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Soft Decision-Making Methods Employing Multiple *ifpifs*-Matrices and Their Application

Burak Arslan, Tuğçe Aydın, Samet Memiş, and Serdar Enginoğlu

ABSTRACT: The present study generalizes 36 soft decision-making (SDM) methods employing multiple fuzzy parameterized fuzzy soft matrices (*fpfs*-matrices) to operable in the intuitionistic fuzzy parameterized intuitionistic fuzzy soft matrices (*ifpifs*-matrices) space and obtains 44 SDM methods containing several variants of the aforesaid methods. It then compares all the SDM methods herein using three *ifpifs*-matrices for each test case proposed by the authors' previous study. The comparison shows that 23 of 44 passed all the test cases. Moreover, this study applies the 23 SDM methods to a performance-based value assignment (PVA) problem in which seven well-known salt-and-pepper noise removal filters used in digital images are considered. The results manifest that 10 of 23 produce the same ranking order at the high noise density, just as 8 of 23 do at the low noise density. Finally, this study discusses SDM methods' applications and the need for further research.

Key Words: Fuzzy sets, intuitionistic fuzzy sets, soft sets, *ifpifs*-matrices, soft decision making.

Contents

1. Introduction

The concept of intuitionistic fuzzy parameterized intuitionistic fuzzy soft matrices (*ifpifs*-matrices) [8] is an up-to-date mathematical tool coming to the forefront with its ability to model problems containing intuitionistic fuzzy uncertainties or multiple fuzzy uncertainties. Therefore, for problems with intuitionistic or multiple fuzzy uncertainties, it is worth studying to construct soft decision-making (SDM) methods by *ifpifs*-matrices. For this reason, this paper is a follow-up to [1], which generalizes SDM methods utilizing a single fuzzy parameterized fuzzy soft matrix (*fpfs*-matrix) [12] to *ifpifs*-matrices space. For more details, see [1]. This paper is a pioneering study and is a reference guide on how to generalize SDM methods employing multiple *fpfs*-matrices to *ifpifs*-matrices space.

Section 2 of the present paper, different from [1] that considers the SDM methods working with a single *fpfs*-matrix, generalizes SDM methods [14,15,16,17,18,19,20,21,22,23] employing multiple *fpfs*-matrices to *ifpifs*-matrices space. Section 3 determines successful ones in five test cases provided in [1]. Section 4 applies the determined SDM methods to a performance-based value assignment to seven noise removal filters. Finally, the paper inquires the need for further research.

2. SDM Methods in *ifpifs*-Matrices Space

This section generalizes the SDM methods in [14,15,16,17,18,19,20,21,22,23] utilizing multiple *fpfs*-matrices to *ifpifs*-matrices space. Across this paper, $I_n := \{1,\ 2,\ \cdots,\ n\}$ and $I_n^* := \{0,\ 1,\ 2,\ \cdots,\ n\}$ such that $n \in \mathbb{Z}^+$, the set of all the positive integers. Furthermore, the notation of each method herein is created by inserting the first letter of the word "intuitionistic" at the beginning of the methods' notations in [14,15,16,17,18,19,20,21,22,23]. For example, iCE10 denotes the generalized form of the SDM method CE10 [14].

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Algorithm 1 iCE10

1. Construct two ifpifs-matrices $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$

2. Find AND*-product/*OR*-product ifpifs-matrix* [8] $[c_{ip}]_{m\times n_1 n_2}$ *of* $[a_{ij}]$ *and* $[b_{ik}]$

3. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := {}^{\mu^s_{i1}}_{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$, $p = n_2(j-1)+k$, $I_a := \{j : \mu^a_{0j} \neq 0 \vee \nu^a_{0j} \neq 1\}$, $I_b := \{k : \mu^b_{0k} \neq 0 \vee \nu^b_{0k} \neq 1\}$,

$$\max_j\min_k\left(\mu^c_{ip}\right) := \begin{cases} \max\limits_{j\in I_a}\left\{\min\limits_{k\in I_b}\left\{\mu^c_{0p}\mu^c_{ip}\right\}\right\}, & I_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\max_k\min_j\left(\mu^c_{ip}\right) := \begin{cases} \max\limits_{k\in I_b}\left\{\min\limits_{j\in I_a}\left\{\mu^c_{0p}\mu^c_{ip}\right\}\right\}, & I_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\min_j\max_k\left(\nu^c_{ip}\right) := \begin{cases} \min\limits_{j\in I_a}\left\{\max\limits_{k\in I_b}\left\{\nu^c_{0p}\nu^c_{ip}\right\}\right\}, & I_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

$$\min_k\max_j\left(\nu^c_{ip}\right) := \begin{cases} \min\limits_{k\in I_b}\left\{\max\limits_{j\in I_a}\left\{\nu^c_{0p}\nu^c_{ip}\right\}\right\}, & I_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

$$\mu^s_{i1} := \max\left\{\max_j\min_k\left(\mu^c_{ip}\right), \max_k\min_j\left(\mu^c_{ip}\right)\right\}$$

and

$$\nu^s_{i1} := \min\left\{\min_j\max_k\left(\nu^c_{ip}\right), \min_k\max_j\left(\nu^c_{ip}\right)\right\}$$

4. Obtain the decision set $\left\{{}^{\mu^s_{k1}}_{\nu^s_{k1}}u_k \middle| u_k \in U\right\}$

Here, if AND-product is used in the second step of iCE10, then this method is denoted by iCE10a. Similarly, if OR-product is used in the second step, then it is denoted by iCE10o.

Algorithm 2 iEMO18ao

1. Construct two ifpifs-matrices $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$

2. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := {}^{\mu^s_{i1}}_{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$I_a := \{j : \mu^a_{0j} \neq 0 \vee \nu^a_{0j} \neq 1\}$$

$$I_b := \{k : \mu^b_{0k} \neq 0 \vee \nu^b_{0k} \neq 1\}$$

$$\max_j\min_k\left(\mu^a_{ij}, \mu^b_{ik}\right) := \begin{cases} \min\left\{\max\limits_{j\in I_a}\left\{\mu^a_{0j}\mu^a_{ij}\right\}, \min\limits_{k\in I_b}\left\{\mu^b_{0k}\mu^b_{ik}\right\}\right\}, & I_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\max_k\min_j\left(\mu^a_{ij}, \mu^b_{ik}\right) := \begin{cases} \min\left\{\max\limits_{k\in I_b}\left\{\mu^b_{0k}\mu^b_{ik}\right\}, \min\limits_{j\in I_a}\left\{\mu^a_{0j}\mu^a_{ij}\right\}\right\}, & I_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\max_j\min_k\left(\nu^a_{ij}, \nu^b_{ik}\right) := \begin{cases} \max\left\{\max\limits_{j\in I_a}\left\{\nu^a_{0j}\nu^a_{ij}\right\}, \min\limits_{k\in I_b}\left\{\nu^b_{0k}\nu^b_{ik}\right\}\right\}, & I_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\max_k\min_j\left(\nu^a_{ij},\nu^b_{ik}\right) := \begin{cases} \max\left\{\max\limits_{k\in I_b}\left\{\nu^b_{0k}\nu^b_{ik}\right\},\min\limits_{j\in I_a}\left\{\nu^a_{0j}\nu^a_{ij}\right\}\right\}, & I_a\neq\emptyset \text{ and } I_b\neq\emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\mu^s_{i1} := \min\left\{\max_j\min_k\left(\mu^a_{ij},\mu^b_{ik}\right),\max_k\min_j\left(\mu^a_{ij},\mu^b_{ik}\right)\right\}$$

and

$$\nu^s_{i1} := \max\left\{\max_j\min_k\left(\nu^a_{ij},\nu^b_{ik}\right),\max_k\min_j\left(\nu^a_{ij},\nu^b_{ik}\right)\right\}$$

3. *Obtain the decision set* $\left\{{}^{\mu^s_{k1}}_{\nu^s_{k1}}u_k \middle| u_k\in U\right\}$

Here, the notation "ao" in iEMO18ao represents AND-product and OR-product, respectively. In the second step of iEMO18ao, the membership and non-membership functions of the score matrix $[s_{i1}]$ are obtained using functions in the second steps of EMO18a [22] and EMO18o [21], respectively.

Algorithm 3 iEMO18oa

1. *Construct two ifpifs-matrices* $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$
2. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := {}^{\mu^s_{i1}}_{\nu^s_{i1}}$ *such that* $i\in I_{m-1}$,

$$I_a := \left\{j : \mu^a_{0j}\neq 0\vee\nu^a_{0j}\neq 1\right\}$$

$$I_b := \left\{k : \mu^b_{0k}\neq 0\vee\nu^b_{0k}\neq 1\right\}$$

$$\max_j\min_k\left(\mu^a_{ij},\mu^b_{ik}\right) := \begin{cases} \max\left\{\max\limits_{j\in I_a}\left\{\mu^a_{0j}\mu^a_{ij}\right\},\min\limits_{k\in I_b}\left\{\mu^b_{0k}\mu^b_{ik}\right\}\right\}, & I_a\neq\emptyset \text{ and } I_b\neq\emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\max_k\min_j\left(\mu^a_{ij},\mu^b_{ik}\right) := \begin{cases} \max\left\{\max\limits_{k\in I_b}\left\{\mu^b_{0k}\mu^b_{ik}\right\},\min\limits_{j\in I_a}\left\{\mu^a_{0j}\mu^a_{ij}\right\}\right\}, & I_a\neq\emptyset \text{ and } I_b\neq\emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\max_j\min_k\left(\nu^a_{ij},\nu^b_{ik}\right) := \begin{cases} \min\left\{\max\limits_{j\in I_a}\left\{\nu^a_{0j}\nu^a_{ij}\right\},\min\limits_{k\in I_b}\left\{\nu^b_{0k}\nu^b_{ik}\right\}\right\}, & I_a\neq\emptyset \text{ and } I_b\neq\emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\max_k\min_j\left(\nu^a_{ij},\nu^b_{ik}\right) := \begin{cases} \min\left\{\max\limits_{k\in I_b}\left\{\nu^b_{0k}\nu^b_{ik}\right\},\min\limits_{j\in I_a}\left\{\nu^a_{0j}\nu^a_{ij}\right\}\right\}, & I_a\neq\emptyset \text{ and } I_b\neq\emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\mu^s_{i1} := \max\left\{\max_j\min_k\left(\mu^a_{ij},\mu^b_{ik}\right),\max_k\min_j\left(\mu^a_{ij},\mu^b_{ik}\right)\right\}$$

and

$$\nu^s_{i1} := \min\left\{\max_j\min_k\left(\nu^a_{ij},\nu^b_{ik}\right),\max_k\min_j\left(\nu^a_{ij},\nu^b_{ik}\right)\right\}$$

3. *Obtain the decision set* $\left\{{}^{\mu^s_{k1}}_{\nu^s_{k1}}u_k \middle| u_k\in U\right\}$

Here, the notation "oa" in iEMO18oa indicates OR-product and AND-product, respectively. In the second step of iEMO18oa, the membership and non-membership functions of the score matrix $[s_{i1}]$ are obtained using functions in the second steps of EMO18o [21] and EMO18a [22], respectively.

Algorithm 4 iCE10n

1. *Construct two ifpifs-matrices* $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$
2. *Find* ANDNOT-*product*/ORNOT-*product ifpifs-matrix* [8] $[c_{ip}]_{m\times n_1n_2}$ *of* $[a_{ij}]$ *and* $[b_{ik}]$
3. *Find* ANDNOT-*product*/ORNOT-*product ifpifs-matrix* [8] $[d_{it}]_{m\times n_1n_2}$ *of* $[b_{ik}]$ *and* $[a_{ij}]$

4. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \frac{\mu^s_{i1}}{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$, $p = n_2(j-1)+k$, $t = n_1(k-1)+j$, $I_a := \{j : \mu^a_{0j} \neq 0 \vee \nu^a_{0j} \neq 1\}$, $I_b := \{k : \mu^b_{0k} \neq 0 \vee \nu^b_{0k} \neq 1\}$, $I^*_a := \{j : \nu^a_{0j} \neq 0 \vee \mu^a_{0j} \neq 1\}$, $I^*_b := \{k : \nu^b_{0k} \neq 0 \vee \mu^b_{0k} \neq 1\}$,

$$\text{max}_j\text{min}_k\left(\mu^c_{ip}\right) := \begin{cases} \max\limits_{j\in I_a}\left\{\min\limits_{k\in I^*_b}\left\{\mu^c_{0p}\mu^c_{ip}\right\}\right\}, & I_a \neq \emptyset \text{ and } I^*_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\text{max}_k\text{min}_j\left(\mu^d_{it}\right) := \begin{cases} \max\limits_{k\in I_b}\left\{\min\limits_{j\in I^*_a}\left\{\mu^d_{0t}\mu^d_{it}\right\}\right\}, & I^*_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\text{min}_j\text{max}_k\left(\nu^c_{ip}\right) := \begin{cases} \min\limits_{j\in I_a}\left\{\max\limits_{k\in I^*_b}\left\{\nu^c_{0p}\nu^c_{ip}\right\}\right\}, & I_a \neq \emptyset \text{ and } I^*_b \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

$$\text{min}_k\text{max}_j\left(\nu^d_{it}\right) := \begin{cases} \min\limits_{k\in I_b}\left\{\max\limits_{j\in I^*_a}\left\{\nu^d_{0t}\nu^d_{it}\right\}\right\}, & I^*_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 1, & \text{otherwise} \end{cases}$$

$$\mu^s_{i1} := \max\left\{\text{max}_j\text{min}_k\left(\mu^c_{ip}\right), \text{max}_k\text{min}_j\left(\mu^d_{it}\right)\right\}$$

and

$$\nu^s_{i1} := \min\left\{\text{min}_j\text{max}_k\left(\nu^c_{ip}\right), \text{min}_k\text{max}_j\left(\nu^d_{it}\right)\right\}$$

5. Obtain the decision set $\left\{ {}^{\mu^s_{k1}}_{\nu^s_{k1}} u_k \middle| u_k \in U\right\}$

Here, if ANDNOT-product is used in the second step of iCE10n, then this method is denoted by iCE10an. Similarly, if ORNOT-product is used in the second step, then it is denoted by iCE10on. Here, iCE10n is a generalization of the SDM method CE10n [17].

Algorithm 5 iEMA18anon

1. Construct two ifpifs-matrices $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$

2. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \frac{\mu^s_{i1}}{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$I_a := \{j : \mu^a_{0j} \neq 0 \vee \nu^a_{0j} \neq 1\}$$

$$I_b := \{k : \mu^b_{0k} \neq 0 \vee \nu^b_{0k} \neq 1\}$$

$$I^*_a := \{j : \nu^a_{0j} \neq 0 \vee \mu^a_{0j} \neq 1\}$$

$$I^*_b := \{k : \nu^b_{0k} \neq 0 \vee \mu^b_{0k} \neq 1\}$$

$$\text{max}_j\text{min}_k\left(\mu^a_{ij}, \mu^b_{ik}\right) := \begin{cases} \min\left\{\max\limits_{j\in I_a}\left\{\mu^a_{0j}\mu^a_{ij}\right\}, \min\limits_{k\in I^*_b}\left\{\nu^b_{0k}\nu^b_{ik}\right\}\right\}, & I_a \neq \emptyset \text{ and } I^*_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\text{max}_k\text{min}_j\left(\mu^a_{ij}, \mu^b_{ik}\right) := \begin{cases} \min\left\{\max\limits_{k\in I_b}\left\{\mu^b_{0k}\mu^b_{ik}\right\}, \min\limits_{j\in I^*_a}\left\{\nu^a_{0j}\nu^a_{ij}\right\}\right\}, & I^*_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\text{max}_j\text{min}_k\left(\nu^a_{ij}, \nu^b_{ik}\right) := \begin{cases} \max\left\{\max\limits_{j\in I_a}\left\{\mu^a_{0j}\mu^a_{ij}\right\}, \min\limits_{k\in I^*_b}\left\{\nu^b_{0k}\nu^b_{ik}\right\}\right\}, & I_a \neq \emptyset \text{ and } I^*_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

$$\max_k\min_j\left(\nu_{ij}^a,\nu_{ik}^b\right):=\begin{cases}\max\left\{\max\limits_{k\in I_b}\left\{\mu_{0k}^b\mu_{ik}^b\right\},\min\limits_{j\in I_a^*}\left\{\nu_{0j}^a\nu_{ij}^a\right\}\right\}, & I_a^*\neq\emptyset \text{ and } I_b\neq\emptyset\\ 0, & \text{otherwise}\end{cases}$$

$$\mu_{i1}^s:=\min\left\{\max_j\min_k\left(\mu_{ij}^a,\mu_{ik}^b\right),\max_k\min_j\left(\mu_{ij}^a,\mu_{ik}^b\right)\right\}$$

and

$$\nu_{i1}^s:=\max\left\{\max_j\min_k\left(\nu_{ij}^a,\nu_{ik}^b\right),\max_k\min_j\left(\nu_{ij}^a,\nu_{ik}^b\right)\right\}$$

3. *Obtain the decision set* $\left\{{}^{\mu_{k1}^s}_{\nu_{k1}^s}u_k \middle| u_k\in U\right\}$

Here, the notation "anon" in iEMA18anon signifies ANDNOT-product and ORNOT-product, respectively. In the second step of iEMA18anon, the membership and non-membership functions of the score matrix $[s_{i1}]$ are obtained using functions in the second steps of EMA18an [17] and EMA18on [18], respectively.

Algorithm 6 iEMA18onan

1. *Construct two ifpifs-matrices* $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$

2. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1}:=\begin{smallmatrix}\mu_{i1}^s\\ \nu_{i1}^s\end{smallmatrix}$ *such that* $i\in I_{m-1}$,

$$I_a:=\left\{j:\mu_{0j}^a\neq 0\vee\nu_{0j}^a\neq 1\right\}$$

$$I_b:=\left\{k:\mu_{0k}^b\neq 0\vee\nu_{0k}^b\neq 1\right\}$$

$$I_a^*:=\left\{j:\nu_{0j}^a\neq 0\vee\mu_{0j}^a\neq 1\right\}$$

$$I_b^*:=\left\{k:\nu_{0k}^b\neq 0\vee\mu_{0k}^b\neq 1\right\}$$

$$\max_j\min_k\left(\mu_{ij}^a,\mu_{ik}^b\right):=\begin{cases}\max\left\{\max\limits_{j\in I_a}\left\{\mu_{0j}^a\mu_{ij}^a\right\},\min\limits_{k\in I_b^*}\left\{\nu_{0k}^b\nu_{ik}^b\right\}\right\}, & I_a\neq\emptyset \text{ and } I_b^*\neq\emptyset\\ 0, & \text{otherwise}\end{cases}$$

$$\max_k\min_j\left(\mu_{ij}^a,\mu_{ik}^b\right):=\begin{cases}\max\left\{\max\limits_{k\in I_b}\left\{\mu_{0k}^b\mu_{ik}^b\right\},\min\limits_{j\in I_a^*}\left\{\nu_{0j}^a\nu_{ij}^a\right\}\right\}, & I_a^*\neq\emptyset \text{ and } I_b\neq\emptyset\\ 0, & \text{otherwise}\end{cases}$$

$$\max_j\min_k\left(\nu_{ij}^a,\nu_{ik}^b\right):=\begin{cases}\min\left\{\max\limits_{j\in I_a}\left\{\mu_{0j}^a\mu_{ij}^a\right\},\min\limits_{k\in I_b^*}\left\{\nu_{0k}^b\nu_{ik}^b\right\}\right\}, & I_a\neq\emptyset \text{ and } I_b^*\neq\emptyset\\ 0, & \text{otherwise}\end{cases}$$

$$\max_k\min_j\left(\nu_{ij}^a,\nu_{ik}^b\right):=\begin{cases}\min\left\{\max\limits_{k\in I_b}\left\{\mu_{0k}^b\mu_{ik}^b\right\},\min\limits_{j\in I_a^*}\left\{\nu_{0j}^a\nu_{ij}^a\right\}\right\}, & I_a^*\neq\emptyset \text{ and } I_b\neq\emptyset\\ 0, & \text{otherwise}\end{cases}$$

$$\mu_{i1}^s:=\min\left\{\max_j\min_k\left(\mu_{ij}^a,\mu_{ik}^b\right),\max_k\min_j\left(\mu_{ij}^a,\mu_{ik}^b\right)\right\}$$

and

$$\nu_{i1}^s:=\max\left\{\max_j\min_k\left(\nu_{ij}^a,\nu_{ik}^b\right),\max_k\min_j\left(\nu_{ij}^a,\nu_{ik}^b\right)\right\}$$

3. *Obtain the decision set* $\left\{{}^{\mu_{k1}^s}_{\nu_{k1}^s}u_k \middle| u_k\in U\right\}$

Here, the notation "onan" in iEMA18onan represents ORNOT-product and ANDNOT-product, respectively. In the second step of iEMA18onan, the membership and non-membership functions of the score matrix $[s_{i1}]$ are obtained using functions in the second steps of EMA18on [18] and EMA18an [17], respectively.

Algorithm 7 iCE10/2

1. *Construct two ifpifs-matrices* $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$
2. *Find* AND*-product/*OR*-product ifpifs-matrix* [8] $[c_{ip}]_{m\times n_1 n_2}$ *of* $[a_{ij}]$ *and* $[b_{ik}]$
3. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \frac{\mu^s_{i1}}{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$I_k := \left\{p : \exists i, \left(\mu^c_{0p}\mu^c_{ip} \neq 0 \vee \nu^c_{0p}\nu^c_{ip} \neq 1\right), (k-1)\, n_2 < p \leq kn_2\right\}$$

$$\mu^s_{i1} := \max_{k\in I_{n_2}} \begin{cases} \min\limits_{p\in I_k} \left\{\mu^c_{0p}\mu^c_{ip}\right\}, & I_k \neq \emptyset \\ 0, & I_k = \emptyset \end{cases} \quad \text{and} \quad \nu^s_{i1} := \min_{k\in I_{n_2}} \begin{cases} \max\limits_{p\in I_k} \left\{\nu^c_{0p}\nu^c_{ip}\right\}, & I_k \neq \emptyset \\ 1, & I_k = \emptyset \end{cases}$$

4. *Obtain the decision set* $\left\{ {}^{\mu^s_{k1}}_{\nu^s_{k1}} u_k \middle| u_k \in U \right\}$

Here, if AND-product is used in the second step of iCE10/2, then this method is denoted by iCE10/2a. Similarly, if OR-product is used in the second step, then it is denoted by iCE10/2o. Here, iCE10/2 is a generalization of the SDM method CE10-2 [14].

Algorithm 8 iCE10/2n

1. *Construct two ifpifs-matrices* $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$
2. *Find* ANDNOT*-product/*ORNOT*-product ifpifs-matrix* [8] $[c_{ip}]_{m\times n_1 n_2}$ *of* $[a_{ij}]$ *and* $[b_{ik}]$
3. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \frac{\mu^s_{i1}}{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$I_k := \left\{p : \exists i, \left(\mu^c_{0p}\mu^c_{ip} \neq 0 \vee \nu^c_{0p}\nu^c_{ip} \neq 1\right), (k-1)\, n_2 < p \leq kn_2\right\}$$

$$\mu^s_{i1} := \max_{k\in I_{n_2}} \begin{cases} \min\limits_{p\in I_k} \left\{\mu^c_{0p}\mu^c_{ip}\right\}, & I_k \neq \emptyset \\ 0, & I_k = \emptyset \end{cases} \quad \text{and} \quad \nu^s_{i1} := \min_{k\in I_{n_2}} \begin{cases} \max\limits_{p\in I_k} \left\{\nu^c_{0p}\nu^c_{ip}\right\}, & I_k \neq \emptyset \\ 1, & I_k = \emptyset \end{cases}$$

4. *Obtain the decision set* $\left\{ {}^{\mu^s_{k1}}_{\nu^s_{k1}} u_k \middle| u_k \in U \right\}$

Here, if ANDNOT-product is used in the second step of iCE10/2n, then this method is denoted by iCE10/2an. Similarly, if ORNOT-product is used in the second step, then it is denoted by iCE10/2on. Here, iCE10/2n is a generalization of the mentioned method CE10-2n in [14].

