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Enhancement of modified Logarithmic Product Cum Ratio type estimator of population coefficient of variation

Fazal Hassan, Sohail Akhtar, Aqsa Ali, Zanib Shabir, Khan Bahadar, Muhammad Awais

ABSTRACT: Estimating population parameters has been a problematic component of a sample survey for a particular time, and many attempts have been made to enhance the precision of the parameters of these estimators. The Taylor series approach is used to calculate the suggested estimator mean squared error. We calculated the MSEs of recommended and competing estimators using R programming. The result of this study, a numerical investigation, and figures showed that the suggested estimator outperformed the existing estimators

Key Words: Population coefficient of variation, auxiliary data, Logarithmic-Product-Cum-Ratio-type estimator, MSE (Mean squared error), PRE (Percentage relative efficiency).

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1. Introduction

The coefficient of Variation (C.V.) is a measure of dispersion without regard to units. It is, therefore, often employed in several scientific and social inquiries. C.V. has received much attention in models of infinite populations but has yet to receive more attention in those of finite populations. Numerous uses of C.V. involve finite populations, such as their usage in official statistics and World Bank economic surveys.

The additional statistical data connected to the research variable is referred to in the statistical literature as auxiliary (or supplemental) information. Reports created from records held at service delivery centres, surveys, and data acquired via the recording of actual evidence are examples of auxiliary data. It separates effective sampling tactics from those not, regardless of the data type produced. A common practice in sampling procedures is the use of supplementary data. It has mainly been applied by Watson [1] and Cochran [2] to create a valuable class of estimators. The use of auxiliary data in various applications has recently been the subject of several significant papers that have recently been published by Zaman and Bulut [3] and Shahzad [4].

Several studies illustrate the use of auxiliary information in estimation, including Singh [5], Singh [6], Khoshnevisan [7], Patel [8], Singh and Kumar [9], Malik and Singh [10], and Singh [11]. Over time, the estimation of the population means and variances have been the subject of extensive research by several authors. Still, the estimation of the population coefficient of variation requires more attention. In 1992–93, Das and Tripathi [12] introduced an estimator for the coefficient of variation when samples were selected using the simple random sampling without replacement (SRSWOR) method. (Breunig [13] proposed an almost unbiased estimator of the coefficient of variation. In a normal distribution, Patel and Shah [8] and Mahmoundvand and Hassani [14] proposed an estimate of the population CV

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based on a roughly unbiased estimator. The confidence intervals for C.V. can also be estimated using this estimator and its variance. Panichkitkosolkul [15] Researched this area and recommended better confidence intervals for the C.V. (Sisodia and Dwivedi [16] proposed a modified ratio estimator based on the auxiliary variable coefficient of variation.

2. Nomenclature

The notation will be circulated throughout the paper as described below:
$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$: The sample variance of the studied variable y,
$s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$: The sample variance of the auxiliary variable x,
$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$: The sample covariance of the Y and X.
$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$: Sample mean of the$\bar{x}$,
$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$: sample mean of the$\bar{y}$,
$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(x_i - \bar{X}\right)^2$: Population variance of the auxiliary variate x,
$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2$: Population variance of the study variate y,
$S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)\left(Y_i - \bar{Y}\right)$: Population covariance of the Y and X,
$\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$: Sample mean of$\bar{X}$,
$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$: Sample mean of$\bar{Y}$,
MSE (.): mean square error of the estimator
PRE=$\frac{MSE(t_0)}{MSE(t_p)}\times 100$: Percentage relative efficiency of the estimator $t_p$overt_0.
Now let us define

