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Optimization Method Based on Bio-Inspired Approach For Solving A Robin Inverse Problem

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ABSTRACT: In this paper, we are interested in solving an inverse problem of reconstructing an unknown Robin coefficient. It consists to identify the Robin parameter from some additional measurement in the accessible part of the boundary. This problem is knowing as an ill-posed problem in the Hadamard sense and require regularisation methods to be solved. To this end, we formulate the inverse problem into an optimization problem, where we consider the given data as a control, in addition to a regularizing term known as Tikhonov regularization. Then, we investigate the particle swarm optimization as a stochastic approach, in combination with a finite element method (FEM) to solve the optimization problem. The numerical results indicate that our approach yields accurate, convergent and stable solutions for the considered Robin inverse problem in irregular 2D domain.

Key Words: Inverse problem, robin coefficient, optimization, particle swarm optimization, tikhonov regularization.

Contents

1. Introduction

An inverse problem is a type of scientific and mathematical problem that involves identifying the underlying causes or conditions that resulted in a particular outcome. Instead of predicting the result, the focus is on determining the factors or causes that played a role in a given situation. The field of inverse problems is broad, encompassing various disciplines, including engineering, medicine, geology, and physics. Inverse problems can be classified into several types based on the nature of the problem and the type of data available for solving it. Parameter estimation problems are a class of inverse problem refer to the task of determining the unknown parameters of a model based on observed data. The Robin inverse problem is a class of parameter estimation problems. This problem involves determining the parameters on an inaccessible part of the boundary, based on Cauchy data (measured data) obtained from the accessible boundary.

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The Robin inverse problem is commonly encountered in a variety of physical scenarios, such as the estimation of parameters in thermal models [1,2], the study of corrosion phenomena [3,4], the analysis of Metal-Oxide-Semiconductor Field-Effect Transistor (MOSFET) devices [5], and the investigation of heat protection mechanisms for spacecraft [6].

Extensive mathematical research has been conducted on the Robin inverse problem, addressing both identifiability and stability concerns. In [3], it was demonstrated that a smooth coefficient in a specific class can be identified with a single measurement. This result has been extended to continuous coefficients and bounded functions [7,8]. Additionally, various types of stability, including local Lipschitz, global monotonic Lipschitz, and global logarithmic stability, have been widely investigated [7,8,9].

Similar to many other inverse problems, the considered problem is ill-posed. Indeed, the solution may not exist for all Cauchy data and it may not have a continuous dependence on the data even if it exist. As is widely known, inverse problems are generally unstable, as demonstrated by Hadamard [10], meaning that a slight variation in the Cauchy data can lead to significant changes in the Robin coefficient. Therefore, regularization methods such as sequential function specification methods [11], iterative methods [12,13], Tikhonov regularization [14] and Truncated singular value decomposition [15] are necessary to obtain a reliable, precise and stable solution.

Numerous methods have been developed for solving the Robin coefficient problem, including those designed specifically for corrosion detection [16,17,18,19]. In [16,20], a simple and efficient approach using the thin-plate approximation method is proposed. In [21], the regularization method was combined with conjugate gradient methods (CGM). In [22], the Tikhonov regularization method was combined with a Genetic algorithm (GA) to minimize the cost function. In addition, the L1-tracking functional [23] and the Kohn-Vogelius functional [24] have been explored as potential approach for solving the Robin inverse problem.

There is a group of optimization methods called Metaheuristic methods that should be considered in addition to the deterministic techniques discussed earlier. Metaheuristics, such as artificial bee colony [25], ant colony optimization [26], particle swarm optimization [27], the Bat algorithm [28], Genetic Algorithm (GA) [29], and others, are capable of solving complex problems that are challenging or impossible to tackle with traditional exact optimization methods. These algorithms are usually applied when the problem space is too vast to explore exhaustively, or when there are too many variables or constraints to consider. One of the primary benefits of using metaheuristic algorithms for solving inverse problems is their ability to manage non-linear relationships between variables.

In this work, a stochastic technique known as particle swarm optimization (PSO) with Tikhonov regularization is utilized to determine the Robin coefficient in a two-dimensional inverse problem of Laplace's equation, without any a prior knowledge of the functional form of the coefficient. PSO was initially introduced by Kennedy and Eberhart to mimic social behavior, such as bird flocking and fish schooling. PSO has a simpler implementation and fewer parameters to control compared to GA, and it only requires basic mathematical operations instead of the more complex genetic operators (selection, crossover, and mutation) used in GA.

This study introduces a novel computational algorithm to identify the Robin coefficient on the inaccessible part of the boundary. The method utilizes PSO combined with Tikhonov Regularization and treats the solution on the under-specified Γ_i boundary as a control in a well-posed direct mixed problem. The goal of the proposed approach is to accurately fit the Cauchy data on the over-specified Γ_c boundary by minimizing an objective function that measures the discrepancies between the observed data and the calculated values.