Algorithm 9 iEMC19ao

1. *Construct two ifpifs-matrices* $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$
2. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \frac{\mu^s_{i1}}{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$I_a := \left\{j : \exists i, \left(\mu^a_{0j}\mu^a_{ij} \neq 0 \vee \nu^a_{0j}\nu^a_{ij} \neq 1\right)\right\}$$

$$I_b := \left\{k : \exists i, \left(\mu^b_{0k}\mu^b_{ik} \neq 0 \vee \nu^b_{0k}\nu^b_{ik} \neq 1\right)\right\}$$

$$\mu^s_{i1} := \begin{cases} \min\left\{\max\limits_{j\in I_a}\left\{\mu^a_{0j}\mu^a_{ij}\right\}, \min\limits_{k\in I_b}\left\{\mu^b_{0k}\mu^b_{ik}\right\}\right\}, & I_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu^s_{i1} := \begin{cases} \max\left\{\max\limits_{j\in I_a}\left\{\nu^a_{0j}\nu^a_{ij}\right\}, \min\limits_{k\in I_b}\left\{\nu^b_{0k}\nu^b_{ik}\right\}\right\}, & I_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

3. Obtain the decision set $\left\{ {}^{\mu^s_{k1}}_{\nu^s_{k1}} u_k \middle| u_k \in U \right\}$

Here, the notation "ao" in iEMC19ao indicates AND-product and OR-product, respectively. In the second step of iEMC19ao, the membership and non-membership functions of the score matrix $[s_{i1}]$ are obtained using functions in the second steps of EMC19a [19] and EMC19o [19], respectively.

Algorithm 10 iEMC19oa

1. Construct two ifpifs-matrices $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$

2. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := {}^{\mu^s_{i1}}_{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$I_a := \left\{ j : \exists i, \left(\mu^a_{0j}\mu^a_{ij} \neq 0 \vee \nu^a_{0j}\nu^a_{ij} \neq 1\right)\right\}$$

$$I_b := \left\{ k : \exists i, \left(\mu^b_{0k}\mu^b_{ik} \neq 0 \vee \nu^b_{0k}\nu^b_{ik} \neq 1\right)\right\}$$

$$\mu^s_{i1} := \begin{cases} \max\left\{ \max\limits_{j\in I_a}\left\{\mu^a_{0j}\mu^a_{ij}\right\}, \min\limits_{k\in I_b}\left\{\mu^b_{0k}\nu^b_{ik}\right\}\right\}, & I_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu^s_{i1} := \begin{cases} \min\left\{ \max\limits_{j\in I_a}\left\{\nu^a_{0j}\nu^a_{ij}\right\}, \min\limits_{k\in I_b}\left\{\nu^b_{0k}\nu^b_{ik}\right\}\right\}, & I_a \neq \emptyset \text{ and } I_b \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

3. Obtain the decision set $\left\{ {}^{\mu^s_{k1}}_{\nu^s_{k1}} u_k \middle| u_k \in U \right\}$

Here, the notation "oa" in iEMC19oa signifies OR-product and AND-product, respectively. In the second step of iEMC19oa, the membership and non-membership functions of the score matrix $[s_{i1}]$ are obtained using functions in the second steps of EMC19o [19] and EMC19a [19], respectively.

Definition 2.1 *Let* $[a_{ij}]_{m\times n_1} \in IFPIFS_{E_1}[U]$, $[b_{ik}]_{m\times n_2} \in IFPIFS_{E_2}[U]$, *and, for* $p = n_2(j-1)+k$, $[c_{ip}]_{m\times n_1 n_2} \in IFPIFS_{E_1\times E_2}[U]$. *For all i and p, if* $\mu^c_{ip} := \frac{\mu^a_{ij}+\mu^b_{ik}}{2}$ *and* $\nu^c_{ip} := \frac{\nu^a_{ij}+\nu^b_{ik}}{2}$, *then* $[c_{ip}]$ *is called* MEAN*-product of* $[a_{ij}]$ *and* $[b_{ik}]$ *and is denoted by* $[a_{ij}] \times_m [b_{ik}]$.

Algorithm 11 iVR13

1. Construct two ifpifs-matrices $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$

2. Find AND*-product/*OR*-product ifpifs-matrix* [8] $[c_{ip}]_{m\times n_1 n_2}$ *of* $[a_{ij}]$ *and* $[b_{ik}]$

3. Find MEAN*-product ifpifs-matrix* $[d_{ip}]_{m\times n_1 n_2}$ *of* $[a_{ij}]$ *and* $[b_{ik}]$

4. Obtain $[e_{ip}]_{m\times n_1 n_2}$ *defined by* $e_{ip} := {}^{\mu^e_{ip}}_{\nu^e_{ip}}$ *such that* $i \in I^*_{m-1}$, $p \in I_{n_1 n_2}$,

$$\mu^e_{ip} := \min\left\{\mu^c_{ip}, \mu^d_{ip}\right\} \quad \textit{and} \quad \nu^e_{ip} := \max\left\{\nu^c_{ip}, \nu^d_{ip}\right\}$$

5. Obtain $[f_{ip}]_{m\times n_1 n_2}$ *defined by* $f_{ip} := {}^{\mu^f_{ip}}_{\nu^f_{ip}}$ *such that* $i \in I_{m-1}$, $p \in I_{n_1 n_2}$, $\mu^f_{0p} := \mu^e_{0p}$, $\nu^f_{0p} := \nu^e_{0p}$,

$$\mu^f_{ip} := \begin{cases} 1, & \mu^e_{ip} \geq \max\limits_{k\in I_{m-1}}\left\{\mu^e_{kp}\right\} \\ 0, & \mu^e_{ip} < \max\limits_{k\in I_{m-1}}\left\{\mu^e_{kp}\right\} \end{cases} \quad \textit{and} \quad \nu^f_{ip} := \begin{cases} 0, & \nu^e_{ip} \leq \min\limits_{k\in I_{m-1}}\left\{\nu^e_{kp}\right\} \\ 1, & \nu^e_{ip} > \min\limits_{k\in I_{m-1}}\left\{\nu^e_{kp}\right\} \end{cases}$$

6. Obtain $[g_{i1}]_{(m-1)\times 1}$ *and* $[h_{i1}]_{(m-1)\times 1}$ *defined by*

$$g_{i1} := \sum_{p=1}^{n_1 n_2} \mu_{0p}^{f}\mu_{ip}^{f} \quad \text{and} \quad h_{i1} := \sum_{p=1}^{n_1 n_2} \nu_{0p}^{f}\nu_{ip}^{f}$$

such that $i \in I_{m-1}$

7. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \begin{matrix}\mu_{i1}^{s}\\ \nu_{i1}^{s}\end{matrix}$ *such that* $i \in I_{m-1}$*,*

$$\mu_{i1}^{s} = \begin{cases} \dfrac{g_{i1} + \left|\min\limits_{k\in I_{m-1}}\{g_{k1}\}\right|}{\max\limits_{k\in I_{m-1}}\{g_{k1}+|h_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{g_{k1}\}\right|}, & \max\limits_{k\in I_{m-1}}\{g_{k1}+|h_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{g_{k1}\}\right| \neq 0 \\ 1, & \max\limits_{k\in I_{m-1}}\{g_{k1}+|h_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{g_{k1}\}\right| = 0 \end{cases}$$

and

$$\nu_{i1}^{s} = \begin{cases} 1 - \dfrac{g_{i1} + |h_{i1}| + \left|\min\limits_{k\in I_{m-1}}\{g_{k1}\}\right|}{\max\limits_{k\in I_{m-1}}\{g_{k1}+|h_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{g_{k1}\}\right|}, & \max\limits_{k\in I_{m-1}}\{g_{k1}+|h_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{g_{k1}\}\right| \neq 0 \\ 0, & \max\limits_{k\in I_{m-1}}\{g_{k1}+|h_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{g_{k1}\}\right| = 0 \end{cases}$$

8. Obtain the decision set $\left\{ {}^{\mu_{k1}^{s}}_{\nu_{k1}^{s}} u_k \,\middle|\, u_k \in U \right\}$

Here, if AND-product is used in the second step of iVR13, then this method is denoted by iVR13a. Similarly, if OR-product is used in the second step, then it is denoted by iVR13o. Here, iVR13 is a generalization of the SDM method VR13 [14].

Algorithm 12 iZZ16(λ)

1. Construct two ifpifs-matrices $[a_{ij}]_{m\times n}$ *and* $[b_{ij}]_{m\times n}$

2. Obtain $[c_{ij}]_{m\times n}$ *defined by* $c_{ij} := \begin{matrix}\mu_{ij}^{c}\\ \nu_{ij}^{c}\end{matrix}$ *such that* $i \in I_{m-1}$*,* $j \in I_n$*,* $\mu_{0j}^{c} := \frac{\mu_{0j}^{a}\mu_{0j}^{b}}{2-\left(\mu_{0j}^{a}+\mu_{0j}^{b}-\mu_{0j}^{a}\mu_{0j}^{b}\right)}$*,* $\mu_{ij}^{c} := \min\left\{\mu_{ij}^{a}, \mu_{ij}^{b}\right\}$ *,* $\nu_{0j}^{c} := \frac{\nu_{0j}^{a}+\nu_{0j}^{b}}{1+\nu_{0j}^{a}\nu_{0j}^{b}}$*, and* $\nu_{ij}^{c} := \max\left\{\nu_{ij}^{a}, \nu_{ij}^{b}\right\}$

3. For $\lambda \in [0,1]$*, obtain* $[d_{ij}]_{m\times n}$ *defined by* $d_{ij} := \begin{matrix}\mu_{ij}^{d}\\ \nu_{ij}^{d}\end{matrix}$ *such that* $i \in I_{m-1}$*,* $j \in I_n$*,* $\mu_{0j}^{d} := \mu_{0j}^{c}$*,* $\mu_{ij}^{d} := \begin{cases} \mu_{ij}^{c}, & \mu_{ij}^{c} \geq \lambda \\ 0, & \mu_{ij}^{c} < \lambda \end{cases}$*,* $\nu_{0j}^{d} := \nu_{0j}^{c}$*, and* $\nu_{ij}^{d} := \begin{cases} \nu_{ij}^{c}, & \nu_{ij}^{c} \leq 1-\lambda \\ 1, & \nu_{ij}^{c} > 1-\lambda \end{cases}$

4. Apply iCCE10 [1] *to* $[d_{ij}]$

Here, the variable λ is a fuzzy value, and iZZ16(λ) is a generalization of the SDM method ZZ16 [14].

Algorithm 13 iZZ16 $\left(\begin{matrix}\boldsymbol{\mu}\\ \boldsymbol{\nu}\end{matrix}\right)$

1. Construct two ifpifs-matrices $[a_{ij}]_{m\times n}$ *and* $[b_{ij}]_{m\times n}$

2. Obtain $[c_{ij}]_{m\times n}$ *defined by* $c_{ij} := \begin{matrix}\mu_{ij}^{c}\\ \nu_{ij}^{c}\end{matrix}$ *such that* $i \in I_{m-1}$*,* $j \in I_n$*,* $\mu_{0j}^{c} := \frac{\mu_{0j}^{a}\mu_{0j}^{b}}{2-\left(\mu_{0j}^{a}+\mu_{0j}^{b}-\mu_{0j}^{a}\mu_{0j}^{b}\right)}$*,* $\mu_{ij}^{c} := \min\left\{\mu_{ij}^{a}, \mu_{ij}^{b}\right\}$ *,* $\nu_{0j}^{c} := \frac{\nu_{0j}^{a}+\nu_{0j}^{b}}{1+\nu_{0j}^{a}\nu_{0j}^{b}}$*, and* $\nu_{ij}^{c} := \max\left\{\nu_{ij}^{a}, \nu_{ij}^{b}\right\}$

3. For $\mu, \nu \in [0,1]$ *such that* $0 \leq \mu + \nu \leq 1$*, obtain* $[d_{ij}]_{m\times n}$ *defined by* $d_{ij} := \begin{smallmatrix}\mu^d_{ij}\\ \nu^d_{ij}\end{smallmatrix}$ *such that* $i \in I_{m-1}$, $j \in I_n$,

$$\begin{smallmatrix}\mu^d_{0j}\\ \nu^d_{0j}\end{smallmatrix} := \begin{smallmatrix}\mu^c_{0j}\\ \nu^c_{0j}\end{smallmatrix} \quad \text{and} \quad \begin{smallmatrix}\mu^d_{ij}\\ \nu^d_{ij}\end{smallmatrix} := \begin{cases} \begin{smallmatrix}\mu^c_{ij}\\ \nu^c_{ij}\end{smallmatrix}, & \mu^c_{ij} \geq \mu \text{ and } \nu^c_{ij} \leq \nu \\ \begin{smallmatrix}0\\ 1\end{smallmatrix}, & \text{otherwise} \end{cases}$$

4. Apply iCCE10 [1] *to* $[d_{ij}]$

Here, the variable $\begin{smallmatrix}\mu\\ \nu\end{smallmatrix}$ is an intuitionistic fuzzy value, and iZZ16$\left(\begin{smallmatrix}\mu\\ \nu\end{smallmatrix}\right)$ is a generalization of the SDM method ZZ16 [14].

Algorithm 14 iZZ16/2(λ)

1. Construct two ifpifs-matrices $[a_{ij}]_{m\times n}$ *and* $[b_{ij}]_{m\times n}$

2. Obtain $[c_{ij}]_{m\times n}$ *defined by* $c_{ij} := \begin{smallmatrix}\mu^c_{ij}\\ \nu^c_{ij}\end{smallmatrix}$ *such that* $i \in I_{m-1}$, $j \in I_n$, $\mu^c_{0j} := \frac{\mu^a_{0j}+\mu^b_{0j}}{1+\mu^a_{0j}\mu^b_{0j}}$, $\mu^c_{ij} := \max\left\{\mu^a_{ij}, \mu^b_{ij}\right\}$, $\nu^c_{0j} := \frac{\nu^a_{0j}\nu^b_{0j}}{2-\left(\nu^a_{0j}+\nu^b_{0j}-\nu^a_{0j}\nu^b_{0j}\right)}$, *and* $\nu^c_{ij} := \min\left\{\nu^a_{ij}, \nu^b_{ij}\right\}$

3. For $\lambda \in [0,1]$*, obtain* $[d_{ij}]_{m\times n}$ *defined by* $d_{ij} := \begin{smallmatrix}\mu^d_{ij}\\ \nu^d_{ij}\end{smallmatrix}$ *such that* $i \in I_{m-1}$, $j \in I_n$, $\mu^d_{0j} := \mu^c_{0j}$, $\mu^d_{ij} := \begin{cases} \mu^c_{ij}, & \mu^c_{ij} \geq \lambda \\ 0, & \mu^c_{ij} < \lambda \end{cases}$, $\nu^d_{0j} := \nu^c_{0j}$, *and* $\nu^d_{ij} := \begin{cases} \nu^c_{ij}, & \nu^c_{ij} \leq 1-\lambda \\ 1, & \nu^c_{ij} > 1-\lambda \end{cases}$

4. Apply iCCE10 [1] *to* $[d_{ij}]$

Here, the variable λ is a fuzzy value, and iZZ16/2(λ) is a generalization of the SDM method ZZ16/2 [14].

Algorithm 15 iZZ16/2$\left(\begin{smallmatrix}\mu\\ \nu\end{smallmatrix}\right)$

1. Construct two ifpifs-matrices $[a_{ij}]_{m\times n}$ *and* $[b_{ij}]_{m\times n}$

2. Obtain $[c_{ij}]_{m\times n}$ *defined by* $c_{ij} := \begin{smallmatrix}\mu^c_{ij}\\ \nu^c_{ij}\end{smallmatrix}$ *such that* $i \in I_{m-1}$, $j \in I_n$, $\mu^c_{0j} := \frac{\mu^a_{0j}+\mu^b_{0j}}{1+\mu^a_{0j}\mu^b_{0j}}$, $\mu^c_{ij} := \max\left\{\mu^a_{ij}, \mu^b_{ij}\right\}$, $\nu^c_{0j} := \frac{\nu^a_{0j}\nu^b_{0j}}{2-\left(\nu^a_{0j}+\nu^b_{0j}-\nu^a_{0j}\nu^b_{0j}\right)}$, *and* $\nu^c_{ij} := \min\left\{\nu^a_{ij}, \nu^b_{ij}\right\}$

3. For $\mu, \nu \in [0,1]$ *such that* $0 \leq \mu + \nu \leq 1$*, obtain* $[d_{ij}]_{m\times n}$ *defined by* $d_{ij} := \begin{smallmatrix}\mu^d_{ij}\\ \nu^d_{ij}\end{smallmatrix}$ *such that* $i \in I_{m-1}$, $j \in I_n$,

$$\begin{smallmatrix}\mu^d_{0j}\\ \nu^d_{0j}\end{smallmatrix} := \begin{smallmatrix}\mu^c_{0j}\\ \nu^c_{0j}\end{smallmatrix} \quad \text{and} \quad \begin{smallmatrix}\mu^d_{ij}\\ \nu^d_{ij}\end{smallmatrix} := \begin{cases} \begin{smallmatrix}\mu^c_{ij}\\ \nu^c_{ij}\end{smallmatrix}, & \mu^c_{ij} \geq \mu \text{ and } \nu^c_{ij} \leq \nu \\ \begin{smallmatrix}0\\ 1\end{smallmatrix}, & \text{otherwise} \end{cases}$$

4. Apply iCCE10 [1] *to* $[d_{ij}]$

Here, the variable $\begin{smallmatrix}\mu\\ \nu\end{smallmatrix}$ is an intuitionistic fuzzy value, and iZZ16/2$\left(\begin{smallmatrix}\mu\\ \nu\end{smallmatrix}\right)$ is a generalization of the SDM method ZZ16/2 [14].

Algorithm 16 iICJ17

1. *Construct two ifpifs-matrices* $[a_{ij}]_{m\times n_1}$ *and* $[b_{ik}]_{m\times n_2}$
2. *Find* AND/OR/ANDNOT/ORNOT-*product ifpifs-matrix* [8] $[c_{ip}]_{m\times n_1 n_2}$ *of* $[a_{ij}]$ *and* $[b_{ik}]$
3. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := {}^{\mu^s_{i1}}_{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$I_k := \left\{p : \exists i, \left(\mu^c_{0p}\mu^c_{ip} \neq 0 \vee \nu^c_{0p}\nu^c_{ip} \neq 1\right), (k-1)\, n_2 < p \leq kn_2\right\}$$

$$\mu^s_{i1} := \min_{k\in I_{n_2},} \begin{cases} \max\limits_{p\in I_k} \left\{\mu^c_{0p}\mu^c_{ip}\right\}, & I_k \neq \emptyset \\ 0, & I_k = \emptyset \end{cases} \quad \text{and} \quad \nu^s_{i1} := \max_{k\in I_{n_2},} \begin{cases} \min\limits_{p\in I_k} \left\{\nu^c_{0p}\nu^c_{ip}\right\}, & I_k \neq \emptyset \\ 1, & I_k = \emptyset \end{cases}$$

4. *Obtain the decision set* $\left\{{}^{\mu^s_{k1}}_{\nu^s_{k1}} u_k \middle| u_k \in U\right\}$

Here, if AND-product is used in the second step of iICJ17, then this method is denoted by iICJ17a. Similarly, if the other products are used in the second step, then they are denoted by ICJ17o, ICJ17an, and ICJ17on, respectively. Here, iICJ17 is a generalization of the SDM method ICJ17 [23].

Algorithm 17 iRM11a

1. *Construct three ifpifs-matrices* $[a_{ij}]_{m\times n_1}$, $[b_{ik}]_{m\times n_2}$, *and* $[c_{il}]_{m\times n_3}$
2. *Obtain* $[d_{ij}]_{(m-1)\times n_1}$, $[e_{ik}]_{(m-1)\times n_2}$, *and* $[f_{il}]_{(m-1)\times n_3}$ *defined by* $d_{ij} := {}^{\mu^d_{ij}}_{\nu^d_{ij}}$, $e_{ij} := {}^{\mu^e_{ij}}_{\nu^e_{ij}}$, *and* $f_{ij} := {}^{\mu^f_{ij}}_{\nu^f_{ij}}$ *such that* $i \in I_{m-1}$, $j \in I_{n_1}$, $k \in I_{n_2}$, $l \in I_{n_3}$, $\mu^d_{ij} := \mu^a_{0j}\mu^a_{ij}$, $\nu^d_{ij} := \nu^a_{0j} + \nu^a_{ij} - \nu^a_{0j}\nu^a_{ij}$, $\mu^e_{ik} := \mu^b_{0k}\mu^b_{ik}$, $\nu^e_{ik} := \nu^b_{0k} + \nu^b_{ik} - \nu^b_{0k}\nu^b_{ik}$, $\mu^f_{il} := \mu^c_{0l}\mu^c_{il}$, *and* $\nu^f_{il} := \nu^c_{0l} + \nu^c_{il} - \nu^c_{0l}\nu^c_{il}$
3. *Obtain* $[g_{ip}]_{(m-1)\times n_1 n_2}$ *defined by* $g_{ij} := {}^{\mu^g_{ij}}_{\nu^g_{ij}}$ *such that* $i \in I_{m-1}$, $j \in I_{n_1}$, $k \in I_{n_2}$, $p = n_2\,(j-1) + k$,

$$\mu^g_{ip} := \min\left\{\mu^d_{ij}, \mu^e_{ik}\right\} \quad \text{and} \quad \nu^g_{ip} := \max\left\{\nu^d_{ij}, \nu^e_{ik}\right\}$$

4. *Obtain* $[x_{ik}]_{(m-1)\times n_1}$ *defined by* $x_{ij} := {}^{\mu^x_{ij}}_{\nu^x_{ij}}$ *such that* $i \in I_{m-1}$, $k \in I_{n_1}$,

$$J_k := \left\{p : \exists i, \left(\mu^g_{ip} \neq 0 \vee \nu^g_{ip} \neq 1\right), (k-1)\, n_2 < p \leq kn_2\right\}$$

$$\mu^x_{ik} := \begin{cases} \min\limits_{p\in J_k} \left\{\mu^g_{ip}\right\}, & J_k \neq \emptyset \\ 0, & J_k = \emptyset \end{cases} \quad \text{and} \quad \nu^x_{ik} := \begin{cases} \max\limits_{p\in J_k} \left\{\nu^g_{ip}\right\}, & J_k \neq \emptyset \\ 1, & J_k = \emptyset \end{cases}$$

5. *Obtain* $[h_{it}]_{(m-1)\times n_1 n_3}$ *defined by* $h_{ij} := {}^{\mu^h_{ij}}_{\nu^h_{ij}}$ *such that* $i \in I_{m-1}$, $k \in I_{n_1}$, $l \in I_{n_3}$, $t = n_3\,(k-1) + l$,

$$\mu^h_{it} := \min\left\{\mu^x_{ik}, \mu^f_{il}\right\} \quad \text{and} \quad \nu^h_{it} := \max\left\{\nu^x_{ik}, \nu^f_{il}\right\}$$

6. *Obtain* $[y_{ik}]_{(m-1)\times n_1}$ *defined by* $y_{ij} := {}^{\mu^y_{ij}}_{\nu^y_{ij}}$ *such that* $i \in I_{m-1}$, $k \in I_{n_1}$,

$$J_k := \left\{t : \exists i, \left(\mu^h_{it} \neq 0 \vee \nu^h_{it} \neq 1\right), (k-1)\, n_3 < t \leq kn_3\right\}$$

$$\mu^y_{ik} := \begin{cases} \min\limits_{p\in J_k} \left\{\mu^h_{it}\right\}, & J_k \neq \emptyset \\ 0, & J_k = \emptyset \end{cases} \quad \text{and} \quad \nu^y_{ik} := \begin{cases} \max\limits_{p\in J_k} \left\{\nu^h_{it}\right\}, & J_k \neq \emptyset \\ 1, & J_k = \emptyset \end{cases}$$

7. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \frac{\mu^s_{i1}}{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$\mu^s_{i1} := \max_{k\in I_{n_1}} \{\mu^y_{ik}\} \quad \textit{and} \quad \nu^s_{i1} := \min_{k\in I_{n_1}} \{\nu^y_{ik}\}$$

8. *Obtain the decision set* $\left\{ {}^{\mu^s_{k1}}_{\nu^s_{k1}} u_k \middle| u_k \in U \right\}$

Here, iRM11a is a generalization of the SDM method RM11 [14] using AND-product, i.e., RM11a.