$$E(e_0) = E(e_1) = E(e_2) = E(e_3) = 0,$$
$$E\left(e_0^2\right) = \gamma C_y^2, E\left(e_1^2\right) = \gamma C_x^2, E\left(e_2^2\right) = \gamma(\lambda_{40} - 1), E\left(e_3^2\right) = \gamma(\lambda_{04} - 1),$$
$$E(e_0 e_1) = \gamma\rho C_y C_x, E(e_0 e_2) = \gamma C_y \lambda_{30}, E(e_0 e_3) = \gamma C_y \lambda_{12},$$
$$E(e_1 e_2) = \gamma C_x \lambda_{21}, E(e_1 e_3) = \gamma C_x \lambda_{03}, E(e_2 e_3) = \gamma(\lambda_{22} - 1),$$
$$\bar{y} = \bar{Y}(1 + e_0), \bar{x} = \bar{X}(1 + e_1), s_y = S_y(1 + e_2)^{1/2},$$
$$s_x = S_x(1 + e_3)^{1/2}, s_y^2 = S_y^2(1 + e_2), s_x^2 = S_x^2(1 + e_3)$$

This sample proportion is the population coefficient of variation for the study variable Y and auxiliary variable X. It Also denotes the correlation coefficient between X and Y.

3. Some Existing Estimators in Literature

The standard approach to estimate the population coefficient of variation using auxiliary variable information is through an unbiased estimator, which can be expressed as:

$$t_0 = \hat{C}_y = \frac{s_y}{\bar{y}} \tag{3.1}$$

The MSE is given in (3.2)

$$MSE(t_0) = C_y^2\gamma\left(C_y^2 + \frac{1}{4}(\lambda_{40} - 1) - C_y\lambda_{30}\right) \tag{3.2}$$

Archana & Rao [18] introduced ratio estimators for the population coefficient of variation, utilising data on the sample mean, the population mean, sample variance, and population variance of the auxiliary variable, as described in equations (3.3) and (3.4).

$$t_{AR1} = \hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right) \tag{3.3}$$

$$t_{AR2} = \hat{C}_y\left(\frac{S_y^2}{s_y^2}\right) \tag{3.4}$$

The mean square error (MSE) expression of the estimator t_{AR} is given by:

$$MSE(t_{AR1}) = C_y^2\gamma\left(C_y^2+\frac{1}{4}(\lambda_{40}-1)+C_x^2-C_x\lambda_{21}-C_y\lambda_{30}+2\rho C_y C_x\right) \tag{3.5}$$

$$MSE(t_{AR2}) = C_y^2\gamma\left(C_y^2+\frac{1}{4}(\lambda_{40}-1)+(\lambda_{04}-1)-(\lambda_{22}-1)-C_y\lambda_{30}+2C_y\lambda_{12}\right) \tag{3.6}$$

Singh [11]. Based on data on a single auxiliary variable, the population mean, the following estimators for the coefficient of variation are available:

$$t_1=\hat{C}_y\left(\frac{\bar{X}}{\bar{x}}\right)^{\alpha} \tag{3.7}$$

$$t_2=\hat{C}_y exp\left\{\beta\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right)\right\} \tag{3.8}$$

$$t_3=\hat{C}_y+d_1\left(\bar{X}-\bar{x}\right) \tag{3.9}$$

The estimators' MSE expressions are provided by,

$$MSE\left(t_1\right)= C_y^2\gamma\left(C_y^2+\frac{1}{4}(\lambda_{40}-1)+\alpha^2 C_x^2-C_y\lambda_{30}+2\alpha\rho C_y C_x-\alpha C_x\lambda_{21}\right) \tag{3.10}$$

$$MSE\left(t_2\right)= C_y^2\gamma\left(C_y^2+\frac{1}{4}(\lambda_{40}-1)+\frac{\beta^2 C_x^2}{4}-C_y\lambda_{30}+\beta\rho C_y C_x-\frac{\beta}{2}C_x\lambda_{21}\right) \tag{3.11}$$

$$MSE\left(t_3\right)=\gamma\left[C_y^2\left(C_y^2-C_y\lambda_{30}+\frac{1}{4}(\lambda_{40}-1)\right)+d_1^2\bar{X}^2C_x^2+2d_1\bar{X}\rho C_y^2C_x-d_1\bar{X}C_yC_x\lambda_{21}\right] \tag{3.12}$$

