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Laplace residual series method for solving fractional Newell–Whitehead–Segel Model

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ABSTRACT: In this paper, we present certain Laplace residual power series method as a novel numeric- analytic approach to create approximate and analytic solutions for a fractional Newell-Whitehead-Segel model equipped with appropriate initial space functions. The aforesaid approach depends on coupling the Laplace integral operator together with the generalized Taylor's formula, where the coefficients of the fractional expansion in the Laplace space is produced by using the limit notion. Numerical experiments are done to guarantee and illustrate the theoretical methodology of the Laplace residual power series LRPS approach. They, also demonstrate its performance, applicability and superiority to solve different kinds of non-linear time and space fractional differential models. The obtained analytical solutions by the present approach agree with other approaches and are compatible with the exact solutions. Numerical simulations of the given results reveal that the LRPS approach is effective, simple and harmonious with the complexity of the non-linear problems.

Key Words: Fractional series expansion, Caputo derivative, fractional Newell-Whitehead-Segel model, Laplace residual power series, fractional derivatives.

Contents

1. Introduction

As certain realistic modeling of various physical phenomena uses instantaneous and past times, several phenomena in the fields of biology, chemistry, psychology and fluid mechanics are effectively modeled by employing the theory of fractional calculus [1-7]. The idea of fractional calculus has gained interest by many scientists from different branches of science. Various concepts of fractional derivatives and integrals are discussed by Riemann, Liouville, Hadamard and Caputo [8-14]. Despite various definitions of the fractional derivatives are given in literature [15,16], the definition of Caputo still the very well liked definition as it applies to various real-world applications.

Recently, an attentiveness has been made to the solution of fractional and partial differential equations which represent usual physical phenomena. Consequently, numerical and analytical approaches for finding approximate and exact solutions of fractional differential equations are discussed by various methods such as the Laplace transform method [17], AD method(ADM) [18,19], VI method (VIM) [20,21], HP method (HPM) [21,22], RPS method (RPSM), HA method (HAM) [23-25] and others [25-29].

The method of the residual power series (RPSM) is a technique which proposes an approximate series solution to differential equations (fractional and non-fractional) which has succeeded in extracting various types of differential equations including nonlinear time-fractional Schrodinger equations [27], equations of neutron diffusion type [26], systems of fractional multi-pantographs type [25], equations of fractional KdV-Burgers [28], and certain class of higher-order non-homogeneous matrix fractional differential equations [29]. The residual power series method provides an easy and fast mechanism in estimating the coefficients

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of a solution of a power series. Despite that, in [30] the authors use the Laplace transform for discussing approximate and exact series solutions to neutral fractional differential equations. The authors use their thoughts to determine the coefficients of the series solution other than the motif of its derivative. This indeed ease the process of estimating the coefficients and increases the speed. It also provides solutions in a form of convergent series without implementing perturbation, discretization and linearization techniques. In fact, it is worth noting that the LRPS transforms differential equations to a Laplace space and produces Laurent expansion as a solution, and then uses the inverse Laplace transform to convert the new solution to an initial problem.

In this study, we will extend the application of the Laplace residual power series (LRPS) method to derive the approximate solution of the Fractional Newell–Whitehead–Segel equation (FNWS) model in two-dimensional space of the following shape:

$$\mathfrak{D}_t^{\delta}\vartheta(x,t) = a\mathfrak{D}_x^{2}\vartheta(x,t) + b\vartheta(x,t) - c\vartheta^{p}(x,t), \tag{1.1}$$

Equipped with the initial data

$$\vartheta(x,0) = \vartheta_0(x) \tag{1.2}$$

where $x \in \mathbb{R}$, $t \geq 0$, $1 \geq \delta > 0$ and $p \in \mathbb{Z}^{+}$. Herein, ϑ is an arbitrary function to be determined and D_t^{δ} is a δ-th time-Caputo fractional derivative.

As a classical model, the model of Newell–Whitehead–Segel types (NWS) is a popular amplitude model which explains recurrence of arbitrary stationary spatial stripe patterns in a dynamic behavior very close to the point of bifurcation of Rayleigh–Benard convection having binary fluid mixtures and two-dimensional systems [7]. In fact, it can be observed that two kinds of patterns are there: the roll and the hexagonal patterns. In the first kind, all cylinders are constituted by employing fluid stream lines forming spirals like patterns, whereas in the hexagonal pattern, all stripes cells and honey combs are constituted by dividing the liquid flow. It worth to be mentioned here that all mentioned hexagonal patterns may be derived in diffusion models and chemical reactions and by applying propagations [8].

Several mathematical formations are found to attain certain nonlinear fractional partial differential equations. However, the solution of such equations is known to be difficult. So, efficient algorithms are in demand to establish their analytical or approximate equations. Nowadays, several techniques are developed to solve the FNWSE including the methods of variational iterations [10], Adomian decompositions [11], sumudu transforms [9] and the fractional complex transforms [12], see also [13–18] for further techniques.

2. Basic Concepts

In this section, we recall some fundamental definitions from the fractional calculus theory and the Laplace operators. Further, we review the most fundamental properties related to the Laplace transform, Caputo operators and fractional series expansion in the Laplace space.

