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## On Some New Scenario of Almost Boundedness Using Matrices

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ABSTRACT: The authors M. F. Rahman and A. B. M. R. Karim have structured and studied the space  $r_g^w(t,s)$  and have computed its various properties like completeness, duals and many others as can be seen in [24]. The basic structure of this paper is to further study it and investigate for the characterization with sequences of almost bounded  $f_{\infty}$ , almost convergent f and almost sequences converging to zero  $f_0$ . Also, we will prove that  $\widetilde{F}$  is not solid, where symbol  $\widetilde{F}$  represents space having Riesz transform in f.

Key Words:  $\Delta$ -operator, almost convergence, matrices.

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# 1. Preliminary and Introduction

Let all sequences be represented by  $\Omega$ . We call a sequence space as subspace of  $\Omega$ . Throughout the paper,  $\mathbb{N}$ ,  $\mathbb{R}$  and  $\mathbb{C}$  acts as set of whole numbers, the set of real numbers and the set of complex numbers, respectively. Let  $\ell_{\infty}$  acts as the set of all bounded sequences and  $\ell(t)$  represents the following:

$$\ell(t) = \left\{ v = (v_j) \in \Omega : \sum_j |v_j|^{t_j} < \infty \right\},\,$$

for  $0 < t_j \le \mathcal{H} = \sup_j t_j < \infty$  as can be seen in [1], [15], [25], [31].

Consider the spaces  $\mathcal{U}$  and  $\mathcal{V}$  and let  $\mathcal{B} = (b_{nk})$  represent as an infinite matrix. Then, the matrix  $\mathcal{B}$  expresses the  $\mathcal{B}$ -transformation from  $\mathcal{U}$  into  $\mathcal{V}$ , if corresponding to all  $v = (v_j) \in \mathcal{U}$ ,  $\mathcal{B}v = \{(\mathcal{B}v)_n\}$  exists and belongs to  $\mathcal{V}$ ; where  $(\mathcal{B}v)_n = \sum_k b_{nk}v_k$ . We represent the symbol without limits as runs from 0 to  $\infty$ . The symbol  $\mathcal{B} \in (\mathcal{U} : \mathcal{V})$  signifies every matrix from  $\mathcal{U}$  to  $\mathcal{V}$  i.e.,  $\mathcal{B} : \mathcal{U} \to \mathcal{V}$ . A sequence v is known as  $\mathcal{B}$ -summable to v if v approaches to v and it is known as the v-limit of v as can be seen in [3], [9], [10], [26] and many more as can be seen in text.

Thus, for the space  $\mathcal{U}$ , the matrix domain  $\mathcal{U}_{\mathcal{B}}$  of  $\mathcal{B}$  is

$$\mathcal{U}_{\mathcal{B}} = \{ v = (v_j) \in \Omega : \mathcal{B}v \in \mathcal{U} \}. \tag{1.1}$$

As in [27], let  $\mathcal{T}$  represents the shift operator on  $\Omega$ , which means,  $\mathcal{T}v = \{v_n\}_{n=1}^{\infty}$ ,  $\mathcal{T}^2v = \{v_n\}_{n=2}^{\infty}$  and so on. By Banach limit  $\mathcal{L}$  on  $\ell_{\infty}$  represents as a non-negative linear functional so that  $\mathcal{L}$  is invariant under the shift operator on  $\ell_{\infty}$ , which means,  $\mathcal{L}(\mathcal{T}v) = \mathcal{L}(v)$  for each  $v \in \ell_{\infty}$  and  $\mathcal{L}(e) = 1$ , e = (1, 1, 1, ...).

According to author in [20], a sequence  $v = \{v_n\} \in \ell_{\infty}$  is almost convergent having  $\mathcal{F}$ -lim  $v = \lambda$  if and only if

$$\lim_{m \to \infty} t_{mn}(v) = \lambda \quad \text{uniformly in } n \in \mathbb{N},$$

where,  $\Psi_{mn}(v) = \frac{1}{m+1} \sum_{j=0}^{m} v_{n+j}$ . The work on such have been studied by various authors as in [12], [14], [21], [23], [26]-[29], and many others.

