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Modelling Test Matches Outcome Using Bradley Terry Model

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ABSTRACT: In cricket, the ranking of players or teams has a significant impact on determining the top player or team. The goal of this study is to forecast Test Cricket Match Outcomes and then create a team rating based on these predictions. In general, in any sport, the team with the most rating points is regarded as the greatest, and the teams are ranked based on their ratings. The Bradley-Terry model is used to forecast match outcomes using data from January 3rd, 2010 through August 25th, 2020, for a total of 444 matches. The teams of the top ten cricket-playing nations have been ranked using model rating. The rating of the underlying model is also compared to the updated 25th August 2020 official ICC ranking, which yields roughly similar findings. To improve model accuracy, the home ground effect is added, which has a substantial impact on team performance. After the home factor is added as a potential co variate, the model ranking is compared to the ICC ranking, which yields more closed results. Later, when these two results are compared with or without the home ground component, there is a significant improvement between them. To assess team performance, winning probabilities and confidence intervals are produced for each participating nation using model estimate coefficients. Three performance indicators are computed for the top ten playing teams, taking away and home ground factors into account. When these real data were compared, the home ground factor produced more precise results than removing the specified co variate. The outcomes of the suggested approach are also compared to predetermined odds that differ marginally.

Contents

1. Introduction

Cricket is one of the most extensively watched and played sports in the world [1,2,3,4,5]. It is believed to have begun at the beginning of the late 16th century) [6,7,8]. The longest type of cricket match is unlimited over cricket, often known as test cricket [9,10,11]. The game is played in the innings. Each test cricket match has four innings [12,13,14,15]. Each team has the option to bat and bowl twice, alternately. One team bat while the other team fields during an inning. The team that bats first in the first inning is determined by a coin flip conducted on the first day of a Test cricket match between the captains of the two teams and the match referee. The team that wins the toss chooses whether to bat or bowl first [16,17,18]. Based on the rating points determined by the ICC, players and teams are rated. When ranking players for all cricket formats, the players' selection criteria are taken into account [19]. Rating is significant in most sports, whether it is for teams or individual players [20,21,22]. The rating of teams or players is used to choose them for events [21,23]. Several studies have been undertaken in

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order to construct models for rating purposes. In order to create modified Bradley-Terry and Plackett-Luce models and other models that support tied ranks, Baker & Scarf [24] exploited discrete structures of probability distributions, particularly the geometric distribution. Test-match cricket is used to exemplify their suggested methodology, which is introduced for some mathematically tractable and other less tractable distributions. Abbas & Aslam [25] reconciled the qualitative and the quantitative paired comparisons by assigning quantitative weights to treatments having qualitative merits using/extending the Bradley–Terry (BT) model. Behaviours of the existing BT model and the proposed weighted BT model are studied through the test of goodness-of-fit. Experimental and simulated data sets are used for illustration. The moderated paired comparisons approach, which Stern [26] suggested, is based on fitting a penalised likelihood model to the observed margins of victory. Importantly, the model can differentiate between the information contained in the dichotomous win-loss outcome of a game and the information contained in the actual margin of victory thanks to the structure of the punishment function that was chosen. Islam [27] compared the performance of the top 10 cricket teams in one-day international (ODI) matches. They considered the Bradley-Terry model, a widely used pairwise comparison model. They studied the home and away effect to highlight how these teams' home advantages differed. Dewart and Gillard [28] used the Bradley Terry model to anticipate the outcomes of a selected number of test cricket matches and found that their forecasts outperformed bookmaker predictions. They provided individual player ratings and used these ratings to forecast the outcomes of several recent matches. Their findings were highly correlated with those of the ICC. Aisha [29] applied the Bradley-Terry model to Tennis data to predict match results. Model coefficients are estimated using 3439 matches from January 2019 to September 2020. They successfully compared model rankings to ATP rankings.

2. Materials and Methods

The goal of the study is to predict how two rival teams will fare in their test matches. To determine a ranking of teams for the future, the relevant data was examined using the Bradley-Terry modelling framework and the BradleyTerry2 package in R. The Bradley-Terry model is used when we have to do a comparison of entities in pairs. In sports tournaments, this method is mainly known for calculating probabilities. The model was suggested by Bradley and Terry in 1952 and it is applied when individuals are to be compared in pairs with one another. In mathematical form it can be written as:

$$P(i \text{ beats } j) \;=\; \frac{\theta_i}{\theta_i \;+\; \theta_j} \tag{i}$$

In the above model, θ_i is a positive parameter assigned to each entity, representing the skill of entities. As an example, let's consider the individuals i and j to be the cricket teams, then θ_i and θ_j represent team i's and team j's overall skills. In the case, when$\lambda_i = \lambda_j$, then team i's winning probability becomes 0.5 and this probability becomes greater than 0.5 when$\lambda_i \;>\; \lambda_j$. The estimated winning probability obtained from the model can also be calculated using the following expression

$$\hat{p}_{ij} = \frac{exp(\hat{\lambda}_i - \hat{\lambda}_j)}{1 \;+\; exp(\hat{\lambda}_i - \hat{\lambda}_j)} \tag{ii}$$

3. Home advantage

In pairwise comparisons, the order of presenting items may cause bias. For example, while doing taste evaluations in pairs, the item which is tasted first gets advantage. If we take the example of sports, particularly in cricket, team playing at home gets advantage, and it is considered as home advantage effect. It is assumed that the performance of team playing at home performs better as compared to the team playing away.