Definition 2.1 *The time-fractional Riemann-Liouville integral operator J_t^{δ} of order $\delta \geq 0$ is defined by*

$$J_t^{\delta}\vartheta(x,t) = \begin{cases} \frac{1}{\Gamma(\delta)}\int_0^t (t-\tau)^{\delta-1}\vartheta(x,\tau)\,d\tau, & \delta > 0\ ,\ t > \tau \geq 0 \\ \vartheta(x,t), & \delta = 0. \end{cases}, \tag{2.1}$$

Definition 2.2 *For $m \in \mathbb{N}$, the time-fractional derivative $\mathfrak{D}_t^{\delta}$ of an arbitrary function ϑ of order $\delta > 0$, is defined, in the meaning of Caputo,*

$$\mathfrak{D}_t^{\delta}\vartheta(x,t) = J_t^{m-\gamma}\partial_t^{m}\vartheta(x,t),\ m-1 < \delta \leq m,\ x \in I,\ t \geq 0. \tag{2.2}$$

Definition 2.3 *Let ϑ be a function defined on $I \times [0,\infty)$. If ϑ is piecewise continuous and having exponential order η, then the Laplace transform of ϑ, denoted by V, is given by*

$$V(x,s) = \mathcal{L}[\vartheta(x,t)] := \int_0^{\infty} e^{-st}\vartheta(x,t)\,dt,\ s > \eta. \tag{2.3}$$

The inversion formula of the Laplace operator of is defined by

$$\vartheta(x,t) = \mathcal{L}^{-1}[V(x,s)] := \int_{z-i\infty}^{z+i\infty} e^{st} V(x,s)\, ds, \quad z = Re(s) > z_0. \tag{2.4}$$

Several important properties of the Laplace transform operator and its inversion formula are enumerated in the following lemma.

Lemma 2.1 *Let the functions ϑ and ω be of exponential orders η_1 and η_2, respectively, where $\eta_1 < \eta_2$ and piecewise continuous functions on $I \times [0,\infty)$. If $V = \mathcal{L}[\vartheta]$, $\mathrm{W} = \mathcal{L}[w]$ and a, b are constants, then we have*

(i) $\mathcal{L}[a\vartheta(x,t) + b\omega(x,t)] = a\,V(x,s) + b\mathrm{W}(x,s),\ x \in I,\ s > \eta_1.$
(ii) $\mathcal{L}^{-1}[aV(x,s) + bW(x,s)] = a\vartheta(x,t) + b\omega(x,t), x \in I, t \geq 0.$
(iii) $\lim_{s\to\infty} s\,V(x,s) = \vartheta(x,0), x \in I.$
(iv) $\mathcal{L}\left[\mathfrak{D}_t^{\delta}\vartheta(x,t)\right] = s^{\delta}V(x,s) - \sum_{k=0}^{m-1} s^{\delta-k-1}\partial_t^k \vartheta(x,0),\ m-1 < \delta < m.$
(v) $\mathcal{L}\left[\mathfrak{D}_t^{n\delta}\vartheta(x,t)\right] = s^{n\delta}V(x,s) - \sum_{k=0}^{n-1} s^{(n-k)\delta-1} D_t^{k\delta}\vartheta(x,0), 0 < \delta < 1.$

Theorem 2.1 *Assume the function ϑ has a fractional power series expansion about $t = 0$ given as*

$$\vartheta(x,t) = \sum_{n=0}^{\infty} \vartheta_n(x) t^{n\delta},\ 0 \leq m-1 < \delta \leq m\ ,\ x \in I,\ 0 \leq t < R. \tag{2.5}$$

Then, if $\mathfrak{D}_t^{n\delta}\vartheta$ is continuous on $I \times (0,R)$, $m = 0,1,2,\ldots$, then the coefficients ϑ_n of the expansion (2.5) *are given as $\vartheta_n(x) = \frac{\mathfrak{D}_t^{n\delta}\vartheta(x,0)}{\Gamma(n\delta+1)}$, $n = 0,1,2,\ldots$, where $\mathfrak{D}_t^{n\delta} = \mathfrak{D}_t^{\delta}.\mathfrak{D}_t^{\delta}\ldots\mathfrak{D}_t^{\delta}$ (n-times). R being the radius of convergence of the series.*

Theorem 2.2 *Suppose that the Laplace transform of a continuous function ϑ is V. Then, the function V assumes the fractional series expansion*

$$V(x,s) = \sum_{n=0}^{\infty} \frac{\vartheta_n(x)}{s^{1+n\delta}}, 0 < \delta \leq 1,\ x \in I,\ s > \eta. \tag{2.6}$$

Theorem 2.3 *Let $V(x,s) = \mathcal{L}[\vartheta(x,t)]$ be written in the fractional series expansion* (2.6)*.*
If $\left|\ s\ \mathcal{L}[\mathfrak{D}_t^{(n+1)\delta}\vartheta]\right| \leq \mathcal{M}$ on $I \times (\eta, d]$ where $0 < \delta \leq 1$, then the reminder $\mathcal{R}_n$ of the expansion (2.6) *satisfies the following inequality*

$$|\mathcal{R}_n(x,s)| \leq \frac{\mathcal{M}(x)}{s^{1+(n+1)\delta}},\ x \in I,\ \eta < s \leq d\ . \tag{2.7}$$

3. Certain Principle of the LRPS Method

This section explains the principle of our algorithm to derive series solutions for the fractional IVPs (1.1) and (1.2) via coupling the Laplace transform operator following the fractional residual power series approach.

Following is a procedure explaining the basic principle of our proposed method.
First: Let $\mathcal{L}\{\vartheta(x,t)\} = V(x,s)$ and use the property (v) of Lemma 2.1. Then, one can employ the Laplace integral transform to the Fractional Newell–Whitehead–Segel problem to get

$$V(x,s) = \frac{V(x,0)}{s} + \frac{a}{s^{\delta}} V_{xx}(x,s) + \frac{b}{s^{\delta}} V(x,s) - \frac{c}{s^{\delta}} \mathcal{L}\left\{\left[\mathcal{L}^{-1}V(x,s)\right]^p\right\}. \tag{3.1}$$

Since $V(x,0) = \vartheta_0(x)$, then we can reformulate (3.1) as

$$V(x,s) = \frac{\vartheta_0(x)}{s} + \frac{a}{s^{\delta}} V_{xx}(x,s) + \frac{b}{s^{\delta}} V(x,s) - \frac{c}{s^{\delta}} \mathcal{L}\left\{\left[\mathcal{L}^{-1}V(x,s)\right]^p\right\}. \tag{3.2}$$