Where $\alpha=\frac{\lambda_{21}-2\rho C_y}{2C_x}$, $\beta=\frac{\lambda_{21}-2\rho C_y}{C_x}$, $d_1=\frac{C_y\lambda_{21}-2\rho C_y^2}{2\bar{X}C_x}$
Based on information on a single auxiliary variable, population variance, Singh [11] developed the following estimators for the coefficient of variation as

$$t_4=\hat{C}_y\left(\frac{S_x^2}{s_x^2}\right)^{\alpha} \tag{3.13}$$

$$t_5=\hat{C}_y exp\left\{\beta\left(\frac{S_x^2-s_x^2}{S_x^2+s_x^2}\right)\right\} \tag{3.14}$$

$$t_6=\hat{C}_y+d_2\left(S_x^2-s_x^2\right) \tag{3.15}$$

The estimators' MSE expressions are provided by,

$$MSE\left(t_4\right)=\gamma C_y^2\left[C_y^2+\frac{1}{4}(\lambda_{40}-1)+\alpha^2(\lambda_{04}-1)-C_y\lambda_{30}+2\alpha C_y\lambda_{12}-\alpha(\lambda_{22}-1)\right] \tag{3.16}$$

$$MSE\left(t_5\right)=\gamma C_y^2\left[C_y^2+\frac{1}{4}(\lambda_{40}-1)+\frac{\beta^2(\lambda_{04}-1)}{4}-C_y\lambda_{30}+\beta C_y\lambda_{12}-\frac{\beta(\lambda_{22}-1)}{2}\right] \tag{3.17}$$

$$MSE\left(t_6\right)=\gamma\left[\begin{array}{l} C_y^2\left(C_y^2-C_y\lambda_{30}+\frac{1}{4}(\lambda_{40}-1)\right)+2C_y^2d_2S_x^2\lambda_{12}+d_2^2S_x^4(\lambda_{04}-1)- \\ C_yd_2S_x^2(\lambda_{22}-1)\end{array}\right], \tag{3.18}$$

where $\alpha=\frac{(\lambda_{22}-1)-2C_y\lambda_{12}}{2(\lambda_{04}-1)}$,$\beta=\frac{(\lambda_{22}-1)-2C_y\lambda_{12}}{(\lambda_{04}-1)}$, $d_2=\frac{C_y(\lambda_{22}-1)-2C_y^2\lambda_{12}}{2S_x^2(\lambda_{04}-1)}$

As described in equation (3.7), Yunusa [18] developed a logarithmic ratio type estimator for calculating the population coefficient of variation using the sample mean, the population mean, sample variance, and the population variance of the auxiliary variable.

$$t_y = \hat{C}_y \left(\frac{Ln\left(S_y^2\right)}{Ln\left(s_y^2\right)} \right) \tag{3.19}$$

The mean square error (MSE) expression of the estimator is given by:

$$MSE(t_y) = C_y^2 \gamma \left(C_y^2 + \frac{1}{4}\left(\lambda_{40} - 1\right) + \frac{\left(\lambda_{04} - 1\right)}{\left(Ln\left(S_x^2\right)\right)^2} - \frac{\left(\lambda_{22} - 1\right)}{Ln\left(S_x^2\right)} - C_y \lambda_{30} + \frac{2C_y \lambda_{12}}{Ln\left(S_x^2\right)} \right) \tag{3.20}$$

Using the logarithmic-product-cum-ratio estimator, Mojeed [19] determined the variance coefficient of a population by using the following estimator:

$$T_{am} = \hat{C}_y \left(\frac{\ln\left(\bar{x}\right)}{\ln\left(\bar{X}\right)} \right) \left(\frac{\ln\left(S_x^2\right)}{\ln\left(s_x^2\right)} \right) \tag{3.21}$$

