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A pair of non-self mappings and their fixed point satisfying integral type contraction condition in convex spaces

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ABSTRACT: A common fixed point theorem is proved for a pair of non-self mappings by using contraction condition of integral type on non-empty closed subset K of a metrically convex metric space X. Result generalizing and unifying the previous results due to Branciari [2], Ćirić [4], Rhoades [19], Khan [12, 13] and others.

Key Words: Fixed point, Integral type contraction condition, Metric convexity, Weakly commuting mappings, Compatible mappings.

Contents

1. Introduction

There exists a variety of an enormous literature over fixed point theoerem. So far various researchers proved fixed point theorems by using different kinds of related concepts and ideas over self mappings, nonself mappings, set valued mappings and hybrid mappings on a closed subset of a metrically convex spaces. Here, we recall Assad and Kirk [1], Nadler [17], Hadzic [5], Hadzic and Gajic [6], Rhoades [18], Ćirić and Ume [3], Imdad et al. [9], Imdad and Khan [7, 8], Khan and Imdad [14, 15, 16], Khan [11] and many others.

Now, I am going to discuss a different type of contraction condition known as the contraction condition of integral type. In 2002, Branciari [2] introduced this very new and interesting contraction condition and proved a fixed point theorem. After this result, many authors have been used this interesting technique in their own research work and proved many productive and high quality research articles which includes Rhoades[19], Khan [12, 13] and others.

In this paper, we also consider the above said contraction condition of integral type in metrically convex spaces and prove a fixed point theorem for a pair of nonself single valued mappings. The result of this paper generalized and extended the result due to Branciari [2], Rhoades [19], Ćirić [4], Khan [12, 13] and many others.

Here, let us recall the following results for our ongoing discussion.

Theorem 1.1 ([2]). Let (X, d) be a complete metric space, $c \in [0, 1), f : X \to X$ be a mapping such that, for each $x, y \in X$,

$$\int_0^{d(fx,fy)} \phi(t)dt \leq \ c \int_0^{d(x,y)} \phi(t)dt,$$

where $\phi : R^+ \to R^+$ is a Lebesgue-integrable mapping which is summable,non-negative and such that, for each $\epsilon > 0$, $\int_0^{\epsilon} \phi(t)dt > 0$. Then f has a unique fixed point $z \in X$ such that, for each $x \in X$, $\lim_{n\to\infty} f^n x = z$.

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Theorem 1.2 ([19]). Let (X,d) be a complete metric space, $k \in [0,1)$, $f : X \to X$ be a mapping such that, for each $x, y \in X$,

$$\int_0^{d(fx,fy)} \phi(t)dt \leq \ k \int_0^{m(x,y)} \phi(t)dt,$$

where $m(x,y) = \max\left\{d(x,y), d(x,fx), d(y,fy), \frac{d(x,fy)+d(y,fx)}{2}\right\}$ and $\phi : R^+ \to R^+$ is a Lebesgue-integrable mapping which is summable,non-negative and such that, for each $\epsilon > 0$, $\int_0^\epsilon \phi(t)dt > 0$. Then f has a unique fixed point $z \in X$ such that, for each $x \in X$, $\lim\limits_{n\to\infty} f^n x = z$.

Theorem 1.3 ([13]). Let (X,d) be a complete metrically convex metric space and K be a nonempty closed subset of X. Let a mapping $T : K \to X$ be generalized contraction condition on K, if for each $x, y \in K$,

$$\int_0^{d(Tx,Ty)} \phi(t)dt \leq \ c \int_0^{m(x,y)} \phi(t)dt, c \in [0,1)$$

where $m(x,y) = h \max\left\{\frac{1}{2}d(x,y), d(x,Tx), d(y,Ty), \frac{d(x,Ty)+d(y,Tx)}{q}\right\}, 0 < h(1+h) \leq k < 1$ and any real number q satisfying $q \geq 1+2k$, and $\phi : R^+ \to R^+$ is a Lebesgue-integrable mapping which is summable, non-negative and such that

$$\int_0^\epsilon \phi(t)dt > 0 \text{ for each } \epsilon > 0.$$

and for each $x \in \delta K, Tx \in K$. Then T has a unique fixed point $x \in K$ such that, for each $x \in K$, $\lim\limits_{n\to\infty} T^n x = x$.

Before proving the results, we collect the following definitions for future discussion.