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Also, as in [20], the set  $f_{\infty}$  is defined as

$$f_{\infty} = \left\{ v \in \ell_{\infty} : \sup_{mn} |\Psi_{mn}(v)| < \infty \right\}.$$

Define the set

$$f = \left\{ v \in \ell_{\infty} : \lim_{m \to \infty} \Psi_{mn}(v) = \beta \text{ uniformly in } n \in \mathbb{N} \right\},$$

and is called as almost convergent sequences.

Define the sequence  $\tau = (\tau_m)$  by

$$\tau_m = \begin{cases} 1, & \text{if m is even,} \\ 0, & \text{if m is odd.} \end{cases}$$

It is clear that  $\tau$  is almost convergent to  $\frac{1}{2}$  but is not convergent.

We call a matrix  $\mathcal{B} = (b_{rm})$  to be regular (see, [11], [19], [26], [27],) iff it holds the following:

(i) 
$$\lim_{r \to \infty} \sum_{m=0}^{\infty} b_{rm} = 1,$$

(ii) 
$$\lim_{r \to \infty} b_{rm} = 0$$
,  $(m = 0, 1, 2, \cdots)$ 

(i) 
$$\lim_{r \to \infty} \sum_{m=0}^{\infty} b_{rm} = 1,$$
  
(ii)  $\lim_{r \to \infty} b_{rm} = 0, \quad (m = 0, 1, 2, \cdots),$   
(iii)  $\sum_{m=0}^{\infty} |b_{rm}| < \mathcal{D}, \quad (\mathcal{D} > 0, \ r = 0, 1, 2, \cdots).$ 

As in [11], define sequence of positive numbers by  $w = (w_m)$  and denote  $W_i = \sum_{n=0}^{\infty} w_m$  for  $i \in \mathbb{N}$ . So,  $R^w = (r_{im}^w)$  of  $(R, w_i)$ - mean is given by

$$r_{im}^{w} = \begin{cases} \frac{w_m}{W_i}, & \text{if } 0 \le m \le i, \\ 0, & \text{if } m > i, \end{cases}$$

and  $(R, w_i)$  mean is regular iff  $W_i \to \infty$  as  $i \to \infty$ .

For  $0 < t_m \le \mathcal{H} = \sup t_m < \infty$ , the author in [23] have defined the space  $r^w(g,t)$  as follows:

$$r^{w}(g,t) = \left\{ v = (v_m) \in \Omega : \sum_{m} \left| \frac{1}{W_m} \sum_{j=0}^{m} g_j w_j v_j \right|^{t_m} < \infty \right\},\,$$

where,  $g_j \neq 0 \ \forall \ j \in \mathbb{N}$ . Represent  $\mathcal{U}^{\beta}$  the  $\beta$ - dual of  $\mathcal{U}$  and is the set of every sequence  $v = (v_m)$  so that  $v\lambda = (v_m\lambda_m) \in cs$ for each  $\lambda = (\lambda_m) \in \mathcal{U}$ , where cs acts as set of all convergent series.

# 2. Main Results

Here we will define the space  $r_g^w(t,s)$  and compute matrix classes  $\left(r_g^w(t,s):f_\infty\right),\,\left(r_g^w(t,s):f\right)$  and  $(r_a^w(t,s):f_0)$ ; where  $f_\infty$ , f and  $f_0$  have been defined as before.

Following AlBaidani [4]-[7], Boss [18], Dowlath et al [8], Hamid et al [13,16], Jalal et al [17,28], Mursaleen et al [22], Rahman et al [24], Sheikh and Ganie [26], Tarray et al [30], and others, the sequence space  $r_q^w(t,s)$  is defined as the space whose  $R_{s,q}^w$ -transform is in  $\ell(t)$ , that is,

$$r_g^w(t,s) = \left\{ v = (v_m) \in \Omega : \sum_m \left| \frac{1}{W_m^{s+1}} \sum_{i=0}^m g_i w_i v_i \right|^{t_m} < \infty \right\}, \tag{2.1}$$

where,  $0 < t_m \le \mathcal{H} = \sup t_m < \infty$ ,  $s \ge 0$  and for each  $m \in \mathbb{N}$ , we have  $g_m \ne 0$ ...