The estimators' MSE expressions derive from,

$$MSE\left(T_{am}\right) = C_y^2 \gamma \left(\begin{array}{l} \frac{\left(\lambda_{40} - 1\right)}{4} + C_y^2 + \theta_1^2 C_x^2 + \theta_2^2 \left(\lambda_{04} - 1\right) - C_y \lambda_{30} + \theta_1 C_x \lambda_{21} - \theta_2 \left(\lambda_{22} - 1\right) - \\ 2\theta_1 \rho C_y C_x + 2\theta_2 C_y \lambda_{12} - 2\theta_1 \theta_2 C_x \lambda_{03} \end{array} \right) \tag{3.22}$$

4. Proposed Estimator:

In order to calculate the coefficient of variation, we presented the following modified logarithmic-product-cum-ratio type estimator. This approach was motivated by the work of Mojeed [19].

$$T_F = \hat{C}_y \left(\frac{\ln\left(\bar{x}\right)}{\ln\left(\bar{X}\right)} \right) \left(\frac{\ln\left(S_x^2\right)}{\ln\left(s_x^2\right)} \right)^{\varpi} \tag{4.1}$$

The estimator mentioned above is defined based on the suppositions that.

$$E\left(e_0\right) = E\left(e_1\right) = E\left(e_2\right) = E\left(e_3\right) = 0,$$

$$E\left(e_0^2\right) = \gamma C_y^2, E\left(e_1^2\right) = \gamma C_x^2, E\left(e_2^2\right) = \gamma\left(\lambda_{40} - 1\right), E\left(e_3^2\right) = \gamma\left(\lambda_{04} - 1\right),$$
$$E\left(e_0 e_1\right) = \gamma \rho C_y C_x, E\left(e_0 e_2\right) = \gamma C_y \lambda_{30}, E\left(e_0 e_3\right) = \gamma C_y \lambda_{12},$$
$$E\left(e_1 e_2\right) = \gamma C_x \lambda_{21}, E\left(e_1 e_3\right) = \gamma C_x \lambda_{03}, E\left(e_2 e_3\right) = \gamma\left(\lambda_{22} - 1\right),$$

$$\bar{y} = \bar{Y}\left(1 + e_0\right), \bar{x} = \bar{X}\left(1 + e_1\right), s_y = S_y\left(1 + e_2\right)^{1/2},$$
$$s_x = S_x\left(1 + e_3\right)^{1/2}, s_y^2 = S_y^2\left(1 + e_2\right), s_x^2 = S_x^2\left(1 + e_3\right)$$

Using the error terms from section 1 to express (4.1) gives us,

$$T_F = \frac{S_y\left(1 + e_2\right)^{1/2}}{\bar{Y}\left(1 + e_0\right)} \left(\frac{\ln\left(\bar{X}\left(1 + e_1\right)\right)}{\ln\left(\bar{X}\right)} \right) \left(\frac{\ln\left(S_x^2\right)}{+\ln\left(S_x^2\left(1 + e_3\right)\right)} \right)^{\varpi} \tag{4.2}$$

Using the rule of the logarithm to expand (4.2), we arrived at (4.3)

$$T_F = \frac{S_y\left(1 + e_2\right)^{1/2}}{\bar{Y}\left(1 + e_0\right)} \left(\frac{\ln\left(\bar{X}\right) + \ln\left(1 + e_1\right)}{\ln\left(\bar{X}\right)} \right) \left(\frac{\ln\left(S_x^2\right)}{\ln\left(S_x^2\right) + \ln\left(1 + e_3\right)} \right)^{\varpi} \tag{4.3}$$

$$T_F = C_y\left(1+e_2\right)^{1/2}\left(1+e_0\right)^{-1}\left(1+\zeta_1\ln\left(1+e_1\right)\right)\left(1+\zeta_2\ln\left(1+e_3\right)\right)^{-\varpi} \tag{4.4}$$

Where $\zeta_1 = \frac{1}{\ln(\bar{X})}$,$\zeta_2 = \frac{1}{\ln(S_x^2)}$

$$\ln\left(1+e_1\right), \left(1+e_2\right)^{\frac{1}{2}}, \ln\left(1+e_3\right), \left(1+e_0\right)^{-1}$$

