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On Pythagorean fuzzy subgroup

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ABSTRACT: The notion of a fuzzy Pythagorean was introduced by R.R. Yager as a generalization of an intuitionistic fuzzy sets. In this paper, we introduced the notion of a fuzzy Pythagorean point to study for the first time the notion of Pythagorean fuzzy subgroups and its properties. This new concept allowed us to reformulate all the mathematical properties of a fuzzy Pythagorean subgroups.

Key Words: Fuzzy group, Pythagorean fuzzy set, Pythagorean fuzzy point, Pythagorean fuzzy subgroup.

Contents

1. Introduction

The notion of fuzzy sets was introduced by Lotfi Zadeh in 1965 [25], as a generalized concept of the classical definition of set, which has a membership function μ defined from the universe X to $[0, 1]$. The value returned by this function represents the degree to which an element belongs to this fuzzy set, where a value of 0 indicates that an element does not belong to a class, a value of 1 indicates that it does, and additional values denote the degree of class membership. In other hand, by allowing for scenarios in between the whole and nothing, this idea broadens the application of conventional set theory. We can characterise a crisp set by his characteristic function, the thing that makes it possible to see that a classic set is so fuzzy. In his paper [25], L. Zadeh gives the characteristics of fuzzy sets, He defined the concept of α-level, which is the crisp part, note here that we can find a relation between the classical case and the fuzzy case. According to Zadeh a fuzzy subset A of X is defined by:

$$A = \{(x, \mu_A(x)) \ ; \ x \in X\} \ with \ \mu_A(x) \in [0, 1].$$

In [10] G. Gerla, gives necessary and sufficient conditions for the existence of a good definition of fuzzy point. In turn M.W. Warner [19] relied on this concept to study some topological properties in the fuzzy frame. For more informations we skinny readers on [20]. Later, in the same context, in 1986, Krassimiri Atassanov was able to highlight the existence of some sets in which the addition between membership and non-membership is less than one. From this idea, trough this discovery he was able to generaliz the theory of fuzzy sets, he introduce the notion of intuitionistic fuzzy sets (IFS) [3]. An IFS A of X is defined as follows:

$$A = \{(x, \mu_A(x), \nu_A(x)) \ ; \ x \in X\} \ with \ \mu_A(x), \nu_A(x) \in [0, 1].$$

With the condition : $\mu_A(x) + \nu_A(x) \leq 1$. In [4,3] K. Atanassov presented many caracteristics related to this theory through which it appears as a genralized fuzzy properties. In this context the intuitionistic fuzzy point, which a generalized of fuzzy point, introduced by Y.B. Jun and S.Z. Song in [24]. The authors in [17] study some relations between the intuitionistic fuzzy ideals of a semigroup and the set of all it s

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intuitionistic fuzzy points. Things did not stop there, but that Yager in 2013 [22], which generalized the last notion to fuzzy subsets of Pythagoras (PFS) [22] by keeping the notion of membership grade and non-membership grade, and by changing the condition on them.
An PFS according to Yager is defined by:

$$A = \{(x, \mu_A(x), \nu_A(x))\ ;\ x \in X\}\ with\ \mu_A(x), \nu_A(x) \in [0, 1].$$

With : $\mu_A^2(x) + \nu_A^2(x) \leq 1$.
Here $\mu_A^2(x) = \{\mu_A(x)\}^2$ and $\nu_A^2(x) = \{\nu_A(x)\}^2$.

