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Nörlund and Riesz Mean of Sequences of Bi-complex Numbers

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ABSTRACT: In this article we have investigated some properties of the Nörlund and Riesz mean of sequences of bi-complex numbers. We establish necessary and sufficient conditions for the Nörlund and Riesz means to transform convergent sequences of bi-complex numbers into convergent sequences of bi-complex numbers.

Key Words: Bi-complex, Nörlund mean, Riesz mean.

Contents

1. Introduction

The concept of bi-complex numbers has been investigated from different aspects by Segre [9], Wagh [15], Sager and Sağir [7]. The concept of bi-complex numbers have been applied in various field of mathematics like topology, functional analysis, sequence space etc. Many researchers [1,2,3,4,6,10,12] investigated the concept of bi-complex numbers and applied it to different fields of mathematics.

The Nörlund mean and Riesz mean play a important role in the field of sequence spaces. It has been studied from different aspects. Tripathy and Baruah [11] have studied Nörlund and Riesz mean for sequences of fuzzy numbers. Tripathy and Dowari [14] have studied Nörlund and Riesz mean for sequences of Complex Uncertain Variables. Saha et. al [8] studied on Riesz mean of complex uncertain sequences. Motivated by these researchers in this article we have investigate Nörlund and Riesz mean for sequences of bi-complex numbers.
Throughout C_0 and C_1 denote the set of real and complex numbers respectively.

2. Preliminaries

2.1. Bi-complex Numbers

The concept of bi-complex numbers was introduced by Segre [9]. The bi-complex numbers is defined by

$$\xi = z_1 + i_2 z_2 = x_1 + i_1 x_2 + i_2 x_3 + i_1 i_2 x_4,$$

where $x_1, x_2, x_3, x_4 \in C_0, z_1, z_2 \in C_1$ and the independent units i_1, i_2 are such that $i_1^2 = i_2^2 = -1$ and $i_1 i_2 = i_2 i_1$, we denote the set of bi-complex numbers C_2 is defined as:

$$C_2 = \{\xi : \xi = x_1 + i_1 x_2 + i_2 x_3 + i_1 i_2 x_4; x_1, x_2, x_3, x_4 \in C_0\}$$

$$i.e., C_2 = \{\xi : \xi = z_1 + i_2 z_2; z_1, z_2 \in C_1\}.$$

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There are four idempotent elements in C_2, these are $0, 1, e_1 = \frac{1+i_1i_2}{2}$ and $e_2 = \frac{1-i_1i_2}{2}$ out of which e_1 and e_2 are nontrivial such that $e_1 + e_2 = 1$ and $e_1e_2 = 0$.
Every bi-complex number $\xi = z_1 + i_2z_2$ can be uniquely expressed as the combination of e_1 and e_2, namely

$$\xi = z_1 + i_2z_2 = (z_1 - i_1z_2)e_1 + (z_1 + i_1z_2)e_2 = \mu_1e_1 + \mu_2e_2,$$

where $\mu_1 = (z_1 - i_1z_2)$ and $\mu_2 = (z_1 + i_1z_2)$.
The Euclidean norm $\|\cdot\|$ on C_2 is defined by

$$\begin{aligned}\|\xi\|_{C_2} &= \sqrt{x_1^2 + x_2^2 + x_3^2 + x_4^2} \\ &= \sqrt{|z_1|^2 + |z_2|^2} \\ &= \sqrt{\frac{|\mu_1|^2 + |\mu_2|^2}{2}},\end{aligned}$$

where $\xi = x_1 + i_1x_2 + i_2x_3 + i_1i_2x_4 = z_1 + i_2z_2 = \mu_1e_1 + \mu_2e_2$ and $\mu_1 = z_1 - i_1z_2, \mu_2 = z_1 + i_1z_2$, $e_1 = \frac{1+i_1i_2}{2}, e_2 = \frac{1-i_1i_2}{2}$. With this norm C_2 is a Banach space, also C_2 is a commutative algebra. Product of two bi-complex numbers are connected by the following inequality:

$$\|\xi.\eta\|_{C_2} \leq \sqrt{2}\|\xi\|_{C_2}.\|\eta\|_{C_2}.$$

We shall use the notation $C(i_1)$ and $C(i_2)$ for the following sets:

$$C(i_1) = \{x_1 + i_1x_2 : x_1, x_2 \in C_0\},$$

$$C(i_2) = \{x_1 + i_2x_2 : x_1, x_2 \in C_0\}.$$

Definition 2.1 *A bi-complex numbers $\xi = z_1 + i_2z_2$ is said to be singular if $|z_1^2 + z_2^2| = 0$ and otherwise it is called non-singular.*

Three types of conjugations for the bi-complex numbers as follows:

Definition 2.2 *The i_1-conjugation of a bi-complex number $\xi = z_1 + i_2z_2$ is denoted by ξ^* and is defined by*
$\xi^* = \bar{z_1} + i_2\bar{z_2}$, for all $z_1, z_2 \in C(i_1)$ and $\bar{z_1}, \bar{z_2}$ are the complex conjugates of z_1, z_2 respectively, where $i_1^2 = i_2^2 = -1$.

Definition 2.3 *The i_2-conjugation of a bi-complex number $\xi = z_1 + i_2z_2$ is denoted by $\bar{\xi}$ and is defined by*
$\bar{\xi} = z_1 - i_2z_2$, for all $z_1, z_2 \in C(i_1)$, where $i_1^2 = i_2^2 = -1$.

Definition 2.4 *The i_3-conjugation of a bi-complex number $\xi = z_1 + i_2z_2$ is denoted by $\xi^{'}$ and is defined by*
$\xi^{'} = \bar{z_1} + i_2\bar{z_2}$, for all $z_1, z_2 \in C(i_1)$ and $\bar{z_1}, \bar{z_2}$ are the complex conjugates of z_1, z_2 respectively, where $i_1^2 = i_2^2 = -1$.

Based on the above three types of conjugations, there are three moduli as given below. Each of the three conjugations three moduli are given by

$$\begin{aligned}|\xi|_{i_1} &= \sqrt{\xi \cdot \bar{\xi}} = \sqrt{z_1^2 + z_2^2}, \\ |\xi|_{i_2} &= \sqrt{\xi \cdot \xi^*} = \sqrt{|z_1|^2 - |z_2|^2 + 2Re(z_1\bar{z_2})i_2}, \\ |\xi|_{i_3} &= \sqrt{\xi \cdot \xi'} = \sqrt{|z_1|^2 + |z_2|^2 - 2Im(z_1\bar{z_2})i_1i_2}.\end{aligned}$$

A bi-complex numbers ξ with $|\xi|_{i_1} \neq 0$ is invertible and its inverse is given by

$$\xi^{-1} = \frac{\bar{\xi}}{|\xi|_{i_1}^2}.$$

Definition 2.5 *A sequence of bi-complex number (ξ_k) is said to be convergent to $\xi \in C_2$ with respect to the Euclidean norm on C_2 if for every $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that*

$$\|\xi_k - \xi\|_{C_2} < \varepsilon, \text{ for all } n \geq n_0(\varepsilon).$$

We written as $\lim \xi_k = \xi$.

2.2. Matrix Map

Let $A = (a_{nk})$ be an infinite matrix mapping from a sequence space E into a sequence space F, then for $\xi = (\xi_n) \in E$, the A-transform of ξ_n is the sequence $(A_n(\xi))$, where $A_n\xi = \sum_{k=1}^{\infty} a_{nk}$ for each $n \in \mathbb{N}$, provided the summation exists for each $n \in \mathbb{N}$.
The following well known result contains the necessary and sufficient conditions for the regularity of a matrix map known as Silverman-Toeplitz conditions (one may refer to Petersen [5]).