Algorithm 18 iRM11o

1. *Construct three ifpifs-matrices* $[a_{ij}]_{m\times n_1}$, $[b_{ik}]_{m\times n_2}$, *and* $[c_{il}]_{m\times n_3}$

2. *Obtain* $[d_{ij}]_{(m-1)\times n_1}$, $[e_{ik}]_{(m-1)\times n_2}$, *and* $[f_{il}]_{(m-1)\times n_3}$ *defined by* $d_{ij} := \frac{\mu^d_{ij}}{\nu^d_{ij}}$, $e_{ij} := \frac{\mu^e_{ij}}{\nu^e_{ij}}$, *and* $f_{ij} := \frac{\mu^f_{ij}}{\nu^f_{ij}}$ *such that* $i \in I_{m-1}$, $j \in I_{n_1}$, $k \in I_{n_2}$, $l \in I_{n_3}$, $\mu^d_{ij} := \mu^a_{0j}\mu^a_{ij}$, $\nu^d_{ij} := \nu^a_{0j} + \nu^a_{ij} - \nu^a_{0j}\nu^a_{ij}$, $\mu^e_{ik} := \mu^b_{0k}\mu^b_{ik}$, $\nu^e_{ik} := \nu^b_{0k} + \nu^b_{ik} - \nu^b_{0k}\nu^b_{ik}$, $\mu^f_{il} := \mu^c_{0l}\mu^c_{il}$, *and* $\nu^f_{il} := \nu^c_{0l} + \nu^c_{il} - \nu^c_{0l}\nu^c_{il}$

3. *Obtain* $[g_{ip}]_{(m-1)\times n_1 n_2}$ *defined by* $g_{ij} := \frac{\mu^g_{ij}}{\nu^g_{ij}}$ *such that* $i \in I_{m-1}$, $j \in I_{n_1}$, $k \in I_{n_2}$, $p = n_2(j-1)+k$,

$$\mu^g_{ip} := \max\{\mu^d_{ij}, \mu^e_{ik}\} \quad \textit{and} \quad \nu^g_{ip} := \min\{\nu^d_{ij}, \nu^e_{ik}\}$$

4. *Obtain* $[x_{ik}]_{(m-1)\times n_1}$ *defined by* $x_{ij} := \frac{\mu^x_{ij}}{\nu^x_{ij}}$ *such that* $i \in I_{m-1}$, $k \in I_{n_1}$,

$$J_k := \{p : \exists i, \left(\mu^g_{ip} \neq 0 \vee \nu^g_{ip} \neq 1\right), (k-1)\, n_2 < p \le kn_2\}$$

$$\mu^x_{ik} := \begin{cases} \min\limits_{p\in J_k} \{\mu^g_{ip}\}, & J_k \neq \emptyset \\ 0, & J_k = \emptyset \end{cases} \quad \textit{and} \quad \nu^x_{ik} := \begin{cases} \max\limits_{p\in J_k} \{\nu^g_{ip}\}, & J_k \neq \emptyset \\ 1, & J_k = \emptyset \end{cases}$$

5. *Obtain* $[h_{it}]_{(m-1)\times n_1 n_3}$ *defined by* $h_{ij} := \frac{\mu^h_{ij}}{\nu^h_{ij}}$ *such that* $i \in I_{m-1}$, $k \in I_{n_1}$, $l \in I_{n_3}$, $t = n_3(k-1)+l$,

$$\mu^h_{it} := \max\left\{\mu^x_{ik}, \mu^f_{il}\right\} \quad \textit{and} \quad \nu^h_{it} := \min\left\{\nu^x_{ik}, \nu^f_{il}\right\}$$

6. *Obtain* $[y_{ik}]_{(m-1)\times n_1}$ *defined by* $y_{ij} := \frac{\mu^y_{ij}}{\nu^y_{ij}}$ *such that* $i \in I_{m-1}$, $k \in I_{n_1}$,

$$J_k := \{t : \exists i, \left(\mu^h_{it} \neq 0 \vee \nu^h_{it} \neq 1\right), (k-1)\, n_3 < t \le kn_3\}$$

$$\mu^y_{ik} := \begin{cases} \min\limits_{p\in J_k} \{\mu^h_{it}\}, & J_k \neq \emptyset \\ 0, & J_k = \emptyset \end{cases} \quad \textit{and} \quad \nu^y_{ik} := \begin{cases} \max\limits_{p\in J_k} \{\nu^h_{it}\}, & J_k \neq \emptyset \\ 1, & J_k = \emptyset \end{cases}$$

7. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \frac{\mu^s_{i1}}{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$\mu^s_{i1} := \max_{k\in I_{n_1}} \{\mu^y_{ik}\} \quad \textit{and} \quad \nu^s_{i1} := \min_{k\in I_{n_1}} \{\nu^y_{ik}\}$$

8. *Obtain the decision set* $\left\{ {}^{\mu^s_{k1}}_{\nu^s_{k1}} u_k \middle| u_k \in U \right\}$

Here, iRM11o is a generalization of the SDM method RM11 [14] using OR-product, i.e., RM11o.

Algorithm 19 iEM20ao

1. *Construct three ifpifs-matrices* $[a_{ij}]_{m\times n_1}$, $[b_{ik}]_{m\times n_2}$, *and* $[c_{il}]_{m\times n_3}$

2. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \frac{\mu^s_{i1}}{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$I_a := \left\{j : \exists i, \left(\mu^a_{0j}\mu^a_{ij} \neq 0 \vee \nu^a_{0j}\nu^a_{ij} \neq 1\right)\right\}$$

$$I_b := \left\{k : \exists i, \left(\mu^b_{0k}\mu^b_{ik} \neq 0 \vee \nu^b_{0k}\nu^b_{ik} \neq 1\right)\right\}$$

$$I_c := \{l : \exists i, (\mu^c_{0l}\mu^c_{il} \neq 0 \vee \nu^c_{0l}\nu^c_{il} \neq 1)\}$$

$$\mu^s_{i1} := \begin{cases} \min\left\{\min\left\{\max\limits_{j\in I_a}\left\{\mu^a_{0j}\mu^a_{ij}\right\}, \min\limits_{k\in I_b}\left\{\mu^b_{0k}\mu^b_{ik}\right\}\right\}, \min\limits_{l\in I_c}\left\{\mu^c_{0l}\mu^c_{il}\right\}\right\}, & I_a \neq \emptyset,\ I_b \neq \emptyset,\ \text{and } I_c \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu^s_{i1} := \begin{cases} \max\left\{\max\left\{\max\limits_{j\in I_a}\left\{\nu^a_{0j}\nu^a_{ij}\right\}, \min\limits_{k\in I_b}\left\{\nu^b_{0k}\nu^b_{ik}\right\}\right\}, \min\limits_{l\in I_c}\left\{\nu^c_{0l}\nu^c_{il}\right\}\right\}, & I_a \neq \emptyset,\ I_b \neq \emptyset,\ \text{and } I_c \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

3. *Obtain the decision set* $\left\{{}^{\mu^s_{k1}}_{\nu^s_{k1}} u_k \middle| u_k \in U\right\}$

Here, the notation "ao" in iEM20ao represents AND-product and OR-product, respectively. In the second step of iEM20ao, the membership and non-membership functions of the score matrix $[s_{i1}]$ are obtained using functions in the second steps of EM20a [16] and EM20o [16], respectively.

Algorithm 20 iEM20oa

1. *Construct three ifpifs-matrices* $[a_{ij}]_{m\times n_1}$, $[b_{ik}]_{m\times n_2}$, *and* $[c_{il}]_{m\times n_3}$

2. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \frac{\mu^s_{i1}}{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$I_a := \left\{j : \exists i, \left(\mu^a_{0j}\mu^a_{ij} \neq 0 \vee \nu^a_{0j}\nu^a_{ij} \neq 1\right)\right\}$$

$$I_b := \left\{k : \exists i, \left(\mu^b_{0k}\mu^b_{ik} \neq 0 \vee \nu^b_{0k}\nu^b_{ik} \neq 1\right)\right\}$$

$$I_c := \{l : \exists i, (\mu^c_{0l}\mu^c_{il} \neq 0 \vee \nu^c_{0l}\nu^c_{il} \neq 1)\}$$

$$\mu^s_{i1} := \begin{cases} \max\left\{\max\left\{\max\limits_{j\in I_a}\left\{\mu^a_{0j}\mu^a_{ij}\right\}, \min\limits_{k\in I_b}\left\{\mu^b_{0k}\mu^b_{ik}\right\}\right\}, \min\limits_{l\in I_c}\left\{\mu^c_{0l}\mu^c_{il}\right\}\right\}, & I_a \neq \emptyset,\ I_b \neq \emptyset,\ \text{and } I_c \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

and

$$\nu^s_{i1} := \begin{cases} \min\left\{\min\left\{\max\limits_{j\in I_a}\left\{\nu^a_{0j}\nu^a_{ij}\right\}, \min\limits_{k\in I_b}\left\{\nu^b_{0k}\nu^b_{ik}\right\}\right\}, \min\limits_{l\in I_c}\left\{\nu^c_{0l}\nu^c_{il}\right\}\right\}, & I_a \neq \emptyset,\ I_b \neq \emptyset,\ \text{and } I_c \neq \emptyset \\ 0, & \text{otherwise} \end{cases}$$

3. *Obtain the decision set* $\left\{{}^{\mu^s_{k1}}_{\nu^s_{k1}} u_k \middle| u_k \in U\right\}$

Here, the notation "oa" in iEM20oa indicates OR-product and AND-product, respectively. In the second step of iEM20oa, the membership and non-membership functions of the score matrix $[s_{i1}]$ are obtained using functions in the second steps of EM20o [16] and EM20a [16], respectively.

Algorithm 21 iZ14(R)

1. *Construct ifpifs-matrices* $[a^1_{ij}]_{m\times n}$, $[a^2_{ij}]_{m\times n}$, $\cdots$, $[a^t_{ij}]_{m\times n}$

2. *Obtain* $[b_{ij}]_{m\times n}$ *defined by* $b_{ij} := \begin{smallmatrix}\mu^b_{ij}\\ \nu^b_{ij}\end{smallmatrix}$ *such that* $i \in I^*_{m-1}$, $j \in I_n$,

$$\mu^b_{ij} := \max_{k\in I_t}\left\{\mu^{a^k}_{ij}\right\} \quad \text{and} \quad \nu^b_{ij} := \min_{k\in I_t}\left\{\nu^{a^k}_{ij}\right\}$$

3. *Apply* iMRB02(R) [1] *to* $[b_{ij}]$ *such that* $R \subseteq I_n$

Here, the variable R signifies a set of indices concerning parameters, and iZ14(R) is a generalization of the SDM methods Z14/2 [10] and Z14 [23].

Algorithm 22 iDB12

1. *Construct ifpifs-matrices* $[a^1_{ij}]_{m\times n}$, $[a^2_{ij}]_{m\times n}$, $\cdots$, $[a^t_{ij}]_{m\times n}$

2. *Obtain* $[b_{ij}]_{m\times n}$ *defined by* $b_{ij} := \begin{smallmatrix}\mu^b_{ij}\\ \nu^b_{ij}\end{smallmatrix}$ *such that* $i \in I^*_{m-1}$, $j \in I_n$,

$$\mu^b_{ij} := \frac{1}{t}\sum_{k=1}^{t}\mu^{a^k}_{ij} \quad \text{and} \quad \nu^b_{ij} := \frac{1}{t}\sum_{k=1}^{t}\nu^{a^k}_{ij}$$

3. *Apply* iMBR01 [1] *to* $[b_{ij}]$

Here, iDB12 is a generalization of the SDM method DB12 [14].

Algorithm 23 isDB12

1. *Construct ifpifs-matrices* $[a^1_{ij}]_{m\times n}$, $[a^2_{ij}]_{m\times n}$, $\cdots$, $[a^t_{ij}]_{m\times n}$

2. *Obtain* $[b_{ij}]_{m\times n}$ *defined by* $b_{ij} := \begin{smallmatrix}\mu^b_{ij}\\ \nu^b_{ij}\end{smallmatrix}$ *such that* $i \in I^*_{m-1}$, $j \in I_n$,

$$\mu^b_{ij} := \frac{1}{t}\sum_{k=1}^{t}\mu^{a^k}_{ij} \quad \text{and} \quad \nu^b_{ij} := \frac{1}{t}\sum_{k=1}^{t}\nu^{a^k}_{ij}$$

3. *Apply* isMBR01 [1] *to* $[b_{ij}]$

Here, isDB12 is a generalization of the SDM method sDB12 [15].

Algorithm 24 iCD12(q)

1. *Construct ifpifs-matrices* $[a^1_{ij}]_{m\times n}$, $[a^2_{ij}]_{m\times n}$, $\cdots$, $[a^t_{ij}]_{m\times n}$

2. *For* $q \in \mathbb{Z}^+$, *obtain* $[b_{ij}]_{m\times n}$ *defined by* $b_{ij} := \begin{smallmatrix}\mu^b_{ij}\\ \nu^b_{ij}\end{smallmatrix}$ *such that* $i \in I_{m-1}$, $j \in I_n$, $\mu^b_{0j} := \left(\frac{1}{t}\sum_{k=1}^{t}\left(\mu^{a^k}_{0j}\right)^q\right)^{\frac{1}{q}}$, $\mu^b_{ij} := \min_{k\in I_t}\left\{\mu^{a^k}_{ij}\right\}$, $\nu^b_{0j} := \left(\frac{1}{t}\sum_{k=1}^{t}\left(\nu^{a^k}_{0j}\right)^q\right)^{\frac{1}{q}}$, *and* $\nu^b_{ij} := \max_{k\in I_t}\left\{\nu^{a^k}_{ij}\right\}$

3. *Apply* iCCE10 [1] *to* $[b_{ij}]$

Here, the variable q is a positive integer, and iCD12(q) is a generalization of the SDM method CD12 [14].

Algorithm 25 iCD12/2(q)

1. *Construct ifpifs-matrices* $[a^1_{ij}]_{m\times n}, [a^2_{ij}]_{m\times n}, \cdots, [a^t_{ij}]_{m\times n}$

2. *For* $q \in \mathbb{Z}^+$, *obtain* $[b_{ij}]_{m\times n}$ *defined by* $b_{ij} := {}^{\mu^b_{ij}}_{\nu^b_{ij}}$ *such that* $i \in I_{m-1}$, $j \in I_n$, $\mu^b_{0j} := \left(\frac{1}{t}\sum_{k=1}^{t}\left(\mu^{a^k}_{0j}\right)^q\right)^{\frac{1}{q}}$, $\mu^b_{ij} := \max_{k\in I_t}\left\{\mu^{a^k}_{ij}\right\}$, $\nu^b_{0j} := \left(\frac{1}{t}\sum_{k=1}^{t}\left(\nu^{a^k}_{0j}\right)^q\right)^{\frac{1}{q}}$, *and* $\nu^b_{ij} := \min_{k\in I_t}\left\{\nu^{a^k}_{ij}\right\}$

3. *Apply* iCCE10 [1] *to* $[b_{ij}]$

Here, the variable q is a positive integer, and iCD12/2(q) is a generalization of the SDM method CD12/2 [14].

Algorithm 26 iE15

1. *Construct ifpifs-matrices* $[a^1_{ij}]_{m\times n}, [a^2_{ij}]_{m\times n}, \cdots, [a^t_{ij}]_{m\times n}$

2. *Obtain* $[b_{ij}]_{(m-1)\times n}$ *defined by* $b_{ij} := {}^{\mu^b_{ij}}_{\nu^b_{ij}}$ *such that* $i \in I_{m-1}$, $j \in I_n$, $k \in I_t$,

$$\mu^b_{ij} := \sum_{k=1}^{t}\mu^{a^k}_{0j}\mu^{a^k}_{ij} \quad \text{and} \quad \nu^b_{ij} := \sum_{k=1}^{t}\nu^{a^k}_{0j}\nu^{a^k}_{ij}$$

3. *Obtain* $[c_{i1}]_{(m-1)\times 1}$ *and* $[d_{i1}]_{(m-1)\times 1}$ *defined by*

$$c_{i1} := \sum_{j=1}^{n}\mu^b_{ij} \quad \text{and} \quad d_{i1} := \sum_{j=1}^{n}\nu^b_{ij}$$

such that $i \in I_{m-1}$ *and* $j \in I_n$

4. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := {}^{\mu^s_{i1}}_{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$\mu^s_{i1} = \begin{cases} \dfrac{c_{i1}+\left|\min\limits_{k\in I_{m-1}}\{c_{k1}\}\right|}{\max\limits_{k\in I_{m-1}}\{c_{k1}+|d_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{c_{k1}\}\right|}, & \max\limits_{k\in I_{m-1}}\{c_{k1}+|d_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{c_{k1}\}\right| \neq 0 \\ 1, & \max\limits_{k\in I_{m-1}}\{c_{k1}+|d_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{c_{k1}\}\right| = 0 \end{cases}$$

and

$$\nu^s_{i1} = \begin{cases} 1-\dfrac{c_{i1}+|d_{i1}|+\left|\min\limits_{k\in I_{m-1}}\{c_{k1}\}\right|}{\max\limits_{k\in I_{m-1}}\{c_{k1}+|d_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{c_{k1}\}\right|}, & \max\limits_{k\in I_{m-1}}\{c_{k1}+|d_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{c_{k1}\}\right| \neq 0 \\ 0, & \max\limits_{k\in I_{m-1}}\{c_{k1}+|d_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{c_{k1}\}\right| = 0 \end{cases}$$

5. *Obtain the decision set* $\left\{{}^{\mu^s_{k1}}_{\nu^s_{k1}}u_k \middle| u_k \in U\right\}$

Here, iE15 is a generalization of the SDM method E15 [14].

Algorithm 27 iEK15

1. *Construct ifpifs-matrices* $[a^1_{ij}]_{m\times n}, [a^2_{ij}]_{m\times n}, \cdots, [a^t_{ij}]_{m\times n}$

2. Obtain $\left[b^1_{kj}\right]_{t\times n}$ *and* $\left[b^2_{kj}\right]_{t\times n}$ *defined by* $b^1_{kj} := \mu^{a^k}_{0j}$ *and* $b^2_{kj} := \nu^{a^k}_{0j}$ *such that* $k \in I_t$ *and* $j \in I_n$

3. Obtain $\left[c^1_{kj}\right]_{t\times n}$ *and* $\left[c^2_{kj}\right]_{t\times n}$ *defined by*

$$c^1_{kj} := \begin{cases} \frac{b^1_{kj}}{\sqrt{\sum\limits_{l=1}^{t}\left(b^1_{lj}\right)^2}}, & \sqrt{\sum\limits_{l=1}^{t}\left(b^1_{lj}\right)^2} \neq 0 \\ 0, & \sqrt{\sum\limits_{l=1}^{t}\left(b^1_{lj}\right)^2} = 0 \end{cases} \quad and \quad c^2_{kj} := \begin{cases} \frac{b^2_{kj}}{\sqrt{\sum\limits_{l=1}^{t}\left(b^2_{lj}\right)^2}}, & \sqrt{\sum\limits_{l=1}^{t}\left(b^2_{lj}\right)^2} \neq 0 \\ 0, & \sqrt{\sum\limits_{l=1}^{t}\left(b^2_{lj}\right)^2} = 0 \end{cases}$$

such that $k \in I_t$

4. Obtain $\left[d^1_{j1}\right]_{n\times 1}$ *and* $\left[d^2_{j1}\right]_{n\times 1}$ *defined by*

$$d^1_{j1} := \frac{1}{t}\sum_{k=1}^{t} c^1_{kj} \quad and \quad d^2_{j1} := \frac{1}{t}\sum_{k=1}^{t} c^2_{kj}$$

such that $j \in I_n$

5. Obtain $\left[e^1_{j1}\right]_{n\times 1}$ *and* $\left[e^2_{j1}\right]_{n\times 1}$ *defined by*

$$e^1_{j1} := \begin{cases} \frac{d^1_{j1}}{\sum\limits_{l=1}^{n} d^1_{l1}}, & \sum\limits_{l=1}^{n} d^1_{l1} \neq 0 \\ 0, & \sum\limits_{l=1}^{n} d^1_{l1} = 0 \end{cases} \quad and \quad e^2_{j1} := \begin{cases} \frac{d^2_{j1}}{\sum\limits_{l=1}^{n} d^2_{l1}}, & \sum\limits_{l=1}^{n} d^2_{l1} \neq 0 \\ 0, & \sum\limits_{l=1}^{n} d^2_{l1} = 0 \end{cases}$$

such that $j \in I_n$

6. Obtain $\left[f^1_{ij}\right]_{(m-1)\times n}$ *and* $\left[f^2_{ij}\right]_{(m-1)\times n}$ *defined by*

$$f^1_{ij} := \frac{1}{t}\sum_{k=1}^{t} \mu^{a^k}_{ij} \quad and \quad f^2_{ij} := \frac{1}{t}\sum_{k=1}^{t} \nu^{a^k}_{ij}$$

such that $i \in I_{m-1}$ *and* $j \in I_n$

7. Obtain $\left[g^1_{ij}\right]_{(m-1)\times n}$ *and* $\left[g^2_{ij}\right]_{(m-1)\times n}$ *defined by* $g^1_{ij} := e^1_{j1} f^1_{ij}$ *and* $g^2_{ij} := e^2_{j1} f^2_{ij}$ *such that* $i \in I_{m-1}$ *and* $j \in I_n$

8. Obtain $\left[{g^1_{1j}}^+\right]_{1\times n}$, $\left[{g^1_{1j}}^-\right]_{1\times n}$, $\left[{g^2_{1j}}^+\right]_{1\times n}$, *and* $\left[{g^2_{1j}}^-\right]_{1\times n}$ *defined by*

$${g^1_{1j}}^+ := \max_{i\in I_{m-1}} \left\{g^1_{ij}\right\},\ {g^1_{1j}}^- := \min_{i\in I_{m-1}} \left\{g^1_{ij}\right\},\ {g^2_{1j}}^+ := \min_{i\in I_{m-1}} \left\{g^2_{ij}\right\}, \quad and \quad {g^2_{1j}}^- := \max_{i\in I_{m-1}} \left\{g^2_{ij}\right\}$$

such that $j \in I_n$

9. Obtain $\left[{s^1_{i1}}^+\right]_{(m-1)\times 1}$, $\left[{s^1_{i1}}^-\right]_{(m-1)\times 1}$, $\left[{s^2_{i1}}^+\right]_{(m-1)\times 1}$, *and* $\left[{s^2_{i1}}^-\right]_{(m-1)\times 1}$ *defined by*

$${s^1_{i1}}^+ := \sqrt{\sum_{j=1}^{n}\left(g^1_{ij} - {g^1_{1j}}^+\right)^2},\ {s^1_{i1}}^- := \sqrt{\sum_{j=1}^{n}\left(g^1_{ij} - {g^1_{1j}}^-\right)^2},$$

$${s^2_{i1}}^+ := \sqrt{\sum_{j=1}^{n}\left(g^2_{ij} - {g^2_{1j}}^+\right)^2}, \quad and \quad {s^2_{i1}}^- := \sqrt{\sum_{j=1}^{n}\left(g^2_{ij} - {g^2_{1j}}^-\right)^2}$$

such that $i \in I_{m-1}$

10. Obtain $\left[s^1_{i1}\right]_{(m-1)\times 1}$ *and* $\left[s^2_{i1}\right]_{(m-1)\times 1}$ *defined by*

$$s^1_{i1} := \begin{cases} \frac{{s^1_{i1}}^-}{{s^1_{i1}}^- + {s^1_{i1}}^+}, & {s^1_{i1}}^- + {s^1_{i1}}^+ \neq 0 \\ 0, & {s^1_{i1}}^- + {s^1_{i1}}^+ = 0 \end{cases} \quad \text{and} \quad s^2_{i1} := \begin{cases} \frac{{s^2_{i1}}^-}{{s^2_{i1}}^- + {s^2_{i1}}^+}, & {s^2_{i1}}^- + {s^2_{i1}}^+ \neq 0 \\ 0, & {s^2_{i1}}^- + {s^2_{i1}}^+ = 0 \end{cases}$$

such that $i \in I_{m-1}$

11. Obtain the score matrix $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := {}^{\mu^s_{i1}}_{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$\mu^s_{i1} = \begin{cases} \frac{s^1_{i1} + \left|\min\limits_{k\in I_{m-1}}\{s^1_{k1}\}\right|}{\max\limits_{k\in I_{m-1}}\{s^1_{k1} + |s^2_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{s^1_{k1}\}\right|}, & \max\limits_{k\in I_{m-1}}\{s^1_{k1} + |s^2_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{s^1_{k1}\}\right| \neq 0 \\ 1, & \max\limits_{k\in I_{m-1}}\{s^1_{k1} + |s^2_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{s^1_{k1}\}\right| = 0 \end{cases}$$

and

$$\nu^s_{i1} = \begin{cases} 1 - \frac{s^1_{i1} + |s^2_{i1}| + \left|\min\limits_{k\in I_{m-1}}\{s^1_{k1}\}\right|}{\max\limits_{k\in I_{m-1}}\{s^1_{k1} + |s^2_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{s^1_{k1}\}\right|}, & \max\limits_{k\in I_{m-1}}\{s^1_{k1} + |s^2_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{s^1_{k1}\}\right| \neq 0 \\ 0, & \max\limits_{k\in I_{m-1}}\{s^1_{k1} + |s^2_{k1}|\} + \left|\min\limits_{k\in I_{m-1}}\{s^1_{k1}\}\right| = 0 \end{cases}$$

12. Obtain the decision set $\left\{{}^{\mu^s_{k1}}_{\nu^s_{k1}} u_k \middle| u_k \in U\right\}$

Here, iEK15 is a generalization of the SDM method EK15 [14].

