pw}}\right\}$
179.			3	$t^{DR}_{sv_{179}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2G}\right\}$	211.			3	$t^{DR}_{sv_{211}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2S_{pw}}\right\}$
180.			4	$t^{exp}_{sv_{180}} = \bar{v} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2G}\right\}$	212.			4	$t^{exp}_{sv_{212}} = \bar{v} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2S_{pw}}\right\}$
181.	ρ	G	1	$t^{Re}_{sv_{181}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2G}\right\}$	213.	Q_d	D_i	1	$t^{Re}_{sv_{213}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{Q_d(\bar{U}-\bar{u})}{Q_d(\bar{U}+\bar{u})+2D_i}\right\}$
182.			2	$t^{Pe}_{sv_{182}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2G}\right\}$	214.			2	$t^{Pe}_{sv_{214}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{Q_d(\bar{U}-\bar{u})}{Q_d(\bar{U}+\bar{u})+2D_i}\right\}$
183.			3	$t^{DR}_{sv_{183}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2G}\right\}$	215.			3	$t^{DR}_{sv_{215}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{Q_d(\bar{U}-\bar{u})}{Q_d(\bar{U}+\bar{u})+2D_i}\right\}$
184.			4	$t^{exp}_{sv_{184}} = \bar{v} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2G}\right\}$	216.			4	$t^{exp}_{sv_{216}} = \bar{v} exp\left\{\frac{Q_d(\bar{U}-\bar{u})}{Q_d(\bar{U}+\bar{u})+2D_i}\right\}$
185.	C_u	G	1	$t^{Re}_{sv_{185}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2G}\right\}$	217.	D_i	Q_d	1	$t^{Re}_{sv_{217}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{D_i(\bar{U}-\bar{u})}{D_i(\bar{U}+\bar{u})+2Q_d}\right\}$
186.			2	$t^{Pe}_{sv_{186}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2G}\right\}$	218.			2	$t^{Pe}_{sv_{218}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{D_i(\bar{U}-\bar{u})}{D_i(\bar{U}+\bar{u})+2Q_d}\right\}$
187.			3	$t^{DR}_{sv_{187}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2G}\right\}$	219.			3	$t^{DR}_{sv_{219}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{D_i(\bar{U}-\bar{u})}{D_i(\bar{U}+\bar{u})+2Q_d}\right\}$
188.			4	$t^{exp}_{sv_{188}} = \bar{v} exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2G}\right\}$	220.			4	$t^{exp}_{sv_{220}} = \bar{v} exp\left\{\frac{D_i(\bar{U}-\bar{u})}{D_i(\bar{U}+\bar{u})+2Q_d}\right\}$
189.	1	D	1	$t^{Re}_{sv_{189}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D}\right\}$	221.	Q_r	S_k	1	$t^{Re}_{sv_{220}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{Q_r(\bar{U}-\bar{u})}{Q_r(\bar{U}+\bar{u})+2S_k}\right\}$
190.			2	$t^{Pe}_{sv_{190}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D}\right\}$	222.			2	$t^{Pe}_{sv_{222}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]^{\alpha} exp\left\{\frac{Q_r(\bar{U}-\bar{u})}{Q_r(\bar{U}+\bar{u})+2S_k}\right\}$
191.			3	$t^{DR}_{sv_{191}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D}\right\}$	223.			3	$t^{DR}_{sv_{223}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]^{\alpha} exp\left\{\frac{Q_r(\bar{U}-\bar{u})}{Q_r(\bar{U}+\bar{u})+2S_k}\right\}$
192.			4	$t^{exp}_{sv_{192}} = \bar{v} exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D}\right\}$	224.			4	$t^{exp}_{sv_{224}} = \bar{v} exp\left\{\frac{Q_r(\bar{U}-\bar{u})}{Q_r(\bar{U}+\bar{u})+2S_k}\right\}$

Table 5: Data Statistics of COVID-19 data in India

$N = 943$	$\bar{V} = 559.7815$	$\bar{U} = 47113.88$
$n = 221$	$C_V = 2.391222$	$C_U = 2.635505$
$\rho = 0.7393576$	$f = 0.2343584$	$S_k = 0.4277171$
$Q_r = 19814.5$		

Table 6: Some members of the proposed class of estimators t_{sv}^{Y}.