Example 1.1. *Let* $\Phi = \{\phi_1, \phi_2\}$*,* $\Phi_1 = \{(\phi_1, \mu_{\Phi_1}(\phi_1), \nu_{\Phi_1}(\phi_1)|\ \phi_1 \in \Phi\}$ *and*

$$\Phi_2 = \{(\phi_2, \mu_{\Phi_2}(\phi_2), \nu_{\Phi_2}(\phi_2)|\ \phi_2 \in \Phi\},$$

with the membership degrees and non-membership degrees of the elements of Φ *are defined by*

$$\mu_{\Phi_1}(u) = 0.35, \quad \nu_{\Phi_1}(u) = 0.5;$$

and

$$\mu_{\Phi_2}(u) = 0.5, \quad \nu_{\Phi_1} = 0.7;$$

Since $\mu_{\Phi_1}(u) + \nu_{\Phi_1}(u) \leq 1$*, for all* $u \in \Phi$*, then* Φ_1 *is an IFS of* Φ*.*
On the other hand $\mu_{\Phi_2}(u) + \nu_{\Phi_2}(u) > 1$*. So* Φ_2 *is not an IFS of* Φ*,*
but we can easily varify that , $\mu_{\Phi_2}^2(u) + \nu_{\Phi_2}^2(u) \leq 1$*,*
thereforen, Φ_2 *can be considered as a PFS of* Φ

To have the difference between the definitions, see the following figure [22].

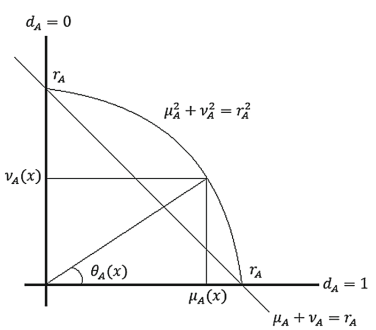

Figure 1: Comparison of IFSs and PFSs

After researchers started to study fuzzy algebra theory, in 1982 Anthony and Sherwood created an article on fuzzy subgroups and these characteristics [6]. In [9] Biswas presented the notion of the fuzzy subgroup in the intuitionnistic case (IFSG). Bhunia, Ghorai and Xin in [8] are the first who thought about the idea of the Pythagorean fuzzy subgroup (PFSG), they created an article presenting the definition and the properties of the Pythagorean fuzzy subgroups [8]. To our surprise, it appeared that no results are available on the Pythagorean fuzzy points, the thing that led us to discuss in this paper to present a new definition of Pythagorean fuzzy subgroup using Pythagorean fuzzy points, then to show some properties of Pythagorean fuzzy subgroups based on this new definition.
This paper is devised in two parts, the first part concerning the preliminaries and the necessary results which help us in this article, the second part is the most important in which we give the new definition and the properties of PFSG. And we end with a short conclusion.

2. Preliminaries

In this section we will present some notions and results to use them in the sequel.

Definition 2.1. *[3] Let X be a non−empty set. An intuitionistic fuzzy set (IFS for short) of X defined as an object having the form $A = \{\langle x,\ \mu(x),\ \nu(x)\rangle | x \in X\}$, where $\mu : X \to [0,\ 1]$ and $\nu : X \to [0,\ 1]$ denote the degree of membership (namely $\mu(x)$) and the degree of non-membership (namely $\nu(x)$) of each element $x \in X$ it to the set A, respectively and $0 \le \mu(x) + \nu(x) \le 1$ for each $x \in X$. For the sake of simplicity we shall use the symbol $A = (\mu, \nu)$ for the intuitionistic fuzzy set $A = \{\langle x,\ \mu(x),\ \nu(x)\rangle | x \in X\}$.*

In this paper we use the symbols $a \wedge b = \min\{a, b\}$ *and* $a \vee b = \max\{a, b\}$.

Definition 2.2. *[22] Let X be a non−empty set. A Pythagorean fuzzy set (PFS) A of an universal set X is of the form $A = \{\langle x,\ \mu(x),\ \nu(x)\rangle | x \in X\}$, where $\mu : X \to [0,\ 1]$ and $\nu : X \to [0,\ 1]$ and $0 \le \mu^2(x) + \nu^2(x) \le 1$ for each $x \in X$.*

Proposition 2.3. *[22] Let X be a non−empty set. Let (μ_1, ν_1) and (μ_2, ν_2) be two Pythagorean fuzzy subsets of X, then :*