Second: According to Theorem 2.1, assume that Laplace equation (3.2) has an approximate solution given by the series expansion

$$V\left(x,s\right)=\sum_{m=0}^{\infty}\frac{\vartheta_m(x)}{s^{m\delta+1}},\ s>0. \tag{3.3}$$

In order to approximate the coefficients $\vartheta_m(x)$, let $V_i\left(x,s\right)$ be the i-th series solution of (3.3). i.e.

$$V_i\left(x,s\right)=\sum_{m=0}^{i}\frac{\vartheta_m(x)}{s^{m\delta+1}},\ s>0. \tag{3.4}$$

For $i=0$, we have $\lim_{s\to\infty}sV_0\left(x,s\right)\ =\vartheta_0(x)$. This yields

$$V_i\left(x,s\right)=\frac{\vartheta_0(x)}{s}+\sum_{m=1}^{i}\frac{\vartheta_m(x)}{s^{m\delta+1}},\ s>0. \tag{3.5}$$

Third: By identifying the i-th Laplace residual function of (3.1), one can obtain a form of the i-th series solution after determining the coefficients ϑ_m, for $m=1,2,3,\ldots,i$ as

$$\mathcal{L}\left\{ResV_i\left(x,s\right)\right\}=V_i\left(x,s\right)-\frac{\vartheta_0\left(x\right)}{s}-\frac{a}{s^\delta}(V_i)_{xx}\left(x,s\right)-\frac{b}{s^\delta}V_i\left(x,s\right)+\frac{c}{s^\delta}\mathcal{L}\left\{\left[\mathcal{L}^{-1}V_i\left(x,s\right)\right]^p\right\}. \tag{3.6}$$

The ∞-th Laplace residual function of (3.6) may be expressed as

$$\begin{aligned}\lim_{i\to\infty}\mathcal{L}\left\{ResV_i\left(x,s\right)\right\}\ &=\mathcal{L}\left\{ResV\left(x,s\right)\right\}=V\left(x,s\right)-\frac{\vartheta_0\left(x\right)}{s}-\frac{a}{s^\delta}V_{xx}\left(x,s\right)-\frac{b}{s^\delta}V\left(x,s\right)\\&+\frac{c}{s^\delta}\mathcal{L}\left\{\left[\mathcal{L}^{-1}V\left(x,s\right)\right]^p\right\}.\end{aligned} \tag{3.7}$$

Evidently, as in [Elajo], we have $\mathcal{L}\left\{ResV\left(x,s\right)\right\}=0$, and $\lim_{i\to\infty}\mathcal{L}\left\{ResV_i\left(x,s\right)\right\}\ =\mathcal{L}\left\{ResV\left(x,s\right)\right\}$, for $x\in\mathbb{R}$ and $s>0$. Further, we have $\lim_{s\to\infty}s\mathcal{L}\left\{ResV\left(x,s\right)\right\}=0$. This, indeed, implies $\lim_{s\to\infty}s\mathcal{L}\left\{ResV_i\left(x,s\right)\right\}\ =0$.

Fourth: After substituting the i-th series solution (3.4) into the i-th Laplace residual function of (3.1), we multiply the resultant equation by the factor $s^{i\delta+1}$.

Fifth: To determine the coefficients ϑ_m, $m=1,2,3,\ldots,i$, we solve $\lim_{s\to\infty}s^{i\delta+1}\mathcal{L}\left\{ResV_i\left(x,s\right)\right\}\ =0$.

Thereafter, the i-th Laplace series solution V_i of (3.2) is derived after collecting the obtained coefficients in term of the expansion series (3.4).

Finally: We apply the inversion formula of the Laplace integral on the i-th Laplace series solution V_i to obtain the approximate solution ϑ_i of the IVP's (1.1) and (1.2).

Now, to determine the first coefficient $\vartheta_1\left(x\right)$, in the expansion (3.5), we substitute $V_1\left(x,s\right)=\frac{\vartheta_0(x)}{s}+\frac{\vartheta_1(x)}{s^{\delta+1}}$ (see (3.5)) into the 1-st Laplace residual function to obtain

$$\begin{aligned}&\mathcal{L}\left\{ResV_1\left(x,s\right)\right\}=V_1\left(x,s\right)-\frac{\vartheta_0\left(x\right)}{s}-\frac{a}{s^\delta}(V_1)_{xx}\left(x,s\right)-\frac{b}{s^\delta}V_1\left(x,s\right)+\frac{c}{s^\delta}\mathcal{L}\left[\left(\mathcal{L}^{-1}V_1\left(x,s\right)\right)^p\right]\\&=\frac{\vartheta_1\left(x\right)}{s^{1+\delta}}-a\left(\frac{\vartheta_0''\left(x\right)}{s^{1+\delta}}+\frac{\vartheta_1''\left(x\right)}{s^{1+2\delta}}\right)-b\left(\frac{\vartheta_0\left(x\right)}{s^{1+\delta}}+\frac{\vartheta_1\left(x\right)}{s^{1+2\delta}}\right)\\&+\frac{c}{s^\delta}\mathcal{L}\left(\sum_{n=0}^{p}\left\{\sum_{i_1=0}^{n}\sum_{i_2=0}^{i_1}\cdots\sum_{i_{p-1}=0}^{i_{p-2}}\vartheta_{i_{p-1}}\vartheta_{i_{p-2}-i_{p-1}}\vartheta_{i_{p-3}-i_{p-2}}\cdots\vartheta_{i_1-i_2}\vartheta_{n-i_1}\right\}\frac{t^{n\delta}}{\Gamma^n\left(\gamma+1\right)}\right)\\&=\frac{\vartheta_1\left(x\right)}{s^{1+\delta}}-a\left(\frac{\vartheta_0''\left(x\right)}{s^{1+\delta}}+\frac{\vartheta_1''\left(x\right)}{s^{1+2\delta}}\right)-b\left(\frac{\vartheta_0\left(x\right)}{s^{1+\delta}}+\frac{\vartheta_1\left(x\right)}{s^{1+2\delta}}\right)\\&+c\left(\sum_{n=0}^{p}\left\{\sum_{i_1=0}^{n}\sum_{i_2=0}^{i_1}\cdots\sum_{i_{p-1}=0}^{i_{p-2}}\vartheta_{i_{p-1}}\vartheta_{i_{p-2}-i_{p-1}}\vartheta_{i_{p-3}-i_{p-2}}\cdots\vartheta_{i_1-i_2}\vartheta_{n-i_1}\right\}\frac{\Gamma\left(n\delta+1\right)}{\Gamma^n\left(\delta+1\right)s^{(n+1)\delta+1}}\right)\end{aligned}$$

