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Convergence of solutions of a $p-$dimensional system of $sk-$order rational recursive equations*

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ABSTRACT: We extend an exciting result on the convergence of positive solutions to a $p-$dimensional system of higher-order rational difference equations, which can also be considered as an extension of the one presented by Şimşek and Doğan (MANAS Journal of Engineering, 2014, 2(1), $16-22$). Our results popularize some findings in the literature. We provide illustrative examples of the solutions that have been calculated and plotted.

Key Words: Convergence; Periodic solution; System of nonlinear difference equations.

Contents

1. Introduction

Systems of nonlinear difference equations are always of great interest because these systems appear themselves as mathematical models of some problems in biology, stochastic time series, physics, probability theory and numerical solutions of differential equations, etc. (see, [1], [7] − [9], [10] − [16], [27] − [29], [34] − [36], [39]). Among these systems is the rational system of difference equations, which gets a lot of attention for the periodic nature and the global asymptotic behavior of all positive solutions, [2] − [31], [37] − [40], [42]. On the other hand, we can see that the system of difference equations is natural extension of the difference equation, an example of this is the open problem that Stević [36] solved, for the following equation $x_{n+1}^{(1)} = x_{n-1}^{(1)} \Big/ h\left(x_n^{(1)}\right)$, $x_{-i}^{(1)} > 0$, $i = 1, 2$, $n \in \mathbb{N}_0$, when the function h satisfies some regularity conditions, which has been developed by Şimşek et al. [33], [34] and [35], for higher-order, $x_{n+1}^{(1)} = x_{n-(2k+1)}^{(1)} \Big/ \left(1 + x_{n-k}^{(1)}\right)$, $x_{n+1}^{(1)} = x_{n-(4k+3)}^{(1)} \Big/ \left(1 + x_{n-k}^{(1)} x_{n-(2k+1)}^{(1)} x_{n-(3k+2)}^{(1)}\right)$ and $x_{n+1}^{(1)} = x_{n-(k+1)}^{(1)} \Big/ \left(1 + x_n^{(1)} x_{n-1}^{(1)} ... x_{n-k}^{(1)}\right)$.
Through the motives provided by these works [2] − [42], and especially [33], [34] and [35], we introduce a natural extension of [35] equation through the following system

$$x_{n+1}^{(i)} = \frac{x_{n-(sk-1)}^{(i)}}{1 + \prod_{j=1}^{s-1} x_{n-(jk-1)}^{(i)}}, i \in \{1, ..., p\}, n \geq 0, k \geq 1, s \geq 2, \tag{1.1}$$

where the initials values are positive.

2. On the system (1.1)

In this paper, we will present some very important results of the System (1.1), which is an extension of Şimşek's papers [32], [33], [34] and [35].

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Theorem 2.1 *Consider the system of difference equations (1.1) and let p, s, k be fixed integers. Assume that the initial values $x_{-j}^{(i)}$, $j \in \{0, ..., sk-1\}$, $i \in \{1, ..., p\}$ satisfy the following condition*

$$\min\left\{x_{-j}^{(i)}, j \in \{0, ..., sk-1\}, i \in \{1, ..., p\}\right\} > 0, \tag{2.1}$$

then for every solution $\left\{\left(x_n^{(1)}, ..., x_n^{(p)}\right)\right\}_n$ to system (1.1) satisfying condition (2.1), then the following statements are true.

(i) *The subsequences $\left\{\left(x_{skn+j}^{(1)}, ..., x_{skn+j}^{(p)}\right)\right\}_n$, $j \in \{1, ..., sk\}$ are decreasing and then there exist non-negative constants $a_i^{(j)}, j \in \{1, ..., sk\}$, $i \in \{1, ..., p\}$ such that*

$$\lim_n \left(x_{skn+j}^{(1)}, ..., x_{skn+j}^{(p)}\right) = \left(a_1^{(j)}, ..., a_p^{(j)}\right), \ j \in \{1, ..., sk\}. \tag{2.2}$$

(ii) *If $a_i^{(j)}, j \in \{1, ..., sk\}$, $i \in \{1, ..., p\}$ are the constants defined in (2.2), then the following sequence*

$$\left(x_{skn+j}^{(1)}, ..., x_{skn+j}^{(p)}\right) = \left(a_1^{(j)}, ..., a_p^{(j)}\right), \ j \in \{1, ..., sk\}, n \in \mathbb{N}_0,$$

is a solution of system (1.1) of period sk.