Definition 1.4. Let (X,d) be a metric space and K be a nonempty subset of a metric space X. Let the mapping $F, T : K \to X$ is said to be contraction condition on K, if for each $x, y \in K$,

$$\int_0^{d(F(x),F(y))} \phi(t)dt \leq \ k \int_0^{m(x,y)} \phi(t)dt, \tag{1.1}$$

where $m(x,y) = h \max\left\{\frac{1}{2}d(Tx,Ty), d(Tx,Fx), d(Ty,Fy), \frac{d(Tx,Fy)+d(Ty,Fx)}{q}\right\}, 0 < h(1+h) \leq k < 1$ and any real number q satisfying $q \geq 1+2k$, and $\phi : R^+ \to R^+$ is a Lebesgue-integrable mapping which is summable, non-negative and such that

$$\int_0^\epsilon \phi(t)dt > 0 \text{ for each } \epsilon > 0. \tag{1.2}$$

Definition 1.5 ([5]). Let K be a nonempty subset of a metric space (X,d) and the mappings $F, T : K \to X$ be single valued. The pair (F,T) is said to be weakly commuting if for every $x, y \in K$ with $x = Fy$ and $Ty \in K$ we have

$$d(Tx, FTy) \leq d(Ty, Fy).$$

Notice that for $K = X$, this definition reduces to that of Sessa [20].

Definition 1.6 ([6]). Let K be a nonempty subset of a metric space (X,d) and the mappings $F, T : K \to X$ be single valued. The pair (F,T) is said to be compatible if for every sequence $\{x_n\} \in K$ and from the relation

$$\lim_{n\to\infty} d(Fx_n, Tx_n) = 0$$

and $Tx_n \in K$ (for every $n \in N$) it follows that

$$\lim_{n\to\infty} d(Ty_n, FTx_n) = 0$$

for every sequence $\{y_n\} \in K$ such that $y_n = Fx_n, n \in N$.

Notice that for $K = X$, this definition reduces to that of Jungck [10].

Definition 1.7 ([1]). A metric space (X, d) is said to be metrically convex if for any $x, y \in X$ with $x \neq y$ there exists a point $z \in X, x \neq z \neq y$ such that

$$d(x, z) + d(z, y) = d(x, y).$$

2. Results

The result of this paper runs as follows.

Theorem 2.1. Let (X, d) be a complete metrically convex metric space and K be a nonempty closed subset of X with $F, T : K \to X$. Assume that F be a generalized T contraction mappings of K into X which satisfy the conditions:

(i) $\delta K \subset TK, FK \cap K \subset TK$

(ii) $Tx \in \delta K \Rightarrow Fx \in K$

(iii) F and T are weakly commuting and

(iv) T is continuous on K

Then F and T have a common unique fixed point.

Proof. Firstly, we proceed to construct two sequences $\{x_n\}$ and $\{y_n\}$ in the following way. Let $x \in \delta K$. Then due to $\delta K \subset TK$ there exists a point $x_0 \in K$ such that $x = Tx_0$, which implies that $Tx_0 \in \delta K$. Since $Tx_0 \in \delta K$ and $Tx \in \delta K \Rightarrow Fx \in K$, we conclude that $Fx_0 \in FK \cap K \subset TK$. Let $x_1 \in K$ be such that $y_1 = Tx_1 = Fx_0 \in K$. Let $y_2 = Fx_1$. Suppose $y_2 \in K$ then $y_2 \in FK \cap K \subset TK$, which implies that there exists a point $x_2 \in K$ such that $y_2 = Tx_2$. Suppose $y_2 \notin K$, then there exists a point $p \in \delta K$ such that

$$d(Tx_1, p) + d(p, y_2) = d(Tx_1, y_2).$$

Since $p \in \delta K \subset TK$, there exists a point $x_2 \in K$ with $p = Tx_2$ so that

$$d(Tx_1, Tx_2) + d(Tx_2, y_2) = d(Tx_1, y_2).$$

Write $y_3 = Fx_2$. Thus, repeating the foregoing arguments, one obtains two sequences $\{x_n\}$ and $\{y_n\}$ such that

(i) $y_{n+1} = Fx_n$,

(ii) $y_n = Tx_n$ if $y_n \in K$,

(iii) if $y_n \notin K \Rightarrow Tx_n \in \delta K$ and

$$d(Tx_{n-1}, Tx_n) + d(Tx_n, y_n) = d(Tx_{n-1}, y_n).$$

Here, one obtains two types of sets we denote as follows:

$$P = \{Tx_i \in \{Tx_n\} : Tx_i = y_i\} \text{ and } Q = \{Tx_i \in \{Tx_n\} : Tx_i \neq y_i\}.$$

One can note that if $Tx_n \in Q$ then Tx_{n-1} and $Tx_{n+1} \in P$. We, wish to estimate $d(Tx_n, Tx_{n+1})$. Now, we distinguish the following three cases.