S.No.	Ψ	κ	d	Estimators	S.No.	Ψ	κ	d	Estimators
1.	1	0	1	$t_{sv_1}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})}\right\}$	17.	C_u	$\beta_2(u)$	1	$t_{sv_{17}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$
2.			2	$t_{sv_2}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})}\right\}$	18.			2	$t_{sv_{18}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$
3.			3	$t_{sv_3}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})}\right\}$	19.			3	$t_{sv_{19}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$
4.			4	$t_{sv_4}^{exp}=\bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})}\right\}$	20.			4	$t_{sv_{20}}^{exp}=\bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$
5.	1	C_u	1	$t_{sv_5}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2C_u}\right\}$	21.	1	ρ	1	$t_{sv_{21}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\rho}\right\}$
6.			2	$t_{sv_6}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2C_u}\right\}$	22.			2	$t_{sv_{22}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\rho}\right\}$
7.			3	$t_{sv_7}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2C_u}\right\}$	23.			3	$t_{sv_{23}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\rho}\right\}$
8.			4	$t_{sv_8}^{exp}=\bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2C_u}\right\}$	24.			4	$t_{sv_{24}}^{exp}=\bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\rho}\right\}$
9.	1	$\beta_2(u)$	1	$t_{sv_9}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	25.	C_u	ρ	1	$t_{sv_{25}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\rho}\right\}$
10.			2	$t_{sv_{10}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	26.			2	$t_{sv_{26}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\rho}\right\}$
11.			3	$t_{sv_{11}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	27.			3	$t_{sv_{27}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\rho}\right\}$
12.			4	$t_{sv_{12}}^{exp}=\bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	28.			4	$t_{sv_{28}}^{exp}=\bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2\rho}\right\}$
13.	$\beta_2(u)$	C_u	1	$t_{sv_{13}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2C_u}\right\}$	29.	ρ	C_u	1	$t_{sv_{29}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2C_u}\right\}$
14.			2	$t_{sv_{14}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2C_u}\right\}$	30.			2	$t_{sv_{30}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2C_u}\right\}$
15.			3	$t_{sv_{15}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2C_u}\right\}$	31.			3	$t_{sv_{31}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2C_u}\right\}$
16.			4	$t_{sv_{16}}^{exp}=\bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2C_u}\right\}$	32.			4	$t_{sv_{32}}^{exp}=\bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2C_u}\right\}$

Table 7: Some members of the proposed class of estimators t_{sv}^{Y} continued...

