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An improved ratio-type estimator of population coefficient of variation

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ABSTRACT: This study discusses the challenges associated with estimating the population coefficient of variation using sampling survey techniques. Yunusa et al. (2021) motivate this study. We further improve these estimators to estimate this study's unknown population coefficient of variation. Using two populations as examples, empirical results indicate that the proposed estimator provides more accurate estimates of population variability since it is more efficient. Further research and validation are recommended to establish the robustness and generalizability of the proposed estimator.

Key Words: Coefficient variation, survey sampling techniques, mean square error (MSE), efficiency conditions, auxiliary information, and study variables.

Contents

1. Introduction

The coefficient of variation is one of the most important scale invariants estimators that has been used to compare datasets on different scales. Numerous fields have used the conventional coefficient of variation, including biology, biochemistry, medical physics, neurology, engineering, psychology, sociology, and economics [1,2]. It is used to determine the level of volatility or risk compared to the expected return on investments [3], express the precision and repeatability of an assay in analytical chemistry, and conduct quality assurance studies in engineering and physics [4]. Furthermore, the Coefficient of variation is used to compare the variation or depression in two or more data sets even though they are measured in different units [5,6], such as weight and height in kilograms and centimeters. Recent advances in sampling theory have centred on developing estimators for estimating the population coefficient of variation with greater precision by utilising auxiliary characters [7,8]. In order to estimate population parameters, a number of authors have employed auxiliary data on the auxiliary variable, including Singh [9], Sahai [10], and Srivastava [11]. For a very long time, the coefficient of variance has been underappreciated. However, some authors, such as Das and Tripathi [12], proposed an estimator for the coefficient of variation when SRSWOR was used to select samples. The population C.V. is estimated to be approximately unbiased in a normal distribution by Patel and Shah [13] and Mahmoudvand [14]. Another work in this area was by Panichkitkosolkul [15], who proposed improved confidence intervals for the C.V. of normal distribution based on this estimator and its variance. Yadav and Kadilar [16], Singh cite17, Kadilar [18], Singh [19], Ahmed [20], Audu and Adewara [21], Audu [22], Muili [23], Khoshnevisan [24], Singh and Audu [25], Audu and Singh [26], Rajyaguru and Gupta [27], Archana and Rao [28], Singh [29] have made significant

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contributions in this area by attempting to estimate the C.V. under both simple and stratified random sampling.

2. Nomenclature

The notation will be circulated throughout the paper as described below:
$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$: Sample variance of the study variate y,
$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$: Sample variance of the auxiliary variate x,
$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$: Sample covariance of the Y and X.
$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$: Sample mean of the$\bar{x}$,
$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$: sample standard of the$\bar{y}$,
$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{X})^2$: Population variance of the auxiliary variate x,
$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$: Population variance of the study variate y,
$S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})(Y_i - \bar{Y})$: Population covariance of the Y and X,
$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$: Sample mean of$\bar{X}$,
$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$: Sample standard of$\bar{Y}$,
MSE () stands for mean square error
PRE$=\frac{MSE(t_0)}{MSE(t_p)}\times 100$: Relative efficiency of the estimator in percentage $t_p$overt_0.
Now let us define

$$E(e_0) = E(e_1) = E(e_2) = E(e_3) = 0,$$

$$E(e_0^2) = \gamma C_y^2, E(e_1^2) = \gamma C_x^2, E(e_2^2) = \gamma(\lambda_{40} - 1), E(e_3^2) = \gamma(\lambda_{04} - 1),$$

$$E(e_0 e_1) = \gamma\rho C_y C_x, E(e_0 e_2) = \gamma C_y \lambda_{30}, E(e_0 e_3) = \gamma C_y \lambda_{12},$$

$$E(e_1 e_2) = \gamma C_x \lambda_{21}, E(e_1 e_3) = \gamma C_x \lambda_{03}, E(e_2 e_3) = \gamma(\lambda_{22} - 1),$$

$$\bar{y} = \bar{Y}(1 + e_0), \bar{x} = \bar{X}(1 + e_1), s_y = S_y(1 + e_2)^{1/2},$$
$$s_x = S_x(1 + e_3)^{1/2}, s_y^2 = S_y^2(1 + e_2), s_x^2 = S_x^2(1 + e_3)$$

The followings are the $\gamma = \frac{(1-f)}{n}, f = \frac{n}{N}$ sampling fractions, along with the population coefficients of variation for auxiliary variable X and study variable Y. Also, r is the correlation coefficient between X and Y.