(iii) *The following relations $\prod_{j=1}^{s} a_i^{(jk+l)} = 0$, $l \in \{1, ..., k\}$, $i \in \{1, ..., p\}$ holds, where $a_i^{(sk+l)} = a_i^{(l)}$, $l \in \{1, ..., k\}$.*

(iv) *If there exists $n_0 \in \mathbb{N}_0$, such that the subsequences $\left(x_n^{(i)}\right)_n$, $i \in \{1, ..., p\}$ are satisfied $x_{n-(s-1)k}^{(i)} \geq x_n^{(i)}$, $i \in \{1, ..., p\}$ for $n \geq n_0$, then $\lim_n \left(x_n^{(1)}, ..., x_n^{(p)}\right) = \underline{O}'_{(p)}$, where $\underline{O}_{(p)}$ is a null vector of dimension p.*

(v) *The following formulas, for $i \in \{1, ..., p\}$, $j \in \{1, ..., sk\}$,*

$$x_{skn+j}^{(i)} = x_{-(sk-j)}^{(i)} \left(1 + \left(1 + \prod_{l=1}^{s-1} x_{-(lk-j)}^{(i)}\right)^{-1} - \sum_{m=0}^{n} \prod_{l=1}^{sm} \left(1 + \prod_{u=1}^{s-1} x_{(l-u)k+j}^{(i)}\right)^{-1}\right),$$

hold.

Proof.

(i) First, from system (1.1), we have $1 + \prod_{j=1}^{s-1} x_{n-(jk-1)}^{(i)} \in (1, +\infty)$, for each n and $x_{n-(sk-1)}^{(i)} > x_{n+1}^{(i)}$ for each n, $i \in \{1, ..., p\}$. Since the sequences $\left\{\left(x_{skn+j}^{(1)}, ..., x_{skn+j}^{(p)}\right)\right\}_n$, $j \in \{1, ..., sk\}$ are decreasing and bounded below by 0, then they are convergent. Therefore, there exist non-negative constants $a_i^{(j)}, j \in \{1, ..., sk\}$, $i \in \{1, ..., p\}$, such that (2.2) is verified.

(ii) $\left(a_1^{(j)}, ..., a_p^{(j)}\right)_{j \in \{0, ..., k+1\}}$ is a solution of system (1.1) of period sk.

(iii) In view of the system (1.1), we obtain,

$$x_{skn+l}^{(i)} = \frac{x_{sk(n-1)+l}^{(i)}}{1 + \prod_{j=1}^{s-1} x_{skn-(jk-l)}^{(i)}} = \frac{x_{sk(n-1)+l}^{(i)}}{1 + \prod_{j=1}^{s-1} x_{k(sn-j)+l}^{(i)}}, \ i \in \{1, ..., p\}, l \in \{1, ..., k\}.$$

Take the limits on both sides of these equalities, and using

$$a_i^{(l)} = a_i^{(sk+l)} = \lim_n x_{sk(n+1)+l}^{(i)} = \lim_n x_{skn+l}^{(i)} = \lim_n x_{sk(n-1)+l}^{(i)}, l \in \{1, ..., sk\},$$

we get

$$\lim_n x_{skn+l}^{(i)} = \frac{\lim_n x_{sk(n-1)+l}^{(i)}}{1 + \prod_{j=1}^{s-1} \lim_n x_{k(sn-j)+l}^{(i)}}, \ i \in \{1, ..., p\}, l \in \{1, ..., k\},$$

then $\prod_{j=1}^{s} \lim_n x_{k(sn-j)+l}^{(i)} = 0$, i.e., $\prod_{j=1}^{s} a_i^{(jk+l)} = 0$, $i \in \{1, ..., p\}$, $l \in \{1, ..., k\}$.

(iv) If there exists $n_0 \in \mathbb{N}_0$, such that $x_{n-k}^{(i)} \geq x_{n+1}^{(i)}$ for $n \geq n_0$, $i \in \{1, ..., p\}$, then

$$x_{skn+l}^{(i)} \geq x_{skn+(s-1)k+l}^{(i)} \geq x_{skn+2(s-1)k+l}^{(i)} \geq \cdots \geq x_{skn+s(s-1)k+l}^{(i)}, \ i \in \{1, ..., p\}$$

for $n \geq n_0$. Limiting on both sides as $n \longrightarrow \infty$, we get

$$a_i^{(l)} \geq a_i^{((s-1)k+i)} \geq a_i^{(2(s-1)k+i)} \geq ... \geq a_i^{(s(s-1)k+i)} \geq a_i^{(l)} \geq 0 \text{ for } i \in \{1, ..., p\}, l \in \{1, ..., k\}.$$