1- The intersection set $(\mu, \nu) = (\mu_1, \nu_1) \cap (\mu_2, \nu_2)$ is defined by :

$$\mu = \mu_1 \wedge \mu_2 \text{ and } \nu = \nu_1 \vee \nu_2.$$

2- The union $(\mu, \nu) = (\mu_1, \nu_1) \cup (\mu_2, \nu_2)$ is defined by :

$$\mu = \mu_1 \vee \mu_2 \text{ and } \nu = \nu_1 \wedge \nu_2.$$

Definition 2.4. [8] *Let $(G, +)$ be a group and (μ, ν) be a PFS of G. Then (μ, ν) is said to be a Pythagorean fuzzy subgroup (PFSG) of G if for all $x, y \in G$ the following conditions hold:*

1. $\mu^2(x + y) \ge \mu^2(x) \wedge \mu^2(y)$
2. $v^2(x + y) \le \nu^2(x) \vee \nu^2(y)$,
3. $\mu^2(-x) \ge \mu^2(x)$.
4. $\nu^2(-x) \le \nu^2(x)$.

Proposition 2.5. [8] *Let $\psi = (\mu, \nu)$ be a PFSG of a group $(G, +)$. Then the following holds:*

i) $\mu^2(e) \ge \mu^2(x)$ *and* $\nu^2(e) \le \nu^2(x),\ \forall x \in G$.

ii) $\mu^2(-x) = \mu^2(x)$ *and* $\nu^2(-x) = \nu^2(x),\quad \forall x \in G$ *where, e is the identity element in G.*

Proposition 2.6. [8] *Let $\psi = (\mu, \nu)$ be a PFS of a group $(G, +)$. Then ψ is a PFSG of $(G, +)$ iff $\mu^2(x - y) \ge \mu^2(x) \wedge \mu^2(y)$ and $\nu^2(x - y) \le \nu^2(x) \vee \nu^2(y),\quad \forall x, y \in G$.*

Definition 2.7. [15] *let E be a set, let $x \in X$, $r \in [0, 1]$ and $\alpha, \beta \in (0, 1]$ such that: $\alpha^2 + \beta^2 = r^2 \le 1$. Then , the Pythagorean fuzzy subset $x_{(\alpha,\beta)} = (c_\alpha, 1 - c_{1-\beta})$ is called a Pythagorean fuzzy point in X with:*

$$c_\alpha(y) = \begin{cases} \alpha\ ,\ if\ x = y; \\ 0\ ,\ if\ x \ne y. \end{cases}$$

and,

$$(1 - c_{1-\beta})(y) = \begin{cases} \beta\ ,\ if\ x = y; \\ 1\ ,\ if\ x \ne y. \end{cases}$$

with the function $r_{x_{(\alpha,\beta)}} : X \longrightarrow [0, 1]$ given by :

$$r_{x_{(\alpha,\beta)}}(y) = \begin{cases} r\ ,\ if\ x = y; \\ 1\ ,\ if\ x \ne y. \end{cases}$$

3. Main results

For reasons of simplicity, we can define a Pythagorean fuzzy point in the following way:

Definition 3.1. *Let $\alpha, \beta \in [0,1]$ with $\alpha^2+\beta^2 \leq 1$. A Pythagorean fuzzy point, written as $x_{(\alpha,\beta)}$ is defined to be an Pythagorean fuzzy subset of X, given by*

$$x_{(\alpha,\beta)}(y) = \begin{cases} (\alpha,\ \beta)\ ,\ if\ x = y; \\ (0,1)\ ,\ if\ x \neq y. \end{cases}$$

A Pythagorean fuzzy point $x_{(\alpha,\beta)}$ is said to belong in PFS (μ,ν) denoted by $x_{(\alpha,\beta)} \in (\mu,\nu)$ if $\mu(x) \geq \alpha$ and $\nu(x) \leq \beta$.