3. Some Existing Estimators in Literature

The population C.V. (coefficient of variation) can be estimated using auxiliary variables by using an unbiased estimator such as:

$$t_0 = \hat{C}_y = \frac{s_y}{\bar{y}} \tag{3.1}$$

The MSE of t_0 is given in (3.2)

$$MSE(t_0) = C_y^2 \gamma\left(C_y^2 + \frac{1}{4}(\lambda_{40} - 1) - C_y \lambda_{30}\right) \tag{3.2}$$

A ratio estimator for the population C.V. was developed by Archana and Rao [28] As described in equations (3.3) and (3.4), we used the sample mean, the population mean, the sample variance, and the population variance of the auxiliary variable.

$$t_{AR1} = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right) \tag{3.3}$$

$$t_{AR2} = \hat{C}_y\left(\frac{S_y^2}{s_y^2}\right) \tag{3.4}$$

An expression for the mean square error (MSE) for estimator t_{AR} is as follows:

$$MSE(t_{AR1}) = C_y^2\gamma\left(C_y^2+\frac{1}{4}(\lambda_{40}-1)+C_x^2-C_x\lambda_{21}-C_y\lambda_{30}+2\rho C_y C_x\right) \tag{3.5}$$

$$MSE(t_{AR2}) = C_y^2\gamma\left(C_y^2+\frac{1}{4}(\lambda_{40}-1)+(\lambda_{04}-1)-(\lambda_{22}-1)-C_y\lambda_{30}+2C_y\lambda_{12}\right) \tag{3.6}$$

The logarithmic ratio type estimator described in equation (3.7) by Yunusa [30] uses data on the sample mean, the population mean, sample variance, and population variance of the auxiliary variable to estimate the population coefficient of variation.

$$t_y=\hat{C}_y\left(\frac{Ln\left(S_y^2\right)}{Ln\left(s_y^2\right)}\right) \tag{3.7}$$

An estimator's mean square error (MSE) is expressed as follows:

$$MSE(t_y) = C_y^2\gamma\left(C_y^2+\frac{1}{4}(\lambda_{40}-1)+\frac{(\lambda_{04}-1)}{\left(Ln\left(S_x^2\right)\right)^2}-\frac{(\lambda_{22}-1)}{Ln\left(S_x^2\right)}-C_y\lambda_{30}+\frac{2C_y\lambda_{12}}{Ln\left(S_x^2\right)}\right) \tag{3.8}$$

4. Proposed Estimator

We developed estimators based on Yunusa [30] study to estimate the unknown population C.V. using the following information. The details and discussion are as follows:

The mean squared error of the T_z

$$T_z=\hat{C}_y+\eta\left(\frac{\bar{X}-\bar{x}}{\bar{X}}\right)exp\left(\bar{X}-\bar{x}\right) \tag{4.1}$$

The constant note should be determined so as to minimize the mean squared error of the estimator T_z. The following error terms and notations explain how to obtain the MSEs expressions for various estimators:

$$E(e_0)=E(e_1)=E(e_2)=E(e_3)=0,$$
$$E\left(e_0^2\right)=\gamma C_y^2, E\left(e_1^2\right)=\gamma C_x^2, E\left(e_2^2\right)=\gamma(\lambda_{40}-1), E\left(e_3^2\right)=\gamma(\lambda_{04}-1),$$
$$E(e_0e_1)=\gamma\rho C_y C_x, E(e_0e_2)=\gamma C_y\lambda_{30}, E(e_0e_3)=\gamma C_y\lambda_{12},$$
$$E(e_1e_2)=\gamma C_x\lambda_{21}, E(e_1e_3)=\gamma C_x\lambda_{03}, E(e_2e_3)=\gamma(\lambda_{22}-1),$$
$$\bar{y}=\bar{Y}(1+e_0), \bar{x}=\bar{X}(1+e_1), s_y=S_y(1+e_2)^{1/2},$$
$$s_x=S_x(1+e_3)^{1/2}, s_y^2=S_y^2(1+e_2), s_x^2=S_x^2(1+e_3)$$