Write $u_n = \sum_{i_1=0}^{n}\sum_{i_2=0}^{i_1}\cdots\sum_{i_{p-1}=0}^{i_{p-2}}\vartheta_{i_{p-1}}\vartheta_{i_{p-2}-i_{p-1}}\vartheta_{i_{p-3}-i_{p-2}}\cdots\vartheta_{i_1-i_2}\vartheta_{n-i_1}$. Then, the above expansion becomes

$$\begin{aligned}\mathcal{L}\left\{ResV_1\left(x,s\right)\right\} = &\frac{\vartheta_1(x)}{s^{1+\delta}} - a\left(\frac{\vartheta_0''\left(x\right)}{s^{1+\delta}}+\frac{\vartheta_1''\left(x\right)}{s^{1+2\delta}}\right) - b\left(\frac{\vartheta_0\left(x\right)}{s^{1+\delta}}+\frac{\vartheta_1\left(x\right)}{s^{1+2\delta}}\right)+ \\ &c\left(\frac{\vartheta_0^p\left(x\right)}{s^{1+\delta}}+\frac{u_1\left(x\right)}{s^{1+2\delta}}+\cdots+\frac{u_p\left(x\right)\Gamma\left(p\delta{+}1\right)}{\Gamma^p\left(\delta{+}1\right)s^{(p+1)\delta+1}}\right).\end{aligned} \tag{3.8}$$

Multiplying both sides of by $s^{\delta+1}$ gives

$$\begin{aligned}s^{1+\delta}\mathcal{L}\left\{ResV_1\left(x,s\right)\right\} = &\vartheta_1\left(x\right) - a\left(\vartheta_0''\left(x\right)+\frac{\vartheta_1''\left(x\right)}{s^{\delta}}\right) - b\left(\vartheta_0\left(x\right)+\frac{\vartheta_1\left(x\right)}{s^{\delta}}\right)+ \\ &c\left(\vartheta_0^p\left(x\right)+\frac{u_1\left(x\right)}{s^{\delta}}+\cdots+\frac{u_p\left(x\right)\Gamma\left(p\delta{+}1\right)}{\Gamma^p\left(\delta{+}1\right)s^{p\delta+1}}\right).\end{aligned} \tag{3.9}$$

Taking into account the fact that $\lim_{s\to\infty} s^{1+\delta}\mathcal{L}\left\{ResV_1\left(x,s\right)\right\} = 0$, we obtain $\vartheta_1\left(x\right) = a\vartheta_0''\left(x\right) + b\vartheta_0\left(x\right) - c\vartheta_0^p\left(x\right)$. Hence, the first Laplace series solution $V_1\left(x,s\right)$ of (3.2) has the form $V_1\left(x,s\right) = \frac{\vartheta_0(x)}{s} + \frac{a\vartheta_0''(x)+b\vartheta_0(x)-c\vartheta_0^p(x)}{s^{\delta+1}}$.

For $i=2$, the second Laplace series solution of equation (3.2) can be written as

$$V_2\left(x,s\right) = \frac{\vartheta_0(x)}{s} + \frac{\vartheta_1(x)}{s^{\delta+1}} + \frac{\vartheta_2\left(x\right)}{s^{2\delta+1}}. \tag{3.10}$$

Write the expansion series (3.10) in terms of the $\mathcal{L}\left\{ResV_2\left(x,s\right)\right\}$ of (3.6) in the form

$$\begin{aligned}\mathcal{L}\left\{ResV_2\left(x,s\right)\right\} = &\frac{\vartheta_1\left(x\right)}{s^{1+\delta}} + \frac{\vartheta_2\left(x\right)}{s^{1+2\delta}} - a\left(\frac{\vartheta_0''\left(x\right)}{s^{1+\delta}}+\frac{\vartheta_1''\left(x\right)}{s^{1+2\delta}}+\frac{\vartheta_2''\left(x\right)}{s^{1+3\delta}}\right) - b\left(\frac{\vartheta_0\left(x\right)}{s^{1+\delta}}+\frac{\vartheta_1\left(x\right)}{s^{1+2\delta}}+\frac{\vartheta_2\left(x\right)}{s^{1+3\delta}}\right)+ \\ &c\left(\frac{\vartheta_0^p\left(x\right)}{s^{1+\delta}}+\frac{u_1\left(x\right)}{s^{1+2\delta}}+\cdots+\frac{u_{2p}\left(x\right)\Gamma\left(n\delta{+}1\right)}{\Gamma^n\left(\delta{+}1\right)s^{(n+2)\delta+1}}\right) \\ = &\frac{\vartheta_2\left(x\right)}{s^{1+2\delta}} - a\left(\frac{\vartheta_1''\left(x\right)}{s^{1+2\delta}}+\frac{\vartheta_2''\left(x\right)}{s^{1+3\delta}}\right) - b\left(\frac{\vartheta_1\left(x\right)}{s^{1+2\delta}}+\frac{\vartheta_2\left(x\right)}{s^{1+3\delta}}\right)+ \\ &c\left(\frac{u_1\left(x\right)}{s^{1+2\delta}}+\cdots+\frac{u_{2p}\left(x\right)\Gamma\left(2p\delta{+}1\right)}{\Gamma^{2p}\left(\delta{+}1\right)s^{(2p+1)\delta+1}}\right).\end{aligned} \tag{3.11}$$