Remark 3.2. *Definitions 2.7 and 3.1 are equivalent.*

Proposition 3.3. *Let G be a group and $H \subset G$, we note by χ_H the characteristic function of the set H. Then, H is a subgroup of G, if and only if $(\chi_H, 1-\chi_H)$ is a nonzero Pythagorean fuzzy subgroup of G.*

Proof. Suppose that H is a subgroup of G.
Let e be the neutral element of G, then $e \in H$ and $\chi_H(e) = 1$.
So, $(\chi_H, 1-\chi_H) \neq \emptyset$.
let $x, y \in G$. If $x \in H$ and $y \in H$, as H is a subgroup of G,
then $-y \in H$ and $x + y \in H$ as a result $x - y \in H$.
Therefore,

$$\chi_H^2(x-y) = 1 = \chi_H^2(x) \wedge \chi_H^2(y).$$

If $x \notin H$ or $y \notin H$, we have $\chi_H(x) = 0$ or $\chi_H(y) = 0$,
hence,

$$\chi_H^2(x) \wedge \chi_H^2(y) = 0 \leq \chi_H^2(x-y).$$

In the same way we show that

$$(1-\chi_H)^2(x-y) \leq (1-\chi_H)^2(x) \vee (1-\chi_H)^2(y).$$

Consequently, $(\chi_H, 1-\chi_H)$ is a Pythagorean fuzzy subgroup of G.
Conversely,
suppose that $(\chi_H, 1-\chi_H)$ is a nonzero Pythagorean fuzzy subgroup of G.
There exists $x \in G$ such as $\chi_H(x) = 1$, which proves that $H \neq \emptyset$,
moreover, if $x, y \in H$ $\chi_H^2(x) = \chi_H^2(y) = 1$,
hence, as $\chi_H^2(x-y) \geq \chi_H^2(x) \wedge \chi_H^2(y)$, $\chi_H^2(x-y) = 1$,
and as a result, $x - y \in H$.
Therefore H is a subgroup of G. □

Using the definition 3.1, we will present the following theorem.

Theorem 3.4. *Let $(G,+)$ be a group and A a Pythagorean fuzzy subset of G, then A is a $PFSG$ of G, if and only if*

1. $x_{(\alpha,\beta)} \in A$, $y_{(\lambda,\gamma)} \in A \Rightarrow (x+y)_{(\alpha\wedge\lambda,\beta\vee\gamma)} \in A$,
2. $x_{(\alpha,\beta)} \in A \Rightarrow (-x)_{(\alpha,\beta)} \in A$.

Proof. $\Leftarrow$)
let's show that the conditions of the definition of the $PFSG$ are verified.
Let $\alpha = \min\{\mu_A(x), \mu_A(y)\}$, $\beta = \max\{\nu_A(x), \nu_A(y)\}$, then $\alpha^2 + \beta^2 \leq 1$ and

$$\mu_A(x) \geq \alpha, \mu_A(y) \geq \alpha \text{ and } \nu_A(x) \leq \beta, \nu_A(y) \leq \beta.$$

So,

$$x_{(\alpha,\beta)} \in A \text{ et } y_{(\alpha,\beta)} \in A,$$

then $(x+y)_{(\alpha,\beta)} \in A$,
therefore,

$$\mu_A^2(x+y) \geq \alpha^2 = \mu_A^2(x) \wedge \mu_A^2(y) \text{ et } \nu_A^2(x+y) \leq \beta^2 = \nu_A^2(x) \vee \nu_A^2(y).$$