The set given by (2.1) can be redefined by using (1.1) as

$$\bigg\{\ell(t)\bigg\}_{R^w_{s,g}} = \bigg\{v \in \Omega: R^w_{s,g}v \in \ell(t)\bigg\}.$$

Set  $\eta = (\eta_m)$  as  $R_{s,q}^w$ -transform of  $v = (v_m)$ , which means,

$$\eta_m = \frac{1}{W_m^{s+1}} \sum_{i=0}^m g_i w_i v_i. \tag{2.2}$$

Now define the following lemma required for proving the main theorems.

**Lemma 2.1** As in [24], define  $\mathcal{D}_1$ ,  $\mathcal{D}_2$  as follows:

$$\mathcal{D}_1 = \left\{ b = (b_m) \in \Omega : \right.$$

$$\sup_{m} \left| \triangle \left( \frac{b_m}{g_m w_m} \right) W_m^{s+1} \right|^{t_m} < \infty \ \ and \ \sup_{m} \left| \frac{b_m}{g_m w_m} W_m^{s+1} \right|^{t_m} < \infty \right\}$$

and 
$$\mathcal{D}_2 = \bigcup_{C>1} \left\{ b = (b_k) \in \Omega : \right.$$

$$\sum_{m} \left| \triangle \left( \frac{b_m}{g_m w_m} \right) W_m^{s+1} C^{-1} \right|^{t_m'} < \infty \text{ and } \left\{ \left( \frac{b_m}{g_m w_m} W_m^{s+1} C^{-1} \right)^{t_m'} \right\} \in \ell_{\infty} \right\}.$$

Then, for 
$$1 < t_m \le \mathcal{H} < \infty$$
, we have  $\left[r_g^w(t,s)\right]^{\beta} = \mathcal{D}_1; \ (0 < t_m \le 1) \ and \ \left[r_g^w(t,s)\right]^{\beta} = \mathcal{D}_2.$ 

It is important to note for s = 0, the space  $r_g^w(t, s)$  reduces to  $r^q(g, t)$  as can be searched in [14]. To get easiness in notations, we set

$$\Psi_{ir}(\mathcal{B}v) = \frac{1}{i+1} \sum_{j=0}^{i} \mathcal{B}_{r+j}(v) = \sum_{k} b(r, k, i) v_{k}$$

where,

$$b(r, k, i) = \frac{1}{i+1} \sum_{j=0}^{i} b_{r+j,k}; \ (r, k, i \in \mathbb{N}).$$

Also,

$$\widehat{b}(r,k,i) = \triangle \left[ \frac{b(r,k,i)}{g_k w_k} \right] W_k^{s+1}$$

where,

$$\triangle \left[\frac{b(r,k,i)}{g_k w_k}\right] W_k^{s+1} = \left[\frac{b(r,k,i)}{g_k w_k} - \frac{b(r,k+1,i)}{g_{k+1} w_{k+1}}\right] W_k^{s+1},$$

where  $t_{k}^{'}$  denotes the Holder conjugate and

$$\triangle \left[ \frac{b(r,k,i)}{g_k w_k} \right] = \left[ \frac{b(r,k,i)}{g_k w_k} \right] - \left[ \frac{b(r,k+1,i)}{g_{k+1} w_{k+1}} \right]$$

and it was further considered in [19], [17] and many more.