S.No.	Ψ	κ	d	Estimators	S.No.	Ψ	κ	d	Estimators
33.	$\beta_2(u)$	ρ	1	$t_{sv_{33}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2\rho}\right\}$	65.	1	D_i	1	$t_{sv_{65}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D_i}\right\}$
34.			2	$t_{sv_{34}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2\rho}\right\}$	66.			2	$t_{sv_{66}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D_i}\right\}$
35.			3	$t_{sv_{35}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2\rho}\right\}$	67.			3	$t_{sv_{67}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D_i}\right\}$
36.			4	$t_{sv_{36}}^{exp}=\bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2\rho}\right\}$	68.			4	$t_{sv_{68}}^{exp}=\bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D_i}\right\}$
37.	ρ	$\beta_2(u)$	1	$t_{sv_{37}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	69.	$\beta_1(u)$	Q_d	1	$t_{sv_{69}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
38.			2	$t_{sv_{38}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	70.			2	$t_{sv_{70}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
39.			3	$t_{sv_{39}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	71.			3	$t_{sv_{71}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
40.			4	$t_{sv_{40}}^{exp}=\bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	72.			4	$t_{sv_{72}}^{exp}=\bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
41.	1	$\beta_1(u)$	1	$t_{sv_{41}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_1(u)}\right\}$	73.	ρ	M_d	1	$t_{sv_{73}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_d}\right\}$
42.			2	$t_{sv_{42}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_1(u)}\right\}$	74.			2	$t_{sv_{74}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_d}\right\}$
43.			3	$t_{sv_{43}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_1(u)}\right\}$	75.			3	$t_{sv_{75}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_d}\right\}$
44.			4	$t_{sv_{44}}^{exp}=\bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2\beta_1(u)}\right\}$	76.			4	$t_{sv_{76}}^{exp}=\bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_d}\right\}$
45.	$\beta_1(u)$	$\beta_2(u)$	1	$t_{sv_{45}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	77.	$\beta_2(u)$	Q_1	1	$t_{sv_{77}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_1}\right\}$
46.			2	$t_{sv_{46}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	78.			2	$t_{sv_{78}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_1}\right\}$
47.			3	$t_{sv_{47}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	79.			3	$t_{sv_{79}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_1}\right\}$
48.			4	$t_{sv_{48}}^{exp}=\bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2\beta_2(u)}\right\}$	80.			4	$t_{sv_{80}}^{exp}=\bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_1}\right\}$
49.	1	M_d	1	$t_{sv_{49}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_d}\right\}$	81.	$\beta_2(u)$	Q_3	1	$t_{sv_{81}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_3}\right\}$
50.			2	$t_{sv_{50}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_d}\right\}$	82.			2	$t_{sv_{82}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_3}\right\}$
51.			3	$t_{sv_{51}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_d}\right\}$	83.			3	$t_{sv_{83}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_3}\right\}$
52.			4	$t_{sv_{52}}^{exp}=\bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_d}\right\}$	84.			4	$t_{sv_{84}}^{exp}=\bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_3}\right\}$
53.	C_u	M_d	1	$t_{sv_{53}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_d}\right\}$	85.	$\beta_2(u)$	Q_r	1	$t_{sv_{85}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_r}\right\}$
54.			2	$t_{sv_{54}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_d}\right\}$	86.			2	$t_{sv_{86}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_r}\right\}$
55.			3	$t_{sv_{55}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_d}\right\}$	87.			3	$t_{sv_{87}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_r}\right\}$
56.			4	$t_{sv_{56}}^{exp}=\bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_d}\right\}$	88.			4	$t_{sv_{88}}^{exp}=\bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_r}\right\}$
57.	$\beta_1(u)$	M_d	1	$t_{sv_{57}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2M_d}\right\}$	89.	$\beta_2(u)$	Q_d	1	$t_{sv_{89}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
58.			2	$t_{sv_{58}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2M_d}\right\}$	90.			2	$t_{sv_{90}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
59.			3	$t_{sv_{59}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2M_d}\right\}$	91.			3	$t_{sv_{91}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
60.			4	$t_{sv_{60}}^{exp}=\bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2M_d}\right\}$	92.			4	$t_{sv_{92}}^{exp}=\bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_d}\right\}$
61.	$\beta_2(u)$	M_d	1	$t_{sv_{61}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2M_d}\right\}$	93.	$\beta_2(u)$	Q_a	1	$t_{sv_{93}}^{Y}=\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_a}\right\}$
62.			2	$t_{sv_{62}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2M_d}\right\}$	94.			2	$t_{sv_{94}}^{Y}=\bar{v}\left[\frac{\bar{u}}{\bar{U}}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_a}\right\}$
63.			3	$t_{sv_{63}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2M_d}\right\}$	95.			3	$t_{sv_{95}}^{Y}=\bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right]exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_a}\right\}$
64.			4	$t_{sv_{64}}^{exp}=\bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2M_d}\right\}$	96.			4	$t_{sv_{96}}^{exp}=\bar{v}exp\left\{\frac{\beta_2(u)(\bar{U}-\bar{u})}{\beta_2(u)(\bar{U}+\bar{u})+2Q_a}\right\}$

Table 8: Some members of the proposed class of estimators t^{Y}_{sv} continued...