Expand to the first order of approximation, giving us,

$$T_F = C_y\left(1+\frac{e_2}{2}-\frac{e_2^2}{8}\right)\left(1-e_0+e_0^2\right)\left(1+\zeta_1\left(e_1-\frac{e_1^2}{2}\right)\right)\left(1-\varpi\zeta_2\left(e_3-\frac{e_3^2}{2}\right)\right)$$

Ignoring the higher order terms.

$$T_F = C_y\left(1-e_0+\frac{e_2}{2}\right)\left(1+\zeta_1 e_1-\varpi\zeta_2 e_3\right)$$

$$T_F = C_y\left(1-e_0+\frac{e_2}{2}+\zeta_1 e_1-\varpi\zeta_2 e_3\right)$$

By simplifying, subtracting from both sides and looking at terms of degree one, we have$T_F - C_y = C_y\left(-e_0+\frac{e_2}{2}+\zeta_1 e_1-\varpi\zeta_2 e_3\right)$
Equation is squared on both sides, and the right-side relation is expanded to the closest approximation.

$$\left(T_F - C_y\right)^2 = \left(C_y\left(\frac{e_2}{2}-e_0+\zeta_1 e_1-\varpi\zeta_2 e_3\right)\right)^2 \tag{4.5}$$

$$\left(T_F - C_y\right)^2 = C_y^2\left(\begin{array}{l}\frac{e_2^2}{4}+e_0^2+\left(\zeta_1 e_1\right)^2+\left(\varpi\zeta_2\right)^2 e_3^2-e_0e_2+\zeta_1e_1e_2-\varpi\zeta_2e_2e_3-2\zeta_1e_0e_1+\\ 2\varpi\zeta_2e_0e_3-2\varpi\zeta_1\zeta_2e_1e_3\end{array}\right) \tag{4.6}$$

To get the MSE of the suggested estimator, take the expectation on both sides of equation (4.6) as follows:

$$MSE\left(T_F\right) = C_y^2\gamma\left(\begin{array}{l}\frac{\left(\lambda_{40}-1\right)}{4}+C_y^2+\zeta_1^2C_x^2+\varpi^2\zeta_2^2\left(\lambda_{04}-1\right)-C_y\lambda_{30}+\zeta_1C_x\lambda_{21}-\\ \varpi\zeta_2\left(\lambda_{22}-1\right)-2\zeta_1\rho C_xC_y+2\varpi\zeta_2C_y\lambda_{12}-2\varpi\zeta_1\zeta_2C_x\lambda_{03}\end{array}\right) \tag{4.7}$$

Differentiate w.r.t, $\varpi$and we obtain (4.8).

$$\varpi = \frac{\left(\lambda_{22}-1\right)-2C_y\lambda_{12}+2\zeta_1C_x\lambda_{03}}{\zeta^2\left(\lambda_{04}-1\right)} \tag{4.8}$$

5. Numerical Analysis

A numerical analysis was done to understand the suggested estimator's accuracy further. Below are descriptions of the population.
Population 1: [Source: Murthy [20], p.399]
In 1963, X represented the area under wheat,
In 1964, Y represented the area under wheat

Table 1: Data Statistics 1

$N = 34$	$n = 15$	$\hat{X} = 208.88$	$\bar{\boldsymbol{Y}}$=199.44	$C_x = 0.72$
$C_y = 0.75$	$\boldsymbol{\rho} = 0.98$	$\boldsymbol{\lambda}_{21} = 1.0045$	$\boldsymbol{\lambda}_{12} = 0.9406$	$\boldsymbol{\lambda}_{40} = 3.6161$
$\boldsymbol{\lambda}_{04} = 2.8266$	$\boldsymbol{\lambda}_{30} = 1.1128$	$\boldsymbol{\lambda}_{03} = 0.9206$	$\boldsymbol{\lambda}_{22} = 3.0133$	