Case 1. If Tx_n and $Tx_{n+1} \in P$, then

$$\int_0^{d(Tx_n,Tx_{n+1})} \phi(t)dt = \int_0^{d(Fx_{n-1},Fx_n)} \phi(t)dt \leq \ k \int_0^{m(y_{n-1},y_n)} \phi(t)dt.$$

Since

$$d(Tx_{n+1},Tx_n) = d(Fx_{n-1},Fx_n)$$

$$\leq h \ \max \left\{\tfrac{d(Tx_{n-1},Tx_n)}{2}, d(Tx_{n-1},Fx_{n-1}), d(Tx_n,Fx_n), \tfrac{d(Tx_{n-1},Fx_n)+d(Tx_n,Fx_{n-1})}{q}\right\}$$

$$\leq h \ \max\left\{\frac{d(y_{n-1},y_n)}{2}, d(y_{n-1},y_n), d(y_n,y_{n+1}), \frac{d(y_{n-1},y_{n+1})}{q}\right\}. \quad (1.3)$$

Here, if $d(y_{n-1},y_n)$ is maximum, then we have

$$d(Tx_{n+1},Tx_n) \leq h \ d(y_{n-1},y_n) < d(Tx_{n-1},Tx_n).$$

Otherwise, if we suppose that $d(y_{n-1},y_n) < d(y_n,y_{n+1})$ then we obtain

$$d(Tx_n,Tx_{n+1}) = d(Fx_{n-1},Fx_n) \leq d(Tx_n,Tx_{n+1}),$$

which is a contradiction.

Now, if $\frac{d(y_{n-1},y_{n+1})}{q}$ is maximum then from equation (1.3) we have

$$d(Tx_n,Tx_{n+1}) = d(Fx_{n-1},Fx_n) \leq \frac{h}{q}\left\{d(y_{n-1},y_n) + d(y_n,y_{n+1})\right\}$$

$$\leq \frac{h}{q-h} d(y_{n-1},y_n) < hd(y_{n-1},y_n) < kd(y_{n-1},y_n) < d(y_{n-1},y_n).$$

Therefore

$$d(Tx_n,Tx_{n+1}) = d(Fx_{n-1},Fx_n) \leq d(y_{n-1},y_n).$$

Hence

$$\int_0^{d(Tx_n,Tx_{n+1})} \phi(t)dt \leq \ k \int_0^{m(y_{n-1},y_n)} \phi(t)dt.$$

Case 2. If $Tx_n \in P$ and $Tx_{n+1} \in Q$, then

$$d(Tx_n,Tx_{n+1}) + d(Tx_{n+1},y_{n+1}) = d(Tx_n,y_{n+1})$$

which in turn yields

$$d(Tx_n,Tx_{n+1}) \leq d(Tx_n,y_{n+1}).$$

Now, proceeding as in case 1, we have

$$\int_0^{d(Tx_n,Tx_{n+1})} \phi(t)dt \leq \ k \int_0^{m(y_{n-1},y_n)} \phi(t)dt.$$

Case 3. If $Tx_n \in Q$ and $Tx_{n+1} \in P$. Since $Tx_n \in Q$ and is a convex linear combination of Tx_{n-1} and y_n, it follows that

$$d(Tx_n,Tx_{n+1}) \leq \max\{d(Tx_{n-1},Tx_{n+1}), d(Tx_{n+1},y_n)\}.$$

If $d(Tx_{n-1},Tx_{n+1}) \leq d(Tx_{n+1},y_n)$, then proceeding as in case 1, we have

$$\int_0^{d(Tx_n,Tx_{n+1})} \phi(t)dt \leq \ k \int_0^{m(y_{n-1},y_n)} \phi(t)dt.$$