Algorithm 28 iEMK19

1. Construct ifpifs-matrices $\left[a^1_{ij}\right]_{m\times n}, \left[a^2_{ij}\right]_{m\times n}, \cdots, \left[a^t_{ij}\right]_{m\times n}$

2. Obtain $[b_{ij}]_{m\times n}$ *and* $[c_{ij}]_{m\times n}$ *defined by*

$$b_{ij} := \frac{1}{t}\sum_{k=1}^{t} \mu^{a^k}_{ij} \quad \text{and} \quad c_{ij} := \frac{1}{t}\sum_{k=1}^{t} \nu^{a^k}_{ij}$$

such that $i \in I^*_{m-1}$ *and* $j \in I_n$

3. Obtain $[d_{ij}]_{(m-1)\times n}$ *and* $[e_{ij}]_{(m-1)\times n}$ *defined by* $d_{ij} := b_{0j}b_{ij}$ *and* $e_{ij} := c_{0j}c_{ij}$ *such that* $i \in I_{m-1}$ *and* $j \in I_n$

4. Obtain $\left[{d_{1j}}^+\right]_{1\times n}$, $\left[{d_{1j}}^-\right]_{1\times n}$, $\left[{e_{1j}}^+\right]_{1\times n}$, *and* $\left[{e_{1j}}^-\right]_{1\times n}$ *defined by*

$${d_{1j}}^+ := \max_{i\in I_{m-1}}\{d_{ij}\},\ {d_{1j}}^- := \min_{i\in I_{m-1}}\{d_{ij}\},\ {e_{1j}}^+ := \min_{i\in I_{m-1}}\{e_{ij}\}, \quad \text{and} \quad {e_{1j}}^- := \max_{i\in I_{m-1}}\{e_{ij}\}$$

such that $j \in I_n$

5. Obtain $\left[{s^1_{i1}}^+\right]_{(m-1)\times 1}$, $\left[{s^1_{i1}}^-\right]_{(m-1)\times 1}$, $\left[{s^2_{i1}}^+\right]_{(m-1)\times 1}$, *and* $\left[{s^2_{i1}}^-\right]_{(m-1)\times 1}$ *defined by*

$${s^1_{i1}}^+ := \sqrt{\sum_{j=1}^{n}\left(d_{ij} - {d_{1j}}^+\right)^2}, \quad {s^1_{i1}}^- := \sqrt{\sum_{j=1}^{n}\left(d_{ij} - {d_{1j}}^-\right)^2},$$

$${s^2_{i1}}^+ := \sqrt{\sum_{j=1}^{n}\left(e_{ij} - {e_{1j}}^+\right)^2}, \quad \text{and} \quad {s^2_{i1}}^- := \sqrt{\sum_{j=1}^{n}\left(e_{ij} - {e_{1j}}^-\right)^2}$$

such that $i \in I_{m-1}$

6. *Obtain* $\left[s_{i1}^1\right]_{(m-1)\times 1}$ *and* $\left[s_{i1}^2\right]_{(m-1)\times 1}$ *defined by*

$$s_{i1}^1 := \begin{cases} \frac{{s_{i1}^1}^-}{{s_{i1}^1}^- + {s_{i1}^1}^+}, & {s_{i1}^1}^- + {s_{i1}^1}^+ \neq 0 \\ 0, & {s_{i1}^1}^- + {s_{i1}^1}^+ = 0 \end{cases} \quad \text{and} \quad s_{i1}^2 := \begin{cases} \frac{{s_{i1}^2}^-}{{s_{i1}^2}^- + {s_{i1}^2}^+}, & {s_{i1}^2}^- + {s_{i1}^2}^+ \neq 0 \\ 0, & {s_{i1}^2}^- + {s_{i1}^2}^+ = 0 \end{cases}$$

such that $i \in I_{m-1}$

7. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := {}^{\mu_{i1}^s}_{\nu_{i1}^s}$ *such that* $i \in I_{m-1}$,

$$\mu_{i1}^s = \begin{cases} \dfrac{s_{i1}^1 + \left|\min\limits_{k\in I_{m-1}}\left\{s_{k1}^1\right\}\right|}{\max\limits_{k\in I_{m-1}}\left\{s_{k1}^1 + \left|s_{k1}^2\right|\right\} + \left|\min\limits_{k\in I_{m-1}}\left\{s_{k1}^1\right\}\right|}, & \max\limits_{k\in I_{m-1}}\left\{s_{k1}^1 + \left|s_{k1}^2\right|\right\} + \left|\min\limits_{k\in I_{m-1}}\left\{s_{k1}^1\right\}\right| \neq 0 \\ 1, & \max\limits_{k\in I_{m-1}}\left\{s_{k1}^1 + \left|s_{k1}^2\right|\right\} + \left|\min\limits_{k\in I_{m-1}}\left\{s_{k1}^1\right\}\right| = 0 \end{cases}$$

and

$$\nu_{i1}^s = \begin{cases} 1 - \dfrac{s_{i1}^1 + \left|s_{i1}^2\right| + \left|\min\limits_{k\in I_{m-1}}\left\{s_{k1}^1\right\}\right|}{\max\limits_{k\in I_{m-1}}\left\{s_{k1}^1 + \left|s_{k1}^2\right|\right\} + \left|\min\limits_{k\in I_{m-1}}\left\{s_{k1}^1\right\}\right|}, & \max\limits_{k\in I_{m-1}}\left\{s_{k1}^1 + \left|s_{k1}^2\right|\right\} + \left|\min\limits_{k\in I_{m-1}}\left\{s_{k1}^1\right\}\right| \neq 0 \\ 0, & \max\limits_{k\in I_{m-1}}\left\{s_{k1}^1 + \left|s_{k1}^2\right|\right\} + \left|\min\limits_{k\in I_{m-1}}\left\{s_{k1}^1\right\}\right| = 0 \end{cases}$$

8. *Obtain the decision set* $\left\{{}^{\mu_{k1}^s}_{\nu_{k1}^s}u_k \middle| u_k \in U\right\}$

Here, iEMK19 is a generalization of the SDM method EMK19 [20].

Algorithm 29 iYJ11

1. *Construct ifpifs-matrices* $\left[a_{ij}^1\right]_{m\times n}, \left[a_{ij}^2\right]_{m\times n}, \cdots, \left[a_{ij}^t\right]_{m\times n}$

2. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := {}^{\mu_{i1}^s}_{\nu_{i1}^s}$ *such that* $i \in I_{m-1}$,

$$\mu_{i1} := \frac{1}{n}\sum_{j=1}^{n}\min_{k\in I_t}\left\{\mu_{0j}^{a^k}\mu_{ij}^{a^k}\right\} \quad \text{and} \quad \nu_{i1} := \frac{1}{n}\sum_{j=1}^{n}\max_{k\in I_t}\left\{\nu_{0j}^{a^k}\nu_{ij}^{a^k}\right\}$$

3. *Obtain the decision set* $\left\{{}^{\mu_{k1}^s}_{\nu_{k1}^s}u_k \middle| u_k \in U\right\}$

Here, iYJ11 is a generalization of the SDM method YJ11 [23].

Algorithm 30 iYJ11/2

1. *Construct ifpifs-matrices* $\left[a_{ij}^1\right]_{m\times n}, \left[a_{ij}^2\right]_{m\times n}, \cdots, \left[a_{ij}^t\right]_{m\times n}$

2. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := {}^{\mu_{i1}^s}_{\nu_{i1}^s}$ *such that* $i \in I_{m-1}$,

$$\mu_{i1}^s := \frac{1}{n}\sum_{j=1}^{n}\max_{k\in I_t}\left\{\mu_{0j}^{a^k}\mu_{ij}^{a^k}\right\} \quad \text{and} \quad \nu_{i1}^s := \frac{1}{n}\sum_{j=1}^{n}\min_{k\in I_t}\left\{\nu_{0j}^{a^k}\nu_{ij}^{a^k}\right\}$$

3. *Obtain the decision set* $\left\{{}^{\mu_{k1}^s}_{\nu_{k1}^s}u_k \middle| u_k \in U\right\}$

Here, iYJ11/2 is a generalization of the SDM method YJ11/2 [23].

Algorithm 31 iBNS12

1. *Construct ifpifs-matrices* $[a^1_{ij}]_{m\times n}$, $[a^2_{ij}]_{m\times n}$, $\cdots$, $[a^t_{ij}]_{m\times n}$
2. *Obtain* $[b_{i1}]_{(m-1)\times 1}$ *and* $[c_{i1}]_{(m-1)\times 1}$ *defined by*

$$b_{i1} := \sum_{j=1}^{n}\left(\prod_{k=1}^{t}\mu^{a^k}_{0j}\mu^{a^k}_{ij}\right) \quad and \quad c_{i1} := \sum_{j=1}^{n}\left(\prod_{k=1}^{t}\nu^{a^k}_{0j}\nu^{a^k}_{ij}\right)$$

such that $i \in I_{m-1}$

3. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \frac{\mu^s_{i1}}{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$\mu^s_{i1} = \begin{cases} \dfrac{b_{i1}+\left|\min\limits_{k\in I_{m-1}}\{b_{k1}\}\right|}{\max\limits_{k\in I_{m-1}}\{b_{k1}+|c_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{b_{k1}\}\right|}, & \max\limits_{k\in I_{m-1}}\{b_{k1}+|c_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{b_{k1}\}\right| \neq 0 \\ 1, & \max\limits_{k\in I_{m-1}}\{b_{k1}+|c_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{b_{k1}\}\right| = 0 \end{cases}$$

and

$$\nu^s_{i1} = \begin{cases} 1-\dfrac{b_{i1}+|c_{i1}|+\left|\min\limits_{k\in I_{m-1}}\{b_{k1}\}\right|}{\max\limits_{k\in I_{m-1}}\{b_{k1}+|c_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{b_{k1}\}\right|}, & \max\limits_{k\in I_{m-1}}\{b_{k1}+|c_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{b_{k1}\}\right| \neq 0 \\ 0, & \max\limits_{k\in I_{m-1}}\{b_{k1}+|c_{k1}|\}+\left|\min\limits_{k\in I_{m-1}}\{b_{k1}\}\right| = 0 \end{cases}$$

4. *Obtain the decision set* $\left\{{}^{\mu^s_{k1}}_{\nu^s_{k1}}u_k \middle| u_k \in U\right\}$

Here, iBNS12 is a generalization of the SDM method BNS12 [23].

Algorithm 32 iS12($R_1, R_2, \cdots, R_t$)

1. *Construct ifpifs-matrices* $[a^1_{ij_1}]_{m\times n_1}$, $[a^2_{ij_2}]_{m\times n_2}$, $\cdots$, $[a^t_{ij_t}]_{m\times n_t}$
2. *Determine* $R_k \subseteq I_{n_k}$, *for all* $k \in I_t$
3. *Obtain the score matrix* $[s_{i1}]_{(m-1)\times 1}$ *defined by* $s_{i1} := \frac{\mu^s_{i1}}{\nu^s_{i1}}$ *such that* $i \in I_{m-1}$,

$$\mu^s_{i1} := \min_{k\in I_t}\left\{\min_{j\in R_k}\left\{\mu^{a^k}_{0j}\mu^{a^k}_{ij}\right\}\right\} \quad and \quad \nu^s_{i1} := \max_{k\in I_t}\left\{\max_{j\in R_k}\left\{\nu^{a^k}_{0j}\nu^{a^k}_{ij}\right\}\right\}$$

4. *Obtain the decision set* $\left\{{}^{\mu^s_{k1}}_{\nu^s_{k1}}u_k \middle| u_k \in U\right\}$

Here, the variables $R_1, R_2, \cdots, R_t$ represent sets of indices concerning parameters, and iS12($R_1, R_2, \cdots, R_t$) is a generalization of the SDM method S12 [23].

Algorithm 33 iMR13

1. *Construct ifpifs-matrices* $[a^1_{ij}]_{m\times n}$, $[a^2_{ij}]_{m\times n}$, $\cdots$, $[a^t_{ij}]_{m\times n}$
2. *Obtain* $[b_{ij}]_{m\times n}$ *defined by* $b_{ij} := \frac{\mu^b_{ij}}{\nu^b_{ij}}$ *such that* $i \in I^*_{m-1}$, $j \in I_n$,

$$\mu^b_{ij} := \min_{k\in I_t}\left\{\mu^{a^k}_{ij}\right\} \quad and \quad \nu^b_{ij} := \max_{k\in I_t}\left\{\nu^{a^k}_{ij}\right\}$$

3. Apply iCCE10 [1] *to* $[b_{ij}]$

Here, iMR13 is a generalization of the SDM method MR13 [23].

Algorithm 34 iMR13/2

1. Construct ifpifs-matrices $\left[a^1_{ij}\right]_{m\times n}, \left[a^2_{ij}\right]_{m\times n}, \cdots, \left[a^t_{ij}\right]_{m\times n}$

2. Find ALGEBRAIC-*product ifpifs-matrix* [2] $[b_{ij}]_{m\times n}$ *of* $\left[a^1_{ij}\right], \left[a^2_{ij}\right], \cdots, \left[a^t_{ij}\right]$

3. Apply iCCE10 [1] *to* $[b_{ij}]$

Here, iMR13/2 is a generalization of the SDM method MR13/2 [23].

Algorithm 35 iMR13/3

1. Construct ifpifs-matrices $\left[a^1_{ij}\right]_{m\times n}, \left[a^2_{ij}\right]_{m\times n}, \cdots, \left[a^t_{ij}\right]_{m\times n}$

2. Obtain $[b_{ij}]_{m\times n}$ *defined by* $b_{ij} := \begin{smallmatrix}\mu^b_{ij}\\ \nu^b_{ij}\end{smallmatrix}$ *such that* $i \in I^*_{m-1}$, $j \in I_n$,

$$\mu^b_{ij} := \max\left\{\left(\sum_{k=1}^{t}\mu^{a^k}_{ij}\right) - t + 1, 0\right\} \quad \textit{and} \quad \nu^b_{ij} := \min\left\{-\left(\sum_{k=1}^{t}\nu^{a^k}_{ij}\right) + t, 1\right\}$$

3. Apply iCCE10 [1] *to* $[b_{ij}]$

Here, iMR13/3 is a generalization of the SDM method MR13/3 [23].

Algorithm 36 iNKY17(γ)

1. Construct ifpifs-matrices $\left[a^1_{ij}\right]_{m\times n}, \left[a^2_{ij}\right]_{m\times n}, \cdots, \left[a^t_{ij}\right]_{m\times n}$

2. Obtain $[b_{ij}]_{m\times n}$ *defined by* $b_{ij} := \begin{smallmatrix}\mu^b_{ij}\\ \nu^b_{ij}\end{smallmatrix}$ *such that* $i \in I^*_{m-1}$, $j \in I_n$,

$$\mu^b_{ij} := \frac{1}{t}\sum_{k=1}^{t}\mu^{a^k}_{ij} \quad \textit{and} \quad \nu^b_{ij} := \frac{1}{t}\sum_{k=1}^{t}\nu^{a^k}_{ij}$$

3. Construct an intuitionistic fuzzy valued column matrix $[\gamma_{i1}]_{(m-1)\times 1}$ *defined by* $\gamma_{i1} := \begin{smallmatrix}\mu_{i1}\\ \nu_{i1}\end{smallmatrix}$ *such that* $0 \le \mu_{i1} + \nu_{i1} \le 1$ *and* $\mu_{i1}, \nu_{i1} \in [0, 1]$, *for all* $i \in I_{m-1}$

4. Obtain $[c_{ij}]_{m\times n}$ *defined by* $c_{ij} := \begin{smallmatrix}\mu^c_{ij}\\ \nu^c_{ij}\end{smallmatrix}$ *such that* $i \in I_{m-1}$, $j \in I_n$, $\mu^c_{0j} := \mu^b_{0j}$, $\mu^c_{ij} := \mu_{i1}\mu^b_{ij}$, $\nu^c_{0j} := \nu^b_{0j}$, *and* $\nu^c_{ij} := \nu_{i1} + \nu^b_{ij} - \nu_{i1}\nu^b_{ij}$

5. Apply iMBR01/2 [1] *to* $[c_{ij}]$

Here, the variable γ indicates an intuitionistic fuzzy valued column matrix related to alternatives, and iNKY17(γ) is a generalization of the SDM method NKY17 [23].

3. Results of Test Cases

This section tests the generalized SDM methods herein by using five test cases provided in [1]. To determine the methods that are successful in all test cases, three *ifpifs*-matrices provided in Tables 1-5 for each test case are utilized. Moreover, Table 6 presents the test results of these methods.

Table 1: *ifpifs*-Matrices Employed in the Test Case 1

$$[a^1_{ij}] := \begin{bmatrix} \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} \\ \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} \\ \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} \\ \substack{0.9\\0.05} & \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} \\ \substack{1\\0} & \substack{0.9\\0.05} & \substack{0.8\\0.1} & \substack{0.7\\0.15} \end{bmatrix} \quad [a^2_{ij}] := \begin{bmatrix} \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} \\ \substack{0.3\\0.35} & \substack{0.2\\0.4} & \substack{0.1\\0.45} & \substack{0\\0.5} \\ \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} & \substack{0.1\\0.45} \\ \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} \\ \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} \end{bmatrix} \quad [a^3_{ij}] := \begin{bmatrix} \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} & \substack{0.4\\0.3} \\ \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} \\ \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} \\ \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} \\ \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} \end{bmatrix}$$

Table 2: *ifpifs*-Matrices Employed in the Test Case 2

$$[b^1_{ij}] := \begin{bmatrix} \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} \\ \substack{1\\0} & \substack{0.9\\0.05} & \substack{0.8\\0.1} & \substack{0.7\\0.15} \\ \substack{0.9\\0.05} & \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} \\ \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} \\ \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} \end{bmatrix} \quad [b^2_{ij}] := \begin{bmatrix} \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} \\ \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} \\ \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} \\ \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} & \substack{0.1\\0.45} \\ \substack{0.3\\0.35} & \substack{0.2\\0.4} & \substack{0.1\\0.45} & \substack{0\\0.5} \end{bmatrix} \quad [b^3_{ij}] := \begin{bmatrix} \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} & \substack{0.8\\0.1} \\ \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} \\ \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} \\ \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} \\ \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} \end{bmatrix}$$

Table 3: *ifpifs*-Matrices Employed in the Test Case 3

$$[c^1_{ij}] := \begin{bmatrix} \substack{0.6\\0.2} & \substack{0.7\\0.15} & \substack{0.8\\0.1} & \substack{0.9\\0.05} \\ \substack{1\\0} & \substack{0\\1} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{1\\0} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{1\\0} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{0\\1} & \substack{1\\0} \end{bmatrix} \quad [c^2_{ij}] := \begin{bmatrix} \substack{0.4\\0.3} & \substack{0.5\\0.25} & \substack{0.6\\0.2} & \substack{0.7\\0.15} \\ \substack{1\\0} & \substack{0\\1} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{1\\0} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{1\\0} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{0\\1} & \substack{1\\0} \end{bmatrix} \quad [c^3_{ij}] := \begin{bmatrix} \substack{0.2\\0.4} & \substack{0.3\\0.35} & \substack{0.4\\0.3} & \substack{0.5\\0.25} \\ \substack{1\\0} & \substack{0\\1} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{1\\0} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{1\\0} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{0\\1} & \substack{1\\0} \end{bmatrix}$$

Table 4: *ifpifs*-Matrices Employed in the Test Case 4

$$[d^1_{ij}] := \begin{bmatrix} \substack{0.9\\0.05} & \substack{0.8\\0.1} & \substack{0.7\\0.15} & \substack{0.6\\0.2} \\ \substack{1\\0} & \substack{0\\1} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{1\\0} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{1\\0} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{0\\1} & \substack{1\\0} \end{bmatrix} \quad [d^2_{ij}] := \begin{bmatrix} \substack{0.7\\0.15} & \substack{0.6\\0.2} & \substack{0.5\\0.25} & \substack{0.4\\0.3} \\ \substack{1\\0} & \substack{0\\1} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{1\\0} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{1\\0} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{0\\1} & \substack{1\\0} \end{bmatrix} \quad [d^3_{ij}] := \begin{bmatrix} \substack{0.5\\0.25} & \substack{0.4\\0.3} & \substack{0.3\\0.35} & \substack{0.2\\0.4} \\ \substack{1\\0} & \substack{0\\1} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{1\\0} & \substack{0\\1} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{1\\0} & \substack{0\\1} \\ \substack{0\\1} & \substack{0\\1} & \substack{0\\1} & \substack{1\\0} \end{bmatrix}$$

Table 5: *ifpifs*-Matrices Employed in the Test Case 5

$$\left[e^1_{ij}\right] := \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \end{bmatrix} \qquad \left[e^2_{ij}\right] := \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \end{bmatrix} \qquad \left[e^3_{ij}\right] := \begin{bmatrix} 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \\ 0.4 & 0.4 & 0.4 & 0.4 \end{bmatrix}$$

Table 6: Performance of the Generalized SDM Methods in the Test Cases

	Algorithms\Test Cases	**Test Case 1**	**Test Case 2**	**Test Case 3**	**Test Case 4**	**Test Case 5**	**Numbers of Tests Passed**
1.	iCE10a	✓	✓	–	–	✓	3
2.	iCE10o	✓	✓	✓	✓	✓	**5**
3.	iEMO18ao	✓	✓	–	–	✓	3
4.	iEMO18oa	✓	✓	✓	✓	✓	**5**
5.	iCE10an	–	–	–	–	✓	1
6.	iCE10on	✓	✓	✓	✓	✓	**5**
7.	iEMA18anon	–	–	–	–	✓	1
8.	iEMA18onan	✓	✓	✓	✓	✓	**5**
9.	iCE10/2a	✓	✓	–	–	✓	3
10.	iCE10/2o	✓	✓	✓	✓	✓	**5**
11.	iCE10/2an	–	–	–	–	✓	1
12.	iCE10/2on	✓	✓	✓	✓	✓	**5**
13.	iEMC19ao	✓	✓	–	–	✓	3
14.	iEMC19oa	✓	✓	✓	✓	✓	**5**
15.	iVR13a	–	–	✓	✓	✓	3
16.	iVR13o	–	–	✓	✓	✓	3
17.	iZZ16(0.4)	✓	✓	✓	✓	✓	**5**
18.	iZZ16$\binom{0.4}{0.4}$	✓	✓	✓	✓	✓	**5**
19.	iZZ16/2(0.4)	✓	✓	✓	✓	✓	**5**
20.	iZZ16/2$\binom{0.4}{0.4}$	✓	✓	✓	✓	✓	**5**
21.	iCJ17a	✓	✓	–	–	✓	3
22.	iCJ17o	✓	✓	–	–	✓	3
23.	iCJ17an	–	–	–	–	✓	1
24.	iCJ17on	–	–	–	–	✓	1
25.	iRM11a	✓	✓	–	–	✓	3
26.	iRM11o	✓	✓	✓	✓	✓	**5**
27.	iEM20ao	✓	✓	–	–	✓	3
28.	iEM20oa	✓	✓	✓	✓	✓	**5**
29.	iZ14(I_4)	✓	✓	✓	✓	✓	**5**
30.	iDB12	✓	✓	✓	✓	✓	**5**
31.	isDB12	✓	✓	✓	✓	✓	**5**
32.	iCD12(2)	✓	✓	✓	✓	✓	**5**
33.	iCD12/2(2)	✓	✓	✓	✓	✓	**5**
34.	iE15	✓	✓	✓	✓	✓	**5**
35.	iEK15	✓	✓	–	–	✓	3
36.	iEMK19	✓	✓	–	–	✓	3
37.	iYJ11	✓	✓	✓	✓	✓	**5**
38.	iYJ11/2	✓	✓	✓	✓	✓	**5**
39.	iBNS12	–	✓	–	–	✓	2
40.	iS12(I_4, I_4, I_4)	✓	✓	–	–	✓	3
41.	iMR13	✓	✓	✓	✓	✓	**5**
42.	iMR13/2	✓	✓	–	–	✓	3
43.	iMR13/3	–	–	–	–	✓	1
44.	iNKY17$\left(\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}^T\right)$	✓	✓	✓	✓	✓	**5**

*Bold values in the last column indicate the SDM methods passing all the test cases (✓: Successful, –: Unsuccessful)

Here, since all the parameters should be considered in all the test cases, $R = R_1 = R_2 = R_3 = I_4$ are used in the SDM methods iZ14(R) and iS12(R_1, R_2, R_3). Furthermore, the other variables are tuned so that the related SDM methods can pass the largest number of test cases. The test results of the SDM methods are obtained by MATLAB R2021b. Moreover, the numbers of the passed tests are presented in last column of Table 6. The results show that 23 of 44 SDM methods are successful in all the test cases: iCE10o, iEMO18oa, iCE10on, iEMA18onan, iCE10/2o, iCE10/2on, iEMC19oa, iZZ16(0.4), iZZ16$\binom{0.4}{0.4}$, iZZ16/2(0.4), iZZ16/2$\binom{0.4}{0.4}$, iRM11o, iEM20oa, iZ14(I_4), iDB12, isDB12, iCD12(2), iCD12/2(2), iE15, iYJ11, iYJ11/2, iMR13, and iNKY17$\left(\begin{bmatrix}1&1&1&1\\0&0&0&0\end{bmatrix}^T\right)$. Here, $[\cdot]^T$ stands for the transpose of $[\cdot]$.