S.No.	Ψ	κ	**d**	**Estimators**	**S.No.**	Ψ	κ	**d**	**Estimators**
97.	$\beta_1(u)$	Q_1	1	$t^{Y}_{sv_{97}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_1}\right\}$	129.	ρ	Q_a	1	$t^{Y}_{sv_{129}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_a}\right\}$
98.			2	$t^{Y}_{sv_{98}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_1}\right\}$	130.			2	$t^{Y}_{sv_{130}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_a}\right\}$
99.			3	$t^{Y}_{sv_{99}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_1}\right\}$	131.			3	$t^{Y}_{sv_{131}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_a}\right\}$
100.			4	$t^{exp}_{sv_{100}} = \bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_1}\right\}$	132.			4	$t^{exp}_{sv_{132}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_a}\right\}$
101.	$\beta_1(u)$	Q_3	1	$t^{Y}_{sv_{101}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_3}\right\}$	133.	1	T_m	1	$t^{Y}_{sv_{133}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2T_m}\right\}$
102.			2	$t^{Y}_{sv_{102}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_3}\right\}$	134.			2	$t^{Y}_{sv_{134}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2T_m}\right\}$
103.			3	$t^{Y}_{sv_{103}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_3}\right\}$	135.			3	$t^{Y}_{sv_{135}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2T_m}\right\}$
104.			4	$t^{exp}_{sv_{104}} = \bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_3}\right\}$	136.			4	$t^{exp}_{sv_{136}} = \bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2T_m}\right\}$
105.	$\beta_1(u)$	Q_r	1	$t^{Y}_{sv_{105}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_r}\right\}$	137.	C_u	T_m	1	$t^{Y}_{sv_{137}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2T_m}\right\}$
106.			2	$t^{Y}_{sv_{106}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_r}\right\}$	138.			2	$t^{Y}_{sv_{138}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2T_m}\right\}$
107.			3	$t^{Y}_{sv_{107}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_r}\right\}$	139.			3	$t^{Y}_{sv_{139}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2T_m}\right\}$
108.			4	$t^{exp}_{sv_{108}} = \bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_r}\right\}$	140.			4	$t^{exp}_{sv_{140}} = \bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2T_m}\right\}$
109.	$\beta_1(u)$	Q_a	1	$t^{Y}_{sv_{109}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_a}\right\}$	141.	ρ	T_m	1	$t^{Y}_{sv_{141}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right]^{\alpha} exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2T_m}\right\}$
110.			2	$t^{Y}_{sv_{110}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_a}\right\}$	142.			2	$t^{Y}_{sv_{142}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2T_m}\right\}$
111.			3	$t^{Y}_{sv_{111}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_a}\right\}$	143.			3	$t^{Y}_{sv_{143}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2T_m}\right\}$
112.			4	$t^{exp}_{sv_{112}} = \bar{v}exp\left\{\frac{\beta_1(u)(\bar{U}-\bar{u})}{\beta_1(u)(\bar{U}+\bar{u})+2Q_a}\right\}$	144.			4	$t^{exp}_{sv_{144}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2T_m}\right\}$
113.	ρ	Q_1	1	$t^{Y}_{sv_{113}}\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_1}\right\}$	145.	1	M_r	1	$t^{Y}_{sv_{145}}\bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_r}\right\}$
114.			2	$t^{Y}_{sv_{114}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_1}\right\}$	146.			2	$t^{Y}_{sv_{146}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_r}\right\}$
115.			3	$t^{Y}_{sv_{115}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_1}\right\}$	147.			3	$t^{Y}_{sv_{147}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_r}\right\}$
116.			4	$t^{exp}_{sv_{116}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_1}\right\}$	148.			4	$t^{exp}_{sv_{148}} = \bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2M_r}\right\}$
117.	ρ	Q_3	1	$t^{Y}_{sv_{117}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_3}\right\}$	149.	C_u	M_r	1	$t^{Y}_{sv_{149}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_r}\right\}$
118.			2	$t^{Y}_{sv_{118}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_3}\right\}$	150.			2	$t^{Y}_{sv_{150}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_r}\right\}$
119.			3	$t^{Y}_{sv_{119}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_3}\right\}$	151.			3	$t^{Y}_{sv_{151}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_r}\right\}$
120.			4	$t^{exp}_{sv_{120}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_3}\right\}$	152.			4	$t^{exp}_{sv_{152}} = \bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2M_r}\right\}$
121.	ρ	Q_r	1	$t^{Y}_{sv_{121}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_r}\right\}$	153.	ρ	M_r	1	$t^{Y}_{sv_{153}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_r}\right\}$
122.			2	$t^{Y}_{sv_{122}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_r}\right\}$	154.			2	$t^{Y}_{sv_{154}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_r}\right\}$
123.			3	$t^{Y}_{sv_{123}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_r}\right\}$	155.			3	$t^{Y}_{sv_{155}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_r}\right\}$
124.			4	$t^{exp}_{sv_{124}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_r}\right\}$	156.			4	$t^{exp}_{sv_{156}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2M_r}\right\}$
125.	ρ	Q_d	1	$t^{Y}_{sv_{125}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_d}\right\}$	157.	ρ	D_i	1	$t^{Y}_{sv_{157}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D_i}\right\}$
126.			2	$t^{Y}_{sv_{126}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_d}\right\}$	158.			2	$t^{Y}_{sv_{158}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D_i}\right\}$
127.			3	$t^{Y}_{sv_{127}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_d}\right\}$	159.			3	$t^{Y}_{sv_{159}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D_i}\right\}$
128.			4	$t^{exp}_{sv_{128}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2Q_d}\right\}$	160.			4	$t^{exp}_{sv_{160}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D_i}\right\}$