Population 2: [Source: Singh [21], p.1116]
In 1993, X represents the number of fish caught,
In 1995, Y represented the number of fish caught,

Table 2: Data Statistics 2

$N = 69$	$n = 40$	$\hat{X} = 4591.07$	$\bar{\boldsymbol{Y}}$=4514.89	$C_x = 1.38$
$C_y = 1.35$	$\boldsymbol{\rho} = 0.96$	$\boldsymbol{\lambda}_{21} = 2.19$	$\boldsymbol{\lambda}_{12} = 2.30$	$\boldsymbol{\lambda}_{40} = 7.66$
$\boldsymbol{\lambda}_{04} = 9.84$	$\boldsymbol{\lambda}_{30} = 1.11$	$\boldsymbol{\lambda}_{03} = 2.52$	$\boldsymbol{\lambda}_{22} = 8.19$	

According to Table 1 above, the suggested estimator's mean square error is relatively low compared to the estimators considered in this study. Given that the proposed estimator is more excellent, the eater's fairly efficient indicates that the suggested estimator has been improved.

Estimators	**Data set 1**		**Data set 2**	
	M S E	P R E	M S E	P R E
t_0	0.008	100	0.038	100
t_{AR1}	0.026	30.9	0.085	44.7
t_{AR2}	0.034	23.8	0.188	20.2
t_1	0.007	116.5	0.037	102.1
t_2	0.007	116.5	0.037	102.1
t_3	0.007	116.5	0.037	102.1
t_4	0.006	114.9	0.038	101.4
t_5	0.006	114.9	0.038	101.4
t_6	0.006	114.9	0.038	101.4
T_Y	0.007	112.3	0.037	101.4
T_{am}	0.005	141.1	0.035	106.0
T_F	0.004	147.8	0.036	107.1

Figures 1 and 2 display the MSE of various estimators and the PRE of the competing estimators.

Figure 1: MSE for different estimators

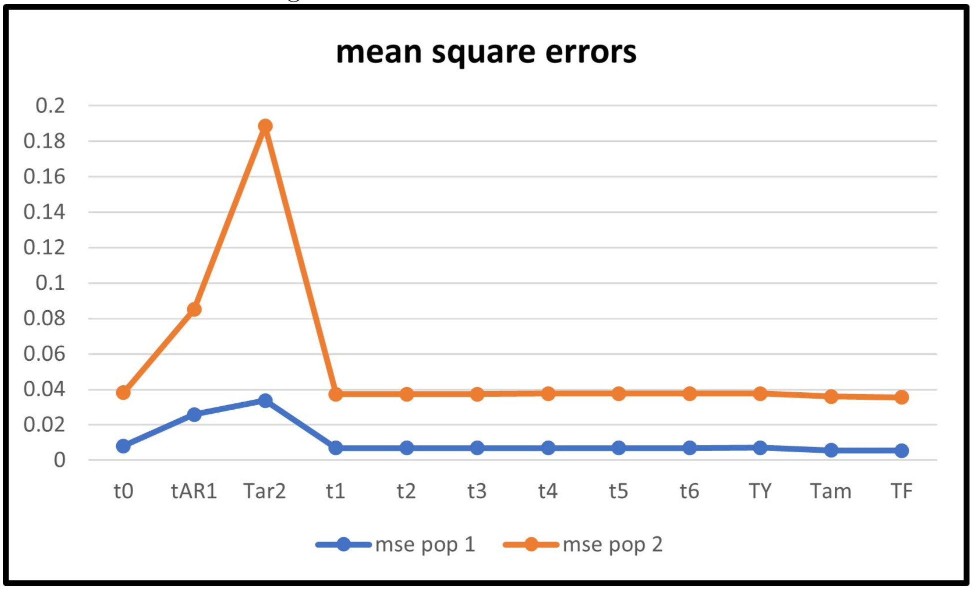
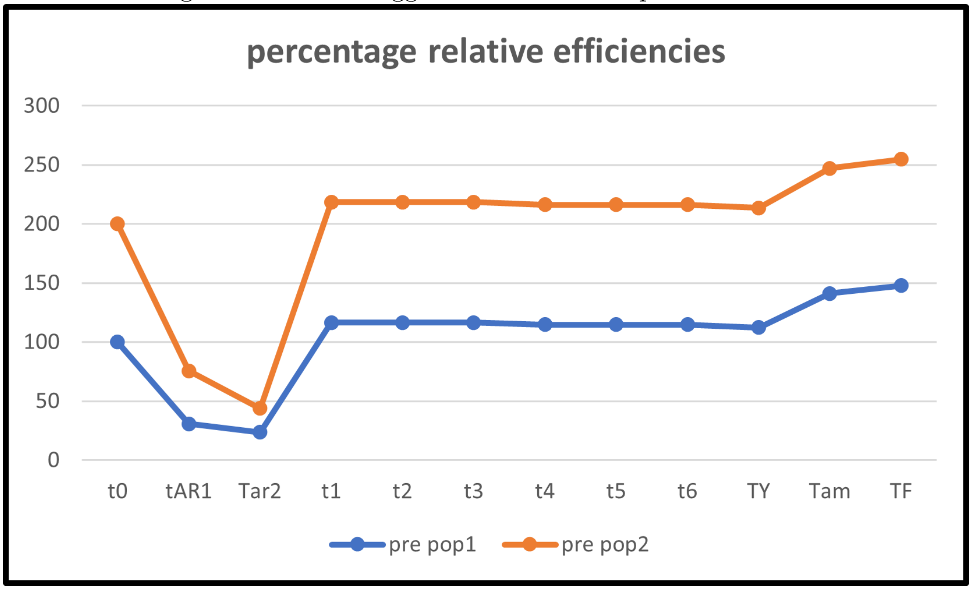