4. An Application of the Successful SDM Methods to a PVA Problem

This section applies the generalized SDM methods that are successful in all test cases in Section 3 to performance-based value assignment (PVA) to seven noise-removal filters, namely, Based on Pixel Density Filter (BPDF) [24], Decision-Based Algorithm (DBAIN) [42], Modified Decision-Based Unsymmetrical Trimmed Median Filter (MDBUTMF) [26], Noise Adaptive Fuzzy Switching Median Filter (NAFSMF) [44], Different Applied Median Filter (DAMF) [25], Adaptive Weighted Mean Filter (AWMF) [43], and Adaptive Riesz Mean Filter (ARmF) [13].

Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ be a set of alternatives such that $u_1 =$"BPDF", $u_2 =$"DBAIN", $u_3 =$"MDBUTMF", $u_4 =$"NAFSMF", $u_5 =$"DAMF", $u_6 =$"AWMF", and $u_7 =$"ARmF". Moreover, let $E = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9\}$ be a set of parameters such that $x_1 =$ "noise density 10%", $x_2 =$ "noise density 20%", $x_3 =$ "noise density 30%", $x_4 =$ "noise density 40%", $x_5 =$ "noise density 50%", $x_6 =$ "noise density 60%", $x_7 =$ "noise density 70%", $x_8 =$ "noise density 80%", and $x_9 =$ "noise density 90%". Suppose that the noise removal success of these filters at high noise densities is more significant than those of at the other densities. Then, the *ifpifs*-matrix $[a_{ij}]$ obtained by using Structural Similarity (SSIM) [45] results of aforesaid seven filters and provided in [1] is as follows:

$$[a_{ij}] = \begin{bmatrix}
\begin{smallmatrix}0.05\\0.9\end{smallmatrix} & \begin{smallmatrix}0.15\\0.8\end{smallmatrix} & \begin{smallmatrix}0.25\\0.7\end{smallmatrix} & \begin{smallmatrix}0.35\\0.6\end{smallmatrix} & \begin{smallmatrix}0.5\\0.5\end{smallmatrix} & \begin{smallmatrix}0.65\\0.3\end{smallmatrix} & \begin{smallmatrix}0.75\\0.2\end{smallmatrix} & \begin{smallmatrix}0.85\\0.1\end{smallmatrix} & \begin{smallmatrix}0.9\\0.05\end{smallmatrix} \\
\begin{smallmatrix}0.9657\\0.0062\end{smallmatrix} & \begin{smallmatrix}0.9335\\0.0142\end{smallmatrix} & \begin{smallmatrix}0.8856\\0.0270\end{smallmatrix} & \begin{smallmatrix}0.8269\\0.0450\end{smallmatrix} & \begin{smallmatrix}0.7503\\0.0759\end{smallmatrix} & \begin{smallmatrix}0.6452\\0.1165\end{smallmatrix} & \begin{smallmatrix}0.5159\\0.1887\end{smallmatrix} & \begin{smallmatrix}0.3648\\0.2998\end{smallmatrix} & \begin{smallmatrix}0.1259\\0.4895\end{smallmatrix} \\
\begin{smallmatrix}0.9666\\0.0031\end{smallmatrix} & \begin{smallmatrix}0.9424\\0.0080\end{smallmatrix} & \begin{smallmatrix}0.9047\\0.0168\end{smallmatrix} & \begin{smallmatrix}0.8552\\0.0297\end{smallmatrix} & \begin{smallmatrix}0.7917\\0.0478\end{smallmatrix} & \begin{smallmatrix}0.7104\\0.0762\end{smallmatrix} & \begin{smallmatrix}0.6060\\0.1223\end{smallmatrix} & \begin{smallmatrix}0.4880\\0.1858\end{smallmatrix} & \begin{smallmatrix}0.3518\\0.2766\end{smallmatrix} \\
\begin{smallmatrix}0.9642\\0.0050\end{smallmatrix} & \begin{smallmatrix}0.9228\\0.0509\end{smallmatrix} & \begin{smallmatrix}0.7833\\0.1319\end{smallmatrix} & \begin{smallmatrix}0.7539\\0.1593\end{smallmatrix} & \begin{smallmatrix}0.7855\\0.1167\end{smallmatrix} & \begin{smallmatrix}0.7572\\0.0551\end{smallmatrix} & \begin{smallmatrix}0.6950\\0.0575\end{smallmatrix} & \begin{smallmatrix}0.6000\\0.1359\end{smallmatrix} & \begin{smallmatrix}0.3492\\0.5228\end{smallmatrix} \\
\begin{smallmatrix}0.9606\\0.0086\end{smallmatrix} & \begin{smallmatrix}0.9216\\0.0169\end{smallmatrix} & \begin{smallmatrix}0.8767\\0.0267\end{smallmatrix} & \begin{smallmatrix}0.8305\\0.0357\end{smallmatrix} & \begin{smallmatrix}0.7800\\0.0465\end{smallmatrix} & \begin{smallmatrix}0.7211\\0.0595\end{smallmatrix} & \begin{smallmatrix}0.6540\\0.0790\end{smallmatrix} & \begin{smallmatrix}0.5766\\0.1082\end{smallmatrix} & \begin{smallmatrix}0.4578\\0.2173\end{smallmatrix} \\
\begin{smallmatrix}0.9700\\0.0018\end{smallmatrix} & \begin{smallmatrix}0.9518\\0.0045\end{smallmatrix} & \begin{smallmatrix}0.9270\\0.0088\end{smallmatrix} & \begin{smallmatrix}0.8953\\0.0139\end{smallmatrix} & \begin{smallmatrix}0.8563\\0.0204\end{smallmatrix} & \begin{smallmatrix}0.8072\\0.0291\end{smallmatrix} & \begin{smallmatrix}0.7465\\0.0423\end{smallmatrix} & \begin{smallmatrix}0.6667\\0.0624\end{smallmatrix} & \begin{smallmatrix}0.5415\\0.1148\end{smallmatrix} \\
\begin{smallmatrix}0.9551\\0.0067\end{smallmatrix} & \begin{smallmatrix}0.9440\\0.0076\end{smallmatrix} & \begin{smallmatrix}0.9209\\0.0095\end{smallmatrix} & \begin{smallmatrix}0.8948\\0.0122\end{smallmatrix} & \begin{smallmatrix}0.8611\\0.0166\end{smallmatrix} & \begin{smallmatrix}0.8148\\0.0240\end{smallmatrix} & \begin{smallmatrix}0.7551\\0.0370\end{smallmatrix} & \begin{smallmatrix}0.6736\\0.0574\end{smallmatrix} & \begin{smallmatrix}0.5469\\0.1052\end{smallmatrix} \\
\begin{smallmatrix}0.9718\\0.0013\end{smallmatrix} & \begin{smallmatrix}0.9532\\0.0030\end{smallmatrix} & \begin{smallmatrix}0.9272\\0.0054\end{smallmatrix} & \begin{smallmatrix}0.8971\\0.0087\end{smallmatrix} & \begin{smallmatrix}0.8630\\0.0137\end{smallmatrix} & \begin{smallmatrix}0.8239\\0.0214\end{smallmatrix} & \begin{smallmatrix}0.7663\\0.0348\end{smallmatrix} & \begin{smallmatrix}0.6819\\0.0554\end{smallmatrix} & \begin{smallmatrix}0.5515\\0.1038\end{smallmatrix}
\end{bmatrix}$$

Furthermore, Tables 7-13 and Tables 14-20 present the Peak Signal-to-Noise Ratio (PSNR) and Visual Information Fidelity (VIF) [41] results of aforesaid seven filters, obtained by MATLAB R2021b, for 20 traditional images, i.e., "Lena", "Cameraman", "Barbara", "Baboon", "Peppers", "Living Room", "Lake", "Plane", "Hill", "Pirate", "Boat", "House", "Bridge", "Elaine", "Flintstones", "Flower", "Parrot", "Dark-Haired Woman", "Blonde Woman", and "Einstein", at noise densities ranging from 10% to 90%, respectively.

Table 7: PSNR Results of BPDF for 20 Traditional Images

Images/Noise Densities	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
Lena	40.1207	35.8460	32.8329	30.5127	28.3562	26.0197	23.0085	18.2969	10.7110
Cameraman	39.3852	35.0055	32.2111	29.8339	27.5254	25.1452	22.2919	18.0011	11.6495
Barbara	32.7983	29.3573	27.1694	25.5811	24.0656	22.4769	20.2863	16.1678	9.4944
Baboon	35.4468	31.5060	28.7908	26.5725	24.5238	22.7308	20.6291	17.0444	8.9981
Peppers	38.2033	34.8804	32.4517	30.5309	28.2914	25.9504	23.1313	18.2481	9.0780
Living Room	35.9376	32.1485	29.6862	27.7004	25.8133	23.8195	21.3705	17.5999	10.9149
Lake	36.6876	32.3391	29.7439	27.3752	25.1171	22.4346	19.7057	15.5530	8.7834
Plane	38.4801	34.5421	31.6290	28.9265	26.4190	24.0467	21.2177	16.4245	7.9709
Hill	37.8776	34.2986	31.9029	29.7889	27.7335	25.5693	23.0180	19.3774	13.8182
Pirate	37.5385	33.8626	31.2111	28.9833	26.7575	24.7837	21.8897	17.2893	10.9445
Boat	36.4823	32.6041	29.9755	27.8538	25.7916	23.7528	20.9115	17.0311	11.1577
House	43.5902	39.7504	36.3429	33.4103	30.3397	27.7133	24.1114	19.8625	13.1290
Bridge	33.8422	30.2166	27.7101	25.7299	23.7898	21.7617	19.4641	16.1564	10.5119
Elaine	39.5501	36.1133	33.4275	31.4015	29.1653	26.6918	23.6521	18.1038	11.0628
Flintstones	31.0347	27.3956	24.9060	22.7339	20.7054	18.7883	16.4151	12.9912	6.9176
Flower	33.4606	31.5380	29.5794	27.7809	25.8168	23.7743	21.4026	18.0230	11.2483
Parrot	31.8533	31.0869	29.0831	28.3085	26.6351	25.3372	23.1418	19.7013	10.7452
Dark-Haired Woman	44.9712	41.1344	38.1960	35.3849	32.9769	30.1407	25.7592	20.1365	10.7867
Blonde Woman	32.1376	30.4787	29.0405	27.6428	26.3547	24.5170	22.2023	17.8902	8.9621
Einstein	41.8411	37.7856	35.2482	33.4590	30.8533	28.4806	25.7568	21.3325	13.7197

Table 8: PSNR Results of DBAIN for 20 Traditional Images

Images/Noise Densities	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
Lena	41.7024	37.4327	34.7209	32.0906	30.2490	28.1340	25.5840	22.9730	19.8600
Cameraman	41.9797	37.4575	34.2022	31.6429	29.2698	27.1595	24.5283	21.9634	18.8275
Barbara	33.0754	29.8123	27.5702	25.9170	24.5338	23.2128	21.6672	20.0471	17.4358
Baboon	36.7356	32.9041	30.2563	28.0196	25.9959	24.1913	22.3661	20.5131	18.3612
Peppers	39.6141	36.4103	33.8338	31.8349	29.7383	27.6079	25.2377	22.6204	19.0465
Living Room	37.5604	33.5608	31.0142	28.9389	27.2588	25.3042	23.5117	21.3496	18.4977
Lake	34.6392	32.4992	30.5092	28.6066	26.7373	24.7123	22.5239	20.1682	17.1273
Plane	39.4614	35.9398	33.3609	30.9328	28.5350	26.3232	24.0754	21.4329	18.2203
Hill	38.9580	35.5585	33.0158	31.0429	29.0009	27.1838	25.2382	23.2213	20.3538
Pirate	38.3700	35.0092	32.5162	30.3586	28.3248	26.4680	24.3707	22.0282	19.1498
Boat	37.1633	33.7982	31.3056	29.0767	27.3376	25.4496	23.3844	21.3486	18.5618
House	46.9856	41.7527	38.3421	35.2990	32.8991	30.3252	27.5612	23.9619	20.1525
Bridge	33.8058	30.6931	28.6066	26.6350	24.8286	23.2263	21.4649	19.4240	16.7823
Elaine	40.3079	37.1622	34.7025	32.7728	31.0557	29.0884	26.8744	23.9132	20.0979
Flintstones	33.0409	29.3856	26.6837	24.1093	22.0558	20.0279	17.8602	15.8061	13.1503
Flower	34.0305	32.5413	30.7202	29.1437	27.2227	25.4142	23.5670	21.5662	18.7590
Parrot	33.0775	32.2262	31.1229	29.8910	28.6025	27.0299	25.4772	23.5876	20.8310
Dark-Haired Woman	46.5696	42.9604	40.5074	37.8290	34.7415	32.6500	30.0161	26.5576	21.9065
Blonde Woman	31.8655	30.7231	29.6103	28.3882	27.0572	25.7529	23.9740	22.2877	19.5317
Einstein	43.3906	39.5696	36.8779	34.6650	31.9730	30.1821	27.9514	25.6926	22.4661

Table 9: PSNR Results of MDBUTMF for 20 Traditional Images

Images/Noise Densities	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
Lena	40.5308	35.6049	31.4577	30.3932	31.0401	30.7714	29.4379	25.7451	16.4201
Cameraman	39.9202	34.6721	30.2708	29.3007	30.0276	29.9566	28.4352	24.9551	16.3161
Barbara	32.5645	29.1775	26.7458	25.5024	24.9391	24.1714	23.2195	21.5984	15.9936
Baboon	34.0621	30.6396	28.1182	26.9155	26.3222	25.5323	24.4306	22.3776	15.7092
Peppers	40.3550	35.3655	31.0817	30.1102	30.8208	30.7592	29.4946	25.8831	16.7819
Living Room	36.0229	32.0736	29.2912	28.0867	27.8125	26.9336	25.8927	23.5614	16.4881
Lake	36.4854	32.1291	28.6167	27.6502	27.6596	27.1770	26.0597	23.4397	15.7501
Plane	38.7981	33.7165	29.7909	28.7364	29.3232	29.1695	27.7305	23.6289	13.7697
Hill	38.2958	34.1233	30.3720	29.4182	29.6018	29.2819	28.0745	25.4791	17.1378
Pirate	38.0641	33.8219	30.1577	29.1395	29.1292	28.6336	27.5312	24.7293	17.0249
Boat	36.7751	32.6277	29.4759	28.3392	28.1407	27.4659	26.2543	23.6388	16.0319
House	47.2603	38.0746	32.3237	31.4540	33.7888	35.0290	33.3810	26.8883	15.8360
Bridge	33.8124	30.0923	27.4450	26.3055	25.7404	25.0269	23.8888	21.9899	16.0124
Elaine	40.9631	36.1123	31.8615	30.9447	31.7410	31.8485	30.5476	25.9854	16.0459
Flintstones	32.0016	28.3444	25.3973	24.0838	23.5164	22.5560	21.2350	19.1389	13.7476
Flower	33.2940	31.3942	29.1486	28.3189	28.2421	27.7531	26.6697	24.3583	17.4342
Parrot	26.1186	25.6994	24.9436	24.7026	24.6451	24.4481	24.0226	22.6789	16.0025
Dark-Haired Woman	46.6049	37.7460	31.6361	31.2219	33.8768	36.0648	34.6230	28.1550	17.4094
Blonde Woman	31.0419	29.8258	28.1923	27.4624	27.3264	26.9460	26.2408	23.7400	15.6890
Einstein	42.1813	37.0571	32.7757	31.8126	32.7269	32.8236	31.4597	27.5641	18.2882

Table 10: PSNR Results of NAFSMF for 20 Traditional Images

Images/Noise Densities	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
Lena	38.8408	35.7244	33.6318	32.2185	31.1273	29.8105	28.6769	27.0939	23.4284
Cameraman	36.6409	34.0358	31.9416	30.4022	29.2852	28.3415	27.1063	25.5442	22.3382
Barbara	33.1453	30.0555	28.1911	26.8663	25.8246	24.7839	23.8217	22.7794	20.6211
Baboon	32.3912	29.4572	27.5740	26.3060	25.2660	24.3347	23.4304	22.4486	20.4587
Peppers	39.3057	36.3208	34.3630	32.8275	31.4453	30.3638	29.0903	27.4512	23.4095
Living Room	34.5237	31.3634	29.5777	28.3018	27.2286	26.1486	25.2361	24.1445	21.6589
Lake	34.6971	31.5187	29.6086	28.3242	27.1148	26.0774	25.1345	23.7908	21.1496
Plane	36.5648	33.4430	31.6712	30.2027	29.0520	27.9017	26.6729	25.3963	22.1633
Hill	37.1007	34.0850	32.1277	30.8328	29.6385	28.6481	27.5382	26.4624	23.3972
Pirate	36.8338	33.6607	31.6719	30.3928	29.0368	27.9088	26.9095	25.5246	22.6413
Boat	35.1108	31.8502	29.9946	28.5836	27.5256	26.5269	25.4945	24.2185	21.8833
House	44.5313	41.3926	38.9129	37.2766	35.6826	34.0870	32.3048	30.2131	24.9187
Bridge	32.7144	29.5726	27.7940	26.4789	25.3542	24.3868	23.3923	22.3196	20.2711
Elaine	40.6090	37.5737	35.6043	34.1432	32.9299	31.7911	30.4155	28.7362	24.6677
Flintstones	29.2074	26.3347	24.4995	23.1593	21.9779	20.9786	19.8595	18.7026	16.8283
Flower	32.1706	30.6349	29.4385	28.5534	27.6052	26.7642	25.8321	24.6199	22.0578
Parrot	31.8475	30.7047	29.7051	28.9493	27.8794	27.0998	26.2466	25.2348	22.4053
Dark-Haired Woman	45.2755	42.0290	40.1639	38.7253	37.3922	35.5937	34.0374	31.8919	25.4782
Blonde Woman	30.4752	29.4436	28.5275	27.7051	26.9854	26.3050	25.6730	24.5875	21.9160
Einstein	40.8248	37.9051	35.9893	34.4461	33.1982	32.1267	30.9430	29.1608	25.3858

Table 11: PSNR Results of DAMF for 20 Traditional Images

Images/Noise Densities	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
Lena	43.0722	39.1806	36.7452	34.7359	33.3103	31.7894	30.3164	28.5806	25.7651
Cameraman	43.6192	39.5829	36.7552	34.6482	32.8886	31.3170	29.6416	27.5994	24.7318
Barbara	33.8636	30.5432	28.4400	26.9131	25.6976	24.5875	23.5081	22.4145	21.0437
Baboon	37.9607	34.4894	32.2077	30.4524	28.8845	27.4818	25.9342	24.1806	21.8066
Peppers	41.2494	37.7822	35.6095	33.8821	32.3639	31.1079	29.8201	28.3911	25.8038
Living Room	38.6765	34.9505	32.6009	30.8271	29.2886	27.9849	26.7079	25.2042	23.1147
Lake	39.1066	35.4611	33.0569	31.4528	29.8093	28.3363	26.9560	25.1767	22.8061
Plane	42.3995	38.3668	35.9131	33.7627	32.0727	30.6427	29.0713	27.2153	24.3841
Hill	40.0487	36.6582	34.3673	32.6891	31.2202	29.9333	28.5978	27.3184	25.3023
Pirate	40.0788	36.5961	34.1419	32.4554	30.7721	29.4173	28.1059	26.5300	24.2527
Boat	39.0948	35.4225	33.0451	31.1233	29.7102	28.3002	26.9122	25.3603	23.2930
House	50.7381	46.0099	42.4789	39.9874	38.2217	36.5861	34.7749	32.6088	28.8869
Bridge	35.6434	32.2425	30.1506	28.5076	27.0818	25.7379	24.4374	23.0009	21.1559
Elaine	41.2573	38.0581	35.9929	34.3900	33.0205	31.7831	30.5781	29.1659	27.0981
Flintstones	35.4137	31.6940	29.1574	27.1173	25.4545	23.8569	22.3090	20.3632	17.8944
Flower	34.4083	33.2306	31.9779	30.8688	29.7092	28.5705	27.3606	25.8127	23.6013
Parrot	32.8146	32.1056	31.1059	30.5176	29.3763	28.4918	27.6130	26.4329	24.3276
Dark-Haired Woman	48.9694	45.1735	42.7511	40.8499	39.2145	37.6213	36.0276	34.1981	30.6662
Blonde Woman	31.9018	31.2015	30.2450	29.2191	28.3794	27.4408	26.5836	25.3487	23.5741
Einstein	44.5768	41.0258	38.7129	36.8561	35.2529	33.8460	32.3472	30.6530	28.3551

Table 12: PSNR Results of AWMF for 20 Traditional Images

Images/Noise Densities	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
Lena	39.1561	37.5021	36.1234	34.7751	33.6929	32.1790	30.6845	28.8292	26.0906
Cameraman	37.9716	37.3066	36.2143	35.0479	33.6967	32.1769	30.2475	27.9841	25.0121
Barbara	32.5713	30.4338	28.8274	27.4757	26.3150	25.1313	23.8951	22.6659	21.2493
Baboon	34.0175	32.6942	31.4953	30.4832	29.2673	27.9141	26.2827	24.4108	21.9899
Peppers	37.7660	36.8735	35.5794	34.3687	32.9569	31.6398	30.2108	28.6895	26.1339
Living Room	35.1127	33.3847	32.0812	30.9182	29.7218	28.4319	27.0841	25.4519	23.2958
Lake	35.6087	33.9485	32.6987	31.5054	30.1904	28.7826	27.3012	25.4475	23.0572
Plane	37.6156	36.2006	35.1936	33.9226	32.6311	31.2022	29.5165	27.5341	24.6750
Hill	37.3690	35.5461	34.1448	32.8744	31.6142	30.3612	28.9238	27.5469	25.5130
Pirate	36.6959	34.8282	33.5362	32.3863	31.0174	29.6935	28.3712	26.7371	24.4863
Boat	35.6108	33.7086	32.4205	31.1648	30.0042	28.7156	27.2543	25.5869	23.4884
House	44.5539	44.1149	42.9999	41.4964	39.7598	37.9130	35.6366	33.2106	29.6470
Bridge	33.0631	31.1503	29.7851	28.6056	27.4055	26.0757	24.7391	23.1963	21.3033
Elaine	39.3248	37.5328	36.0340	34.6495	33.3807	32.1213	30.8349	29.3584	27.3686
Flintstones	30.0982	29.1923	28.1912	27.0103	25.7642	24.3258	22.6890	20.6340	18.0690
Flower	32.5330	32.0272	31.3771	30.7335	29.8521	28.8265	27.5929	26.0116	23.8087
Parrot	32.1664	31.8779	31.2809	30.9221	30.2468	29.4604	28.4976	27.2598	25.1985
Dark-Haired Woman	45.5301	43.9980	42.6852	41.4225	39.9602	38.2895	36.5186	34.5991	31.4464
Blonde Woman	30.9504	30.6212	29.9981	29.2354	28.5513	27.6710	26.7552	25.5371	23.9030
Einstein	41.0321	39.4445	38.1746	36.8740	35.6258	34.2236	32.6971	30.8923	28.6541

Table 13: PSNR Results of ARmF for 20 Traditional Images

Images/Noise Densities	10%	20%	30%	40%	50%	60%	70%	80%	90%
Lena	43.1580	39.7282	37.5635	35.6664	34.2712	32.5274	30.8789	28.9058	26.1196
Cameraman	44.6038	41.0662	38.6506	36.5366	34.5724	32.6766	30.5298	28.1184	25.0654
Barbara	34.8083	31.5844	29.5445	28.0079	26.6725	25.3451	24.0153	22.7069	21.2355
Baboon	39.1855	35.9332	33.6522	31.8380	30.0907	28.4000	26.5327	24.5148	22.0104
Peppers	41.6044	38.4530	36.3822	34.7735	33.1802	31.7906	30.3074	28.7399	26.1455
Living Room	38.7920	35.6140	33.5774	31.9044	30.3921	28.8746	27.3436	25.5954	23.3628
Lake	39.6953	36.3422	34.1816	32.4972	30.7621	29.1288	27.5077	25.5517	23.0998
Plane	42.7624	39.3626	37.2593	35.1758	33.3924	31.6736	29.7830	27.6577	24.7243
Hill	40.5507	37.3953	35.3108	33.6869	32.1567	30.7051	29.1468	27.6591	25.5582
Pirate	40.1030	36.8744	34.7412	33.2099	31.5298	30.0225	28.5651	26.8251	24.5093
Boat	39.2023	35.7773	33.8393	32.0587	30.5992	29.0785	27.4691	25.7065	23.5358
House	52.4247	48.5287	45.3646	42.8265	40.4613	38.3474	35.8799	33.3543	29.7020
Bridge	36.1575	32.9129	30.9348	29.3513	27.9065	26.3840	24.9089	23.2821	21.3272
Elaine	41.2637	38.1574	36.2041	34.6569	33.3120	32.0313	30.7701	29.3243	27.3601
Flintstones	35.4229	32.2400	30.0330	28.1460	26.4628	24.7365	22.9234	20.7470	18.1079
Flower	34.3498	33.3375	32.3341	31.4062	30.3025	29.1281	27.7651	26.0971	23.8387
Parrot	33.0991	32.5886	31.8427	31.3716	30.5589	29.6963	28.6376	27.3240	25.2090
Dark-Haired Woman	49.6752	46.2660	44.0037	42.2059	40.4246	38.5998	36.7028	34.6823	31.4796
Blonde Woman	32.2503	31.4649	30.5499	29.6267	28.8134	27.8587	26.8804	25.5925	23.9200
Einstein	45.0130	41.6818	39.6201	37.8252	36.2129	34.5852	32.9004	30.9867	28.6707