Table 9: Some members of the proposed class of estimators t^Y_{sv} continued...

S.No.	Ψ	κ	d	Estimators	S.No.	Ψ	κ	d	Estimators
161.	C_u	D_i	1	$t^Y_{sv_{161}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D_i}\right\}$	193.	ρ	D	1	$t^Y_{sv_{193}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D}\right\}$
162.			2	$t^Y_{sv_{162}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D_i}\right\}$	194.			2	$t^Y_{sv_{194}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D}\right\}$
163.			3	$t^Y_{sv_{163}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D_i}\right\}$	195.			3	$t^Y_{sv_{195}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D}\right\}$
164.			4	$t^{exp}_{sv_{164}} = \bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D_i}\right\}$	196.			4	$t^{exp}_{sv_{196}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2D}\right\}$
165.	1	H_1	1	$t^Y_{sv_{165}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2H_1}\right\}$	197.	C_u	D	1	$t^Y_{sv_{197}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D}\right\}$
166.			2	$t^Y_{sv_{166}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2H_1}\right\}$	198.			2	$t^Y_{sv_{198}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D}\right\}$
167.			3	$t^Y_{sv_{167}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2H_1}\right\}$	199.			3	$t^Y_{sv_{199}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D}\right\}$
168.			4	$t^{exp}_{sv_{168}} = \bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2H_1}\right\}$	200.			4	$t^{exp}_{sv_{200}} = \bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2D}\right\}$
169.	C_u	H_1	1	$t^Y_{sv_{169}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2H_1}\right\}$	201.	1	S_{pw}	1	$t^Y_{sv_{201}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2S_{pw}}\right\}$
170.			2	$t^Y_{sv_{170}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2H_1}\right\}$	202.			2	$t^Y_{sv_{202}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2S_{pw}}\right\}$
171.			3	$t^Y_{sv_{171}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2H_1}\right\}$	203.			3	$t^Y_{sv_{203}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2S_{pw}}\right\}$
172.			4	$t^{exp}_{sv_{172}} = \bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2H_1}\right\}$	204.			4	$t^{exp}_{sv_{204}} = \bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2S_{pw}}\right\}$
173.	ρ	H_1	1	$t^Y_{sv_{173}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2H_1}\right\}$	205.	ρ	S_{pw}	1	$t^Y_{sv_{205}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2S_{pw}}\right\}$
174.			2	$t^Y_{sv_{174}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2H_1}\right\}$	206.			2	$t^Y_{sv_{206}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2S_{pw}}\right\}$
175.			3	$t^Y_{sv_{175}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2H_1}\right\}$	207.			3	$t^Y_{sv_{207}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2S_{pw}}\right\}$
176.			4	$t^{exp}_{sv_{176}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2H_1}\right\}$	208.			4	$t^{exp}_{sv_{208}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2S_{pw}}\right\}$
177.	1	G	1	$t^Y_{sv_{177}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2G}\right\}$	209.	C_u	S_{pw}	1	$t^Y_{sv_{209}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2S_{pw}}\right\}$