(**3.** and **4.**). Evident.
As a result A is a $PFSG$ of G.
$\Rightarrow$)
Suppose that A is a $PFSG$ of G. Let $x_{(\alpha,\beta)} \in A$, $y_{(\lambda,\gamma)} \in A$,
so, $\mu_A(x) \geq \alpha$, $\mu_A(y) \geq \lambda$, and $\nu_A(x) \leq \beta$, $\nu_A(y) \leq \gamma$
hence,

$$\mu_A(x+y) \geq \mu_A(x) \wedge \mu_A(y) \geq \alpha \wedge \lambda \text{ et } \nu_A(xy) \leq \nu_A(x) \vee \nu_A(y) \leq \beta \vee \gamma,$$

then,

$$(x+y)_{(\alpha\wedge\lambda,\beta\vee\gamma)} \in A.$$

moreover

$$\mu_A^2(-x) \geq \mu_A^2(x) \;\text{ et } \nu_A^2(-x) \leq \nu_A^2(x),$$

And because $x_{(\alpha,\beta)} \in A$
so,

$$\mu_A(-x) \geq \mu_A(x) \geq \alpha \text{ et } \nu_A(-x) \leq \nu_A(x) \leq \beta,$$

as a result,

$$(-x)_{(\alpha,\beta)} \in A.$$

□

Now, we are going to propose some properties on the PFSG using theorem 3.4.

Proposition 3.5. *let $(G,+)$ be a fuzzy group, and let $A(\mu,\nu)$ be a Pythagorean fuzzy subgroup of G. Then the following properties:*

1- Let $x_{(\alpha,\beta)} \in A(\mu,\nu)$ and $y_{(\lambda,\gamma)} \in A(\mu,\nu)$ be two Pythagorean fuzzy points, then :

$$(x-y)_{(\alpha\wedge\lambda,\beta\vee\gamma)} \in A(\mu,\nu).$$

2- let e be the neutral element of G , then:

$$\forall x_{(\alpha,\beta)} \in A(\mu,\nu)\ ,\ \mu_A(e) \geq \alpha \text{ and } \nu_A(e) \leq \beta.$$

Proof. let $(G,+)$ be a fuzzy group, and let $A(\mu,\nu)$ be a Pythagorean fuzzy subgroup of G.

1- Let $x_{(\alpha,\beta)} \in A(\mu,\nu)$ and $y_{(\lambda,\gamma)} \in A(\mu,\nu)$ be two Pythagorean fuzzy points.
Then, according to theorem 3.4 we have :

$$(-y)_{(\lambda,\gamma)} \in A(\mu,\nu) \text{ and } (x+y)_{(\alpha\wedge\lambda,\beta\vee\gamma)} \in A(\mu,\nu).$$

We have : $x-y = x+(-y)$.
So,

$$(x-y)_{(\alpha\wedge\lambda,\beta\vee\gamma)} \in A(\mu,\nu).$$

2- let e be the neutral element of G ,let $x_{(\alpha,\beta)} \in A(\mu,\nu)$ be a Pythagorean fuzzy point,then:

$$\mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq \beta.$$

Then, according to proposition 2.5 we have : $\mu_A^2(e) \geq \mu_A^2(x)$ and $\nu_A^2(e) \leq \nu_A^2(x)$.
So,

$$\mu_A(e) \geq \mu_A(x) \geq \alpha,$$

and,

$$\nu_A(e) \leq \nu_A(x) \leq \beta.$$

□

Remark 3.6. *Note that we have* $\mu_A(e) = 1, \nu_A(e) = 0$.

Let $L = \{(\alpha, \beta)/\alpha, \beta \in [0, 1]/\alpha^2 + \beta^2 \leq 1\}$.

Lemma 3.7. *Let A a Pythagorean fuzzy subset of E. for $(\alpha, \beta) \in L$, let*

$$A_\alpha = \{x \in E,\ \mu_A(x) \geq \alpha\},\ \textit{and}\ \ A^\beta = \{x \in E,\ \nu_A(x) \leq \beta\}.$$

Then, $(\mu_A(x), \nu_A(x)) = (\vee\{\alpha/\ x \in A_\alpha\}, \wedge\{\beta \mid x \in A^\beta\})$, $\forall x \in E$.