Lemma 2.1 *The matrix $A = (a_{nk})$ is regular or limit preserving if and only if it satisfies the following conditions:*

i. there exists a constant M such that $\sup_n \sum_{k=1}^{\infty} |a_{nk}| < M$;

ii. for every k, $\lim_{n\to\infty} a_{nk} = 0$;

iii. $\lim_{n\to\infty} \sum_{k=1}^{\infty} a_{nk} = 1$.

3. Nörlund Mean of Sequences of Bi-complex Numbers

Let (ξ_n) be a sequence of bi-complex numbers with

$$|P_n|_{i_1} \neq 0, \text{ where } P_n = \xi_1 + ... + \xi_n, \text{ for all } n \in \mathbb{N}.$$

Remark 3.1 In the case for the sequence (p_n) of non-negative real numbers with $p_1 > 0$, we have $P_n \neq 0$, for all $n \in \mathbb{N}$. In this article we consider the sequence (ξ_n) to be the sequence of bi-complex numbers. Thus in this case the sequence (ξ_n) of bi-complex numbers is such that $|P_n|_{i_1} \neq 0$, for all $n \in \mathbb{N}$, so that P_n will be inevitable for each $n \in \mathbb{N}$.

Definition 3.1 *Let (ξ_n) be sequence of bi-complex numbers with*

$$P_n = \sum_{k=1}^{n} \xi_k, |P_n|_{i_1} \neq 0, \text{ for all } n \in \mathbb{N}.$$

Then the transformation (t_n) of the sequence (η_k) is given by

$$t_n = \frac{\xi_n\eta_1 + \xi_{n-1}\eta_2 + ... + \xi_1\eta_n}{P_n}$$

is called bi-complex Nörlund mean (N, ξ_n) or simply the (N, ξ_n) mean of the bi-complex sequence (η_k).

Remark 3.2 The matrix of the Nörlund mean (N, ξ_n) is given by

$$a_{nk} = \begin{cases} \frac{\xi_{n-k+1}}{P_n}, & \text{for } k \leq n; \\ 0, & \text{for } k > n. \end{cases}$$

Theorem 3.1 *The Nörlund mean (N, ξ_n), where $\xi_n = \mu_{1n}e_1 + \mu_{2n}e_2$ is expressed as $(N, \mu_{1n})e_1 + (N, \mu_{2n})e_2$, where (N, μ_{jn}) are the Nörlund mean of the complex sequences $(\mu_{jn}), j = 1, 2$.*

Proof: Let (ξ_n) be a sequence of bi-complex numbers with

$$P_n = \sum_{k=1}^{n} \xi_k = \sum_{k=1}^{n} (\mu_{1k}e_1 + \mu_{2k}e_2) = e_1 \sum_{k=1}^{n} \mu_{1k} + e_2 \sum_{k=1}^{n} \mu_{2k},$$

and $|P_n|_{i_1} \neq 0$. Then P_n^{-1} exists and
Now,

$$\bar{P_n} = e_2 \sum_{k=1}^{n} \mu_{1k} + e_1 \sum_{k=1}^{n} \mu_{2k}$$

and

$$\begin{aligned} |P_n|_{i_1}^2 &= P_n \cdot \bar{P_n} \\ &= \left(\sum_{k=1}^{n} \mu_{1k} \right) \left(\sum_{k=1}^{n} \mu_{2k} \right) \neq 0 \end{aligned}$$

$$\implies \left(\sum_{k=1}^{n} \mu_{1k} \right) \neq 0 \text{ and } \left(\sum_{k=1}^{n} \mu_{2k} \right) \neq 0.$$

Hence,

$$P_n^{-1} = \frac{\bar{P_n}}{|P_n|_{i_1}^2} = \frac{1}{\sum_{k=1}^{n} \mu_{1k}} e_1 + \frac{1}{\sum_{k=1}^{n} \mu_{2k}} e_2. \tag{3.1}$$