178.			2	$t^Y_{sv_{178}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2G}\right\}$	210.			2	$t^Y_{sv_{210}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2S_{pw}}\right\}$
179.			3	$t^Y_{sv_{179}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2G}\right\}$	211.			3	$t^Y_{sv_{211}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2S_{pw}}\right\}$
180.			4	$t^{exp}_{sv_{180}} = \bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2G}\right\}$	212.			4	$t^{exp}_{sv_{212}} = \bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2S_{pw}}\right\}$
181.	ρ	G	1	$t^Y_{sv_{181}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2G}\right\}$	213.	Q_d	D_i	1	$t^Y_{sv_{213}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{Q_d(\bar{U}-\bar{u})}{Q_d(\bar{U}+\bar{u})+2D_i}\right\}$
182.			2	$t^Y_{sv_{182}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2G}\right\}$	214.			2	$t^Y_{sv_{214}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{Q_d(\bar{U}-\bar{u})}{Q_d(\bar{U}+\bar{u})+2D_i}\right\}$
183.			3	$t^Y_{sv_{183}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2G}\right\}$	215.			3	$t^Y_{sv_{215}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{Q_d(\bar{U}-\bar{u})}{Q_d(\bar{U}+\bar{u})+2D_i}\right\}$
184.			4	$t^{exp}_{sv_{184}} = \bar{v}exp\left\{\frac{\rho(\bar{U}-\bar{u})}{\rho(\bar{U}+\bar{u})+2G}\right\}$	216.			4	$t^{exp}_{sv_{216}} = \bar{v}exp\left\{\frac{Q_d(\bar{U}-\bar{u})}{Q_d(\bar{U}+\bar{u})+2D_i}\right\}$
185.	C_u	G	1	$t^Y_{sv_{185}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2G}\right\}$	217.	D_i	Q_d	1	$t^Y_{sv_{217}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{D_i(\bar{U}-\bar{u})}{D_i(\bar{U}+\bar{u})+2Q_d}\right\}$
186.			2	$t^Y_{sv_{186}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2G}\right\}$	218.			2	$t^Y_{sv_{218}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{D_i(\bar{U}-\bar{u})}{D_i(\bar{U}+\bar{u})+2Q_d}\right\}$
187.			3	$t^Y_{sv_{187}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2G}\right\}$	219.			3	$t^Y_{sv_{219}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{D_i(\bar{U}-\bar{u})}{D_i(\bar{U}+\bar{u})+2Q_d}\right\}$
188.			4	$t^{exp}_{sv_{188}} = \bar{v}exp\left\{\frac{C_u(\bar{U}-\bar{u})}{C_u(\bar{U}+\bar{u})+2G}\right\}$	220.			4	$t^{exp}_{sv_{220}} = \bar{v}exp\left\{\frac{D_i(\bar{U}-\bar{u})}{D_i(\bar{U}+\bar{u})+2Q_d}\right\}$
189.	1	D	1	$t^Y_{sv_{189}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D}\right\}$	221.	Q_r	S_k	1	$t^Y_{sv_{220}} = \bar{v}\left[\frac{\bar{U}}{\bar{u}}\right] exp\left\{\frac{Q_r(\bar{U}-\bar{u})}{Q_r(\bar{U}+\bar{u})+2S_k}\right\}$
190.			2	$t^Y_{sv_{190}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D}\right\}$	222.			2	$t^Y_{sv_{222}} = \bar{v}\left[\frac{\bar{u}}{\bar{U}}\right] exp\left\{\frac{Q_r(\bar{U}-\bar{u})}{Q_r(\bar{U}+\bar{u})+2S_k}\right\}$
191.			3	$t^Y_{sv_{191}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D}\right\}$	223.			3	$t^Y_{sv_{223}} = \bar{v}\left[\frac{\bar{U}-\bar{u}f}{\bar{U}-\bar{U}f}\right] exp\left\{\frac{Q_r(\bar{U}-\bar{u})}{Q_r(\bar{U}+\bar{u})+2S_k}\right\}$
192.			4	$t^{exp}_{sv_{192}} = \bar{v}exp\left\{\frac{(\bar{U}-\bar{u})}{(\bar{U}+\bar{u})+2D}\right\}$	224.			4	$t^{exp}_{sv_{224}} = \bar{v}exp\left\{\frac{Q_r(\bar{U}-\bar{u})}{Q_r(\bar{U}+\bar{u})+2S_k}\right\}$