Figure 2: PRE of suggested estimate compared to others



6. Discussion and Conclusion:

We recommend a modified LogarithmicProductCum ratio type estimator for estimating the variability coefficient of the study variable. The natural logarithm of the sample and population means, as well as the variance of the auxiliary variable, were used in this estimation. The numerical analysis shows that the suggested estimator is more effective than other current estimators considered in the study. According to Fig. 1, a modified logarithmic-product-cum ratio estimator provides lower MSE than the other estimators. According to Fig. 2, a modified logarithmic-product-cum ratio estimator provides higher relative efficiency. Therefore, the study has found an estimator that can be used in various commercial decision-making contexts, such as life insurance, car insurance, banking, marketing, etc.

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References

1. Watson, D.J., 1937. The estimation of leaf area in field crops. The Journal of Agricultural Science, 27, pp.474-483.
2. Cochran, W.G., 1940. The estimation of the yields of cereal experiments by sampling for the ratio of grain to total produce.The journal of agricultural science, pp.262-275.
3. Zaman, T. and Bulut, H., 2019. Modified ratio estimators using robust regression methods.Communications in Statistics-Theory and Methods, 48, pp.2039-2048.
4. Shahzad, U., Al-Noor, N.H., Hanif, M. and Sajjad, I., 2021. An exponential family of median-based mean estimators with a simple random sampling scheme.Communications in Statistics-Theory and Methods, 50, pp.4890-4899.
5. Singh, H.P. and Tailor, R., 2005. Estimating finite population mean with known coefficient of variation of an auxiliary character. *Statistica*, pp.301-313.
6. SINGH, R., CHAUHAN, P., SAWAN, N., (2007). A family of estimators for estimating population means using a known correlation coefficient in two-phase sampling. Statistics in Transition, 8, pp. 89–96.
7. KHOSHNEVISAN, M., SINGH, R., CHAUHAN, P., SAWAN, N., SMARANDACHE, F. (2007). A general family of estimators for estimating population means using the known value of some population parameter(s), East Journal of Statistics, 22, pp. 181–191.
8. PATEL, P. A., RINA, S., (2009). A Monte Carlo comparison of some suggested estimators of co-efficient of variation in a finite population, Journal of Statistics sciences, 1, pp. 137–147.