Multiplying both sides of (3.11) by the factor $s^{1+2\delta}$ gives

$$\begin{aligned}s^{1+2\delta}\mathcal{L}\left\{ResV_2\left(x,s\right)\right\} = &\vartheta_2\left(x\right) - a\left(\vartheta_1''\left(x\right)+\frac{\vartheta_2''\left(x\right)}{s^{\delta}}\right) - b\left(\vartheta_1\left(x\right)+\frac{\vartheta_2\left(x\right)}{s^{\delta}}\right)+ \\ &c\left(u_1\left(x\right)+\cdots+\frac{u_{2p}\left(x\right)\Gamma\left(2p\delta{+}1\right)}{\Gamma^{2p}\left(\delta{+}1\right)s^{(2p-1)\delta+1}}\right).\end{aligned} \tag{3.12}$$

Next, by solving $\lim_{s\to\infty} s^{1+2\delta}\mathcal{L}\left\{ResV_2\left(x,s\right)\right\} = 0$, we obtain $\vartheta_2\left(x\right) = a\vartheta_1''\left(x\right) + b\vartheta_1\left(x\right) - c\,u_1\left(x\right)$. Hence, the 2-nd Laplace series solution of the Laplace equation (3.2) is expressed as

$$V_2\left(x,s\right) = \frac{\vartheta_0\left(x\right)}{s} + \frac{a\vartheta_0''\left(x\right)+b\vartheta_0\left(x\right)-c\vartheta_0^p\left(x\right)}{s^{\delta+1}} + \frac{a\vartheta_1''\left(x\right)+b\vartheta_1\left(x\right)-c\,u_1\left(x\right)}{s^{2\delta+1}}.$$

Again, for $i=3$, the 3-rd residual function of the Laplace equation (3.2), $\mathcal{L}\left\{ResV_3\left(x,s\right)\right\}$, is read as

$$\begin{aligned}\mathcal{L}\{ResV_3(x,s)\} =&\frac{\vartheta_1(x)}{s^{1+\delta}}+\frac{\vartheta_2(x)}{s^{1+2\delta}}+\frac{\vartheta_3(x)}{s^{1+3\delta}}-a\left(\frac{\vartheta''_0(x)}{s^{1+\delta}}+\frac{\vartheta''_1(x)}{s^{1+2\delta}}+\frac{\vartheta''_2(x)}{s^{1+3\delta}}+\frac{\vartheta''_3(x)}{s^{1+4\delta}}\right)\\&-b\left(\frac{\vartheta_0(x)}{s^{1+\delta}}+\frac{\vartheta_1(x)}{s^{1+2\delta}}+\frac{\vartheta_2(x)}{s^{1+3\delta}}+\frac{\vartheta_3(x)}{s^{1+4\delta}}\right)+\\&c\left(\frac{\vartheta_0^p(x)}{s^{1+\delta}}+\frac{u_1(x)}{s^{1+2\delta}}+\frac{u_2(x)}{s^{1+3\delta}}+\cdots+\frac{u_{3p}(x)\,\Gamma(3p\delta+1)}{\Gamma^{3p}(\delta+1)\,s^{(3p+1)\delta+1}}\right)\\&=\frac{\vartheta_3(x)}{s^{1+3\delta}}-a\left(\frac{\vartheta''_2(x)}{s^{1+3\delta}}+\frac{\vartheta''_3(x)}{s^{1+4\delta}}\right)-b\left(\frac{\vartheta_2(x)}{s^{1+3\delta}}+\frac{\vartheta_3(x)}{s^{1+4\delta}}\right)+\\&c\left(\frac{u_2(x)\,\Gamma(2\delta+1)}{\Gamma^2(\delta+1)\,s^{1+3\delta}}+\cdots+\frac{u_{3p}(x)\,\Gamma(3p\delta+1)}{\Gamma^{3p}(\delta+1)\,s^{(3p+1)\delta+1}}\right).\end{aligned} \tag{3.13}$$

Substituting $V_3(x,s)=\frac{\vartheta_0(x)}{s}+\frac{\vartheta_1(x)}{s^{\delta+1}}+\frac{\vartheta_2(x)}{s^{2\delta+1}}+\frac{\vartheta_3(x)}{s^{3\delta+1}}$ into (3.13) and multiplying the obtained fractional equation by $s^{1+3\delta}$ reveals that

$$\begin{aligned}s^{1+3\delta}\mathcal{L}\{ResV_3(x,s)\} =&\vartheta_3(x)-a\left(\vartheta''_2(x)+\frac{\vartheta''_3(x)}{s^{\delta}}\right)-b\left(\vartheta_2(x)+\frac{\vartheta_3(x)}{s^{\delta}}\right)+\\&c\left(\frac{u_2(x)\,\Gamma(2\delta+1)}{\Gamma^2(\delta+1)}+\cdots+\frac{u_{3p}(x)\,\Gamma(n\delta+1)}{\Gamma^n(\delta+1)\,s^{n\delta}}\right).\end{aligned} \tag{3.14}$$

Using the fact $s^{1+3\delta}\mathcal{L}\{ResV_3(x,s)\}=0$ gives $\vartheta_3(x)=a\vartheta''_2(x)+b\vartheta_2(x)-\frac{\Gamma(2\delta+1)\ u_2(x)}{\Gamma^2(\delta+1)}$.