Otherwise if $d(Tx_{n+1}, y_n) \le d(Tx_{n-1}, Tx_{n+1})$, then we have

$$\int_0^{d(Tx_n,Tx_{n+1})} \phi(t)dt \le \int_0^{d(Tx_{n-1},Tx_{n+1})} \phi(t)dt = \int_0^{d(Tx_{n-2},Tx_n)} \phi(t)dt \le \ k \int_0^{m(y_{n-2},y_n)} \phi(t)dt.$$

Here

$$d(Tx_n, Tx_{n+1}) \le$$

$$h \ \max \left\{ \frac{d(Tx_{n-2}, Tx_n)}{2}, d(Tx_{n-2}, Fx_{n-2}), d(Tx_n, Fx_n), \frac{d(Tx_{n-2}, Fx_n) + d(Tx_n, Fx_{n-2})}{q} \right\}$$

$$\le h \ \max \left\{ \frac{d(y_{n-2}, y_n)}{2}, d(y_{n-2}, y_{n-1}), d(y_n, y_{n+1}), \frac{d(y_{n-2}, y_{n+1}) + d(y_n, y_{n-1})}{q} \right\}.$$

Notice that

$$\frac{d(y_{n-2}, y_n)}{2} \le \frac{d(y_{n-2}, y_{n-1}) \ + \ d(y_{n-1}, y_n)}{2} \le \max \left\{d(y_{n-2}, y_{n-1}), d(y_{n-1}, y_n)\right\}.$$

Here, if

$$d(y_{n-2}, y_{n-1}) \le d(y_{n-1}, y_n), \text{then } d(y_{n-2}, y_n) \le d(y_{n-1}, y_n).$$

Otherwise, if

$$d(y_{n-1}, y_n) \le d(y_{n-2}, y_{n-1}), \text{ then } d(y_{n-2}, y_n) \le d(y_{n-2}, y_{n-1}).$$

Therefore, we obtain

$$d(Tx_n, Tx_{n+1}) \le$$

$$h \ \max \left\{ d(y_{n-2}, y_{n-1}), d(y_{n-1}, y_n), d(y_n, y_{n+1}), \frac{d(y_{n-2}, y_{n+1}) + d(y_n, y_{n-1})}{q} \right\}$$

which in turn yields

$$d(Tx_n, Tx_{n+1}) \le \begin{cases} k \ d(Tx_{n-1}, Tx_n) \ \text{ if } \ d(y_{n-1}, y_n) \ge d(y_{n-2}, y_{n-1}) \text{ or} \\ k \ d(Tx_{n-2}, Tx_{n-1}) \ \text{ if } \ d(y_{n-1}, y_n) \le d(y_{n-2}, y_{n-1}). \end{cases}$$

Thus in all the cases, we have

$$d(Tx_n, Tx_{n+1}) \le k \max\{d(Tx_{n-2}, Tx_{n-1}), d(Tx_{n-1}, Tx_n)\}.$$

It can be easily shown by induction that for $n > 1$, we have

$$d(Tx_n, Tx_{n+1}) \le k \max\{d(Tx_0, Tx_1), d(Tx_1, Tx_2)\}.$$

Thus

$$\int_0^{d(Tx_n,Tx_{n+1})} \phi(t)dt \le k \int_0^{\max\{d(Tx_0,Tx_1),d(Tx_1,Tx_2)\}} \phi(t)dt$$

which implies that

$$\int_0^{d(Tx_n,Tx_{n+1})} \phi(t)dt \le k \ \max \left\{ \int_0^{d(Tx_0,Tx_1)} \phi(t)dt, \int_0^{d(Tx_1,Tx_2)} \phi(t)dt \right\}.$$

It follows that the sequence $\{d(Tx_n, Tx_{n+1})\}$ is monotonically decreasing. Hence

$\int_0^{d(Tx_n,Tx_{n+1})}\phi(t)dt \to 0$ as $n\to\infty$. From equation (1.2) it implies that $\lim_{n\to\infty} d(Tx_n,Tx_{n+1})=0$, so that $\{Tx_n\}$ is a Cauchy sequence and hence converges to a point $z\in X$.