By using the above notation, expression in (4.1) and we get

$$T_z=\frac{S_y(1+e_2)^{1/2}}{\bar{Y}(1+e_0)}+\eta\left(\frac{\bar{X}-\bar{X}(1+e_1)}{\bar{X}}\right)exp\left(\bar{X}-\bar{X}(1+e_1)\right) \tag{4.2}$$

$$T_z=C_y(1+e_2)^{1/2}(1+e_0)^{-1}+\eta\left(\frac{\bar{X}-\bar{X}-\bar{X}e_1}{\bar{X}}\right)exp\left(\bar{X}-\bar{X}-\bar{X}e_1\right)$$

$$T_z=C_y(1+e_2)^{1/2}(1+e_0)^{-1}+\eta\left(\frac{-\bar{X}e_1}{\bar{X}}\right)exp\left(-\bar{X}e_1\right)$$

We first expand the above estimators up to 1^{st} order approximations by using the Taylor series to obtain the approximate expressions for mean square errors MSE in (4.1).

$$T_z=C_y\left(1+\tfrac{e_2}{2}-\tfrac{e_2^2}{8}\right)\left(1-e_0+e_0^2\right)+\eta(-e_1)\left(1-\bar{X}e_1\right)$$

Simplify the form of the Taylor series

$$T_z = C_y\left(1 - e_0 + e_0^2 + \frac{e_2}{2} - \frac{e_2^2}{8}\right) + \eta\left(-e_1 + \bar{X}e_1^2\right)$$

They are ignoring the higher order terms.

$$T_z = C_y\left(1 - e_0 + \frac{e_2}{2}\right) + \eta\left(-e_1\right)$$

$$T_z = C_y + C_y\left(-e_0 + \frac{e_2}{2}\right) - \eta e_1$$

After subtracting the population coefficient of variation from both sides, we obtained

$$T_z - C_y = C_y\left(-e_0 + \frac{e_2}{2}\right) - \eta e_1$$

$$\left(T_z - C_y\right)^2 = \left(\left(-C_y e_0 + C_y\frac{e_2}{2} - \eta e_1\right)\right)^2$$

$$MSE\left(T_z\right) = \left(C_y^2 e_0^2 + C_y^2\frac{e_2^2}{4} + \left(\eta e_1\right)^2 - C_y^2 e_0 e_2 + C_y 2\eta e_0 e_1 - \eta C_y e_1 e_2\right)$$

I am applying expectation.

$$MSE\left(T_z\right) = \left(C_y^2 E\left(e_0^2\right) + C_y^2\frac{E\left(e_2^2\right)}{4} + \eta^2 E\left(e_1^2\right) - C_y^2 E\left(e_0 e_2\right) + 2\eta C_y E\left(e_0 e_1\right) - \eta C_y E\left(e_1 e_2\right)\right)$$

$$MSE\left(T_z\right) = \gamma\left(C_y^2\left(C_y^2\right) + C_y^2\frac{\left(\lambda_{40} - 1\right)}{4} + \eta^2 C_x^2 - C_y^2\left(C_y\lambda_{30}\right) + 2\eta C_y\left(\rho C_x C_y\right) - \eta C_y\left(C_x\lambda_{21}\right)\right)$$

$$MSE\left(T_z\right) = \gamma\left(C_y^4 + C_y^2\frac{\left(\lambda_{40} - 1\right)}{4} + \eta^2 C_x^2 - C_y^3\lambda_{30} + 2\eta C_y^2\rho C_x - \eta C_y C_x\lambda_{21}\right) \tag{4.3}$$

Differentiate w.r.t η

$$2\eta C_x^2 + 2\rho C_x C_y^2 - C_y C_x\lambda_{21} = 0$$

$$\eta = \frac{\lambda_{21} C_y - 2\rho C_y^2}{2C_x} \tag{4.4}$$

We obtain the best M.S.E. of the first degree of approximation as by putting (4.3) into (4.4).