By proceeding in the same manner as above and utilizing the fact $\lim_{s\to\infty}s^{1+i\delta}\mathcal{L}\{ResV_i(x,s)\}=0$, for $i=4,5,6\ldots$, the rest of coefficients ϑ_i can be determined as

$$\vartheta_i(x)=a\vartheta''_{i-1}(x)+b\vartheta_{i-1}(x)-\frac{\Gamma((i-1)\delta+1)\ \ u_{i-1}(x)}{\Gamma^{(i-1)}(\delta+1)},\ i=2,3,4,\ldots\ . \tag{3.15}$$

Thus, one can reach to the Laplace series solution V in terms of the expansion series (3.3) as

$$V(x,s)=\frac{\vartheta_0(x)}{s}+\sum_{m=1}^{\infty}\frac{a\vartheta''_{m-1}(x)+b\vartheta_{m-1}(x)-\frac{\Gamma((m-1)\delta+1)\ \ u_{m-1}(x)}{\Gamma^{(m-1)}(\delta+1)}}{s^{m\delta+1}}\ . \tag{3.16}$$

Therefore, the approximate solution of the IVP's (1.1) and (1.2) will be

$$\vartheta(x,t)=\vartheta_0(x)+\sum_{m=1}^{\infty}\frac{a\vartheta''_{m-1}(x)+b\vartheta_{m-1}(x)-\frac{\Gamma((m-1)\delta+1)\ \ u_{m-1}(x)}{\Gamma^{(m-1)}(\delta+1)}}{\Gamma(m\delta+1)}t^{m\delta}\ . \tag{3.17}$$

4. Numerical Examples

In the present section we clarify the excellency of the proposed method in solving two time-fractional Newell–Whitehead–Segel model with appropriate initial conditions. All expected calculations will be implemented by following Mathematica software 12.

Example 4.1 Consider the following IVPs:

$$\begin{cases}\mathfrak{D}_t^{\delta}\vartheta(x,t)=\ \mathfrak{D}_x^{2}\vartheta(x,t)-2\vartheta(x,t)\\ \vartheta(x,0)=e^{x}\end{cases},\quad x\in R\ ,\ \ t\geq0,\ \ \delta\in(0,1]\,. \tag{4.1}$$

Here, we notice that $a=1$, b=-2, c=0, and the initial data is e^x. For $\delta=1$, the exact solution of (4.1) is $\vartheta(x,t)=e^{x-t}$.

As stated in the last section, we can obtain the coefficients $\vartheta_i(x)$ for $i=1,2,3,\ldots$ as

$$\begin{aligned}\vartheta_1(x) =& -e^x,\\ \vartheta_2(x) =& e^x,\\ \vartheta_3(x) =& -e^x,\\ \vartheta_4(x) =& e^x,\\ \vartheta_5(x) =& -e^x.\end{aligned} \tag{4.2}$$

Thus, the Laplace series solution of (4.1) has the series form

$$V(x,s) = e^x \sum_{m=1}^{\infty} \frac{(-1)^m}{s^{m\delta+1}} . \tag{4.3}$$

Consequently, the series solution for the fractional IVPs () can be written as

$$\vartheta(x,t) = e^x \sum_{m=1}^{\infty} \frac{(-1)^m t^{m\delta}}{\Gamma(m\delta+1)} . \tag{4.4}$$

If we replace $\delta = 1$, in the fractional expansion (4.4), we have

$$\vartheta(x,t) = e^x \sum_{m=1}^{\infty} \frac{(-1)^m t^m}{m!} , \tag{4.5}$$

which coincides with the Taylor series of e^{x-t} and it is the same exact solution of (4.1).

Example 4.2 Consider the following IVPs:

$$\begin{cases} \mathfrak{D}_t^{\delta}\vartheta(x,t) = \mathfrak{D}_x^2\vartheta(x,t) + 2\vartheta(x,t) - 3\vartheta^2(x,t) \\ \vartheta(x,0) = q \end{cases} \tag{4.6}$$

Here, we notice that $a = 1$, b=2, c=3, p=2 and the initial data q constant. For $\delta = 1$, the exact solution of (4.6) is $\vartheta(x,t) = \frac{\frac{-2}{3}qe^{2t}}{\frac{-2}{3}+q-qe^{2t}}$.

As stated in the last section, we obtain the coefficients $\vartheta_i(x)$ for $i = 1, 2, 3, \ldots$, as

$$\begin{aligned}\vartheta_0(x) =& q,\\ \vartheta_1(x) =& 2q - 3q^2,\\ \vartheta_2(x) =& 2q(2 - 9q + 9q^2),\\ \vartheta_3(x) =& \frac{2q(-2+3q)((-2+6q)\Gamma^2[1+\alpha] + 3(4-9q)q\Gamma[1+2\alpha])}{\Gamma^2[1+\alpha]},\\ \vartheta_4(x) =& \frac{1}{\Gamma^5(\alpha+1)}\left(4(3q-2)^2q\left((6q-2)\Gamma^5(\alpha+1) - 3q(9q-4)\Gamma^3(\alpha+1)\Gamma(2\alpha+1)\right)\right.\\ &+ \frac{1}{\Gamma^5(\alpha+1)}\left(3\left(9q^2-15q+4\right)q\Gamma(3\alpha+1)\Gamma^2(\alpha+1) + 9(9q-4)q^2\Gamma(2\alpha+1)\Gamma(3\alpha+1)\right).\end{aligned} \tag{4.7}$$

Thus, the 4^{th}- Laplace series solution of (4.7), can be written as

$$V_4(x,s) = \left(\frac{\vartheta_0(x)}{s} + \frac{\vartheta_1(x)}{s^{\delta+1}} + \frac{\vartheta_2(x)}{s^{2\delta+1}} + \frac{\vartheta_3(x)}{s^{3\delta+1}} + \frac{\vartheta_4(x)}{s^{4\delta+1}}\right) . \tag{4.8}$$

Consequently, the series solution for the fractional IVPs (4.6) can be written as

$$\vartheta_4(x,t) = \vartheta_0(x) + \frac{\vartheta_1(x)t^{\delta}}{\Gamma(\delta+1)} + \frac{\vartheta_2(x)t^{2\delta}}{\Gamma(2\delta+1)} + \frac{\vartheta_3(x)t^{3\delta}}{\Gamma(3\delta+1)} + \frac{\vartheta_4(x)t^{4\delta}}{\Gamma(4\delta+1)}. \tag{4.9}$$