Now we assume that there exists a subsequence $\{Tx_{n_k}\}$ of $\{Tx_n\}$, which is contained in P. Since T is continuous $\{TTx_{n_k}\}$ converges to a point $Tz\in X$. On using the weak commutativity of (F,T), we have $Fx_{n_k-1}=Tx_{n_k}$ and $Tx_{n_k-1}\in K$, so

$$d(TTx_{n_k},FTx_{n_k-1})\le d(Fx_{n_k-1},Tx_{n_k-1})=d(Tx_{n_k},Tx_{n_k-1}).$$

On letting $k\to\infty$ we obtain $d(Tz,FTx_{n_k-1})\to 0$, which means that $FTx_{n_k-1}\to Tz$ as $k\to\infty$.

Now using contraction condition, we can write

$$d(FTx_{n_k-1},Fz)$$

$$\le h\ \max\left\{\frac{d(TTx_{n_k-1},Tz)}{2},d(TTx_{n_k-1},FTx_{n_k-1}),d(Tz,Fz),\frac{d(TTx_{n_k-1},Fz)+d(Tz,FTx_{n_k-1})}{q}\right\}$$

which on letting $k\to\infty$, reduces to

$$d(Tz,Fz)\le h\ \max\left\{0,0,d(Tz,Fz),\frac{d(Tz,Fz)}{q}\right\}$$

yielding thereby $Tz=Fz$.

To prove that z is a fixed point of T, consider

$$\int_0^{d(Tx_{n_k},Tz)}\phi(t)dt=\int_0^{d(Fx_{n_k-1},Fz)}\phi(t)dt\le k\int_0^{m(x_{n_k-1},z)}\phi(t)dt$$

Since

$$d(Tx_{n_k},Tz)=d(Fx_{n_k-1},Fz)$$

$$\le h\ \max\left\{\frac{d(Tx_{n_k-1},Tz)}{2},d(Tx_{n_k-1},Fx_{n_k-1}),d(Tz,Fz),\frac{d(Tx_{n_k-1},Fz)+d(Tz,Fx_{n_k-1})}{q}\right\}.$$

On letting $k\to\infty$, then we have,

$$d(Tz,z)\le h\ \max\left\{\frac{d(Tz,z)}{2},d(z,z),d(Tz,Fz),\frac{d(z,Fz)+d(Tz,z)}{q}\right\}$$

Therefore this implies that

$$\int_0^{d(Tz,z)}\phi(t)dt\le k\ \max\left\{\int_0^{\frac{d(Tz,z)}{2}}\phi(t)dt,\int_0^{d(z,z)}\phi(t)dt,\int_0^{d(Tz,Fz)}\phi(t)dt,\int_0^{2d(Tz,z)}\phi(t)dt,\right\}.$$

$$\int_0^{d(Tz,z)}\phi(t)dt\le k\int_0^{d(Tz,z)}\phi(t)dt$$

which implies that

$$\int_0^{d(Tz,z)}\phi(t)dt=0,$$

which implies that $d(Tz,z)=0\Rightarrow Tz=z$. This shows that z is a fixed point of T. This also implies that z is a common fixed point of F and T.

To prove that the uniqueness of fixed points. Let us suppose that z_1 and z_2 are common fixed points of F and T. Then

$$\int_0^{d(z_1,z_2)} \phi(t)dt = \int_0^{d(Fz_1,Fz_2)} \phi(t)dt \leq k \int_0^{m(z_1,z_2)} \phi(t)dt$$
$$= k \max \left\{ \int_0^{d(z_1,z_2)} \phi(t)dt, 0 \right\} = k \int_0^{d(z_1,z_2)} \phi(t)dt$$

which implies that $\int_0^{d(z_1,z_2)} \phi(t)dt = 0$. Also imply that $d(z_1, z_2) = 0$ or $z_1 = z_2$. This shows the uniqueness of fixed point. This completes the proof.

Remark 2.2. By setting $K = X$ and $F = T$ in the Theorem 2.1, then we deduce a sharpened version of the result due to Branciari [2].

Remark 2.3. By setting $K = X$ and $F = T$ in the Theorem 2.1, then we deduce a partial generalization of the result due to Rhoades [19].

Remark 2.4. By setting $F = T$ and with some minor changes in the Theorem 2.1, then we deduce the result due to Khan [12].

Remark 2.5. By setting $F = T$ in the Theorem 2.1, then we deduce the result due to Khan [13].

Remark 2.6. Theorem 2.1 remains true if weakly commuting mappings is replaced by compatible mappings.

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