$$MSE\left(T_z\right) = \gamma\left(C_y^4 + C_y^2\frac{\left(\lambda_{40} - 1\right)}{4} - \frac{\left(\lambda_{21}^2 C_y^2\right)}{4} + \left(\rho C_y^2\right)^2 - C_y^3\lambda_{30} + \rho\lambda_{21} C_y^3 - 2\rho^2 C_y^4\right)$$

5. Efficiency comparisonr

$MSE\left(T_z\right) < MSE(t_0)$

$$\gamma\left(\begin{array}{l} C_y^4 + C_y^2\frac{\left(\lambda_{40} - 1\right)}{4} - \frac{\left(\lambda_{21}^2 C_y^2\right)}{4} + \left(\rho C_y^2\right)^2 - C_y^3\lambda_{30} + \\ \rho\lambda_{21} C_y^3 - 2\rho^2 C_y^4 \end{array}\right) < C_y^2\gamma\left(\begin{array}{l} C_y^2 + \frac{1}{4}\left(\lambda_{40} - 1\right) - \\ C_y\lambda_{30} \end{array}\right)$$

$$\left(\rho C_y^2\right)^2 + \rho\lambda_{21} C_y^3 < \frac{\left(\lambda_{21}^2 C_y^2\right)}{4} + 2\rho^2 C_y^4$$

$$MSE\left(T_z\right) < MSE\left(t_{AR1}\right)$$

$$\gamma\left(\begin{array}{l} C_y^4 + C_y^2\frac{\left(\lambda_{40} - 1\right)}{4} - \frac{\left(\lambda_{21}^2 C_y^2\right)}{4} + \left(\rho C_y^2\right)^2 - C_y^3\lambda_{30} + \\ \rho\lambda_{21} C_y^3 - 2\rho^2 C_y^4 \end{array}\right) < C_y^2\gamma\left(\begin{array}{l} C_y^2 + \frac{1}{4}\left(\lambda_{40} - 1\right) + C_x^2 - C_x\lambda_{21} - \\ C_y\lambda_{30} + 2\rho C_y C_x \end{array}\right)$$

$$\left(\rho C_y^2\right)^2 + \rho\lambda_{21} C_y^3 + C_y^2 C_x \lambda_{21} < 2\rho^2 C_y^4 + \frac{\left(\lambda_{21}^2 C_y^2\right)}{4} + C_y^2 C_x^2 + 2\rho C_y^3 C_x$$

$$MSE\left(T_z\right) < MSE\left(t_{AR2}\right)$$

$$\gamma\left(\begin{array}{l} C_y^4 + C_y^2 \frac{(\lambda_{40}-1)}{4} - \frac{(\lambda_{21}^2 C_y^2)}{4} + \left(\rho C_y^2\right)^2 - \\ C_y^3\lambda_{30} + \rho\lambda_{21} C_y^3 - 2\rho^2 C_y^4 \end{array}\right) < C_y^2\gamma\left(\begin{array}{l} C_y^2 + \frac{1}{4}\left(\lambda_{40}-1\right) + \left(\lambda_{04}-1\right) - \left(\lambda_{22}-1\right) - \\ C_y\lambda_{30} + 2C_y\lambda_{12} \end{array}\right)$$

$$\left(\rho C_y^2\right)^2 + \rho\lambda_{21} C_y^3 + C_y^2\left(\lambda_{22}-1\right) < \frac{\left(\lambda_{21}^2 C_y^2\right)}{4} + 2\rho^2 C_y^4 + C_y^2\left(\lambda_{04}-1\right) + 2C_y^3\lambda_{12}$$

$$MSE\left(T_z\right) < MSE\left(t_y\right)$$

$$\gamma\left(\begin{array}{l} C_y^4 + C_y^2 \frac{(\lambda_{40}-1)}{4} - \frac{(\lambda_{21}^2 C_y^2)}{4} + \left(\rho C_y^2\right)^2 - C_y^3\lambda_{30} + \\ \rho\lambda_{21} C_y^3 - 2\rho^2 C_y^4 \end{array}\right)$$

$$< C_y^2\gamma\left(\begin{array}{l} C_y^2 + \frac{1}{4}\left(\lambda_{40}-1\right) + \frac{(\lambda_{04}-1)}{(Ln(S_x^2))^2} - \frac{(\lambda_{22}-1)}{Ln(S_x^2)} - \\ C_y\lambda_{30} + \frac{2C_y\lambda_{12}}{Ln(S_x^2)} \end{array}\right)$$