Now the transformation (t_n) of the sequence (η_n), where $\eta_n = \mu'_{1n}e_1 + \mu'_{2n}e_2$ is given by

$$\begin{aligned} t_n &= \frac{\xi_n\eta_1 + \xi_{n-1}\eta_2 + ... + \xi_1\eta_n}{P_n} \\ &= P_n^{-1} \sum_{k=1}^{n} \xi_k \cdot \eta_{n-k+1} \\ &= \left(\frac{1}{\sum_{k=1}^{n} \mu_{1k}} e_1 + \frac{1}{\sum_{k=1}^{n} \mu_{2k}} e_2 \right) \cdot \left(e_1 \sum_{k=1}^{n} \mu_{1k}\mu'_{1n-k+1} + e_2 \sum_{k=1}^{n} \mu_{2k}\mu'_{2n-k+1} \right) \\ &= e_1 \frac{\sum_{k=1}^{n} \mu_{1k}\mu'_{1n-k+1}}{\sum_{k=1}^{n} \mu_{1k}} + e_2 \frac{\sum_{k=1}^{n} \mu_{2k}\mu'_{2n-k+1}}{\sum_{k=1}^{n} \mu_{2k}} \\ &= e_1(N, \mu_{1k}) + e_2(N, \mu_{2k}), \end{aligned}$$

where (N, μ_{jk}) are the Nörlund mean of the complex sequences $(\mu_{jk}), j = 1, 2$. □

Theorem 3.2 *The Nörlund mean (N, ξ_n) is regular if and only if*

$$\frac{\xi_n}{P_n} \to 0, \text{ as } n \to \infty.$$

Proof: Let the sequence (η_n) be convergent to η, then there exists a constant $M > 0$ such that $\|\eta_n\|_{C_2} \leq M$ for all $n \in \mathbb{N}$. Since (η_n) is convergent, so for a given $0 < \varepsilon$, there exists an integer n_0 such that

$$\|\eta_n - \eta\|_{C_2} < \varepsilon, \text{ for } n > n_0.$$

Let $\frac{\xi_n}{P_n} \to 0$, as $n \to \infty$. We choose n such that $\left\|\frac{\xi_n}{P_n}\right\|_{C_2} < \frac{\varepsilon}{2n_0 M}$, for $n > n_0$.
We have,

$$\begin{aligned}
\|t_n - \eta\|_{C_2} &= \left\|\frac{\xi_n\eta_1 + \xi_{n-1}\eta_2 + \ldots + \xi_1\eta_n}{P_n} - \eta\right\|_{C_2} \\
&= \left\|\frac{\xi_n\eta_1 + \xi_{n-1}\eta_2 + \ldots + \xi_1\eta_n - \eta(\xi_1 + \xi_2 + \ldots + \xi_n)}{P_n}\right\|_{C_2} \\
&= \left\|\frac{\xi_1(\eta_n - \eta) + \xi_2(\eta_{n-1} - \eta) + \ldots + \xi_n(\eta_1 - \eta)}{P_n}\right\|_{C_2} \\
&\leq \left\|\frac{\xi_1(\eta_n - \eta) + \xi_2(\eta_{n-1} - \eta) + \ldots + \xi_{n-n_0+1}(\eta_{n_0} - \eta)}{P_n}\right\|_{C_2} + \\
&\qquad \left\|\frac{\xi_{n-n_0}(\eta_{n_0+1} - \eta) + \ldots + \xi_n(\eta_1 - \eta)}{P_n}\right\|_{C_2} \\
&\leq \left\|\frac{\xi_n}{P_n}\right\|_{C_2} \|\eta_1 - \eta\|_{C_2} + \left\|\frac{\xi_{n-1}}{P_n}\right\|_{C_2} \|\eta_2 - \eta\|_{C_2} + \ldots + \left\|\frac{\xi_{n-n_0+1}}{P_n}\right\|_{C_2} \|\eta_{n_{n_0}} - \eta\|_{C_2} + \\
&\qquad \left\|\frac{\xi_{n-n_0}}{P_n}\right\|_{C_2} \|\eta_{n_0+1} - \eta\|_{C_2} + \ldots + \left\|\frac{\xi_1}{P_n}\right\|_{C_2} \|\eta_n - \eta\|_{C_2} \\
&\leq \left(\left\|\frac{\xi_n}{P_n}\right\|_{C_2} + \ldots + \left\|\frac{\xi_{n-n_0+1}}{P_n}\right\|_{C_2}\right) M + \left(\left\|\frac{\xi_{n-n_0}}{P_n}\right\|_{C_2} + \ldots + \left\|\frac{\xi_1}{P_n}\right\|_{C_2}\right) \frac{\varepsilon}{2} \\
&< \frac{\varepsilon}{2n_0 M} n_0 M + \frac{\varepsilon}{2} = \varepsilon.
\end{aligned}$$