Our paper is organized as follows: In Section 2, we provide the mathematical formulation of the Robin coefficient problem, in addition to its formulation as an optimization problem. In Section 3, we provide an overview of particle swarm optimization. Section 4 describes the procedure used to minimize the cost function. In Section 5, we present and discuss numerical results obtained for irregular domain showing the effectiveness of the proposed algorithm.

2. Robin coefficient problem

2.1. Problem setting

Let $\Omega \subset \mathbb{R}^2$ be an open bounded set with a Lipschitz continuous boundary $\partial\Omega$. We assume that $\partial\Omega$ consists of two disjoint parts, i.e., $\partial\Omega = \Gamma_c \cup \Gamma_i$, where Γ_c and Γ_i are accessible and inaccessible to boundary measurements, respectively. We consider the following inverse Robin problem:

$$\left\{ \begin{array}{l} \text{Being given a prescribed flux } g \not\equiv 0 \text{ together with measurements } f \text{ on } \Gamma_c, \\ \text{find a function } \gamma \text{ on } \Gamma_i \text{ such that the solution } u \text{ of} \\ \\ \text{(DP)} \left\{ \begin{array}{l} -\Delta u = 0 \text{ in } \Omega \\ \frac{\partial u}{\partial n} = g \text{ on } \Gamma_c \\ \frac{\partial u}{\partial n} + \gamma u = 0 \text{ on } \Gamma_i \end{array} \right. \\ \\ \text{also satisfies } u = f \text{ on } \Gamma_c \end{array} \right. \tag{2.1}$$

The Robin coefficient γ, which is to be identified, belongs to:

$$\mathcal{A} = \{\gamma \in L^\infty(\Gamma_i) \mid 0 < c_{\min} \leq \gamma \leq c_{\max} < \infty\} \tag{2.2}$$

where $\mathcal{A}$ is a bounded, closed, and convex admissible set that includes a priori information about γ. The constants $c_{\min}$ and $c_{\max}$ are distinct positive values. The problem described by Eq.(2.1) has been extensively studied for its existence, uniqueness and stability; see [7].

Direct problem It should be noted that, if γ and g in Eq.(DP) are given, a unique solution u can be determined. This problem is known as the forward problem.

Inverse problem The aim of the Robin inverse problem is to recovering the γ on Γ_i from a partial boundary measurement of function u. In other words, given the partial boundary measurable data f of u on Γ_c (the accessible part of the boundary), we will reconstruct the Robin coefficient γ on Γ_i.

2.2. Least-square minimization

When dealing with practical applications, the Cauchy data is obtained through experimental devices, which may lead to measurement errors. These errors can result in significant deviations of the Robin coefficient from its exact value. To handle the instability of the inverse problem, it is possible to apply the Tikhonov regularization method to modify the original inverse Robin coefficient problem into a least-squares minimization problem given by:
Find $\bar{\gamma} \in \mathcal{A}$, such that

$$\min_{\gamma \in \mathcal{A}} J(\gamma) = \|u(\gamma) - f\|^2_{0,\Gamma_c} + \frac{\eta}{2}\|\gamma\|^2_{0,\Gamma_i}, \quad \eta > 0 \tag{2.3}$$

where $u(\gamma)$ is a weak solution of the Eq. (2.1) in standard Sobolev space $H^1(\Omega)$.
The regularisation parameter η serves the dual purpose of improving the accuracy and stability of the solution. To select the optimal value of the regularization parameter, various effective techniques have been suggested in the literature such as the $L2$ curve method [30], the discrepancy principle [31], etc. These methods eliminate the need for using excessively small or large positive values.

Under appropriate assumptions, Problem (2.3) has a unique stable solution γ_η that converges to γ as $\eta \to 0$, which is a solution of Eq. (2.1) with the minimal L_2-norm. See [32,33] and references therein.

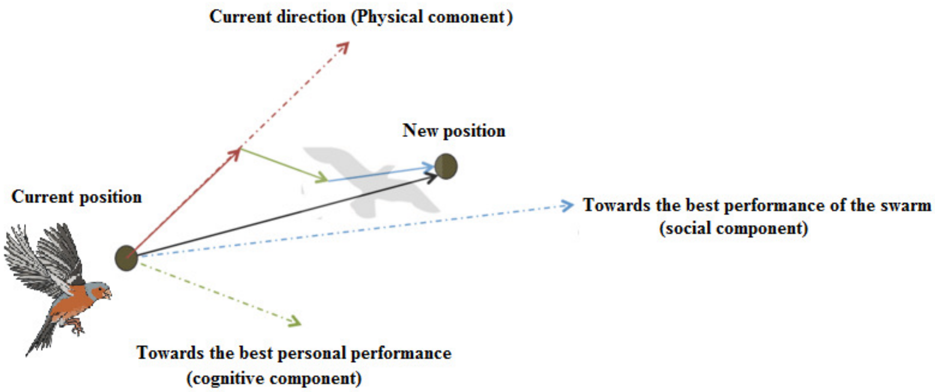


Figure 1: Movement of particle in space

3. Bio-inspired approach for Robin inverse problem

3.1. Overview of Particle Swarm Optimization

The Particle Swarm Optimization (PSO) is a stochastic optimization technique that is inspired by the collective behavior of social organisms such as flock of birds or schools of fish. It is developed by Eberhart and Kennedy in 1995 [27] .
In PSO, a population of candidate solutions, referred to particles, is moved around in the search space in order to find the optimal solution. Each particle is considered as a solution to the problem and has a position $X_k(t)$ and velocity vector $V_k(t)$ at time t, which are updated based on the particle's previous best position p_k and the best position p_{g} of the swarm as a whole. The objective function is used to evaluate the position of each particle. This helps the swarm to converge towards the optimal solution.
The update equations for each particle k, at time t for PSO can be written as follows:

- Update the velocity of the particle:

$$V_k(t+1) = \omega V_k(t) + c_1 r_1 \left(p_k(t) - X_k(t)\right) + c_2 r_2 \left(p_{\mathrm{g}}(t) - X_k(t)\right) \tag{3.1}$$

- Update the position of the particle:

$$X_k(t+1) = X_k(t) + V_k(t+1) \tag{3.2}$$

where w is the inertia weight, c_1 and c_2 are acceleration coefficients, r_1 and r_2 are random positive numbers that follow a uniform distribution over $[0, 1]$, see [34], p_k is the personal best position of particle k, $x_k(t)$ is the current position of particle k, and p_{g} is the global best position among all particles.
It will be noted that the coefficients c_1 and c_2 are constants determined empirical as a function of the relationship $c_1 + c_2 \leqslant 4$, see [35].
The movement strategy of a particle, as shown in Figure 1, is influenced by the following three components:

1. $V_k(t)$: Indicates that the particle tends to continue moving in its current direction.
2. $c_1 r_1 \left(p_k(t) - X_k(t)\right)$: Indicates that the particle has a tendency to move towards its best previously visited position.
3. $c_2 r_2 \left(p_{\mathrm{g}}(t) - X_k(t)\right)$: Indicates that the particle is inclined to move towards the best previously visited position of the swarm as a whole.

PSO parameters adjustment involves providing suitable input values for the inertia constant ω as well as for the local and global accelerations c_1 and c_2, in making it possible to explore the regions with small variance of the cost function. Figure 2 show the basic steps of the PSO.

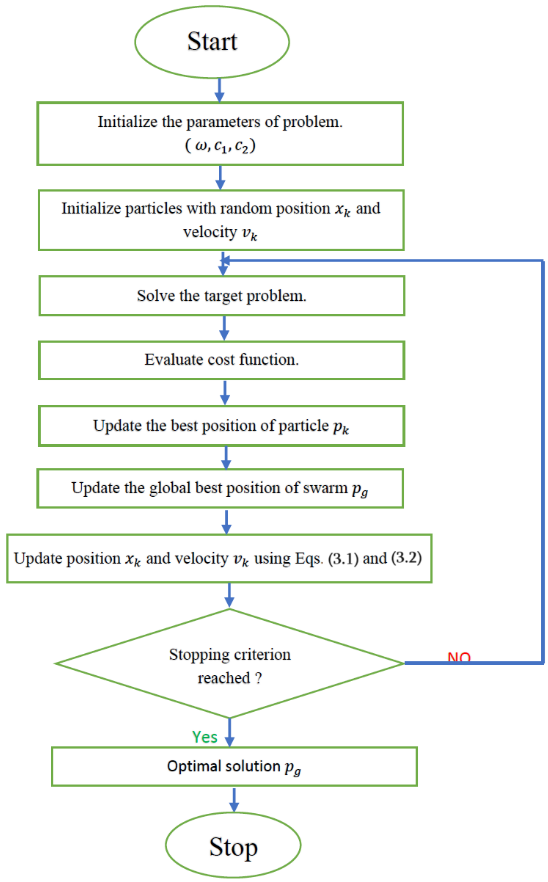


Figure 2: Flowchart of the algorithm

3.2. Computation procedure for the Particle Swarm Optimization

In this section, we present the steps of the PSO algorithm to solve the Robin inverse problem which can be defined as follows:

Step 1 : Parameter setting:

- N Swarm size
- c_1 and c_2 acceleration coefficients
- ω inertia coefficient
- $MaxIt$ Maximum number of iterations
- Lb Lower bound of the search space
- Ub Upper bound of the search space

Step 2 : Initialization: Random starting position $\gamma_k^{(0)}$ for $k = 1, \ldots, N$ are determined for each particle

- Initialize each particle velocity $v_k(0) \sim 0$.

- Initialize p_k to its initial position $p_k(0) = \gamma_k^{(0)}$

Step 3 : Solve the direct problem (3.3), for each given $\gamma_k^{(0)}$ for $k = 1, ..., N$, using the finite element method.

$$\begin{cases} -\Delta u &= 0 \quad \text{in} \quad \Omega \\ \partial_n u &= g \quad \text{on} \quad \Gamma_c \\ \frac{\partial u}{\partial n} + \gamma_k^{(0)} u &= 0 \quad \text{on} \quad \Gamma_i \end{cases} \tag{3.3}$$

Step 4 : Evaluation: Compute the fitness value using $J\left(\gamma_k^{(0)}\right)$ (Eq. (2.3)) for each particle $\gamma_k^{(0)}$ and initialize $p_g^{(0)}$ to the minimal value of the swarm: $p_g(0) = \operatorname{argmin} J\left(\gamma_k^{(0)}\right)$.