9. SINGH, R., KUMAR, M., (2011). A note on transformations on auxiliary variable in survey sampling. *Mod. Assis. Stat. Appl.*, 6:1, pp. 17–19.

10. MALIK, S., SINGH, R., (2013). An improved estimator using two auxiliary attributes, *Appli. Math. Compt.*, 219, pp. 10983–10986

11. SINGH, R., MISHRA, P., BOUZA, C. N., (2018). Estimation of population mean using information on auxiliary attribute: A review, R.G., DOI: 10.13140/RG.2.2.20477.87524.

12. Das, A.K. and Tripathi, T.P., 1992. Use of auxiliary information in estimating the coefficient of variation. Alig. J. of. Statist, 12, pp.51-58.

13. BREUNIG, R., (2001). An almost unbiased estimator of the coefficient of variation. , 70, pp. 15–19.

14. Mahmoudvand, R. and Hassani, H., 2009. Two new confidence intervals for the coefficient of variation in a normal distribution.Journal of applied statistics, 36, pp.429-442.

15. Panichkitkosolkul, W., 2009. Improved confidence intervals for a coefficient of variation of a normal distribution.Thailand statistician, 7, pp.193-199.

16. SISODIA, B. V. S., DWIVEDI, V. K., (1981). Modified ratio estimator using coefficient of variation of auxiliary variable, *Journal-Indian Society of Agricultural Statistics.*

17. Archana, V. and Rao, A., 2014. Some improved estimators of co-efficient of variation from a bi-variate normal distribution: a Monte *Carlo comparison. Pakistan Journal of Statistics and Operation Research*, pp.87-105.

18. Yunusa, M.A., Audu, A., Musa, N., Beki, D.O., Rashida, A., Bello, A.B. and Hairullahi, M.U., 2021. Logarithmic ratio-type estimator of population coefficient of variation. Asian Journal of Probability and Statistics, 14 pp.13-22.

19. Yunusa, Mojeed Abiodun & Audu, Ahmed & Adejumobi, Awwal. (2023). Logarithmic-Product-Cum-Ratio Type Estimator for Estimating Finite Population Coefficient of Variation Oriental *Journal of Physical Sciences.* Oriental Journal of Physical Sciences. 7. 10.13005/OJPS07.02.05.

20. Murthy, M.N., 1967. Sampling theory and methods. *Sampling theory and methods.*

21. Singh, S., 2003. Advanced Sampling Theory with Applications: How Michael”” Selected” Amy (Vol. 2). *Springer Science & Business Media.*

Fazal Hassan,
Assistant Director ORIC at Iqra National University Swat Campus, Khyber Pakhtunkhwa,
Pakistan.
E-mail address: `Fazalhassan0349@gmail.com`

and

Sohail Akhtar,
Department of Mathematics and Statistics,
The University of Haripur, Khyber Pakhtunkhwa,
Pakistan.
E-mail address: `Akhtar013@gmail.com, S.akhtar@uoh.edu.pk`

and

Aqsa Ali,
Department of Statistics,
Gc University Lahore,
Pakistan.
E-mail address: `Aqsaa924@gmail.com`

and

Zanib Shabir,
Department of Mathematics and Statistics,
The University of Haripur, Khyber Pakhtunkhwa,
Pakistan.
E-mail address: `Khanzani613@gmail.com`

and

Khan Bahadar,
Assistant Professor of Statistics at the Higher Education Department of Khyber Pakhtunkhwa,
Pakistan.
E-mail address: Dr.khanbrave@gmail.com

and

Muhammad Awais,
Lecturer in Statistics,
Govt. Graduate College Civil Lines Sheikhupura,
Pakistan.
E-mail address: Ovaisqarni15@gmail.com