Table 14: VIF Results of BPDF for 20 Traditional Images

Images/Noise Densities	10%	20%	30%	40%	50%	60%	70%	80%	90%
Lena	0.8554	0.7147	0.5871	0.4682	0.3596	0.2579	0.1587	0.0767	0.0342
Cameraman	0.8486	0.7162	0.6031	0.4957	0.3915	0.2893	0.1884	0.0982	0.0405
Barbara	0.7205	0.5693	0.4610	0.3643	0.2730	0.1931	0.1236	0.0591	0.0272
Baboon	0.7801	0.6221	0.4877	0.3755	0.2764	0.1915	0.1167	0.0579	0.0217
Peppers	0.8293	0.7048	0.5950	0.4934	0.3896	0.2888	0.1908	0.0942	0.0325
Living Room	0.7860	0.6374	0.5161	0.4073	0.3117	0.2236	0.1402	0.0708	0.0346
Lake	0.8060	0.6448	0.5310	0.4177	0.3098	0.2178	0.1340	0.0656	0.0326
Plane	0.8335	0.7021	0.5839	0.4621	0.3535	0.2497	0.1552	0.0713	0.0292
Hill	0.8243	0.6876	0.5644	0.4472	0.3357	0.2343	0.1448	0.0759	0.0374
Pirate	0.8194	0.6803	0.5537	0.4357	0.3310	0.2332	0.1420	0.0671	0.0309
Boat	0.8020	0.6455	0.5192	0.4085	0.3051	0.2181	0.1324	0.0651	0.0326
House	0.9195	0.8292	0.7155	0.6035	0.4798	0.3597	0.2202	0.1123	0.0456
Bridge	0.7415	0.5824	0.4590	0.3580	0.2658	0.1826	0.1119	0.0587	0.0287
Elaine	0.8386	0.7086	0.5878	0.4760	0.3685	0.2735	0.1754	0.0795	0.0326
Flintstones	0.6854	0.5350	0.4293	0.3425	0.2652	0.1962	0.1278	0.0654	0.0241
Flower	0.8261	0.6817	0.5410	0.4321	0.3181	0.2292	0.1403	0.0704	0.0327
Parrot	0.8225	0.7137	0.6090	0.4916	0.3950	0.2903	0.1807	0.0977	0.0356
Dark-Haired Woman	0.9187	0.8288	0.7289	0.6180	0.4972	0.3662	0.2273	0.1131	0.0444
Blonde Woman	0.7720	0.6399	0.5305	0.4259	0.3304	0.2381	0.1486	0.0690	0.0281
Einstein	0.8825	0.7586	0.6420	0.5261	0.4108	0.2921	0.1822	0.0833	0.0338

Table 15: VIF Results of DBAIN for 20 Traditional Images

Images/Noise Densities	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
Lena	0.8828	0.7577	0.6475	0.5301	0.4298	0.3334	0.2306	0.1417	0.0653
Cameraman	0.8889	0.7733	0.6622	0.5592	0.4508	0.3501	0.2517	0.1537	0.0726
Barbara	0.7456	0.6094	0.5016	0.4006	0.3154	0.2383	0.1682	0.1019	0.0484
Baboon	0.8154	0.6716	0.5459	0.4331	0.3333	0.2440	0.1605	0.0947	0.0395
Peppers	0.8694	0.7493	0.6356	0.5284	0.4286	0.3299	0.2313	0.1452	0.0698
Living Room	0.8306	0.6865	0.5663	0.4601	0.3677	0.2755	0.1957	0.1195	0.0536
Lake	0.8336	0.6981	0.5807	0.4735	0.3709	0.2813	0.1964	0.1204	0.0549
Plane	0.8724	0.7503	0.6415	0.5286	0.4244	0.3256	0.2318	0.1428	0.0641
Hill	0.8553	0.7317	0.6131	0.5043	0.3930	0.2955	0.2071	0.1299	0.0575
Pirate	0.8468	0.7228	0.6057	0.4967	0.3948	0.2957	0.2055	0.1235	0.0558
Boat	0.8300	0.6941	0.5785	0.4619	0.3709	0.2797	0.1940	0.1188	0.0549
House	0.9494	0.8832	0.7814	0.6778	0.5694	0.4608	0.3333	0.2143	0.1025
Bridge	0.7797	0.6234	0.5072	0.4031	0.3102	0.2307	0.1540	0.0929	0.0398
Elaine	0.8671	0.7462	0.6353	0.5250	0.4222	0.3280	0.2352	0.1504	0.0707
Flintstones	0.7567	0.6003	0.4793	0.3713	0.2875	0.2115	0.1452	0.0884	0.0398
Flower	0.8549	0.7346	0.5984	0.4947	0.3826	0.2932	0.2043	0.1255	0.0552
Parrot	0.8636	0.7622	0.6593	0.5563	0.4530	0.3575	0.2554	0.1627	0.0804
Dark-Haired Woman	0.9423	0.8662	0.7848	0.6865	0.5752	0.4561	0.3317	0.2114	0.1026
Blonde Woman	0.8179	0.6930	0.5840	0.4803	0.3808	0.2922	0.2032	0.1300	0.0603
Einstein	0.9094	0.8030	0.6939	0.5824	0.4693	0.3626	0.2553	0.1573	0.0708

Table 16: VIF Results of MDBUTMF for 20 Traditional Images

Images/Noise Densities	**10%**	**20%**	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**
Lena	0.8607	0.6995	0.5187	0.4539	0.4515	0.4263	0.3521	0.2374	0.0727
Cameraman	0.8564	0.6901	0.5169	0.4585	0.4503	0.4330	0.3605	0.2448	0.0963
Barbara	0.7251	0.5688	0.4333	0.3703	0.3513	0.3127	0.2532	0.1703	0.0625
Baboon	0.7425	0.5966	0.4702	0.4007	0.3612	0.3085	0.2439	0.1574	0.0466
Peppers	0.8661	0.7027	0.5253	0.4569	0.4560	0.4222	0.3494	0.2380	0.0806
Living Room	0.7889	0.6355	0.4974	0.4259	0.3913	0.3392	0.2749	0.1853	0.0641
Lake	0.8011	0.6294	0.4695	0.4167	0.3899	0.3590	0.3006	0.2080	0.0816
Plane	0.8367	0.6623	0.4933	0.4315	0.4199	0.4036	0.3380	0.2176	0.0684
Hill	0.8324	0.6795	0.5159	0.4494	0.4246	0.3847	0.3135	0.2202	0.0711
Pirate	0.8280	0.6763	0.5088	0.4501	0.4243	0.3820	0.3138	0.2113	0.0700
Boat	0.8070	0.6474	0.4984	0.4302	0.4041	0.3601	0.2916	0.1925	0.0651
House	0.9461	0.8139	0.6201	0.5682	0.5954	0.5804	0.5044	0.3707	0.1132
Bridge	0.7381	0.5800	0.4507	0.3881	0.3483	0.3024	0.2359	0.1629	0.0590
Elaine	0.8662	0.7123	0.5304	0.4651	0.4638	0.4384	0.3634	0.2359	0.0643
Flintstones	0.7174	0.5640	0.4385	0.3785	0.3427	0.2927	0.2353	0.1673	0.0732
Flower	0.8377	0.6880	0.5254	0.4646	0.4417	0.3998	0.3276	0.2284	0.0802
Parrot	0.8157	0.6559	0.4827	0.4270	0.4373	0.4234	0.3568	0.2344	0.0639
Dark-Haired Woman	0.9270	0.7556	0.5327	0.4930	0.5377	0.5459	0.4804	0.3218	0.0945
Blonde Woman	0.7535	0.6203	0.4700	0.4087	0.3901	0.3577	0.2948	0.1888	0.0543
Einstein	0.8781	0.7283	0.5490	0.4768	0.4701	0.4312	0.3509	0.2241	0.0636

Table 17: VIF Results of NAFSMF for 20 Traditional Images

Images/Noise Densities	10%	20%	30%	40%	50%	60%	70%	80%	90%
Lena	0.8275	0.7144	0.6225	0.5397	0.4629	0.3897	0.3159	0.2327	0.1318
Cameraman	0.7949	0.6862	0.5952	0.5088	0.4382	0.3720	0.2947	0.2166	0.1242
Barbara	0.7198	0.5972	0.5126	0.4400	0.3747	0.3088	0.2476	0.1798	0.1063
Baboon	0.6878	0.5613	0.4659	0.3923	0.3248	0.2604	0.1970	0.1306	0.0683
Peppers	0.8436	0.7329	0.6436	0.5511	0.4729	0.4014	0.3247	0.2423	0.1388
Living Room	0.7467	0.6186	0.5251	0.4464	0.3736	0.3062	0.2411	0.1718	0.0927
Lake	0.7549	0.6242	0.5298	0.4574	0.3779	0.3186	0.2578	0.1896	0.1098
Plane	0.7877	0.6682	0.5846	0.4994	0.4262	0.3538	0.2847	0.2134	0.1223
Hill	0.7992	0.6779	0.5800	0.4993	0.4192	0.3488	0.2744	0.2005	0.1127
Pirate	0.7942	0.6724	0.5737	0.4977	0.4167	0.3433	0.2783	0.2004	0.1191
Boat	0.7634	0.6314	0.5379	0.4550	0.3878	0.3203	0.2529	0.1789	0.1013
House	0.9113	0.8318	0.7417	0.6630	0.5859	0.5052	0.4175	0.3297	0.1942
Bridge	0.6999	0.5626	0.4717	0.3980	0.3320	0.2709	0.2056	0.1453	0.0823
Elaine	0.8556	0.7498	0.6617	0.5766	0.5016	0.4291	0.3479	0.2628	0.1532
Flintstones	0.6356	0.5081	0.4290	0.3624	0.3048	0.2507	0.1969	0.1462	0.0864
Flower	0.7876	0.6636	0.5673	0.4924	0.4205	0.3555	0.2867	0.2175	0.1278
Parrot	0.7942	0.6929	0.6119	0.5407	0.4643	0.3981	0.3182	0.2433	0.1366
Dark-Haired Woman	0.9059	0.8237	0.7445	0.6687	0.5906	0.5024	0.4195	0.3241	0.1890
Blonde Woman	0.7279	0.6240	0.5387	0.4664	0.3943	0.3319	0.2677	0.1926	0.1118
Einstein	0.8518	0.7407	0.6421	0.5537	0.4720	0.3904	0.3053	0.2167	0.1241

Table 18: VIF Results of DAMF for 20 Traditional Images

Images/Noise Densities	10%	20%	30%	40%	50%	60%	70%	80%	90%
Lena	0.9072	0.8137	0.7321	0.6481	0.5732	0.5041	0.4266	0.3293	0.1963
Cameraman	0.9179	0.8333	0.7518	0.6759	0.6060	0.5364	0.4573	0.3594	0.2173
Barbara	0.7742	0.6564	0.5698	0.4916	0.4259	0.3598	0.2985	0.2273	0.1423
Baboon	0.8450	0.7306	0.6309	0.5485	0.4677	0.3939	0.3156	0.2296	0.1258
Peppers	0.8915	0.7951	0.7111	0.6277	0.5544	0.4849	0.4099	0.3219	0.1993
Living Room	0.8535	0.7384	0.6380	0.5533	0.4750	0.4072	0.3383	0.2578	0.1533
Lake	0.8635	0.7538	0.6608	0.5870	0.5075	0.4361	0.3667	0.2791	0.1725
Plane	0.9044	0.8096	0.7291	0.6451	0.5697	0.5054	0.4278	0.3360	0.2038
Hill	0.8755	0.7754	0.6849	0.6035	0.5252	0.4519	0.3789	0.2951	0.1780
Pirate	0.8762	0.7763	0.6809	0.6026	0.5234	0.4471	0.3737	0.2865	0.1727
Boat	0.8648	0.7481	0.6545	0.5688	0.4987	0.4277	0.3522	0.2699	0.1604
House	0.9750	0.9350	0.8785	0.8228	0.7670	0.7028	0.6187	0.5157	0.3330
Bridge	0.7975	0.6739	0.5770	0.4964	0.4246	0.3562	0.2884	0.2164	0.1265
Elaine	0.8812	0.7848	0.7016	0.6160	0.5424	0.4692	0.3995	0.3108	0.1999
Flintstones	0.8112	0.6793	0.5769	0.4896	0.4180	0.3501	0.2847	0.2100	0.1255
Flower	0.8941	0.7954	0.6958	0.6125	0.5353	0.4655	0.3867	0.3001	0.1846
Parrot	0.8760	0.7975	0.7196	0.6509	0.5760	0.5107	0.4341	0.3453	0.2184
Dark-Haired Woman	0.9560	0.9065	0.8534	0.7958	0.7343	0.6656	0.5837	0.4756	0.3104
Blonde Woman	0.7994	0.7078	0.6242	0.5495	0.4783	0.4121	0.3442	0.2631	0.1656
Einstein	0.9221	0.8415	0.7630	0.6826	0.6058	0.5277	0.4421	0.3374	0.2033

Table 19: VIF Results of AWMF for 20 Traditional Images

Images/Noise Densities	10%	20%	30%	40%	50%	60%	70%	80%	90%
Lena	0.8267	0.7706	0.7139	0.6474	0.5836	0.5141	0.4339	0.3326	0.1985
Cameraman	0.8176	0.7780	0.7324	0.6759	0.6175	0.5497	0.4657	0.3632	0.2193
Barbara	0.7056	0.6306	0.5673	0.5023	0.4418	0.3738	0.3076	0.2318	0.1438
Baboon	0.7399	0.6837	0.6169	0.5554	0.4822	0.4073	0.3240	0.2333	0.1273
Peppers	0.7821	0.7558	0.7005	0.6363	0.5693	0.4962	0.4168	0.3254	0.2004
Living Room	0.7557	0.6877	0.6215	0.5574	0.4887	0.4195	0.3462	0.2608	0.1542
Lake	0.7797	0.7118	0.6490	0.5888	0.5166	0.4455	0.3729	0.2814	0.1738
Plane	0.8071	0.7544	0.7047	0.6442	0.5786	0.5139	0.4340	0.3383	0.2054
Hill	0.8038	0.7352	0.6726	0.6059	0.5362	0.4625	0.3858	0.2979	0.1792
Pirate	0.7914	0.7213	0.6596	0.5990	0.5299	0.4544	0.3787	0.2884	0.1736
Boat	0.7711	0.6998	0.6389	0.5743	0.5099	0.4386	0.3601	0.2734	0.1619
House	0.9199	0.9012	0.8701	0.8329	0.7840	0.7208	0.6318	0.5233	0.3387
Bridge	0.7108	0.6316	0.5636	0.5001	0.4354	0.3658	0.2940	0.2191	0.1272
Elaine	0.8245	0.7585	0.6935	0.6193	0.5491	0.4769	0.4032	0.3123	0.2008
Flintstones	0.6396	0.5992	0.5489	0.4861	0.4245	0.3599	0.2895	0.2119	0.1262
Flower	0.7874	0.7329	0.6703	0.6111	0.5461	0.4756	0.3939	0.3025	0.1852
Parrot	0.7963	0.7537	0.7012	0.6468	0.5827	0.5200	0.4406	0.3495	0.2227
Dark-Haired Woman	0.9113	0.8771	0.8390	0.7941	0.7401	0.6735	0.5906	0.4794	0.3158
Blonde Woman	0.7299	0.6743	0.6133	0.5516	0.4865	0.4195	0.3496	0.2656	0.1669
Einstein	0.8544	0.7998	0.7439	0.6807	0.6131	0.5358	0.4490	0.3404	0.2049

Table 20: VIF Results of ARmF for 20 Traditional Images

Images/Noise Densities	10%	20%	30%	40%	50%	60%	70%	80%	90%
Lena	0.9077	0.8262	0.7549	0.6770	0.6048	0.5288	0.4437	0.3377	0.2011
Cameraman	0.9264	0.8580	0.7901	0.7173	0.6482	0.5705	0.4795	0.3714	0.2234
Barbara	0.7908	0.6833	0.6034	0.5275	0.4584	0.3842	0.3139	0.2347	0.1448
Baboon	0.8690	0.7715	0.6778	0.5948	0.5079	0.4233	0.3329	0.2380	0.1294
Peppers	0.8966	0.8123	0.7332	0.6573	0.5835	0.5067	0.4239	0.3298	0.2025
Living Room	0.8540	0.7564	0.6693	0.5907	0.5114	0.4347	0.3557	0.2668	0.1577
Lake	0.8764	0.7775	0.6941	0.6200	0.5367	0.4585	0.3811	0.2865	0.1762
Plane	0.9084	0.8282	0.7598	0.6812	0.6045	0.5319	0.4456	0.3448	0.2089
Hill	0.8824	0.7928	0.7134	0.6355	0.5575	0.4764	0.3953	0.3035	0.1824
Pirate	0.8747	0.7832	0.7007	0.6287	0.5499	0.4681	0.3875	0.2932	0.1756
Boat	0.8656	0.7622	0.6826	0.6021	0.5296	0.4523	0.3679	0.2784	0.1646
House	0.9786	0.9511	0.9116	0.8672	0.8106	0.7417	0.6467	0.5323	0.3431
Bridge	0.8080	0.6946	0.6045	0.5272	0.4533	0.3774	0.3006	0.2232	0.1293
Elaine	0.8815	0.7888	0.7113	0.6299	0.5571	0.4819	0.4069	0.3145	0.2021
Flintstones	0.8045	0.6905	0.6006	0.5186	0.4442	0.3718	0.2966	0.2152	0.1278
Flower	0.8861	0.8017	0.7182	0.6439	0.5679	0.4910	0.4029	0.3073	0.1872
Parrot	0.8801	0.8089	0.7400	0.6771	0.6031	0.5354	0.4509	0.3557	0.2256
Dark-Haired Woman	0.9603	0.9187	0.8737	0.8222	0.7626	0.6912	0.6030	0.4876	0.3194
Blonde Woman	0.8057	0.7202	0.6424	0.5714	0.5004	0.4292	0.3560	0.2690	0.1685
Einstein	0.9277	0.8558	0.7874	0.7140	0.6383	0.5539	0.4610	0.3476	0.2084

In this PVA problem, we construct *ifpifs*-matrices $[b_{ij}]_{8\times 9}$ and $[c_{ij}]_{8\times 9}$ by using the values provided in Tables 7-13 and Tables 14-20, respectively. Since the PSNR values are not in the interval $[0,1]$, they are normalized by dividing the PSNR values produced by all the filters for each image by the maximum of these values to construct the *ifpifs*-matrix $[b_{ij}]$. After, we get intuitionistic fuzzy values in the nonzero-indexed rows of the *ifpifs*-matrix $[b_{ij}]$ using the membership function and non-membership function defined by

$$\mu^b_{ij} := \min_t P^t_{ij} \quad \text{and} \quad \nu^b_{ij} := 1 - \max_t P^t_{ij}$$

such that $i \in I_{m-1}$ and $j \in I_n$. Here, the notation $\left(P^t_{ij}\right)$ states the ordered s-tuples such that P^t_{ij} corresponds to normalized PSNR values originating from t^{th} image for i^{th} filter and j^{th} noise density. Moreover, s is the number of images. That is, $s = 20$ herein. To exemplify, for BPDF ($i = 1$) and 10% noise density ($j = 1$),

$$\begin{aligned}(P^t_{11}) &= \Big(\tfrac{40.1207}{43.1580}, \tfrac{39.3852}{44.6038}, \tfrac{32.7983}{34.8083}, \tfrac{35.4468}{39.1855}, \tfrac{38.2033}{41.6044}, \tfrac{35.9376}{38.7920}, \tfrac{36.6876}{39.6953}, \tfrac{38.4801}{42.7624}, \tfrac{37.8776}{40.5507}, \tfrac{37.5385}{40.1030}, \tfrac{36.4823}{39.2023}, \tfrac{43.5902}{52.4247}, \\ &\quad \tfrac{33.8422}{36.1575}, \tfrac{39.5501}{41.2637}, \tfrac{31.0347}{35.4229}, \tfrac{33.4606}{34.4083}, \tfrac{31.8533}{33.0991}, \tfrac{44.9712}{49.6752}, \tfrac{32.1376}{32.2503}, \tfrac{41.8411}{45.0130}\Big) \\ &= (0.9296, 0.8830, 0.9423, 0.9046, 0.9183, 0.9264, 0.9242, 0.8999, 0.9341, 0.9361, 0.9306, 0.8315, \\ &\quad 0.9360, 0.9585, 0.8761, 0.9725, 0.9624, 0.9053, 0.9965, 0.9295)\end{aligned}$$

Here, for $t = 1$ (Lena), since

$$\begin{aligned}\max\{&40.1207, 35.8460, 32.8329, 30.5127, 28.3562, 26.0197, 23.0085, 18.2969, 10.7110, \\ &41.7024, 37.4327, 34.7209, 32.0906, 30.2490, 28.1340, 25.5840, 22.9730, 19.8600, \\ &40.5308, 35.6049, 31.4577, 30.3932, 31.0401, 30.7714, 29.4379, 25.7451, 16.4201, \\ &38.8408, 35.7244, 33.6318, 32.2185, 31.1273, 29.8105, 28.6769, 27.0939, 23.4284, \\ &43.0722, 39.1806, 36.7452, 34.7359, 33.3103, 31.7894, 30.3164, 28.5806, 25.7651, \\ &39.1561, 37.5021, 36.1234, 34.7751, 33.6929, 32.1790, 30.6845, 28.8292, 26.0906, \\ &43.1580, 39.7282, 37.5635, 35.6664, 34.2712, 32.5274, 30.8789, 28.9058, 26.1196\} = 43.1580\end{aligned}$$

then $P^1_{11} = \frac{40.1207}{43.1580} = 0.9296$. Therefore, $\mu^b_{11} := \min_t P^t_{11} = 0.8315$ and $\nu^b_{11} := 1 - \max_t P^t_{11} = 1 - 0.9965 = 0.0035$. Similarly, the values of the other alternatives can be calculated. Likewise, we get intuitionistic fuzzy values in the nonzero-indexed rows of the *ifpifs*-matrix $[c_{ij}]$ using the membership function and non-membership function defined by

$$\mu^c_{ij} := \min_t V^t_{ij} \quad \text{and} \quad \nu^c_{ij} := 1 - \max_t V^t_{ij}$$

such that $i \in I_{m-1}$ and $j \in I_n$. Here, the notation $\left(V^t_{ij}\right)$ states the ordered s-tuples such that V^t_{ij} corresponds to VIF values originating from t^{th} image for i^{th} filter and j^{th} noise density. Moreover, s is the number of images. That is, $s = 20$ herein. Besides, suppose that the noise removal success of the filters at high noise densities is more significant than at the other densities, it is anticipated that the membership degrees at high noise densities are greater than the non-membership degrees and the former at low noise densities are smaller than the latter. In other words, we consider the first rows of $[b_{ij}]$ and $[c_{ij}]$ to be

$$\begin{bmatrix} 0.05 & 0.15 & 0.25 & 0.35 & 0.5 & 0.65 & 0.75 & 0.85 & 0.9 \\ 0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.3 & 0.2 & 0.1 & 0.05 \end{bmatrix}$$