Step 5 : Update particles position and particles velocity

for $t = 1, \ldots, MaxIt$, do

for $k = 1, \ldots, N$, do

- Pick two random numbers: $r_1, r_2 \sim U\,(Lb, Ub)$
- Update particle's velocity using Eq .(3.1).
- Update particle's position using Eq .(3.2).

if $J\left(\gamma_k^{(t)}\right) < J(p_k^{(t)})$, Update the best known position of particle k $\mathrm{p_k}^{(t)} = \gamma_k^{(t)}$.

end if

if $J\left(\gamma_k^{(t)}\right) < J(p_g^{(it)})$, update the swarm's best known position: $p_g^{(t)} = p_k^{(t)}$.

end if

end for

end for

Step 6 : Output p_g that holds the best found solution.

4. Numerical Results and discussion

The domain under consideration is a unit square domain $\Omega = (0,1)^2$ with the boundary $\partial\Omega$ divided into two parts $\Gamma_i = \{(0, y) \mid 0 < y < 1\}$ and $\Gamma_c = \partial\Omega \setminus \Gamma_i$, (see Figure 3). .

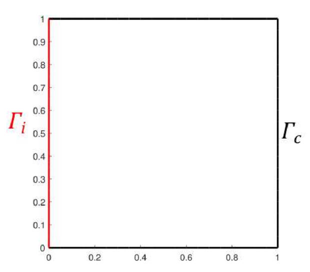


Figure 3: The square Domain

We consider a typical benchmark test example, given by the following:

$$u_{ex} = \cos(x)\cosh(y) + \sin(x)\sinh(y) \quad \text{in} \quad \Omega \tag{4.1}$$

We can easily extract the over-specified Cauchy data f and g on Γ_i from the analytic solution u_{ex}. The exact Robin coefficient to be retrieved on the inaccessible boundary is given by:

$$\gamma_{ex} = -\tanh(y) \quad \text{on} \quad \Gamma_i \tag{4.2}$$

The experiments are done on an Intel(R) Core(TM) i7-8565U CPU @ 1.80GHz (1.99 GHz) machine with 16,0 Go RAM.
It should be noted that the algorithm is implemented using the FreeFem++ software. To this end, we have to write the weak formulation of the direct problem Eq. (3.3). given by:
Find $u \in H^1(\Omega)$ such that for all $v \in H^1(\Omega)$

$$\int_{\Omega} \nabla u \cdot \nabla v dx + \int_{\Gamma_i} \gamma_k^{(0)} uv ds = \int_{\Gamma_c} gv ds$$

The domain mesh is generated via FreeFem++ considering the desired number of nodes on each part of the boundary.

PSO parameters used in numerical simulations are :

- Swarm size : $N = 70$
- Acceleration coefficients : $c_1 = 1.5$ and $c_2 = 1.5$
- Inertia coefficient : $\omega = 0.5$
- Maximum number of iterations: $MaxIt = 300$
- Lower bound of the search space : $Lb = -10$
- Upper bound of the search space : $Ub = 10$

We have set the regularization parameter η to a fixed value of $1e-8$ knowing that the numerical results obtained do not differ much for the different regularization parameters taken between 0 and $1e-5$.

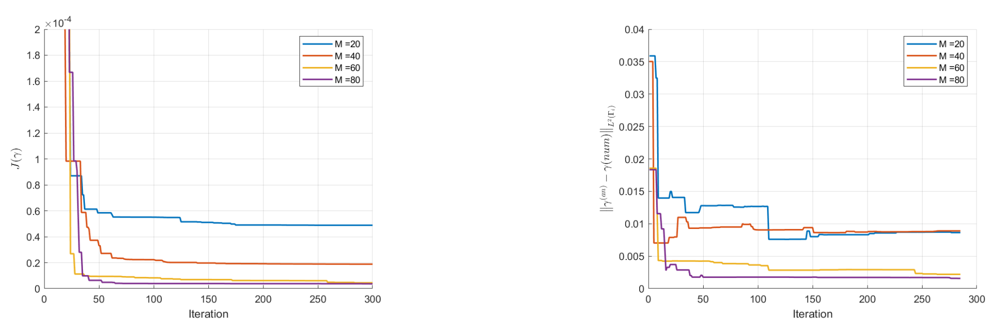
4.1. Convergence of the algorithm

In addition of the cost functional $J(\gamma)$, we examine the convergence of the algorithm by computing the error e_t at each iteration t, defined by :