**Theorem 2.1** (i) For each  $m \in \mathbb{N}$  with  $1 < t_m \le \mathcal{H} < \infty$ . We have  $\mathcal{B} \in (r_g^w(t, s) : f_\infty)$  iff we have an integer  $\varkappa > 1$  so that

$$\sup_{i,r\in\mathbb{N}} \sum_{m} \left| \widehat{b}(i,m,r)\varkappa^{-1} \right|^{t'_{m}} < \infty, \tag{2.3}$$

and

$$\left\{ \left( \frac{b_{im}}{g_m w_m} W_m^{s+1} \varkappa^{-1} \right)^{t_m'} \right\} \in \ell_{\infty} \ \forall \ i \in \mathbb{N}.$$
(2.4)

(ii) If  $\forall m \in \mathbb{N} \text{ and } 0 \leq t_m \leq 1$ , Then  $\mathcal{B} \in (r_q^w(t,s) : \ell_\infty)$  iff

$$\sup_{i,m,r\in\mathbb{N}} \left| \widehat{b}(i,m,r) \right|^{t_m} < \infty \tag{2.5}$$

## **Proof:**

Suppose the conditions (2.3) and (2.4) holds and  $v \in r_g^w(t,s)$ . Then  $\{b_{nm}\}_{m \in \mathbb{N}} \in [r_g^w(t,s)]^{\beta}$ ,  $\forall n \in \mathbb{N}$ , the  $\mathcal{B}$ -transform of v exists. Now using

$$|ab| \le \varkappa \left\{ |a\varkappa^{-1}|^{t'} + |b|^p \right\},$$

where  $\varkappa>0$  ,  $a,\ b\in {\bf C},\ t>1$  along with  $t^{-1}+(t')^{-1}=1$  [2], and we see by utilizing the relation (2.2) that

$$\begin{aligned} |\Psi_{rj}(\mathcal{B}v)| &= \left| \sum_{k} b(j, m, r) v_{m} \right| \\ &= \left| \sum_{m} \widehat{b}(j, m, r) \eta_{m} \right| \\ &\leq \sum_{m} \left| \widehat{b}(j, m, r) \eta_{m} \right| \\ &\leq \sum_{m} \varkappa \left\{ \left| \widehat{b}(j, m, r) \varkappa^{-1} \right|^{t'_{m}} + |\eta_{m}|^{t_{m}} \right\}. \end{aligned}$$

Now, take supremum over r, j on both sides, we see  $\mathcal{B}v \in f_{\infty}$  for every  $v \in r_q^w(t, s)$ .

Conversely, we suppose  $\mathcal{B} \in \left(r_g^w(t,s): f_\infty\right)$  and  $1 < t_m \leq \mathcal{H} < \infty \ \forall \ m \in \mathbb{N}$ . Therefore,  $\mathcal{B}v$  exists for each  $v \in r_g^w(t,s)$  and this implies that  $\{b_{n,m}\}_{m \in \mathbb{N}} \in [r_g^w(t,s)]^\beta \ \forall n \in \mathbb{N}$ , the necessity is obvious for (2.4). But for all r, n,  $\sum_j b(n,j,r)v_j$  exists and  $v \in r_g^w(t,s)$ , so that  $\{b(n,k,m)\}_{k \in \mathbb{N}}$  defines the continuous linear functionals  $\phi_{rn}(v)$  on  $r_g^w(t,s)$  by

$$\phi_{rn}(v) = \sum_{j} b(n, j, r) v_j; \ (n, j, r \in \mathbb{N}).$$

Since  $r_g^w(t,s)$  is complete and  $\sup_{n,r} \left| \sum_j b(n,j,r) v_j \right| < \infty$  on  $r_g^w(t,s)$ , so by Banach–Steinhaus theorem, there exists  $\varkappa > 0$  such that

$$\sup_{r,n} |\phi_{rn}(v)| = \sup_{r,n} \sum_{j} |b(n,j,r)v_j| = \sup_{r,n} \left| \widehat{b}(n,j,r)\eta_j \right| < \infty.$$

So there by giving  $\sup_{r,n} \left| \widehat{b}(n,j,r)\varkappa^{-1} \right|^{t'_j} < \infty$ , yielding the necessity of (2.3) and completed the proof (i). The part (ii) can be proved similarly and the proof of theorem is complete.