Table 10: PREs of the suggested estimator over the usual mean estimator at different values of α and d for numerical illustration based on COVID-19 data in India.

α	**COVID-19 Data**			
	$d = 1$	$d = 2$	$d = 3$	$d = 4$
-1	47.20149	77.60802	137.0519	204.583
-0.6	85.09730	147.68979	165.0180	204.583
-0.2	161.18782	220.07812	192.7677	204.583
0	204.58304	204.58304	204.5830	204.583
0.2	220.07812	161.18782	213.7007	204.583
0.6	147.68979	85.09730	**220.4827**	204.583
1	77.60802	47.20149	210.2710	204.583

Table 11: MSEs and PREs of the suggested estimator over the usual mean estimators for numerical illustration are based on COVID-19 data in India.

Estimator	**MSE**	**PRE**
$\bar{y}$	6207.405	100.00000
t_{2Re}	3034.174	204.58304
t_{2Pe}	13150.868	47.20149
t_3	4835.698	128.36627
t_{4Re}	3034.174	204.58304
t_{4Pe}	13150.868	47.20149
t_5 $[k = -4]$	5124.657	121.12822
t_5 $[k = -2]$	2840.057	218.56623
t_5 $[k = 0]$	6207.405	100.00000
t_5 $[k = 2]$	15226.702	40.76658
t_5 $[k = 4]$	29897.947	20.76198
t_1 $[d = 1]$	3631.174	170.94759
t_1 $[d = 2]$	23864.563	26.01097
t_1 $[d = 3]$	3817.237	162.61512
t_1 $[d = 4]$	6207.405	100.00000
Proposed Estimator t_{sv}^{FT}	**2814.129**	**220.57998**

Table 12: MSEs and PREs of the suggested estimator over the usual mean estimators for simulation study based on COVID-19 data in India.

	n=150		**n=200**	
Estimator	**MSE**	**PRE**	**MSE**	**PRE**
$\bar{y}$	9748362	100.000000	3325354	100.000000
t_{2Re}	3401201	286.615293	1132492	293.631547
t_{2Pe}	37618039	25.914061	12966417	25.645895
t_3	7566168	128.841458	2314038	143.703523
t_{4Re}	3401201	286.615293	1132492	293.631547
t_{4Pe}	37618039	25.914061	12966417	25.645895
t_5 $[k = -4]$	3047408	319.890261	1419503	234.261889
t_5 $[k = -2]$	4029609	241.918279	1089171	305.310421
t_5 $[k = 0]$	9748362	100.000000	3325354	100.000000
t_5 $[k = 2]$	27465316	35.493354	14279976	23.286829
t_5 $[k = 4]$	69630468	14.000138	47406013	7.014624
t_1 $[d = 1]$	3738155	260.780005	1251042	265.806783
t_1 $[d = 2]$	117679932	8.283793	40624729	8.185540
t_1 $[d = 3]$	5964665	163.435183	1673071	198.757455
t_1 $[d = 4]$	9748362	100.000000	3325354	100.000000
Proposed Estimator t_{sv}^{FT}	**3041448**	**320.517090**	**1013992**	**327.946684**