Hence, the *ifpifs*-matrices $[b_{ij}]$ and $[c_{ij}]$ are constructed as follows:

$$[b_{ij}] = \begin{bmatrix} 0.05 & 0.15 & 0.25 & 0.35 & 0.5 & 0.65 & 0.75 & 0.85 & 0.9 \\ 0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.3 & 0.2 & 0.1 & 0.05 \\ 0.8315 & 0.7582 & 0.6932 & 0.6373 & 0.5787 & 0.5286 & 0.4599 & 0.3667 & 0.1864 \\ 0.0035 & 0.0549 & 0.0995 & 0.1429 & 0.1828 & 0.2345 & 0.3008 & 0.4048 & 0.6592 \\ 0.8726 & 0.7964 & 0.7314 & 0.6733 & 0.6226 & 0.5654 & 0.5042 & 0.4462 & 0.3712 \\ 0.0007 & 0.0264 & 0.0597 & 0.0969 & 0.1359 & 0.1834 & 0.2303 & 0.2874 & 0.3706 \\ 0.7891 & 0.7263 & 0.6166 & 0.6000 & 0.6445 & 0.6368 & 0.5995 & 0.5129 & 0.3021 \\ 0.0073 & 0.0752 & 0.1258 & 0.1485 & 0.1527 & 0.1645 & 0.1863 & 0.2639 & 0.4933 \\ 0.8215 & 0.7434 & 0.6916 & 0.6538 & 0.6204 & 0.5922 & 0.5606 & 0.5280 & 0.4751 \\ 0.0159 & 0.0723 & 0.1025 & 0.1254 & 0.1577 & 0.1813 & 0.2039 & 0.2376 & 0.3204 \\ 0.9678 & 0.8775 & 0.8103 & 0.7628 & 0.7186 & 0.6735 & 0.6298 & 0.5749 & 0.5052 \\ 0 & 0.0300 & 0.0602 & 0.0780 & 0.1125 & 0.1392 & 0.1657 & 0.2014 & 0.2650 \\ 0.8497 & 0.8241 & 0.7958 & 0.7625 & 0.7273 & 0.6867 & 0.6405 & 0.5825 & 0.5101 \\ 0.0282 & 0.0369 & 0.0549 & 0.0658 & 0.0862 & 0.1099 & 0.1390 & 0.1764 & 0.2387 \\ 0.9983 & 0.9074 & 0.8478 & 0.7946 & 0.7471 & 0.6983 & 0.6471 & 0.5857 & 0.5112 \\ 0 & 0.0154 & 0.0380 & 0.0522 & 0.0767 & 0.1028 & 0.1348 & 0.1745 & 0.2384 \end{bmatrix}$$

and

$$[c_{ij}] = \begin{bmatrix} 0.05 & 0.15 & 0.25 & 0.35 & 0.5 & 0.65 & 0.75 & 0.85 & 0.9 \\ 0.9 & 0.8 & 0.7 & 0.6 & 0.5 & 0.3 & 0.2 & 0.1 & 0.05 \\ 0.6854 & 0.5350 & 0.4293 & 0.3425 & 0.2652 & 0.1826 & 0.1119 & 0.0579 & 0.0217 \\ 0.0805 & 0.1708 & 0.2711 & 0.3820 & 0.5028 & 0.6338 & 0.7727 & 0.8869 & 0.9544 \\ 0.7456 & 0.6003 & 0.4793 & 0.3713 & 0.2875 & 0.2115 & 0.1452 & 0.0884 & 0.0395 \\ 0.0506 & 0.1168 & 0.2152 & 0.3135 & 0.4248 & 0.5392 & 0.6667 & 0.7857 & 0.8974 \\ 0.7174 & 0.5640 & 0.4333 & 0.3703 & 0.3427 & 0.2927 & 0.2353 & 0.1574 & 0.0466 \\ 0.0539 & 0.1861 & 0.3799 & 0.4318 & 0.4046 & 0.4196 & 0.4956 & 0.6293 & 0.8868 \\ 0.6356 & 0.5081 & 0.4290 & 0.3624 & 0.3048 & 0.2507 & 0.1969 & 0.1306 & 0.0683 \\ 0.0887 & 0.1682 & 0.2555 & 0.3313 & 0.4094 & 0.4948 & 0.5805 & 0.6703 & 0.8058 \\ 0.7742 & 0.6564 & 0.5698 & 0.4896 & 0.4180 & 0.3501 & 0.2847 & 0.2100 & 0.1255 \\ 0.0250 & 0.0650 & 0.1215 & 0.1772 & 0.2330 & 0.2972 & 0.3813 & 0.4843 & 0.6670 \\ 0.6396 & 0.5992 & 0.5489 & 0.4861 & 0.4245 & 0.3599 & 0.2895 & 0.2119 & 0.1262 \\ 0.0801 & 0.0988 & 0.1299 & 0.1671 & 0.2160 & 0.2792 & 0.3682 & 0.4767 & 0.6613 \\ 0.7908 & 0.6833 & 0.6006 & 0.5186 & 0.4442 & 0.3718 & 0.2966 & 0.2152 & 0.1278 \\ 0.0214 & 0.0489 & 0.0884 & 0.1328 & 0.1894 & 0.2583 & 0.3533 & 0.4677 & 0.6569 \end{bmatrix}$$

Table 21 presents decision sets produced by the generalized SDM methods having passed all the test cases in Section 3. 11 of these generalized SDM methods, namely iCE10o, iEMO18oa, iCE10on, iEMA18onan, iCE10/2o, iCE10/2on, iEMC19oa, iZZ16(0.4), iZZ16$\begin{pmatrix}0.4\\0.4\end{pmatrix}$, iZZ16/2(0.4), and iZZ16/2$\begin{pmatrix}0.4\\0.4\end{pmatrix}$, employ two *ifpifs*-matrices. Besides, iRM11o and iEM20oa utilize three *ifpifs*-matrices, and the others work with multiple *ifpifs*-matrices. For this reason, the first 11 SDM methods employ the *ifpifs*-matrices $[a_{ij}]$ and $[b_{ij}]$, and the others utilize the *ifpifs*-matrices $[a_{ij}]$, $[b_{ij}]$, and $[c_{ij}]$.

Table 21: Decision Sets Produced by the Generalized SDM Methods*

Algorithms	**Decision Sets**
iCE10o	$\{{}^{0.4194}_{0.0031}\text{BPDF}, {}^{0.4618}_{0.0006}\text{DBAIN}, {}^{0.5213}_{0.0045}\text{MDBUTMF}, {}^{0.4905}_{0.0078}\text{NAFSMF}, {}^{0.5667}_{0}\text{DAMF}, {}^{0.5726}_{0.0053}\text{AWMF}, {}^{0.5796}_{0}\text{ARmF}\}$
iEMO18oa	$\{{}^{0.4194}_{0.0031}\text{BPDF}, {}^{0.4618}_{0.0006}\text{DBAIN}, {}^{0.5213}_{0.0045}\text{MDBUTMF}, {}^{0.4905}_{0.0078}\text{NAFSMF}, {}^{0.5667}_{0}\text{DAMF}, {}^{0.5726}_{0.0053}\text{AWMF}, {}^{0.5796}_{0}\text{ARmF}\}$
iCE10on	$\{{}^{0.4194}_{0.0056}\text{BPDF}, {}^{0.4618}_{0.0028}\text{DBAIN}, {}^{0.5213}_{0.0045}\text{MDBUTMF}, {}^{0.4905}_{0.0078}\text{NAFSMF}, {}^{0.5667}_{0.0016}\text{DAMF}, {}^{0.5726}_{0.0053}\text{AWMF}, {}^{0.5796}_{0.0012}\text{ARmF}\}$
iEMA18onan	$\{{}^{0.3449}_{0.0056}\text{BPDF}, {}^{0.3793}_{0.0028}\text{DBAIN}, {}^{0.4496}_{0.0066}\text{MDBUTMF}, {}^{0.4488}_{0.0143}\text{NAFSMF}, {}^{0.4886}_{0.0016}\text{DAMF}, {}^{0.4951}_{0.0119}\text{AWMF}, {}^{0.4978}_{0.0012}\text{ARmF}\}$
iCE10/2o	$\{{}^{0.4194}_{0.0050}\text{BPDF}, {}^{0.4618}_{0.0025}\text{DBAIN}, {}^{0.5213}_{0.0045}\text{MDBUTMF}, {}^{0.4905}_{0.0078}\text{NAFSMF}, {}^{0.5667}_{0.0015}\text{DAMF}, {}^{0.5726}_{0.0053}\text{AWMF}, {}^{0.5796}_{0.0011}\text{ARmF}\}$
iCE10/2on	$\{{}^{0.4194}_{0.0056}\text{BPDF}, {}^{0.4618}_{0.0028}\text{DBAIN}, {}^{0.5213}_{0.0045}\text{MDBUTMF}, {}^{0.4905}_{0.0078}\text{NAFSMF}, {}^{0.5667}_{0.0016}\text{DAMF}, {}^{0.5726}_{0.0053}\text{AWMF}, {}^{0.5796}_{0.0012}\text{ARmF}\}$
iEMC19oa	$\{{}^{0.4194}_{0.0031}\text{BPDF}, {}^{0.4618}_{0.0006}\text{DBAIN}, {}^{0.5213}_{0.0066}\text{MDBUTMF}, {}^{0.4905}_{0.0143}\text{NAFSMF}, {}^{0.5667}_{0}\text{DAMF}, {}^{0.5726}_{0.0083}\text{AWMF}, {}^{0.5796}_{0}\text{ARmF}\}$
iZZ16(0.4)	$\{{}^{0.0724}_{0.0945}\text{BPDF}, {}^{0.1132}_{0.0625}\text{DBAIN}, {}^{0.1261}_{0.0815}\text{MDBUTMF}, {}^{0.1643}_{0.0752}\text{NAFSMF}, {}^{0.1836}_{0.0504}\text{DAMF}, {}^{0.1859}_{0.0461}\text{AWMF}, {}^{0.1882}_{0.0362}\text{ARmF}\}$
iZZ16$\binom{0.4}{0.4}$	$\{{}^{0.0724}_{0.1076}\text{BPDF}, {}^{0.1132}_{0.0695}\text{DBAIN}, {}^{0.1261}_{0.0868}\text{MDBUTMF}, {}^{0.1643}_{0.0752}\text{NAFSMF}, {}^{0.1836}_{0.0504}\text{DAMF}, {}^{0.1859}_{0.0461}\text{AWMF}, {}^{0.1882}_{0.0362}\text{ARmF}\}$
iZZ16/2(0.4)	$\{{}^{0.3320}_{0.0074}\text{BPDF}, {}^{0.4087}_{0.0045}\text{DBAIN}, {}^{0.4206}_{0.0186}\text{MDBUTMF}, {}^{0.4721}_{0.0062}\text{NAFSMF}, {}^{0.5229}_{0.0020}\text{DAMF}, {}^{0.5256}_{0.0027}\text{AWMF}, {}^{0.5306}_{0.0014}\text{ARmF}\}$
iZZ16/2$\binom{0.4}{0.4}$	$\{{}^{0.3320}_{0.0079}\text{BPDF}, {}^{0.4087}_{0.0046}\text{DBAIN}, {}^{0.4206}_{0.0187}\text{MDBUTMF}, {}^{0.4721}_{0.0062}\text{NAFSMF}, {}^{0.5229}_{0.0020}\text{DAMF}, {}^{0.5256}_{0.0027}\text{AWMF}, {}^{0.5306}_{0.0014}\text{ARmF}\}$
iRM11o	$\{{}^{0.4194}_{0.3509}\text{BPDF}, {}^{0.4618}_{0.2673}\text{DBAIN}, {}^{0.5213}_{0.2223}\text{MDBUTMF}, {}^{0.4905}_{0.1974}\text{NAFSMF}, {}^{0.5667}_{0.1562}\text{DAMF}, {}^{0.5726}_{0.1500}\text{AWMF}, {}^{0.5796}_{0.1486}\text{ARmF}\}$
iEM20oa	$\{{}^{0.4194}_{0.0031}\text{BPDF}, {}^{0.4618}_{0.0006}\text{DBAIN}, {}^{0.5213}_{0.0066}\text{MDBUTMF}, {}^{0.4905}_{0.0143}\text{NAFSMF}, {}^{0.5667}_{0}\text{DAMF}, {}^{0.5726}_{0.0083}\text{AWMF}, {}^{0.5796}_{0}\text{ARmF}\}$
iZ14(I_9)	$\{{}^{0.8196}_{0.1412}\text{BPDF}, {}^{0.8919}_{0.0837}\text{DBAIN}, {}^{0.9094}_{0.0303}\text{MDBUTMF}, {}^{0.9244}_{0.0514}\text{NAFSMF}, {}^{0.9834}_{0.0066}\text{DAMF}, {}^{0.9870}_{0.0025}\text{AWMF}, {}^{0.9927}_{0}\text{ARmF}\}$
iDB12	$\{{}^{0}_{0.7952}\text{BPDF}, {}^{0.1506}_{0.8402}\text{DBAIN}, {}^{0.2140}_{0.5878}\text{MDBUTMF}, {}^{0.2074}_{0.6420}\text{NAFSMF}, {}^{0.4571}_{0.3699}\text{DAMF}, {}^{0.5284}_{0.4108}\text{AWMF}, {}^{0.6711}_{0}\text{ARmF}\}$
isDB12	$\{{}^{0}_{0.7952}\text{BPDF}, {}^{0.1506}_{0.8402}\text{DBAIN}, {}^{0.2140}_{0.5878}\text{MDBUTMF}, {}^{0.2074}_{0.6420}\text{NAFSMF}, {}^{0.4571}_{0.3699}\text{DAMF}, {}^{0.5284}_{0.4108}\text{AWMF}, {}^{0.6711}_{0}\text{ARmF}\}$
iCD12(2)	$\{{}^{0.0829}_{0.1512}\text{BPDF}, {}^{0.0976}_{0.1232}\text{DBAIN}, {}^{0.1191}_{0.1397}\text{MDBUTMF}, {}^{0.1086}_{0.1299}\text{NAFSMF}, {}^{0.1547}_{0.0700}\text{DAMF}, {}^{0.1540}_{0.0765}\text{AWMF}, {}^{0.1620}_{0.0580}\text{ARmF}\}$
iCD12/2(2)	$\{{}^{0.2622}_{0.0251}\text{BPDF}, {}^{0.3085}_{0.0156}\text{DBAIN}, {}^{0.3197}_{0.0386}\text{MDBUTMF}, {}^{0.3292}_{0.0155}\text{NAFSMF}, {}^{0.3670}_{0.0064}\text{DAMF}, {}^{0.3693}_{0.0067}\text{AWMF}, {}^{0.3729}_{0.0047}\text{ARmF}\}$
iE15	$\{{}^{0.7406}_{0.1066}\text{BPDF}, {}^{0.8087}_{0.0750}\text{DBAIN}, {}^{0.8356}_{0.0126}\text{MDBUTMF}, {}^{0.8441}_{0.0285}\text{NAFSMF}, {}^{0.9236}_{0.0057}\text{DAMF}, {}^{0.9257}_{0}\text{AWMF}, {}^{0.9384}_{0.0059}\text{ARmF}\}$
iYJ11	$\{{}^{0.0829}_{0.1512}\text{BPDF}, {}^{0.0976}_{0.1232}\text{DBAIN}, {}^{0.1191}_{0.1397}\text{MDBUTMF}, {}^{0.1086}_{0.1299}\text{NAFSMF}, {}^{0.1547}_{0.0700}\text{DAMF}, {}^{0.1540}_{0.0765}\text{AWMF}, {}^{0.1620}_{0.0580}\text{ARmF}\}$
iYJ11/2	$\{{}^{0.2622}_{0.0251}\text{BPDF}, {}^{0.3085}_{0.0156}\text{DBAIN}, {}^{0.3197}_{0.0386}\text{MDBUTMF}, {}^{0.3292}_{0.0155}\text{NAFSMF}, {}^{0.3670}_{0.0064}\text{DAMF}, {}^{0.3693}_{0.0067}\text{AWMF}, {}^{0.3729}_{0.0047}\text{ARmF}\}$
iMR13	$\{{}^{0.0829}_{0.1512}\text{BPDF}, {}^{0.0976}_{0.1232}\text{DBAIN}, {}^{0.1191}_{0.1397}\text{MDBUTMF}, {}^{0.1086}_{0.1299}\text{NAFSMF}, {}^{0.1547}_{0.0700}\text{DAMF}, {}^{0.1540}_{0.0765}\text{AWMF}, {}^{0.1620}_{0.0580}\text{ARmF}\}$
iNKY17(γ)	$\{{}^{0}_{0.8171}\text{BPDF}, {}^{0.1671}_{0.8105}\text{DBAIN}, {}^{0.1914}_{0.6243}\text{MDBUTMF}, {}^{0.2276}_{0.5895}\text{NAFSMF}, {}^{0.4546}_{0.3651}\text{DAMF}, {}^{0.5342}_{0.4013}\text{AWMF}, {}^{0.6724}_{0}\text{ARmF}\}$

*In the event that noise removal performance at high noise densities is more important. Here, $\gamma = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$.

The intuitionistic fuzzy values in the decision sets provided in Table 21 are produced on MATLAB R2021b. Further, using the relation presented in Proposition 2.1 [1], the ranking orders of the filters are presented in Table 22. The number(s) of the SDM methods producing the same ranking order is signified in the last column of Table 22. According to the table, iCE10o, iEMO18oa, iCE10on, iCE10/2o, iCE10/2on, iRM11o, iZ14(I_9), iDB12, isDB12, and iE15 have the same ranking orders just as iEMA18onan, iEMC19oa, iEM20oa, iCD12(2), iYJ11, and iMR13 do. Besides, iZZ16(0.4), iZZ16$\binom{0.4}{0.4}$, iZZ16/2(0.4), iZZ16/2$\binom{0.4}{0.4}$, iCD12/2(2), and iYJ11/2 generate the same ranking orders. iNKY17(γ) produces a ranking order different from the others. Although the decision-making abilities of all the SDM methods herein differ, all signify that ARmF has the highest noise removal performance and BPDF has the lowest noise removal performance.

Table 22: Ranking Orders of the Generalized SDM Methods*

Algorithms	**Ranking Orders**	**Frequency**
iCE10o	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺DAMF≺AWMF≺ARmF	10
iEMO18oa	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺DAMF≺AWMF≺ARmF	10
iCE10on	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺DAMF≺AWMF≺ARmF	10
iEMA18onan	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺AWMF≺DAMF≺ARmF	6
iCE10/2o	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺DAMF≺AWMF≺ARmF	10
iCE10/2on	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺DAMF≺AWMF≺ARmF	10
iEMC19oa	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺AWMF≺DAMF≺ARmF	6
iZZ16(0.4)	BPDF≺MDBUTMF≺DBAIN≺NAFSMF≺DAMF≺AWMF≺ARmF	6
iZZ16$\binom{0.4}{0.4}$	BPDF≺MDBUTMF≺DBAIN≺NAFSMF≺DAMF≺AWMF≺ARmF	6
iZZ16/2(0.4)	BPDF≺MDBUTMF≺DBAIN≺NAFSMF≺DAMF≺AWMF≺ARmF	6
iZZ16/2$\binom{0.4}{0.4}$	BPDF≺MDBUTMF≺DBAIN≺NAFSMF≺DAMF≺AWMF≺ARmF	6
iRM11o	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺DAMF≺AWMF≺ARmF	10
iEM20oa	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺AWMF≺DAMF≺ARmF	6
iZ14(I_9)	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺DAMF≺AWMF≺ARmF	10
iDB12	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺DAMF≺AWMF≺ARmF	10
isDB12	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺DAMF≺AWMF≺ARmF	10
iCD12(2)	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺AWMF≺DAMF≺ARmF	6
iCD12/2(2)	BPDF≺MDBUTMF≺DBAIN≺NAFSMF≺DAMF≺AWMF≺ARmF	6
iE15	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺DAMF≺AWMF≺ARmF	10
iYJ11	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺AWMF≺DAMF≺ARmF	6
iYJ11/2	BPDF≺MDBUTMF≺DBAIN≺NAFSMF≺DAMF≺AWMF≺ARmF	6
iMR13	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺AWMF≺DAMF≺ARmF	6
iNKY17(γ)	BPDF≺DBAIN≺MDBUTMF≺NAFSMF≺DAMF≺AWMF≺ARmF	1

*In the event that noise removal performance at high noise densities is more important. Here, $\gamma = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$.

Additionally, suppose that the noise removal success of the filters at low noise densities are more significant than at the other densities, it is anticipated that the membership degrees at high noise densities are smaller than the non-membership degrees and the former at low noise densities are greater than the latter. That is, we consider the first rows of the *ifpifs*-matrices to be

$$\begin{bmatrix} 0.9 & 0.85 & 0.75 & 0.65 & 0.5 & 0.35 & 0.25 & 0.15 & 0.05 \\ 0.05 & 0.1 & 0.2 & 0.3 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \end{bmatrix}$$

Therefore, we present the *ifpifs*-matrix $[d_{ij}]$ ($[b_{ij}]$ as cited in [1]), obtained by SSIM results of aforesaid seven filters, in the event that noise removal performance at low noise densities is more important. In addition, we also present the *ifpifs*-matrices $[e_{ij}]$ and $[f_{ij}]$, derived from the PNSR and VIF value-based *ifpifs*-matrices $[b_{ij}]$ and $[c_{ij}]$, respectively, using the above row matrix.

Moreover, Table 23 presents decision sets produced by the generalized SDM methods having passed all the test cases in Section 3. Here, iCE10o, iEMO18oa, iCE10on, iEMA18onan, iCE10/2o, iCE10/2on, iEMC19oa, iZZ16(0.4), iZZ16$\binom{0.4}{0.4}$, iZZ16/2(0.4), and iZZ16/2$\binom{0.4}{0.4}$ employ the *ifpifs*-matrices $[d_{ij}]$ and $[e_{ij}]$, and the others utilize the *ifpifs*-matrices $[d_{ij}]$, $[e_{ij}]$, and $[f_{ij}]$.