**Theorem 2.2** For  $1 < t_m \le \mathcal{H} < \infty$ ,  $\mathcal{B} \in (r_g^w(t,s):f)$  iff (2.3), (2.4), (2.5) holds for each  $m \in \mathbb{N}$  of Theorem 2.2 and there is  $(\beta_m)$  sequence of scalars so that

$$\lim_{r \to \infty} \widehat{b}(n, m, r) = \beta_m, \text{ uniformly in } n \in \mathbb{N}.$$
(2.6)

**Proof:** Suppose (2.3), (2.4), (2.5) and (2.6) holds good and  $v \in r_g^w(t, s)$ . Then  $\mathcal{B}v$  exists and by (2.6) we see for each  $m \in \mathbb{N}$  that

$$|\widehat{b}(l,m,r)\varkappa^{-1}|^{t'_m} \to |\beta_m \varkappa^{-1}|^{t'_m}$$

as  $r \to \infty$ , uniformly in l, m yielding with (2.3) that

$$\sum_{j=0}^{m} \left| \beta_{j} \varkappa^{-1} \right|^{t'_{j}} = \lim_{r \to \infty} \sum_{j=0}^{m} \left| \widehat{b}(l, j, r) \varkappa^{-1} \right|^{t'_{m}} \quad (uniformly \ in \ l)$$

$$\leq \sup_{l,r\in\mathbb{N}} \sum_{j=0}^{m} \left| \widehat{b}(l,j,r)\varkappa^{-1} \right|^{t'_{m}} < \infty, \tag{2.7}$$

holds for each  $m \in \mathbb{N}$ . But given  $v \in r_g^w(t,s)$  and as in [25]-Theorem 2.2,  $r_g^w(t,s)$  is isomorphic to  $\ell(t)$  linearly, so yielding  $\sigma \in \ell(t)$ . Hence, from (2.7), the series  $\sum_m \beta_m \eta_m$  and  $\sum_m \hat{b}(l,m,r) \eta_m$  converges for each r, l and  $\sigma \in \ell(t)$ . Thus, for a given  $\epsilon > 0$ , fix  $m_0 \in \mathbb{N}$  so that

$$\left(\sum_{m=m_0+1}^{\infty} |\eta_m|^{t_m}\right)^{\frac{1}{t_m}} < \epsilon.$$

Now clearly for some  $r_0 \in \mathbb{N}$ , we see

$$\left| \sum_{m=0}^{m_0} \left[ \widehat{b}(l, m, r) - \beta_m \right] \right| < \epsilon,$$

for every  $r \geq m_0$  and uniformly in l. But (2.6) holds, giving

$$\left| \sum_{m_0+1}^{\infty} \left[ \widehat{b}(l, m, r) - \beta_m \right] \right|$$

is arbitrary small. Therefore, we see

$$\lim_{r} \sum_{m} b(l, m, r) v_{m} = \lim_{r} \sum_{m} \widehat{b}(l, m, r) \eta_{m}$$
$$= \sum_{m} \beta_{m} \eta_{m},$$

uniformly in l. Consequently,  $\mathcal{B}v \in f$  there by proving sufficiency.

We now let  $\mathcal{B} \in (r_g^w(t,s):f)$ . Then, necessities of (2.3) and (2.4) are immediate from Theorem 2.2 since  $f \subset f_{\infty}$ . To establish the necessity of (2.6), we define

$$b_l^m(w) = \begin{cases} (-1)^{l-m} \frac{W_m^{s+1}}{g_l w_l}, & \text{if } m \le l \le m+1, \\ 0, & \text{if } 0 \le l < k \text{ or } l > m+1. \end{cases}$$

But for each  $v \in r_g^w(t,s)$ ,  $\mathcal{B}v$  exists and is in f, yielding obviously that  $\mathcal{B}b^{(m)}(w) = \left\{ \triangle \left( \frac{b_{lm}}{g_m w_m} \right) W_m^{s+1} \right\}_{n \in \mathbb{N}} \in f$  for each  $m \in \mathbb{N}$ , which proves the necessity of (2.6). This concludes the proof.  $\square$ 

**Note** that if we let  $\beta_m \to 0$  for each  $m \in \mathbb{N}$  if above theorem, we have the following result.

**Theorem 2.3** Let  $1 < t_m \le \mathcal{H} < \infty$  for every  $m \in \mathbb{N}$ . Then  $\mathcal{B} \in (r_g^w(t,s):f_0)$  if and only if (2.3), (2.4), (2.5) and (2.6) hold.