Proof. let $A(\mu, \nu)$ be a pythagorean fuzzy subset of E.
We have: $A_\alpha = \{x \in E,\ \mu_A(x) \geq \alpha\}$, and $A^\beta = \{x \in E,\ \nu_A(x) \leq \beta\}$.
Then,

$$\begin{aligned}\vee\{\alpha/\ x \in A_\alpha\} &= \vee\{\alpha \in [0, 1]\ ,\ x \in E\ ,\ \mu_A(x) \geq \alpha\}\\ &= \mu_A(x),\end{aligned}$$

and,

$$\begin{aligned}\wedge\{\beta\ x \in A^\beta\} &= \wedge\{\beta \in [0, 1]\ ,\ x \in E\ ,\ \nu_A(x) \leq \beta\}\\ &= \nu_A(x).\end{aligned}$$

□

Hence, $(\mu_A(x), \nu_A(x)) = (\vee\{\alpha/\ x \in A_\alpha\}, \wedge\{\beta \mid x \in A^\beta\})$, $\forall x \in E$.

Theorem 3.8. *Let G be a group and A a Pythagorean fuzzy subset of G. Then A is a Pythagorean fuzzy subgroup of G if and only if A_α and A^β are subgroups of G for every $(\alpha, \beta) \in L$.*

Proof. Let A be a $PFSG$ of G.
We will show that A^β and A_α are subgroups of G.
Let $x, y \in A^\beta$,
then,

$$\nu_A(x) \leq \beta,\ \text{et}\ \nu_A(y) \leq \beta.$$

Therefore, we have

$$\nu_A^2(x - y) \leq \nu_A^2(x) \vee \nu_A^2(-y) \leq \nu_A^2(x) \vee \nu_A^2(y) \leq \beta^2,$$

so

$$\nu_A(x - y) \leq \beta,$$

as a result,

$$x - y \in A^\beta.$$

Therefore, A^β is a subgroup of G.
Similarly, we can show that A_α is a subgroup of G.
Conversely.
We have

$$\begin{aligned}\mu_A^2(x + y) = \{\vee\{\alpha\ /x + y \in A_\alpha\}\}^2 &\geq \{\vee\{\alpha\ /x \in A_\alpha\ \text{et}\ y \in A_\alpha\}\}^2\\ &= \{\vee\{\alpha\ /\mu_A(x) \geq \alpha\ \text{et}\ \mu_A(y) \geq \alpha\}\}^2\\ &= \{\vee\{\alpha\ /\mu_A(x) \wedge \mu_A(y) \geq \alpha\}\}^2\\ &= \mu_A^2(x) \wedge \mu_A^2(y).\end{aligned}$$

And

$$\begin{aligned}\nu_A^2(x+y)=\{\wedge\{\beta\ /x+y\in A^{\beta}\}\}^2 &\le \{\wedge\{\beta\ /x\in A^{\beta}\text{ et }y\in A^{\beta}\}\}^2\\ &= \{\wedge\{\beta\ /\nu_A(x)\le\beta\text{ et }\nu_A(y)\le\beta\}\}^2\\ &= \{\wedge\{\beta\ /\nu_A(x)\vee\nu_A(y)\le\beta\}\}^2\\ &= \nu_A^2(x)\vee\nu_A^2(y).\end{aligned}$$

And we have

$$\mu_A^2(-x)=\{\vee\{\alpha\ /-x\in A_{\alpha}\}\}^2=\{\vee\{\alpha\ /x\in A_{\alpha}\}\}^2=\mu_A^2(x)$$

$$\nu_A^2(-x)=\{\wedge\{\beta\ /-x\in A^{\beta}\}\}^2=\{\wedge\{\beta\ /x\in A^{\beta}\}\}^2=\nu_A^2(x)$$

Therefore, A is a $PFSG$ of G. □

4. Conclusion

In this article, we have presented a new definition of Pythagorean fuzzy subgroups (PFSG) using the notion of Pythagorean fuzzy points, and with the help of this definition we have tried to show some properties on Pythagorean fuzzy subgroups (PFSG).

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