Moreover, using the result obtained in (**iii**), we have $a_i^{(l)} = 0$, for $i \in \{1, ..., p\}$, $l \in \{1, ..., k\}$ and hence $a_i^{(l)} = 0$, for $i \in \{1, ..., p\}$, $l \in \{1, ..., sk\}$. For these, we have

$$\lim_n \left(x_n^{(1)}, ..., x_n^{(p)}\right) = \underline{O}'_{(p)}.$$

(v) From system (1.1), we get, for $i \in \{1, ..., p\}$, $n \geq 0$,

$$\begin{aligned} x_{n+1}^{(i)} - x_{n-(sk-1)}^{(i)} &= \left(\left(1 + \prod_{j=1}^{s-1} x_{n-(jk-1)}^{(i)}\right)^{-1} - 1\right) x_{n-(sk-1)}^{(i)} \\ &= -\left(1 + \prod_{j=1}^{s-1} x_{n-(jk-1)}^{(i)}\right)^{-1} \prod_{j=1}^{s} x_{n-(jk-1)}^{(i)} \\ &= -\left(1 + \prod_{j=1}^{s-1} x_{n-(jk-1)}^{(i)}\right)^{-1} x_{n-(k-1)}^{(i)} \prod_{j=1}^{s-1} x_{n-((j+1)k-1)}^{(i)}, \end{aligned}$$

thus, we have

$$\begin{aligned} x_{n+1}^{(i)} - x_{n-(sk-1)}^{(i)} &= \left(1 + \prod_{j=1}^{s-1} x_{n-(jk-1)}^{(i)}\right)^{-1} x_{n-(k-1)}^{(i)} \left(1 - \frac{x_{n-((s+1)k-1)}^{(i)}}{x_{n-(k-1)}^{(i)}}\right) \\ &= \left(1 + \prod_{j=1}^{s-1} x_{n-(jk-1)}^{(i)}\right)^{-1} \left(x_{n-(k-1)}^{(i)} - x_{n-((s+1)k-1)}^{(i)}\right), \end{aligned} \tag{2.3}$$

for $n \geq 0$, $i \in \{1, ..., p\}$. We use the change of variables $y_{n,l}^{(i)} = x_{nk+l}^{(i)} - x_{(n-s)k+l}^{(i)}$ and replacing n with $nk + l - 1$, $l \in \{1, ..., sk\}$ in 2.3, we have

$$\begin{aligned} y_{n,l}^{(i)} &= \left(1 + \prod_{j=1}^{s-1} x_{nk-(jk-l)}^{(i)}\right)^{-1} y_{n-1,l}^{(i)} \\ &= y_{0,l}^{(i)} \prod_{j=1}^{n} \left(1 + \prod_{u=1}^{s-1} x_{jk-(uk-l)}^{(i)}\right)^{-1}, \end{aligned}$$

for $n \geq 1$, $i \in \{1, ..., p\}$, $l \in \{1, ..., sk\}$. Now, from the last expression, we obtain

$$\sum_{m=0}^{n} y_{sm,l}^{(i)} = y_{0,l}^{(i)} \sum_{m=0}^{n} \prod_{j=1}^{sm} \left(1 + \prod_{u=1}^{s-1} x_{jk-(uk-l)}^{(i)}\right)^{-1},$$

for $n \geq 1$, $i \in \{1, ..., p\}$, $l \in \{1, ..., sk\}$. From the last result, we obtain

$$x_{skn+l}^{(i)} - x_{-(sk-l)}^{(i)} = \left(x_l^{(i)} - x_{-(sk-l)}^{(i)}\right) \sum_{m=0}^{n} \prod_{j=1}^{sm} \left(1 + \prod_{u=1}^{s-1} x_{jk-(uk-l)}^{(i)}\right)^{-1},$$

for $n \geq 1$, $i \in \{1, ..., p\}$, $l \in \{1, ..., sk\}$, then

$$\begin{aligned} x_{skn+l}^{(i)} - x_{-(sk-l)}^{(i)} &= \left(x_l^{(i)} - x_{-(sk-l)}^{(i)}\right) \sum_{m=0}^{n} \prod_{j=1}^{sm} \left(1 + \prod_{u=1}^{s-1} x_{jk-(uk-l)}^{(i)}\right)^{-1}, \\ &= x_{-(sk-l)}^{(i)} + \left(x_{-(sk-l)}^{(i)} \left(1 + \prod_{j=1}^{s-1} x_{-(jk-l)}^{(i)}\right)^{-1} - x_{-(sk-l)}^{(i)}\right) \\ &\times \sum_{m=0}^{n} \prod_{j=1}^{sm} \left(1 + \prod_{u=1}^{s-1} x_{jk-(uk-l)}^{(i)}\right)^{-1} \\ &= x_{-(sk-l)}^{(i)} \left(1 + \left(1 + \prod_{j=1}^{s-1} x_{-(jk-l)}^{(i)}\right)^{-1} - \sum_{m=0}^{n} \prod_{j=1}^{sm} \left(1 + \prod_{u=1}^{s-1} x_{jk-(uk-l)}^{(i)}\right)^{-1}\right). \end{aligned}$$