$$
[d_{ij}] = \begin{bmatrix}
0.9 & 0.85 & 0.75 & 0.65 & 0.5 & 0.35 & 0.25 & 0.15 & 0.05 \\
0.05 & 0.1 & 0.2 & 0.3 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\
0.9657 & 0.9335 & 0.8856 & 0.8269 & 0.7503 & 0.6452 & 0.5159 & 0.3648 & 0.1259 \\
0.0062 & 0.0142 & 0.0270 & 0.0450 & 0.0759 & 0.1165 & 0.1887 & 0.2998 & 0.4895 \\
0.9666 & 0.9424 & 0.9047 & 0.8552 & 0.7917 & 0.7104 & 0.6060 & 0.4880 & 0.3518 \\
0.0031 & 0.0080 & 0.0168 & 0.0297 & 0.0478 & 0.0762 & 0.1223 & 0.1858 & 0.2766 \\
0.9642 & 0.9228 & 0.7833 & 0.7539 & 0.7855 & 0.7572 & 0.6950 & 0.6000 & 0.3492 \\
0.0050 & 0.0509 & 0.1319 & 0.1593 & 0.1167 & 0.0551 & 0.0575 & 0.1359 & 0.5228 \\
0.9606 & 0.9216 & 0.8767 & 0.8305 & 0.7800 & 0.7211 & 0.6540 & 0.5766 & 0.4578 \\
0.0086 & 0.0169 & 0.0267 & 0.0357 & 0.0465 & 0.0595 & 0.0790 & 0.1082 & 0.2173 \\
0.9700 & 0.9518 & 0.9270 & 0.8953 & 0.8563 & 0.8072 & 0.7465 & 0.6667 & 0.5415 \\
0.0018 & 0.0045 & 0.0088 & 0.0139 & 0.0204 & 0.0291 & 0.0423 & 0.0624 & 0.1148 \\
0.9551 & 0.9440 & 0.9209 & 0.8948 & 0.8611 & 0.8148 & 0.7551 & 0.6736 & 0.5469 \\
0.0067 & 0.0076 & 0.0095 & 0.0122 & 0.0166 & 0.0240 & 0.0370 & 0.0574 & 0.1052 \\
0.9718 & 0.9532 & 0.9272 & 0.8971 & 0.8630 & 0.8239 & 0.7663 & 0.6819 & 0.5515 \\
0.0013 & 0.0030 & 0.0054 & 0.0087 & 0.0137 & 0.0214 & 0.0348 & 0.0554 & 0.1038
\end{bmatrix}
$$

$$
[e_{ij}] = \begin{bmatrix}
0.9 & 0.85 & 0.75 & 0.65 & 0.5 & 0.35 & 0.25 & 0.15 & 0.05 \\
0.05 & 0.1 & 0.2 & 0.3 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 \\
0.8315 & 0.7582 & 0.6932 & 0.6373 & 0.5787 & 0.5286 & 0.4599 & 0.3667 & 0.1864 \\
0.0035 & 0.0549 & 0.0995 & 0.1429 & 0.1828 & 0.2345 & 0.3008 & 0.4048 & 0.6592 \\
0.8726 & 0.7964 & 0.7314 & 0.6733 & 0.6226 & 0.5654 & 0.5042 & 0.4462 & 0.3712 \\
0.0007 & 0.0264 & 0.0597 & 0.0969 & 0.1359 & 0.1834 & 0.2303 & 0.2874 & 0.3706 \\
0.7891 & 0.7263 & 0.6166 & 0.6000 & 0.6445 & 0.6368 & 0.5995 & 0.5129 & 0.3021 \\
0.0073 & 0.0752 & 0.1258 & 0.1485 & 0.1527 & 0.1645 & 0.1863 & 0.2639 & 0.4933 \\
0.8215 & 0.7434 & 0.6916 & 0.6538 & 0.6204 & 0.5922 & 0.5606 & 0.5280 & 0.4751 \\
0.0159 & 0.0723 & 0.1025 & 0.1254 & 0.1577 & 0.1813 & 0.2039 & 0.2376 & 0.3204 \\
0.9678 & 0.8775 & 0.8103 & 0.7628 & 0.7186 & 0.6735 & 0.6298 & 0.5749 & 0.5052 \\
0 & 0.0300 & 0.0602 & 0.0780 & 0.1125 & 0.1392 & 0.1657 & 0.2014 & 0.2650 \\
0.8497 & 0.8241 & 0.7958 & 0.7625 & 0.7273 & 0.6867 & 0.6405 & 0.5825 & 0.5101 \\
0.0282 & 0.0369 & 0.0549 & 0.0658 & 0.0862 & 0.1099 & 0.1390 & 0.1764 & 0.2387 \\
0.9983 & 0.9074 & 0.8478 & 0.7946 & 0.7471 & 0.6983 & 0.6471 & 0.5857 & 0.5112 \\
0 & 0.0154 & 0.0380 & 0.0522 & 0.0767 & 0.1028 & 0.1348 & 0.1745 & 0.2384
\end{bmatrix}
$$

and

$$[f_{ij}] = \begin{bmatrix} \begin{smallmatrix}0.9\\0.05\end{smallmatrix} & \begin{smallmatrix}0.85\\0.1\end{smallmatrix} & \begin{smallmatrix}0.75\\0.2\end{smallmatrix} & \begin{smallmatrix}0.65\\0.3\end{smallmatrix} & \begin{smallmatrix}0.5\\0.5\end{smallmatrix} & \begin{smallmatrix}0.35\\0.6\end{smallmatrix} & \begin{smallmatrix}0.25\\0.7\end{smallmatrix} & \begin{smallmatrix}0.15\\0.8\end{smallmatrix} & \begin{smallmatrix}0.05\\0.9\end{smallmatrix} \\ \begin{smallmatrix}0.6854\\0.0805\end{smallmatrix} & \begin{smallmatrix}0.5350\\0.1708\end{smallmatrix} & \begin{smallmatrix}0.4293\\0.2711\end{smallmatrix} & \begin{smallmatrix}0.3425\\0.3820\end{smallmatrix} & \begin{smallmatrix}0.2652\\0.5028\end{smallmatrix} & \begin{smallmatrix}0.1826\\0.6338\end{smallmatrix} & \begin{smallmatrix}0.1119\\0.7727\end{smallmatrix} & \begin{smallmatrix}0.0579\\0.8869\end{smallmatrix} & \begin{smallmatrix}0.0217\\0.9544\end{smallmatrix} \\ \begin{smallmatrix}0.7456\\0.0506\end{smallmatrix} & \begin{smallmatrix}0.6003\\0.1168\end{smallmatrix} & \begin{smallmatrix}0.4793\\0.2152\end{smallmatrix} & \begin{smallmatrix}0.3713\\0.3135\end{smallmatrix} & \begin{smallmatrix}0.2875\\0.4248\end{smallmatrix} & \begin{smallmatrix}0.2115\\0.5392\end{smallmatrix} & \begin{smallmatrix}0.1452\\0.6667\end{smallmatrix} & \begin{smallmatrix}0.0884\\0.7857\end{smallmatrix} & \begin{smallmatrix}0.0395\\0.8974\end{smallmatrix} \\ \begin{smallmatrix}0.7174\\0.0539\end{smallmatrix} & \begin{smallmatrix}0.5640\\0.1861\end{smallmatrix} & \begin{smallmatrix}0.4333\\0.3799\end{smallmatrix} & \begin{smallmatrix}0.3703\\0.4318\end{smallmatrix} & \begin{smallmatrix}0.3427\\0.4046\end{smallmatrix} & \begin{smallmatrix}0.2927\\0.4196\end{smallmatrix} & \begin{smallmatrix}0.2353\\0.4956\end{smallmatrix} & \begin{smallmatrix}0.1574\\0.6293\end{smallmatrix} & \begin{smallmatrix}0.0466\\0.8868\end{smallmatrix} \\ \begin{smallmatrix}0.6356\\0.0887\end{smallmatrix} & \begin{smallmatrix}0.5081\\0.1682\end{smallmatrix} & \begin{smallmatrix}0.4290\\0.2555\end{smallmatrix} & \begin{smallmatrix}0.3624\\0.3313\end{smallmatrix} & \begin{smallmatrix}0.3048\\0.4094\end{smallmatrix} & \begin{smallmatrix}0.2507\\0.4948\end{smallmatrix} & \begin{smallmatrix}0.1969\\0.5805\end{smallmatrix} & \begin{smallmatrix}0.1306\\0.6703\end{smallmatrix} & \begin{smallmatrix}0.0683\\0.8058\end{smallmatrix} \\ \begin{smallmatrix}0.7742\\0.0250\end{smallmatrix} & \begin{smallmatrix}0.6564\\0.0650\end{smallmatrix} & \begin{smallmatrix}0.5698\\0.1215\end{smallmatrix} & \begin{smallmatrix}0.4896\\0.1772\end{smallmatrix} & \begin{smallmatrix}0.4180\\0.2330\end{smallmatrix} & \begin{smallmatrix}0.3501\\0.2972\end{smallmatrix} & \begin{smallmatrix}0.2847\\0.3813\end{smallmatrix} & \begin{smallmatrix}0.2100\\0.4843\end{smallmatrix} & \begin{smallmatrix}0.1255\\0.6670\end{smallmatrix} \\ \begin{smallmatrix}0.6396\\0.0801\end{smallmatrix} & \begin{smallmatrix}0.5992\\0.0988\end{smallmatrix} & \begin{smallmatrix}0.5489\\0.1299\end{smallmatrix} & \begin{smallmatrix}0.4861\\0.1671\end{smallmatrix} & \begin{smallmatrix}0.4245\\0.2160\end{smallmatrix} & \begin{smallmatrix}0.3599\\0.2792\end{smallmatrix} & \begin{smallmatrix}0.2895\\0.3682\end{smallmatrix} & \begin{smallmatrix}0.2119\\0.4767\end{smallmatrix} & \begin{smallmatrix}0.1262\\0.6613\end{smallmatrix} \\ \begin{smallmatrix}0.7908\\0.0214\end{smallmatrix} & \begin{smallmatrix}0.6833\\0.0489\end{smallmatrix} & \begin{smallmatrix}0.6006\\0.0884\end{smallmatrix} & \begin{smallmatrix}0.5186\\0.1328\end{smallmatrix} & \begin{smallmatrix}0.4442\\0.1894\end{smallmatrix} & \begin{smallmatrix}0.3718\\0.2583\end{smallmatrix} & \begin{smallmatrix}0.2966\\0.3533\end{smallmatrix} & \begin{smallmatrix}0.2152\\0.4677\end{smallmatrix} & \begin{smallmatrix}0.1278\\0.6569\end{smallmatrix} \end{bmatrix}$$

Table 23: Decision Sets Produced by the Generalized SDM Methods*

Algorithms	**Decision Sets**
iCE10o	$\{^{0.8691}_{0.0002}\text{BPDF}, ^{0.8700}_{0}\text{DBAIN}, ^{0.8678}_{0.0003}\text{MDBUTMF}, ^{0.8645}_{0.0004}\text{NAFSMF}, ^{0.8730}_{0}\text{DAMF}, ^{0.8596}_{0.0003}\text{AWMF}, ^{0.8985}_{0}\text{ARmF}\}$
iEMO18oa	$\{^{0.8691}_{0.0002}\text{BPDF}, ^{0.8700}_{0}\text{DBAIN}, ^{0.8678}_{0.0003}\text{MDBUTMF}, ^{0.8645}_{0.0004}\text{NAFSMF}, ^{0.8730}_{0}\text{DAMF}, ^{0.8596}_{0.0003}\text{AWMF}, ^{0.8985}_{0}\text{ARmF}\}$
iCE10on	$\{^{0.8691}_{0.0003}\text{BPDF}, ^{0.8700}_{0.0002}\text{DBAIN}, ^{0.8678}_{0.0003}\text{MDBUTMF}, ^{0.8645}_{0.0004}\text{NAFSMF}, ^{0.8730}_{0.0001}\text{DAMF}, ^{0.8596}_{0.0003}\text{AWMF}, ^{0.8985}_{0.0001}\text{ARmF}\}$
iEMA18onan	$\{^{0.7483}_{0.0003}\text{BPDF}, ^{0.7854}_{0.0002}\text{DBAIN}, ^{0.7102}_{0.0004}\text{MDBUTMF}, ^{0.7393}_{0.0008}\text{NAFSMF}, ^{0.8710}_{0.0001}\text{DAMF}, ^{0.7647}_{0.0014}\text{AWMF}, ^{0.8746}_{0.0001}\text{ARmF}\}$
iCE10/2o	$\{^{0.8691}_{0.0003}\text{BPDF}, ^{0.8700}_{0.0002}\text{DBAIN}, ^{0.8678}_{0.0003}\text{MDBUTMF}, ^{0.8645}_{0.0004}\text{NAFSMF}, ^{0.8730}_{0.0001}\text{DAMF}, ^{0.8596}_{0.0003}\text{AWMF}, ^{0.8746}_{0.0001}\text{ARmF}\}$
iCE10/2on	$\{^{0.8691}_{0.0003}\text{BPDF}, ^{0.8700}_{0.0002}\text{DBAIN}, ^{0.8678}_{0.0003}\text{MDBUTMF}, ^{0.8645}_{0.0004}\text{NAFSMF}, ^{0.8730}_{0.0001}\text{DAMF}, ^{0.8596}_{0.0003}\text{AWMF}, ^{0.8746}_{0.0001}\text{ARmF}\}$
iEMC19oa	$\{^{0.8691}_{0.0002}\text{BPDF}, ^{0.8700}_{0}\text{DBAIN}, ^{0.8678}_{0.0004}\text{MDBUTMF}, ^{0.8645}_{0.0008}\text{NAFSMF}, ^{0.8730}_{0}\text{DAMF}, ^{0.8596}_{0.0014}\text{AWMF}, ^{0.8746}_{0}\text{ARmF}\}$
iZZ16(0.4)	$\{^{0.2002}_{0.2440}\text{BPDF}, ^{0.2112}_{0.1385}\text{DBAIN}, ^{0.1920}_{0.1573}\text{MDBUTMF}, ^{0.2008}_{0.1322}\text{NAFSMF}, ^{0.2356}_{0.1027}\text{DAMF}, ^{0.2210}_{0.0883}\text{AWMF}, ^{0.2419}_{0.0832}\text{ARmF}\}$
iZZ16$\binom{0.4}{0.4}$	$\{^{0.2002}_{0.3093}\text{BPDF}, ^{0.2112}_{0.2080}\text{DBAIN}, ^{0.1920}_{0.2100}\text{MDBUTMF}, ^{0.2008}_{0.1322}\text{NAFSMF}, ^{0.2356}_{0.1027}\text{DAMF}, ^{0.2210}_{0.0883}\text{AWMF}, ^{0.2419}_{0.0832}\text{ARmF}\}$
iZZ16/2(0.4)	$\{^{0.5096}_{0.0854}\text{BPDF}, ^{0.5377}_{0.0511}\text{DBAIN}, ^{0.5213}_{0.0650}\text{MDBUTMF}, ^{0.5383}_{0.0365}\text{NAFSMF}, ^{0.5730}_{0.0195}\text{DAMF}, ^{0.5713}_{0.0176}\text{AWMF}, ^{0.5793}_{0.0170}\text{ARmF}\}$
iZZ16/2$\binom{0.4}{0.4}$	$\{^{0.5096}_{0.1858}\text{BPDF}, ^{0.5377}_{0.1155}\text{DBAIN}, ^{0.5213}_{0.1101}\text{MDBUTMF}, ^{0.5383}_{0.0365}\text{NAFSMF}, ^{0.5730}_{0.0195}\text{DAMF}, ^{0.5713}_{0.0176}\text{AWMF}, ^{0.5793}_{0.0170}\text{ARmF}\}$
iRM11o	$\{^{0.8691}_{0.0559}\text{BPDF}, ^{0.8700}_{0.0530}\text{DBAIN}, ^{0.8678}_{0.0548}\text{MDBUTMF}, ^{0.8645}_{0.0582}\text{NAFSMF}, ^{0.8730}_{0.0517}\text{DAMF}, ^{0.8596}_{0.0564}\text{AWMF}, ^{0.8746}_{0.0513}\text{ARmF}\}$
iEM20oa	$\{^{0.8691}_{0.0002}\text{BPDF}, ^{0.8700}_{0}\text{DBAIN}, ^{0.8678}_{0.0004}\text{MDBUTMF}, ^{0.8645}_{0.0008}\text{NAFSMF}, ^{0.8730}_{0}\text{DAMF}, ^{0.8596}_{0.0014}\text{AWMF}, ^{0.8746}_{0}\text{ARmF}\}$
iZ14(I_9)	$\{^{0.8760}_{0}\text{BPDF}, ^{0.8910}_{0.0342}\text{DBAIN}, ^{0.8752}_{0.0247}\text{MDBUTMF}, ^{0.8865}_{0.0587}\text{NAFSMF}, ^{0.9121}_{0.0593}\text{DAMF}, ^{0.9100}_{0.0642}\text{AWMF}, ^{0.9175}_{0.0581}\text{ARmF}\}$
iDB12	$\{^{0.0603}_{0.5765}\text{BPDF}, ^{0.2015}_{0.6412}\text{DBAIN}, ^{0.0500}_{0.7765}\text{MDBUTMF}, ^{0}_{0.9147}\text{NAFSMF}, ^{0.4515}_{0.4044}\text{DAMF}, ^{0.3176}_{0.4397}\text{AWMF}, ^{0.6074}_{0}\text{ARmF}\}$
isDB12	$\{^{0.0603}_{0.5765}\text{BPDF}, ^{0.2015}_{0.6412}\text{DBAIN}, ^{0.0500}_{0.7765}\text{MDBUTMF}, ^{0}_{0.9147}\text{NAFSMF}, ^{0.4515}_{0.4044}\text{DAMF}, ^{0.3176}_{0.4397}\text{AWMF}, ^{0.6074}_{0}\text{ARmF}\}$
iCD12(2)	$\{^{0.1964}_{0.3430}\text{BPDF}, ^{0.2174}_{0.3024}\text{DBAIN}, ^{0.2163}_{0.2729}\text{MDBUTMF}, ^{0.1978}_{0.2740}\text{NAFSMF}, ^{0.2570}_{0.1901}\text{DAMF}, ^{0.2374}_{0.1862}\text{AWMF}, ^{0.2679}_{0.1770}\text{ARmF}\}$
iCD12/2(2)	$\{^{0.3833}_{0.1084}\text{BPDF}, ^{0.3964}_{0.0653}\text{DBAIN}, ^{0.3826}_{0.0876}\text{MDBUTMF}, ^{0.3925}_{0.0479}\text{NAFSMF}, ^{0.4149}_{0.0250}\text{DAMF}, ^{0.4130}_{0.0226}\text{AWMF}, ^{0.4196}_{0.0214}\text{ARmF}\}$
iE15	$\{^{0.7429}_{0}\text{BPDF}, ^{0.7653}_{0.0355}\text{DBAIN}, ^{0.7498}_{0.0482}\text{MDBUTMF}, ^{0.7510}_{0.0722}\text{NAFSMF}, ^{0.8074}_{0.0693}\text{DAMF}, ^{0.7916}_{0.0924}\text{AWMF}, ^{0.8185}_{0.0711}\text{ARmF}\}$
iYJ11	$\{^{0.1964}_{0.3430}\text{BPDF}, ^{0.2174}_{0.3024}\text{DBAIN}, ^{0.2163}_{0.2729}\text{MDBUTMF}, ^{0.1978}_{0.2740}\text{NAFSMF}, ^{0.2570}_{0.1901}\text{DAMF}, ^{0.2374}_{0.1862}\text{AWMF}, ^{0.2679}_{0.1770}\text{ARmF}\}$
iYJ11/2	$\{^{0.3833}_{0.1084}\text{BPDF}, ^{0.3964}_{0.0653}\text{DBAIN}, ^{0.3826}_{0.0876}\text{MDBUTMF}, ^{0.3925}_{0.0479}\text{NAFSMF}, ^{0.4149}_{0.0250}\text{DAMF}, ^{0.4130}_{0.0226}\text{AWMF}, ^{0.4196}_{0.0214}\text{ARmF}\}$
iMR13	$\{^{0.1964}_{0.3430}\text{BPDF}, ^{0.2174}_{0.3024}\text{DBAIN}, ^{0.2163}_{0.2729}\text{MDBUTMF}, ^{0.1978}_{0.2740}\text{NAFSMF}, ^{0.2570}_{0.1901}\text{DAMF}, ^{0.2374}_{0.1862}\text{AWMF}, ^{0.2679}_{0.1770}\text{ARmF}\}$
iNKY17(γ)	$\{^{0.0015}_{0.6265}\text{BPDF}, ^{0.1875}_{0.6341}\text{DBAIN}, ^{0}_{0.8498}\text{MDBUTMF}, ^{0.0015}_{0.8902}\text{NAFSMF}, ^{0.4223}_{0.4230}\text{DAMF}, ^{0.2881}_{0.4649}\text{AWMF}, ^{0.5930}_{0}\text{ARmF}\}$

*In the event that noise removal performance at low noise densities is more important. Here, $\gamma = \begin{bmatrix} \begin{smallmatrix}1\\0\end{smallmatrix} & \begin{smallmatrix}1\\0\end{smallmatrix} & \begin{smallmatrix}1\\0\end{smallmatrix} & \begin{smallmatrix}1\\0\end{smallmatrix} & \begin{smallmatrix}1\\0\end{smallmatrix} & \begin{smallmatrix}1\\0\end{smallmatrix} & \begin{smallmatrix}1\\0\end{smallmatrix} \end{bmatrix}^T$.

The intuitionistic fuzzy values in the decision sets provided in Table 23 are produced on MATLAB R2021b. Furthermore, using the relation provided in Proposition 2.1 [1], the ranking orders of the filters are presented in Table 24. The number(s) of the SDM methods producing the same ranking order is signified in the last column of Table 24. According to the table, iCE10o, iEMO18oa, iCE10on, iCE10/2o, iCE10/2on, iEMC19oa, iRM11o, and iEM20oa have the same ranking orders just as iZZ16/2(0.4), iZZ16/2$\binom{0.4}{0.4}$, iCD12/2(2), and iYJ11/2 do. Besides, iDB12, isDB12, and iNKY17(γ) generate the same ranking orders, as iCD12(2), iYJ11, and iMR13 do. Others also have different ranking orders.

Table 24: Ranking Orders of the Generalized SDM Methods*

Algorithms	**Ranking Orders**	**Frequency**
iCE10o	AWMF≺NAFSMF≺MDBUTMF≺BPDF≺DBAIN≺DAMF≺ARmF	8
iEMO18oa	AWMF≺NAFSMF≺MDBUTMF≺BPDF≺DBAIN≺DAMF≺ARmF	8
iCE10on	AWMF≺NAFSMF≺MDBUTMF≺BPDF≺DBAIN≺DAMF≺ARmF	8
iEMA18onan	MDBUTMF≺NAFSMF≺BPDF≺AWMF≺DBAIN≺DAMF≺ARmF	1
iCE10/2o	AWMF≺NAFSMF≺MDBUTMF≺BPDF≺DBAIN≺DAMF≺ARmF	8
iCE10/2on	AWMF≺NAFSMF≺MDBUTMF≺BPDF≺DBAIN≺DAMF≺ARmF	8
iEMC19oa	AWMF≺NAFSMF≺MDBUTMF≺BPDF≺DBAIN≺DAMF≺ARmF	8
iZZ16(0.4)	BPDF≺MDBUTMF≺NAFSMF≺DBAIN≺AWMF≺DAMF≺ARmF	1
iZZ16$\binom{0.4}{0.4}$	BPDF≺MDBUTMF≺DBAIN≺NAFSMF≺AWMF≺DAMF≺ARmF	1
iZZ16/2(0.4)	BPDF≺MDBUTMF≺DBAIN≺NAFSMF≺DAMF≺AWMF≺ARmF	4
iZZ16/2$\binom{0.4}{0.4}$	BPDF≺MDBUTMF≺DBAIN≺NAFSMF≺DAMF≺AWMF≺ARmF	4
iRM11o	AWMF≺NAFSMF≺MDBUTMF≺BPDF≺DBAIN≺DAMF≺ARmF	8
iEM20oa	AWMF≺NAFSMF≺MDBUTMF≺BPDF≺DBAIN≺DAMF≺ARmF	8
iZ14(I_9)	NAFSMF≺AWMF≺MDBUTMF≺DAMF≺DBAIN≺ARmF≺BPDF	1
iDB12	NAFSMF≺MDBUTMF≺BPDF≺DBAIN≺AWMF≺DAMF≺ARmF	3
isDB12	NAFSMF≺MDBUTMF≺BPDF≺DBAIN≺AWMF≺DAMF≺ARmF	3
iCD12(2)	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺AWMF≺DAMF≺ARmF	3
iCD12/2(2)	BPDF≺MDBUTMF≺DBAIN≺NAFSMF≺DAMF≺AWMF≺ARmF	4
iE15	NAFSMF≺AWMF≺MDBUTMF≺DBAIN≺DAMF≺BPDF≺ARmF	1
iYJ11	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺AWMF≺DAMF≺ARmF	3
iYJ11/2	BPDF≺MDBUTMF≺DBAIN≺NAFSMF≺DAMF≺AWMF≺ARmF	4
iMR13	BPDF≺DBAIN≺NAFSMF≺MDBUTMF≺AWMF≺DAMF≺ARmF	3
iNKY17(γ)	NAFSMF≺MDBUTMF≺BPDF≺DBAIN≺AWMF≺DAMF≺ARmF	3

*In the event that noise removal performance at low noise densities is more important. Here, $\gamma = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$.

5. Conclusion

The present study generalized 36 SDM methods in [14,15,16,17,18,19,20,21,22,23] constructed by the concept of *fpfs*-matrices to the *ifpifs*-matrices space. It then determined the generalized SDM methods that were successful in five test cases demonstrating the reliability of an SDM method. Subsequently, it applied the successful generalized SDM methods to a PVA problem to rank the state-of-the-art noise removal filters in compliance with their noise removal performance.

Furthermore, this study serves as a source to generalize other methods constructed by the concept of *fpfs*-matrices for future studies. Therefore, researchers can study generalizing SDM methods provided in [4,10,11] to the *ifpifs*-matrices space. Thus, the generalizations of all the configured SDM methods to be operable in *fpfs*-matrices space to *ifpifs*-matrices space will be completed. Besides, using generalized SDM methods, future papers can develop classifiers (for more details, see [9,30,31,32,33,34,35,36,37]). Additionally, generalizations of SDM methods constructed by superstructures of *ifpifs*-sets/matrices, which are not within the scope of this study, to such spaces as interval-valued intuitionistic fuzzy parameterized interval-valued intuitionistic fuzzy soft sets/matrices space [5,6] and the hybrid versions of fuzzy sets [46], intuitionistic fuzzy sets [3], picture fuzzy sets [7,28,29,39], T-spherical fuzzy sets [27], linear Diophantine fuzzy sets [40], and soft sets [38] can be studied.

Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

Data sharing does not apply to this article as no new data were created or analyzed in this study.

Author Contributions

S. Enginoğlu directed the project and supervised the process whereby the findings were obtained. T. Aydın and B. Arslan generalized the SDM methods. S. Memiş and B. Arslan produced the application results of the SDM methods by writing their MATLAB codes. T. Aydın and B. Arslan wrote the paper with the support of S. Enginoğlu and S. Memiş. S. Enginoğlu reviewed and edited the paper. All the authors discussed the results and contributed to the last paper.

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Burak Arslan (Corresponding Author),
Department of Marine Engineering, Faculty of Maritime,
Çanakkale Onsekiz Mart University, Çanakkale, Türkiye.
E-mail address: tburakarslan@gmail.com

and

Tuğçe Aydın,
Department of Mathematics, Faculty of Science,
Çanakkale Onsekiz Mart University, Çanakkale, Türkiye.
E-mail address: aydinttugce@gmail.com

and

Samet Memiş,
[1]*Department of Marine Engineering, Maritime Faculty,*
Bandırma Onyedi Eylül University, Balıkesir, Türkiye.
E-mail address: samettmemis@gmail.com

and

Serdar Enginoğlu,
Department of Mathematics, Faculty of Science,
Çanakkale Onsekiz Mart University, Çanakkale, Türkiye.
E-mail address: serdarenginoglu@gmail.com