$$\left(\rho C_y^2\right)^2 + \rho\lambda_{21} C_y^3 + C_y^2\frac{\left(\lambda_{22}-1\right)}{Ln\left(S_x^2\right)} < \frac{\left(\lambda_{21}^2 C_y^2\right)}{4} + 2\rho^2 C_y^4 + C_y^2\frac{\left(\lambda_{04}-1\right)}{\left(Ln\left(S_x^2\right)\right)^2} + C_y^2\frac{2C_y\lambda_{12}}{Ln\left(S_x^2\right)}$$

6. Empirical Study

The performance of the suggested class of estimators with other estimators is being compared using the following descriptions of two natural population data sets.
Population 1: [Source: Murthy [31]. p.399]
Areas under wheat in 1963 and 1964 are shown as X and Y respectively

Table 1: Data Statistics 1

N= 35	n=15	$\hat{X} = 208.88$	$\hat{Y} = 199.44$	$C_x = 0.72$
$C_y = 0.75$	$\rho = 0.98$	$\lambda_{21} = 1.0045$	$\lambda_{12} = 0.9406$	$\lambda_{40} = 3.6161$
$\lambda_{04} = 2.8266$	$\lambda_{30} = 1.1128$	$\lambda_{03} = 0.9206$	$\lambda_{22} = 3.0133$	

Table 2: Mean square error (M.S.E.) and relative percentage efficiencies of the proposed estimator with other competitor estimators

Estimators	MSE	PRE
t_0	0.0080036	100.00
t_{AR1}	0.0258907	30.91
t_{AR2}	0.0336578	23.78
T_y	0.0071255	112.32
T_Z	0.006868	116.52

Table 3: Theoretical condition of Data 1

Condition	Population1
$T_z vs t_0$	0.7191745< 0.9861357
$T_z vs t_{AR1}$	1.125997<1.636597
$T_z vs t_{AR2}$	1.851656<2.372459
$T_z vs t_y$	0.8321232<0.8390202

2nd Population: [Source: Singh [32]. p.1116]
X: The number of fish caught in 1994,
Y: The number of fish caught in 1995,

Table 4: Table 4: Data Statistics 2

N= 69	n=40	$\hat{X} = 4591.07$	$\hat{Y} = 4514.89$	$C_x = 1.38$
$C_y = 1.35$	$\boldsymbol{\rho}$ =0.96	$\boldsymbol{\lambda}_{21} = 2.19$	$\boldsymbol{\lambda}_{12} = 2.30$	$\boldsymbol{\lambda}_{40} = 7.66$
$\boldsymbol{\lambda}_{04} = 9.84$	$\boldsymbol{\lambda}_{30} = 1.11$	$\boldsymbol{\lambda}_{03} = 2.52$	$\boldsymbol{\lambda}_{22} = 8.19$	

Table 5: Mean square error (M.S.E.) and relative percentage efficiencies of the proposed estimator with other competitor estimators

Estimators	MSE	PRE
$\boldsymbol{t_0}$	0.038088	100.00
$\boldsymbol{t}_{AR1}$	0.0851798	44.71
$\boldsymbol{t}_{AR2}$	0.18860299	20.19
T_y	0.0375686	101.38
$\boldsymbol{T_Z}$	**0.0373146**	**102.0733**

Table 6: Theoretical condition of Data 2

Condition	Population2
$\boldsymbol{T_z vs t_0}$	8.233793< 9.359568
$\boldsymbol{T_z vs t_{AR1}}$	13.74175< 18.2972
$\boldsymbol{T_z vs t_{AR2}}$	21.33757< 30.93733
$\boldsymbol{T_z vs t_y}$	8.982242< 9.006418

7. Conclusion

We created a new estimator for the population coefficient of variation that outperforms numerous current estimators. The main objective of this research is to evaluate the efficiency of our suggested estimator to that of various existing estimators. Our proposed estimator's MSE equation was theoretically derived, and its effectiveness was tested using two real data sets. The empirical results showed that the new suggested estimator batter the study's versions. As a result, the new estimator is meant for usage in real-world applications.

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