$$e_t = \left\| \gamma_{(an)} - \gamma_t \right\|_{L^2(\Gamma_i)} \tag{4.3}$$

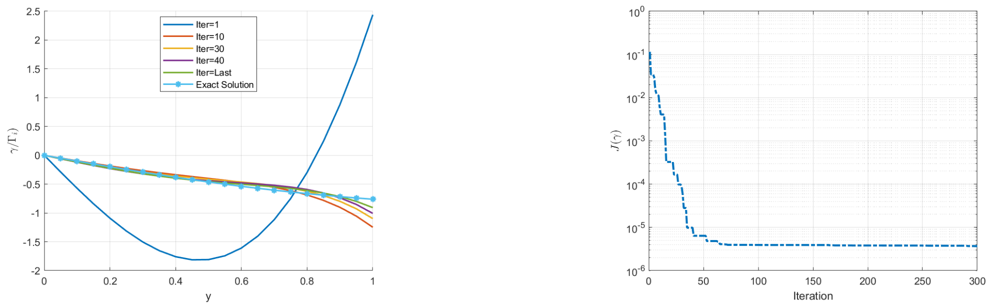
where γ_t is the reconstructed Robin coefficient on the boundary Γ_i at the t^{th} iteration, and $\gamma_{(an)}$ is the analytical Robin coefficient.

The cost functional $J(\gamma)$ and the error e_t for various iterations t are presented in Figure 4, depending on the number of nodes on the boundary $\partial\Omega$ taken as $M = 20, 40, 60, 80$. The figure shows that, both the functional $J(\gamma)$ and the error e_t decrease and the accuracy is better for a refined mesh up to 80 nodes on the domain boundary. Additionally, the convergence rate during the iteration process is slow, but both the residual and error still decrease as the number of iterations increase.



(a) (b)

Figure 4: The functional $J(\gamma)$ and the error e_t as function of the number of iterations t with exact data



(a) (b)

Figure 5: Numerical solution for $\gamma(y)$ in comparison with the exact solution (a) and the cost function (b) during an iterative process

Table 1: Comparing the objective function for the three approaches according iterations

Iteration	1	50	100	150	200
Approach 1	0.1021	2.5651e-06	4.0297e-07	3.8399e-07	1.6853e-07
Approach 2	0.11581	6.3827e-06	3.9195e-06	3.9034e-06	3.8123e-06
Approach 3	0.15519	0.00011587	0.00010174	8.8624e-05	8.5978e-05

Figure 5a displays the numerical solution, while Figure 5b presents the objective function $J(\gamma)$ for the value of γ obtained at $k = 300$ with $M = 80$ using exact data. These figures indicate that we obtain a highly accurate approximations during the iterative process even if the initial approximation is quite far from the initial exact.

4.2. Influence of a priori information

The influence of the knowledge of a priori information on the obtained result is considered. Thus, three approaches are considered, namely: For the first approach we suppose that the form of the Robin coefficient is known and we seek in this case the solution in the form $\gamma(y) = a \times tanh(b \times y)$; for the second approach, we search the solution as a polynomial approximation knowing that it is odd; however, in the third approach a polynomial approximation is sought without any information about the exact solution.

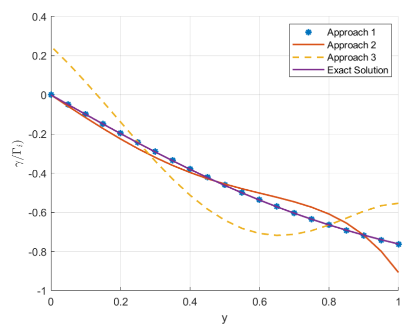
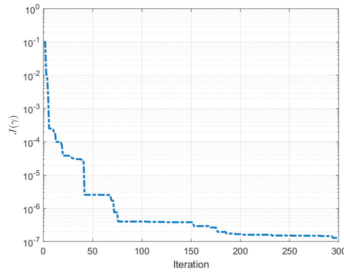
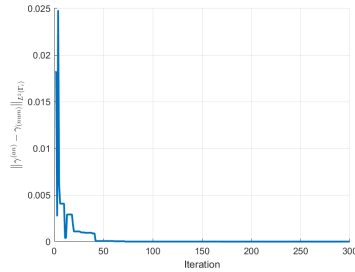


Figure 6: Comparison of the numerical solutions for the three approaches

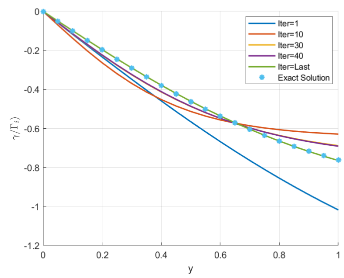
The table 1 present different values of the objective function for the 3 approaches during the iterative process; in particular for iterations 1, 50, 100, 150, and 200. It show that the objective function values decrease as the number of iterations increases for all three approaches which give a good approximation to the solution. Therefor, the first approach has the lowest objective function values across all iterations, followed by Approach 2, and then the Approach 3, which allow to conclude that with a priori information we can obtain a more performing approximation in less iterations.