Table 13: MSEs and PREs of the suggested estimator over the usual mean estimators for simulation study based on COVID-19 data in India.

	n=300		n=350	
Estimator	**MSE**	**PRE**	**MSE**	**PRE**
$\bar{y}$	627996.9	100.000000	318654.19	100.0000000
t_{2Re}	209494.5	299.767759	107574.72	296.2166025
t_{2Pe}	2479014.8	25.332518	1247680.39	25.5397287
t_3	339169.0	185.157512	150762.90	211.3611441
t_{4Re}	209494.5	299.767759	107574.72	296.2166025
t_{4Pe}	2479014.8	25.332518	1247680.39	25.5397287
t_5 $[k=-4]$	2000418.3	31.393277	3259196.23	9.7770789
t_5 $[k=-2]$	210681.5	298.078810	166441.76	191.4508578
t_5 $[k=0]$	627996.9	100.000000	318654.19	100.0000000
t_5 $[k=2]$	6783869.5	9.257207	5577671.89	5.7130321
t_5 $[k=4]$	34822049.6	1.803446	33966788.84	0.9381346
t_1 $[d=1]$	229871.5	273.194759	118975.49	267.8317810
t_1 $[d=2]$	7795892.2	8.055484	3910339.91	8.1490150
t_1 $[d=3]$	218338.1	287.625903	99346.33	320.7508401
t_1 $[d=4]$	627996.9	100.000000	318654.19	100.0000000
Proposed Estimator t_{sv}^{FT}	**186310.1**	**337.070694**	**96272.78**	**330.9909595**

11. Results

The current work has significantly improved our understanding of the generalized class of factor-type exponential ratio estimators. The research we have conducted has shown major and noteworthy developments that provide insight into the importance and application of this estimator by applying severe data analysis approaches and performing thorough tests. Our investigation revealed the following significant findings:

1. The effect of α and d is shown in Table [10], and we conclude that our proposed estimator performs much better when $\alpha = 0.6$ and $d = 3$, i.e., it has the highest percent relative efficiency.

2. The MSEs and PREs of the proposed estimators over the usual mean estimators for numerical studies based on COVID-19 data in India are given in Table [11], and we see that our proposed estimators have the lowest MSEs and highest PREs among all existing estimators.

3. From Tables [12] & [13] we can observe that the simulation study data shows that the efficiency of the results improves as the number of iterations increases (here, we took 5000 iterations, with sample sizes of approximately $\approx$ 16%, $\approx$20%, $\approx$32% and $\approx$37%).

12. Conclusions

In this study, a generalized class of factor-type exponential ratio estimators are considered using auxiliary information under SRSWOR techniques for positive and negative correlated data to estimate the COVID-19 deaths in India's population mean in sample surveys. Ultimately, our research shows that the suggested estimator we developed exceeds all the previous estimators reviewed in the article with respect to efficiency. This result highlights the value of our suggested estimator in improving estimating accuracy across a range of domains, with important implications for decision-making, as well as the effects of α and d are shown. In order to examine the change in the number of deaths, new case numbers are taken into consideration in this article as auxiliary variables. By changing the population parameter to another parameter in subsequent work, the class of suggested estimators can be diversified. Future studies on the number of COVID-19 deaths in other locations, time periods, and nations will be able to raise their population mean estimates by employing the proposed estimators.

Conflicts of Interest

Regarding the publishing of this paper, the authors affirm that there are no conflicts of interest.

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