Corollary 2.1 If we take  $g_m = 1$  for each  $m \in \mathbb{N}$ , then we get results obtained in [1].

Corollary 2.2 If we take s = 0, then we get results obtained in [14].

**Corollary 2.3** If we take  $g_m = 1$  and s = 0 for all  $m \in \mathbb{N}$ , then we get results obtained in [28].

**Definition 2.1** Solid spaces [18]: We call a sequence space  $\mathcal{X}$  to be solid if and only if  $\ell_{\infty}\mathcal{X} \subset \mathcal{X}$ .

**Definition 2.2** Define a space  $\widetilde{F}$  as the sets of all sequences such that its R-transform is in the space f, that is,

$$\widetilde{F} = \left\{ v \in \Omega : \lim_{m \to \infty} \sum_{k=0}^{m} \frac{1}{m+1} \sum_{j=0}^{k} \frac{w_j v_{j+n}}{W_k^{s+1}} = \beta \quad uniformly \ in \ n \in \mathbb{N} \right\}.$$

**Theorem 2.4** The space  $\widetilde{F}$  is not solid space.

**Proof:** To prove the result, choose

$$\varsigma = (\varsigma_j) = \left(\frac{W_0}{w_0}, -\left(\frac{W_0}{w_0} + \frac{W_0}{w_0}\right), \left(\frac{W_1}{w_1} + \frac{W_2}{w_2}\right), \cdots, (-1)^j \left(\frac{W_j}{w_j} + \frac{W_{j+1}}{w_{j+1}}\right), \cdots\right); \ s = 0$$

and

$$\xi = (\xi_j) = \left(1, -1, 1, \cdots, (-1)^j, \cdots\right).$$

Then trivially, we see  $\zeta \in f$  and  $\xi \in \ell_{\infty}$ . Define  $\zeta = v$ , that is,  $(\zeta = v)_j = (v_j)$ . Also, it is obvious that

$$(v_j) = \left(\frac{W_0}{w_0}, \left(\frac{W_0}{w_0} + \frac{W_0}{w_0}\right), \left(\frac{W_1}{w_1} + \frac{W_2}{w_2}\right), \cdots, \left(\frac{W_j}{w_j} + \frac{W_{j+1}}{w_{j+1}}\right), \cdots\right).$$

So, by definition of Riesz matrix, we see that

$$\mathcal{F}_R - \lim \upsilon = \lim_{m \to \infty} \sum_{k=0}^m \frac{1}{m+1} \sum_{j=0}^k \frac{w_j \upsilon_{j+n}}{W_k} = \infty.$$

Therefore, for spaces  $\ell_{\infty}$  and  $\widetilde{F}$ , the multiplication  $\ell_{\infty}\widetilde{F}$  is not a subset of  $\widetilde{F}$  and hence we conclude that the space  $\widetilde{F}$  is not solid.  $\Box$ 

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### References

- 1. Abdul, H. G. and Dowlath, F., Almost convergence property of generalized Riesz spaces. Journal of Applied Mathematics and Computation 4(4), 249-253, (2020).
- 2. Abdul, H. G., Sigma bounded sequence and some matrix transformations. Algebra Letters 3 , 1-7, (2013).
- 3. Abdul, H. G. and Neyaz, S., Infinite matrices and almost bounded sequences. Vietnam J. Math. 42(2), 153-157, (2014).
- 4. AlBaidani, M. M., Notion of new structure of uncertain sequences using  $\Delta$ -spaces. Journal of Mathematics, Volume 2022, Article ID 2615772, 6 pages.
- 5. AlBaidani, M. M., Statistical convergence of Δ-spaces using fractional Order. Symmetry 14(8), 1-8, (2022).
- 6. AlBaidani, M. M. and McDonald, J. J., On the block structure and frobenius normal form of powers of matrices. Electron. J. Linear Algebra 35(1), 297-306, (2019).
- AlBaidani M. M., Srivastava, H. M. and Ganie, A. H., Notion of non-absolute family of spaces. Int. J. Nonlinear Anal. Appl. 14(1), 345-354, (2023).