(a)

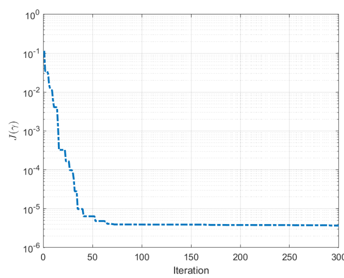


(b)

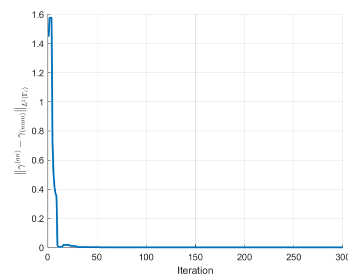


(c)

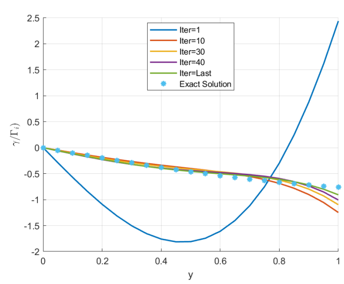
Figure 7: Objective function (a), the error e_t (b) and numerical solution for various iterations (c) with (approach 1)



(a)



(b)



(c)

Figure 8: Objective function (a), the error e_t (b) and numerical solution for various iterations (c) with (approach 2)

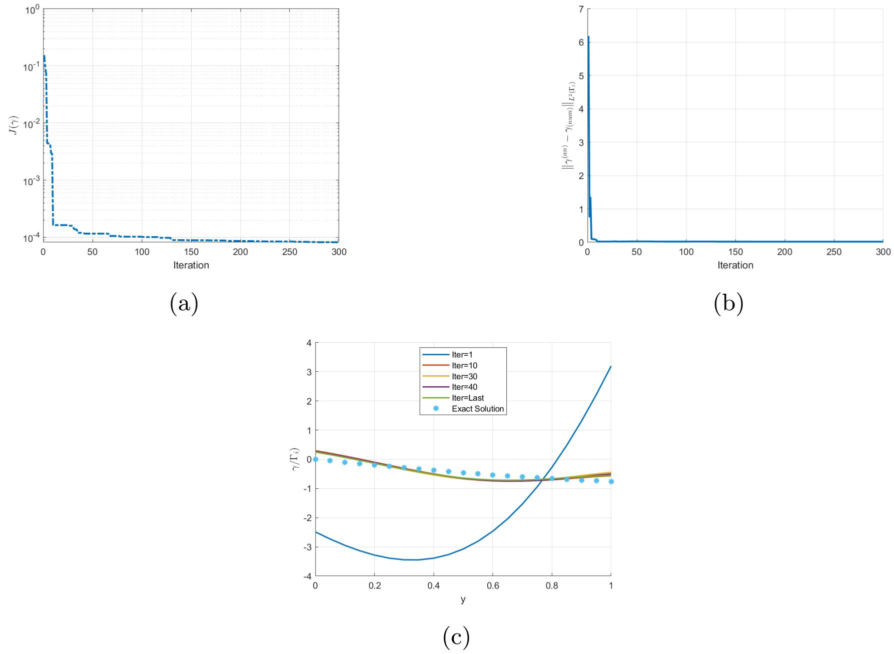


Figure 9: Objective function (a), the error e_t (b) and numerical solution for various iterations (c) with (approach 3)

Figures 7, 8 and 9 show that the proposed approach using PSO makes it possible to obtain good approximations of the solution sought with the three different approaches. However, having a prior information makes it possible to obtain better approximations with better precision in a reduced number of iterations than using a polynomial approximation without a prior information.

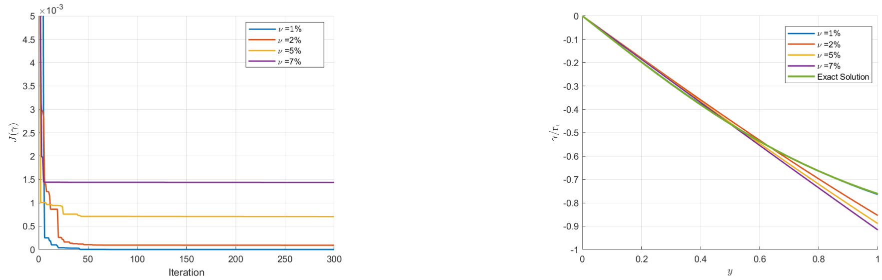
4.3. Stability of the algorithm

When dealing with inverse problems in the real world, the boundary data is obtained through experimental measurements, which makes it susceptible to measurement errors. In our testing scenarios, we create simulated noisy data by applying the following formula:

$$\tilde{T}(x) = T(x)(1 + \nu\zeta) \text{ on } \Gamma_c \tag{4.4}$$

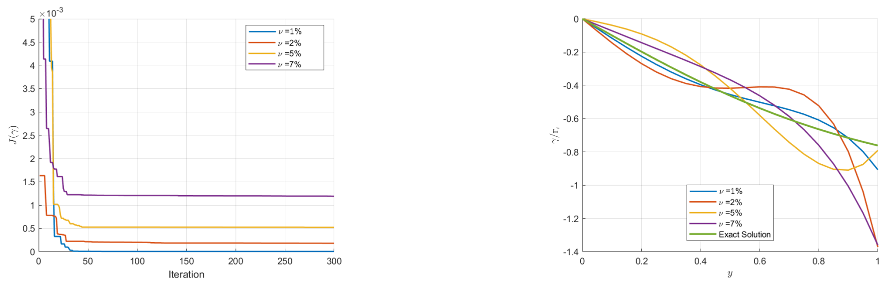
Here, ζ is a uniformly distributed random variable in the interval $[-1, 1]$, and the level of noise is determined by the parameter ν. In our current study, we implement the random variable ζ by utilizing the FreeFem++ function $randreal1()$.
The impact of the noise level on the proposed approach efficiency in reconstructing the Robin coefficient is analyzed. Figures 10, 11 and 12 displays the numerical outcomes and cost functions obtained with different levels of noise in the data.

By referring to Figures 10, 11 and 12 which represent the comparison between the exact Robin coefficient and the numerical approximations with different level of noise (on the left) and the cost function without and with different level of noise (on the right). It can be observed that as the amount of noise in the measurement data increases, it is evident that even with a noise level up to 7%, the approximated value of γ remains in good conformity with the exact value.



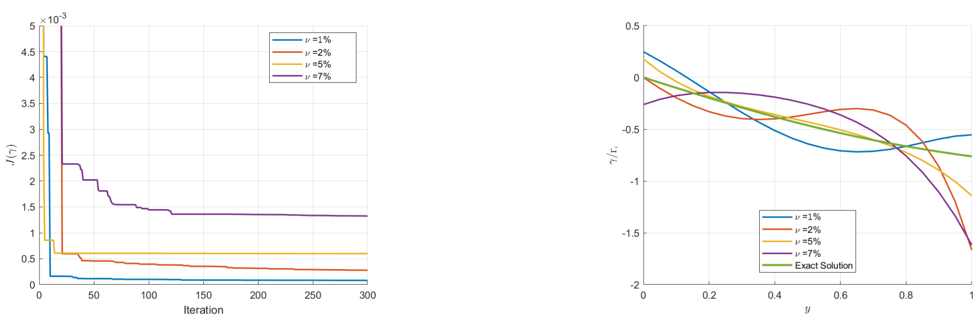
(a) (b)

Figure 10: The cost functions (right) and numerical solution for $\gamma(y)$ in comparison with the exact solution (left) as functions of various levels of noise $\nu = 1\%$, 2%, 5% and 7% (approach 1)



(a) (b)

Figure 11: The cost functions (right) and numerical solution for $\gamma(y)$ in comparison with the exact solution (left) as functions of various levels of noise $\nu = 1\%$, 2%, 5% and 7% (approach 2)



(a) (b)

Figure 12: The cost functions (right) and numerical solution for $\gamma(y)$ in comparison with the exact solution (left) as functions of various levels of noise $\nu = 1\%$, 2%, 5% and 7% (approach 3)

5. Conclusion

Our study focuses on solving an inverse problem of identification of parameters which consists on estimating the unknown Robin coefficient representing for example the damage of corrosion. We propose an optimization approach based on PSO method with Tikhonov regularization. The considered algorithm is implemented using the finite element method. The numerical simulations with irregular domain confirm that the proposed approach is very accurate, efficient and stable for the estimation of the Robin coefficient even if we don't have a priori information. Therefore, we can conclude that this algorithm has a promising potential for solving different inverse problems.