The statement **(v)** is proven, which completes the proof of Theorem 2.1.■

Example 2.1 *We consider interesting numerical example for the following system*

$$x_{n+1}^{(i)} = \frac{x_{n-13}^{(i)}}{1 + x_{n-1}^{(i)} x_{n-3}^{(i)} x_{n-5}^{(i)} x_{n-7}^{(i)} x_{n-9}^{(i)} x_{n-11}^{(i)}}, i \in \{1, 2, 3, 4\}, n \geq 0, \tag{2.4}$$

with $s = 7$, $k = 2$, $p = 4$ *and the initial conditions*

i/j	0	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1.1	1.2	1.32	1.22	1.05	1.11	1.16	1.38	1.31	1.28	1.26	1.21	1.20	1.19
2	0.01	0.13	0.16	0.39	0.36	0.35	0.31	0.24	0.23	0.21	0.19	0.18	0.17	0.2
3	11	12	15	17	9	8	7	6	14	13	19	17	16	9
4	2	3	6	5	1	4	7	7.3	2	3.2	8	3	4.8	0.4

Table 1. The initial conditions.

The plot of the system (2.4) *is shown in Figure* 1.
The plot of the solutions is shown in Figure 2.

Example 2.2 *We consider interesting numerical example for the following system*

$$x_{n+1}^{(i)} = \frac{x_{n-8}^{(i)}}{1 + x_{n-2}^{(i)} x_{n-5}^{(i)}}, i \in \{1, 2, 3\}, n \geq 0, \tag{2.5}$$

with $s = 3$, $k = 3$, $p = 4$ *and the initial conditions*

i/j	0	1	2	3	4	5	6	7	8
1	1.14	1.32	1.46	1.57	1.65	1.72	1.78	1.83	1.91
2	1.47	0.54	0.20	0.07	0.03	0.001	0.004	0.002	0.001
3	3	4	5	6	7	8	9	10	11

Table 2. The initial conditions.

The plot of the system (2.5) *is shown in Figure* 3. *The plot of the solutions is shown in Figure* 4.

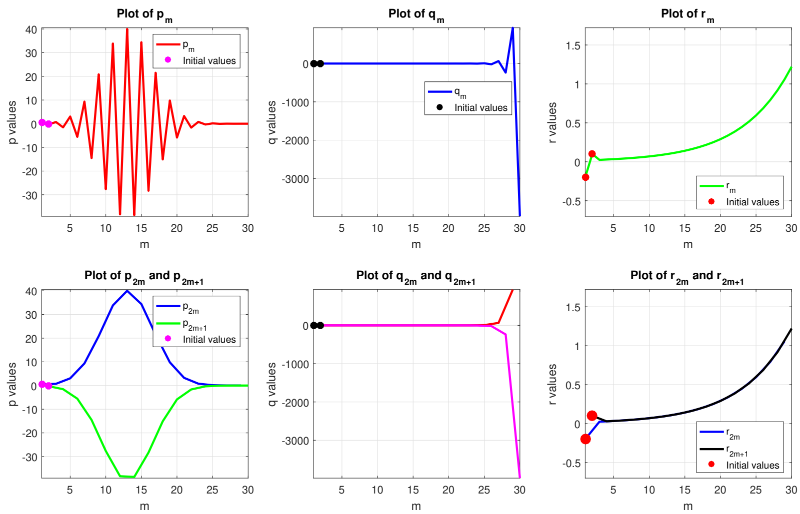

Figure 1: The plot of the system (2.4) when we put the initial conditions in Table 1.