- 8. Dowlath, F. and Ganie, A. H., On some new scenario of Δ-spaces. J. Nonlinear Sci. Appl. 14, 163-167, (2021).
- 9. Ganie, A. H., New Spaces Over Modulus Function . Bol. Soc. Paran. Mat. (3s.) 41, 1-6, (2021).
- Ganie, A. H., Mobin, A., Neyaz, A. S. and Tanweer, J., New type of Riesz sequence space of non-absolute type. J. Appl. Comput. Math. 5(1), 1-4, (2016).
- 11. Gordon, M. P., Regular matrix transformations. McGraw-Hill Publishing Co. Ltd., London-New York-Toronto, (1966).
- 12. Gupkari, S. A., Some new sequence spaces and almost convergence. Filomat 22(2), 59-64, (2008).
- Hamid, G. A. and Albaidan, M. m., Matrix Structure of Jacobsthal numbers. J. Funct. Spaces., 2021 Article ID 2888840, 5pages.
- 14. Hamid, G. A. and Sheikh, N. A., Infinite matrices and almost convergence. Filomat 29(6), 1183-1188, (2015).
- 15. Hamid, G. A. and Ahmad, S. N., On some new sequence space of non-absolute type and matrix transformations. J. Egypt. Math. Soc. 21, 34-40, (2013).
- Hamid, G. A., Tripathy, B. C., Sheikh, N. A. and Sen, M., Invariant means and matrix transformations. Func. Anal.-TMA 2, 28-33, (2016).
- 17. Ishfaq, A. Malik and Tanweer, J., Measures of noncompactness in  $(\overline{N}_{\Delta^{-}}^{q})$  summable difference sequence space. J. Math. Ext., 13(4), 155-171, (2019).
- 18. Johann, B., Classical and modern methods in summability. Oxford University Press, Oxford, UK, (2001).
- 19. Kizmaz, H., On certain sequence spaces. Canad. Math. Bull. 24(2), 169-176, (1981).
- 20. Lorentz, G. G., A contribution to the theory of divergent series. Acta Math. (80), 167-190, (1948).
- 21. Mursaleen, M., Infinite matrices and almost convergent sequences. Southeast Asian Bull. Math., 19(1), 45-48, (1995).
- 22. Mursaleen, M., Abdul, H. G., and Neyaz, A. S., New type of difference sequence space and matrix transformation. FILOMAT 28(7), 1381-1392, (2014).
- 23. Nanda, S., Matrix transformations and almost boundedness. Glas. Mat. 14(34), 99-107, (1979).
- Rahman, M. F. and Karim, A. B. M. R., Generalized Riesz sequence space of non-absolute type and some matrix mappings. Pure Appl. Math. J. 4(3), 90-95, (2015).
- 25. Sheikh, N. A. and Abdul, H. G., A new paranormed sequence space and some matrix transformations. Acta Math. Acad. Paedago. Nygr. 28(1), 47-58, (2012).
- 26. Sheikh, N. A. and Ganie, A. H., On the spaces of  $\lambda$ -convergent sequences and almost convergence. Thai J. Math. 11(2), 393-398, (2013).
- 27. Stefan, B., Theöries des operations linéaries. Warszawa, (1932).
- Tanweer, J. and Abdul, H. G., Almost Convergence and some matrix transformation. Shekhar (New Series)- Int. J. Math. 1(1), 133-138, (2009).
- 29. Tanweer, J., Sameer, A. G. and Abdul, H. G., Infinite matrices and  $\sigma$ -convergent sequences. Southeast Asian Bull. Math. 36(6), 825-830, (2012).
- 30. Tarray, A. T., Naik, P. A. and Najar, R. A., Matrix representation of an all-inclusive Fibonacci sequence. Asian Journal of Mathematics and Statistics, 11(1), 18-26, (2018).
- 31. Wilansky, A., Summability Through Functional Analysis. Amsterdam-New York-Oxford, (Mathematics Studies 85), (1984).

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