References

1. Chantasiriwan, S. Inverse heat conduction problem of determining time-dependent heat transfer coefficient. *International Journal Of Heat And Mass Transfer* **42**, 4275-4285, (1999).
2. Divo, E., Kassab, A., Kapat, J. & Chyu, M. Retrieval of multidimensional heat transfer coefficient distributions using an inverse BEM-based regularized algorithm: numerical and experimental results. *Engineering Analysis With Boundary Elements* **29**, 150-160 (2005).
3. Inglese, G. An inverse problem in corrosion detection. *Inverse Problems* **13**, 977-995, (1997).
4. Alessandrini, G., Del Piero, L., Rondi, L. & Others Stable determination of corrosion by a single electrostatic boundary measurement. *Inverse Problems* **19**, 973-984 (2003).
5. Fang, W. & Cumberbatch, E. Inverse problems for metal oxide semiconductor field-effect transistor contact resistivity. *SIAM Journal On Applied Mathematics* **52**, 699-709, (1992).
6. Beck, J. & Osman, A. Nonlinear inverse problem for the estimation of time-and-space-dependent heat-transfer coefficients. *Journal Of Thermophysics And Heat Transfer* **3**, 146-152, (1989).
7. Chaabane, S. & Jaoua, M. Identification of Robin coefficients by the means of boundary measurements. *Inverse Problems* **15** 1425, (1999).
8. Chaabane, S., Ferchichi, J. & Kunisch, K. Differentiability properties of the L1-tracking functional and application to the Robin inverse problem. *Inverse Problems* **20**, 1083, (2004).
9. Alessandrini, G., Del Piero, L., Rondi, L. & Others Stable determination of corrosion by a single electrostatic boundary measurement. *Inverse Problems* **19**, 973-984, (2003).
10. Hadamard, J. & Morse, P. Lectures on Cauchy's Problem in Linear Partial Differential Equations. *Physics Today* **6(8)**, 18 (1953).
11. Beck, J. & Murio, D. Combined function specification-regularization procedure for solution of inverse heat conduction problem. *AIAA Journal* **24**, 180-185, (1986).
12. Avdonin, S., Kozlov, V., Maxwell, D. & Truffer, M. Iterative methods for solving a nonlinear boundary inverse problem in glaciology. *Journal of Inverse and Ill-posed Problems* 17(3), 239-258, (2009).
13. Alifanov, O. Solution of an inverse problem of heat conduction by iteration methods. *Journal Of Engineering Physics.* **26**, 471-476, (1974).
14. Tikhonov, A. & Arsenin, V. Solutions of Ill-posed Problems: Andrey N. Tikhonov and Vasiliy Y. Arsenin. Translation Editor Fritz John, V. H. Winston and Sons (1977).
15. Hansen, P. Truncated singular value decomposition solutions to discrete ill-posed problems with ill-determined numerical rank. *SIAM Journal On Scientific And Statistical Computing* **11**, 503-518, (1990).
16. Fasino, D. & Inglese, G. An inverse Robin problem for Laplace's equation: theoretical results and numerical methods. *Inverse Problems* **15**, 41, (1999).
17. Chaabane, S., Elhechmi, C. & Jaoua, M. A stable recovery method for the Robin inverse problem. *Mathematics And Computers In Simulation* **66**, 367-383, (2004).
18. Fang, W. & Lu, M. A fast collocation method for an inverse boundary value problem. *International Journal For Numerical Methods In Engineering* **59**, 1563-1585, (2004).
19. Tajani, C., Jouilik, B. & Abouchabaka, J. Numerical identification of Robin coefficient by iterative method. *Palestine Journal Of Mathematics* **7**, 64-72, (2018).
20. Fasino, D. & Inglese, G. Discrete methods in the study of an inverse problem for Laplace's equation. *IMA Journal of Numerical Analysis* **19**, 105-118, (1999).
21. Jin, B. Conjugate gradient method for the Robin inverse problem associated with the Laplace equation. *International Journal For Numerical Methods In Engineering* **71**, 433-453, (2007).
22. Jouilik, B., Daoudi, J., Tajani, C. & Abouchabaka, J. Optimization approach based on genetic algorithm for a Robin coefficient problem. *Palestine Journal Of Mathematics* **11(3)**, 708–718, (2022).
23. Chaabane, S., Ferchichi, J. & Kunisch, K. Differentiability properties of the L1-tracking functional and application to the Robin inverse problem. *Inverse Problems* **20**, 1083, (2004)
24. Chaabane, S., Elhechmi, C. & Jaoua, M. A stable recovery method for the Robin inverse problem. *Mathematics And Computers In Simulation* **66**, 367-383, (2004).
25. Karaboga, D., Gorkemli, B., Ozturk, C. & Karaboga, N. A comprehensive survey: artificial bee colony (ABC) algorithm and applications. *Artificial Intelligence Review* **42**, 21-57, (2014).

26. Socha, K. & Dorigo, M. Ant colony optimization for continuous domains. *European Journal Of Operational Research* **185**, 1155-1173, (2008).

27. Kennedy, J. & Eberhart, R. Particle swarm optimization. *Proceedings Of ICNN'95-international Conference On Neural Networks* **4**, 1942-1948, (1995).

28. Yang, X. & Hossein Gandomi, A. Bat algorithm: a novel approach for global engineering optimization. *Engineering Computations* **29**, 464-483, (2012).

29. De Jong, K. Learning with genetic algorithms: An overview. *Machine Learning* **3**, 121-138, (1988).

30. Vogel, C. Non-convergence of the L-curve regularization parameter selection method. *Inverse Problems* **12**, 535, (1996).

31. Engl, H. Discrepancy principles for Tikhonov regularization of ill-posed problems leading to optimal convergence rates. *Journal Of Optimization Theory And Applications* **52**, 209-215, (1987).

32. Jin, B., & Zou, J. Numerical estimation of piecewise constant Robin coefficient. *SIAM journal on control and optimization* **48**, 1977-2002, (2009).

33. Jin, B., & Zou, J. Numerical estimation of the Robin coefficient in a stationary diffusion equation. *IMA Journal of Numerical Analysis* **3**, 677-701, (2010).

34. Garcıia-Gonzalo, E. & Fernández-Martıinez, J. Convergence and stochastic stability analysis of particle swarm optimization variants with generic parameter distributions. *Applied Mathematics And Computation* **249**, 286-302, (2014).

35. Pedersen, M. Good parameters for particle swarm optimization. *Hvass Lab., Copenhagen, Denmark, Tech. Rep. HL1001*, 1551-3203, (2010).

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