Example 4.3 Consider the following IVPs:

$$\begin{cases} \mathfrak{D}_t^{\delta}\vartheta(x,t) = \mathfrak{D}_x^{2}\vartheta(x,t) + 3\vartheta(x,t) - 4\vartheta^3(x,t) \\ \vartheta(x,0) = \frac{\sqrt{\frac{3}{4}}e^{\sqrt{6}x}}{e^{\sqrt{6}x}+e^{\frac{\sqrt{6}}{2}x}} \end{cases} \tag{4.10}$$

Here, we notice that $a=1$, b=3, c=4, p=3 and the initial data $\frac{\sqrt{\frac{3}{4}}e^{\sqrt{6}x}}{e^{\sqrt{6}x}+e^{\frac{\sqrt{6}}{2}x}}$. For $\delta=1$, the exact solution of (4.10) is $\vartheta(x,t) = \frac{\sqrt{\frac{3}{4}}e^{\sqrt{6}x}}{e^{\sqrt{6}x}+e^{(\frac{\sqrt{6}}{2}x-\frac{9}{2}t)}}$.

As stated in the last section, we can obtain the coefficients $\vartheta_i(x)$ for $i=1,2,3,\ldots$ as

$$\vartheta_0(x) = \frac{\sqrt{\frac{3}{4}}e^{\sqrt{6}x}}{e^{\sqrt{6}x}+e^{\frac{\sqrt{6}}{2}x}}, \vartheta_1(x) = \frac{9\sqrt{3}ee^{\sqrt{\frac{3}{2}}x}}{4(1+e^{\sqrt{\frac{3}{2}}x})^2},$$

$$\vartheta_1(x) = \frac{63\sqrt{3}e^{\sqrt{\frac{3}{2}}x} + 114\sqrt{3}e^{3\sqrt{\frac{3}{2}}x} - 81\sqrt{3}e^{5\sqrt{\frac{3}{2}}x} + 90\sqrt{3}e^{\sqrt{6}x} + 54(-9+\sqrt{3})e^{2\sqrt{6}x}}{8(1+e^{\sqrt{\frac{3}{2}}x})^6},$$

$$\frac{-12(-9+4\sqrt{3})e^{3\sqrt{6}x}}{8(1+e^{\sqrt{\frac{3}{2}}x})^6}$$

$$\vartheta_2(x) = \frac{1}{16(1+e^{\sqrt{\frac{3}{2}}x})^9 \Gamma^2[1+\alpha]} 9e^{\sqrt{\frac{3}{2}}x}((-63\sqrt{3} + 9\sqrt{3}e^{\sqrt{\frac{3}{2}}x} - 9(-324+61\sqrt{3})e^{3\sqrt{\frac{3}{2}}x} +$$
$$(-3312+811\sqrt{3})e^{5\sqrt{\frac{3}{2}}x} + 81\sqrt{3}e^{7\sqrt{\frac{3}{2}}x} - 373\sqrt{3}e^{\sqrt{6}x} + 531\sqrt{3}e^{2\sqrt{6}x} + 9(-36+25\sqrt{3})e^{3\sqrt{6}x} +$$
$$8(-9+4\sqrt{3})e^{4\sqrt{6}x})\Gamma^2[1+\alpha]^2 - 2e^{\sqrt{6}x}(-112\sqrt{3} - 321\sqrt{3}e^{\sqrt{\frac{3}{2}}x} + (612-440\sqrt{3})e^{3\sqrt{\frac{3}{2}}x} +$$
$$(54+57\sqrt{3})e^{5\sqrt{\frac{3}{2}}x} - 474\sqrt{3}e^{\sqrt{6}x} + (702-150\sqrt{3})e^{2\sqrt{6}x} + (-36+5\sqrt{3})e^{3\sqrt{6}x})\Gamma[1+2\alpha]),$$