Hence, (N, ξ_n) is regular method.

Conversely, let (N, ξ_n) is regular method. Consider a sequence of bi-complex numbers $(\zeta_k) = \left(\frac{1+i_1+i_2+i_1i_2}{2}, 0, 0, 0, \ldots\right)$.
Now, $\|t_n - \zeta_k\|_{C_2} = \left\|\frac{\xi_n}{P_n}\right\|_{C_2} \to 0$, as $n \to \infty$.
$\implies \frac{\xi_n}{P_n} \to 0$, as $n \to \infty$.

□

Lemma 3.1 *The Nörlund mean (N, ξ_n) is regular if and only if (N, μ_{1n}) and (N, μ_{2n}) are both regular.*

Proof: Let (N, ξ_n) be regular. Then

$$\frac{\xi_n}{P_n} \to 0, \text{ as } n \to \infty.$$

$$\implies (\mu_{1n}e_1 + \mu_{2n}e_2)\left(\frac{1}{\sum\limits_{k=1}^{n}\mu_{1k}}e_1 + \frac{1}{\sum\limits_{k=1}^{n}\mu_{2k}}e_2\right) \to 0,\ \text{as } n\to\infty[\text{from } (3.1)]$$

$$\implies \frac{\mu_{1n}}{\sum\limits_{k=1}^{n}\mu_{1k}}e_1 + \frac{\mu_{2n}}{\sum\limits_{k=1}^{n}\mu_{2k}}e_2 \to 0e_1 + 0e_2,\ \text{as } n\to\infty$$

$$\implies \frac{\mu_{1n}}{\sum\limits_{k=1}^{n}\mu_{1k}} \to 0,\ \text{as } n\to\infty \text{ and } \frac{\mu_{2n}}{\sum\limits_{k=1}^{n}\mu_{2k}} \to 0,\ \text{as } n\to\infty.$$

Since,

$$\sum_{k=1}^{n}\mu_{jk} \neq 0, j = 1, 2,$$

we have

$$\frac{\mu_{jn}}{\sum\limits_{k=1}^{n}\mu_{jk}} \text{ are the regularity conditions for } (N, \mu_{jk}), j = 1, 2.$$

The converse part can be established using standard technique. □

4. Riesz Mean of Sequences of Bi-complex numbers

Definition 4.1 *Let (ξ_n) be a sequence of bi-complex numbers with*

$$P_n = \sum_{k=1}^{n}\xi_k \text{ and } |P_n|_{i_1} \neq 0,\ \text{for all } n \in \mathbb{N}.$$

Then the transformation (t_n) of the sequence (η_k) is given by

$$t_n = \frac{\xi_1\eta_1 + \xi_2\eta_2 + ... + \xi_n\eta_n}{P_n}$$

is called bi-complex Riesz mean (R, ξ_n) or simply the (R, ξ_n) mean of the sequence (η_k).

Remark 4.1 The matrix of the Riesz mean (R, ξ_n) is given by

$$a_{nk} = \begin{cases} \frac{\xi_k}{P_n}, & \text{for } k \leq n; \\ 0, & \text{for } k > n. \end{cases}$$

In view of Theorem 3.1, we formulate the following theorem.