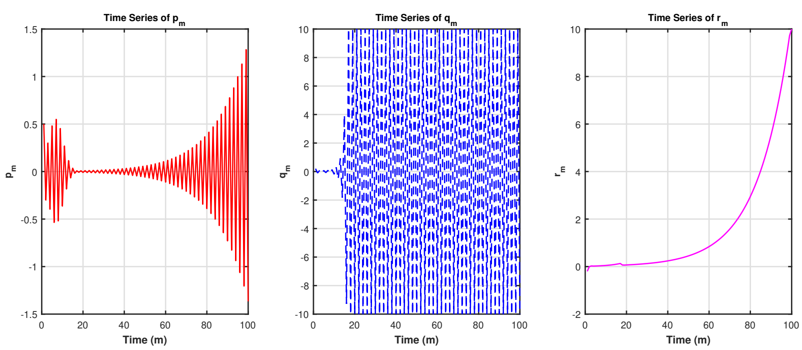

Figure 2: The plot of the solutions of system (2.4) when we put the initial conditions in Table 1.

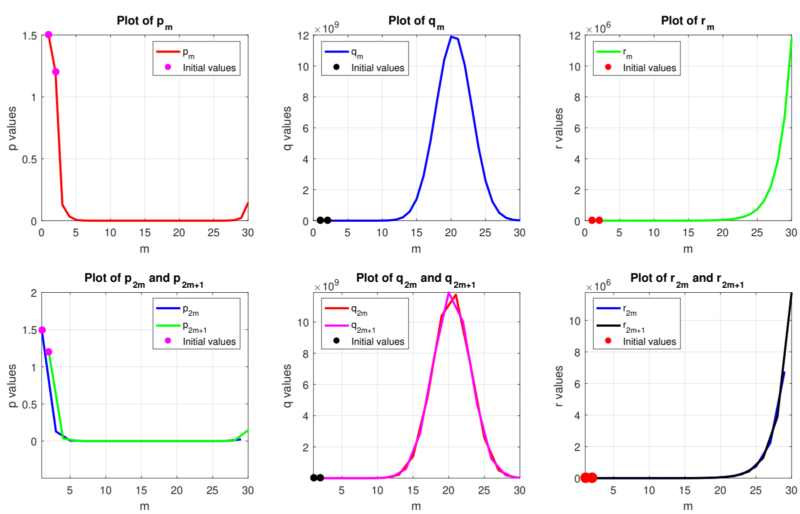

Figure 3: The plot of the system (2.5) when we put the initial conditions in Table 2.

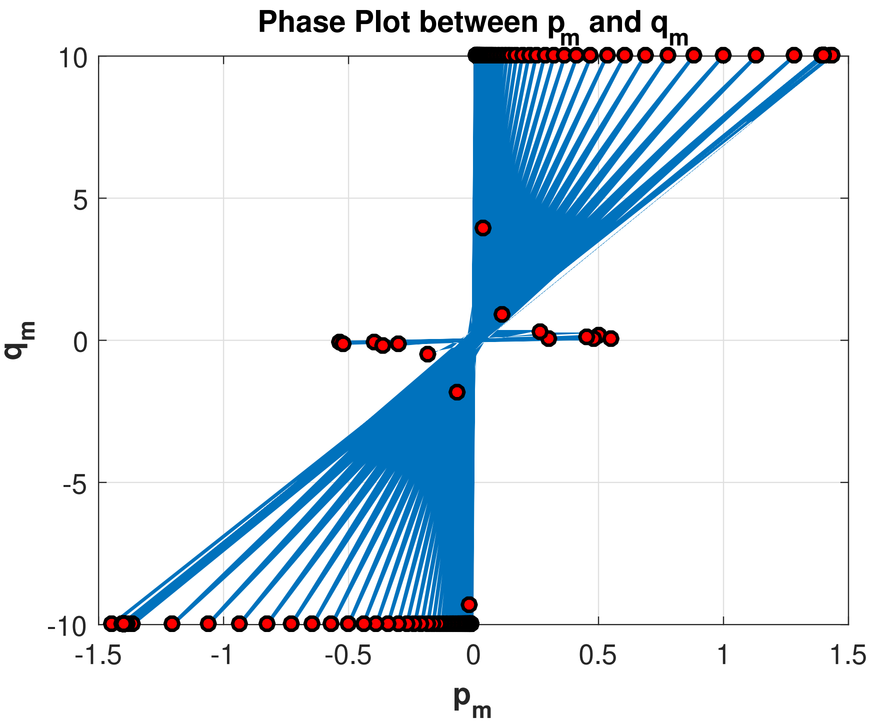

Figure 4: The plot of the solutions of system (2.5) when we put the initial conditions in Table 2.

Author contributions

All authors have contributed equally to the paper.

Data availability statement

This manuscript has no associated data or the data will not be deposited. [Authors'comment: The numerical data generated and analyzed in this paper is available from the corresponding author on reasonable request.]

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