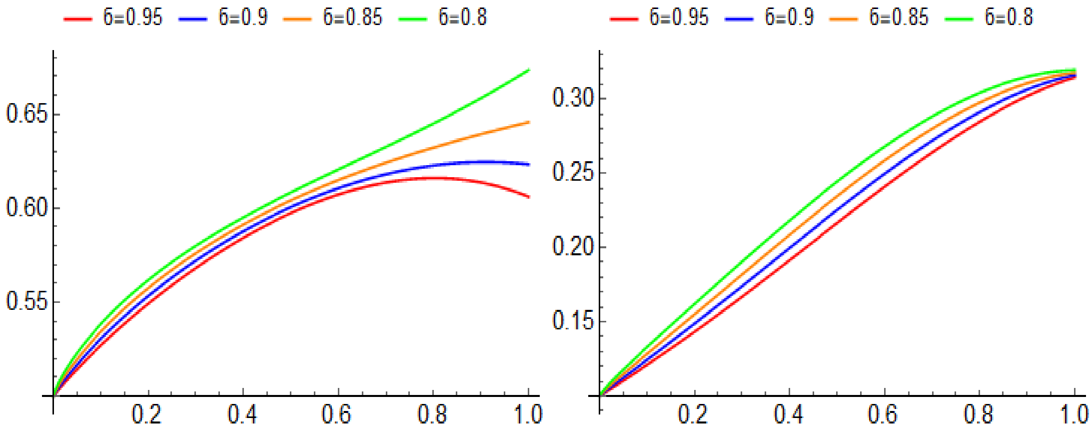
$$\vartheta_3(x) = \frac{1}{32(1+e^{\sqrt{\frac{3}{2}}x})^{12} \Gamma^2[1+\alpha]^4} 3e^{\sqrt{\frac{3}{2}}x}(9(1+e^{\sqrt{\frac{3}{2}}x})^2(-189\sqrt{3} + 1566\sqrt{3}e^{\sqrt{\frac{3}{2}}x} +$$
$$2(26244+7997\sqrt{3})e^{3\sqrt{\frac{3}{2}}x} -$$
$$2(-68292+8387\sqrt{3})e^{5\sqrt{\frac{3}{2}}x} + 18(-468+127\sqrt{3})e^{7\sqrt{\frac{3}{2}}x} + 16(-9+4\sqrt{3})e^{9\sqrt{\frac{3}{2}}x} - 10124\sqrt{3}e^{\sqrt{6}x} +$$
$$18(-10206+493\sqrt{3})e^{2\sqrt{6}x} + 4(-4329+1139\sqrt{3})e^{3\sqrt{6}x} + (504+19\sqrt{3})e^{4\sqrt{6}x})\Gamma^4[1+\alpha] -$$
$$2e^{\sqrt{6}x}(-15778\sqrt{3} - 35946\sqrt{3}e^{\sqrt{\frac{3}{2}}x} + (313173-123258\sqrt{3})e^{3\sqrt{\frac{3}{2}}x} + 198(-171+379\sqrt{3})e^{5\sqrt{\frac{3}{2}}x} -$$
$$9(387+640\sqrt{3})e^{7\sqrt{\frac{3}{2}}x} + (-7101+3662\sqrt{3})e^{9\sqrt{\frac{3}{2}}x} - 71316\sqrt{3}e^{\sqrt{6}x} + (452628-53532\sqrt{3})e^{2\sqrt{6}x} +$$
$$60(-2853+724\sqrt{3})e^{3\sqrt{6}x} + 18(-720+607\sqrt{3})e^{4\sqrt{6}x})\Gamma^2[1+\alpha]\Gamma[1+2\alpha] + 108e^{2\sqrt{6}x}$$
$$(1+e^{\sqrt{\frac{3}{2}}x})(-112\sqrt{3} - 321\sqrt{3}e^{\sqrt{\frac{3}{2}}x} + (612-440\sqrt{3})e^{3\sqrt{\frac{3}{2}}x} + (54+57\sqrt{3})e^{5\sqrt{\frac{3}{2}}x} - 474\sqrt{3}e^{\sqrt{6}x} +$$
$$(702-150\sqrt{3})e^{2\sqrt{6}x} + (-36+5\sqrt{3})e^{3\sqrt{6}x})\Gamma^2[1+2\alpha]). \tag{4.11}$$

Thus, the 3^{rd}- Laplace series solution of (4.10) can be written as

$$V_3(x,s) = \left(\frac{\vartheta_0(x)}{s} + \frac{\vartheta_1(x)}{s^{\delta+1}} + \frac{\vartheta_2(x)}{s^{2\delta+1}} + \frac{\vartheta_3(x)}{s^{3\delta+1}}\right). \tag{4.12}$$

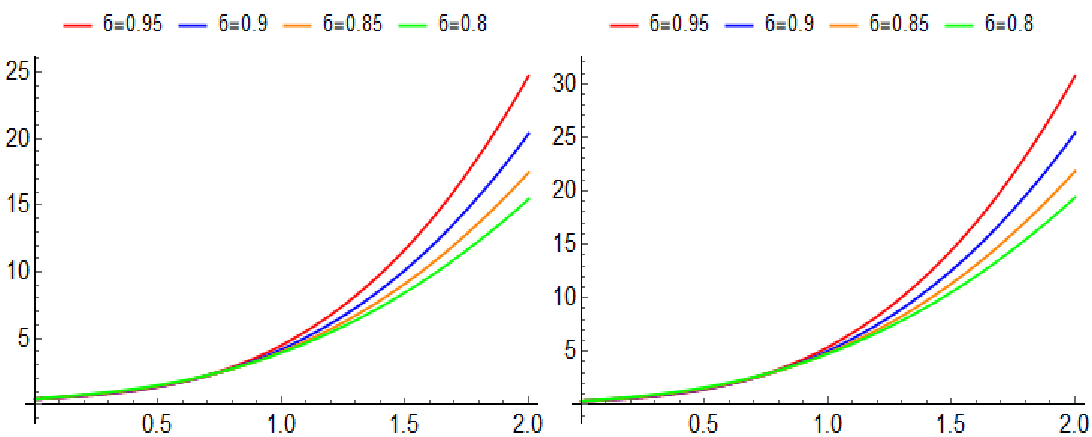
Consequently, the series solution for the fractional IVPs (4.10) can be written as

$$\vartheta_3(x,t) = \vartheta_0(x) + \frac{\vartheta_1(x)t^{\delta}}{\Gamma(\delta+1)} + \frac{\vartheta_2(x)t^{2\delta}}{\Gamma(2\delta+1)} + \frac{\vartheta_3(x)t^{3\delta}}{\Gamma(3\delta+1)}. \tag{4.13}$$



(a) (b)

Figure 1: Behavior of the series solution for the fractional IVPs (4.6) in (4.9): (a) $q = 0.5,\ x = 0$; (b) $q = 0.23,\ x = -0.3$.



(a) (b)

Figure 2: Behavior of the series solution for the fractional IVPs (4.10) in (4.13): (a) $q = 0.5,\ x = 0$; (b) $q = 0.23,\ x = -0.3$.

5. Conclusion

The main object of the current article is to solve the fractional Newell–Whitehead–Segel model by extending the LRPS approach. It's a robust and effective algorithm in obtaining approximate solutions for different classes of linear and nonlinear FDEs. The LRPS algorithm has been used to solve the main equation in the Laplace space, the obtained Laplace solutions are transferred in the original space to get the fractional approximate solutions. Two different initial data of the fractional Newell–Whitehead–Segel model is considered. We have been tested the accuracy of the present approach by computing the absolute errors of the reached approximate for fractional Newell–Whitehead–Segel problem. Analyzing the obtained results reveals that the LRPS approach is a promising tool in fractional calculus theory based on its simplicity, performance and accuracy.

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