Theorem 4.1 *The Riesz mean (R, ξ_n), where $\xi_n = \mu_{1n}e_1 + \mu_{2n}e_2$ is expressed as $(R, \mu_{1n})e_1 + (R, \mu_{2n})e_2$, where (R, μ_{jn}) are the Riesz mean of the complex sequences $(\mu_{jn}), j = 1, 2$.*

Theorem 4.2 *The Riesz mean (R, ξ_n) is regular if and only if P_n is unbounded.*

Proof: Let the sequence (η_n) be convergent to η, then there exists a constant $M > 0$ such that $\|\eta_n\|_{C_2} \leq M$ for all $n \in \mathbb{N}$. Since (η_n) is convergent, so for a given $0 < \varepsilon$, there exists an integer n_0 such that

$$\|\eta_n - \eta\|_{C_2} < \varepsilon,\ \text{for } n > n_0.$$

Let P_n is unbounded. We choose n such that $\left\|\frac{\xi_k}{P_n}\right\|_{C_2} < \frac{\varepsilon}{2n_0M}$, for $n > n_0$.
We have,

$$\begin{aligned}
\|t_n - \eta\|_{C_2} &= \left\|\frac{\xi_1\eta_1 + \xi_2\eta_2 + ... + \xi_n\eta_n}{P_n} - \eta\right\|_{C_2} \\
&= \left\|\frac{\xi_1\eta_1 + \xi_2\eta_2 + ... + \xi_n\eta_n - \eta(\xi_1 + \xi_2 + ... + \xi_n)}{P_n}\right\|_{C_2} \\
&= \left\|\frac{\xi_1(\eta_1 - \eta) + \xi_2(\eta_2 - \eta) + ... + \xi_n(\eta_n - \eta)}{P_n}\right\|_{C_2} \\
&\leq \left\|\frac{\xi_1(\eta_1 - \eta) + \xi_2(\eta_2 - \eta) + ... + \xi_{n_0}(\eta_{n_0} - \eta)}{P_n}\right\|_{C_2} + \\
&\qquad \left\|\frac{\xi_{n_0+1}(\eta_{n_0+1} - \eta) + ... + \xi_n(\eta_n - \eta)}{P_n}\right\|_{C_2} \\
&\leq \left\|\frac{\xi_1}{P_n}\right\|_{C_2}\|\eta_1 - \eta\|_{C_2} + \left\|\frac{\xi_2}{P_n}\right\|_{C_2}\|\eta_2 - \eta\|_{C_2} + ... + \left\|\frac{\xi_{n_0}}{P_n}\right\|_{C_2}\|\eta_{n_0} - \eta\|_{C_2} + \\
&\left\|\frac{\xi_{n_0+1}}{P_n}\right\|_{C_2}\|\eta_{n_0+1} - \eta\|_{C_2} + ... + \left\|\frac{\xi_n}{P_n}\right\|_{C_2}\|\eta_n - \eta\|_{C_2} \\
&\leq \left(\left\|\frac{\xi_1}{P_n}\right\|_{C_2} + ... + \left\|\frac{\xi_{n_0}}{P_n}\right\|_{C_2}\right)M + \left(\left\|\frac{\xi_{n-n_0}}{P_n}\right\|_{C_2} + ... + \left\|\frac{\xi_1}{P_n}\right\|_{C_2}\right)\frac{\varepsilon}{2} \\
&< \frac{\varepsilon}{2n_0M}n_0M + \frac{\varepsilon}{2} = \varepsilon.
\end{aligned}$$

□

Lemma 4.1 *The Riesz mean (R, ξ_n) is regular if and only if (R, μ_{1n}) and (R, μ_{2n}) are both regular.*

Remark 4.2 On considering $\xi_n = e_1$ or e_2, for all $n \in \mathbb{N}$, one can get the Cesàro mean a particular case of the Nörlund mean as well as Riesz mean.

5. Conclusion

In this article we have studied Nörlund and Riesz mean for seqences of bi-complex numbers.. This is the first article on this topic and it is expected that it will attract researcher for further investigation and applications.

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