The pairwise comparison modeled with "home-field advantage" using the Bradley-Terry model is given below

$$\Pr(i \text{ beats } j) \;= \begin{cases} \frac{\delta\theta_i}{\delta\theta_i + \theta_j} \text{ if } i \text{ is at home,} \\ \frac{\theta_i}{\theta_i + \delta\theta_j} \text{ if } j \text{ is at home.} \end{cases} \tag{iii}$$

The parameterδwhich is a positive number, is used to measure how strong the home-field advantage is. If $\delta>$ 1 then it means home-field advantage exists, if δ=1, then it implies that there is no home field advantage and if $\delta < 1$ then it means that home-field is unfavorable. Model 3.3 assumes that all the teams to be compared have a common home-field advantage effect but in fact this home-field advantage effect could be different among teams.

4. Classification accuracy

As the name suggests, classification accuracy is what we mean by the term accuracy. This scoring rule is the percentage of correct predictions. It can be applied in the case when each class contains equal samples. This measure of performance is great and easy to compute but is not good in representing anything regarding quality of the prediction. It can be computed using the formula given below:

$$Accuracy \ = \ \frac{No. \ of \ correctly \ made \ predictions}{Total \ predictions}$$

5. Average probability

Average probability for team winning a match can be computed by dividing the sum of the winning probabilities by total number of matches played and mathematically it can be represented as:

$$m_2 = \ \frac{1}{M} \sum_{i=1}^{M} p_{ij}^{n} \tag{iv}$$

When team i and team j play against each other,p_{ij}^{n} is the probability of wining of a team in n^{th} match and M represents total number of matches.

6. Results and Discussion

For the current study, data on test cricket matches have been obtained from ESPN Cricinfo. The data collection contains 444 test matches from January 3rd, 2010 to August 25th, 2020. The data set includes information about the home and away teams, the match outcome (1: home, 0: away, 0.5: draw), the match venue, the batting first team (1: home team batted first, 0: away team), and the match start date. Data obtained from ESPN Cricinfo is shown in Table 1

Table 1: Extract of data of test match results taken from ESPN Cricinfo website

Home	**Away**	**Result**	**Ground**	**Bat First**	**Start Date**
Australia	Pakistan	1	Sydney	1	03/1/2010
South Africa	England	1	Johannesburg	1	14/01/2010
Australia	Pakistan	1	Hobart	1	14/01/2010
Bangladesh	India	0	Chittagong	0	17/01/2010
Bangladesh	India	0	Dhaka	1	24/01/2010
India	South Africa	0	Nagpur	0	06/2/2010
India	South Africa	1	Kolkata	0	14/02/2010
New Zealand	Bangladesh	1	Hamilton	1	15/02/2010
Bangladesh	England	0	Chittagong	0	12/3/2010
New Zealand	Australia	0	Wellington	0	19/03/2010
Bangladesh	England	0	Dhaka	1	20/03/2010
New Zealand	Australia	0	Hamilton	0	27/03/2010
England	Bangladesh	1	Lord's	1	27/05/2010

Zimbabwe and Bangladesh are regarded as inferior teams when compared to other test-playing nations. It is also obvious from Table.4.2 that the two teams stated above have played fewer games than other

competing teams and have a worse record than other nations. Bangladesh played a total of 57 games, while Zimbabwe played a total of 27 games, winning 11 and 4 games, respectively.

Table 2: Table 2 Summary of team performances over 444 test matches between 2010 and 2020

Team	**Matches**	**Wins**	**Losses**	**Draws**	**Win (%)**	**Loss (%)**	**Draw (%)**
Australia	113	58	38	17	51.33	33.63	15.04
Bangladesh	57	11	36	10	19.30	63.16	17.54
England	134	62	47	25	46.27	35.07	18.66
India	108	55	31	22	50.93	28.70	20.37
New Zealand	86	34	32	20	39.53	37.21	23.26
Pakistan	87	33	38	16	37.93	43.68	18.39
South Africa	93	45	28	20	48.39	30.11	21.51
Sri Lanka	98	32	40	26	32.65	40.82	26.53
West Indies	85	22	45	18	25.88	52.94	21.18
Zimbabwe	27	4	21	2	14.81	77.78	7.41

We are left with 369 test matches that were played between 8 elite international teams after the weakest teams were removed from the data set. Our research revealed significant standard errors corresponding to the relevant parameter estimations since Bangladesh and Zimbabwe performed poorly. After Bangladesh and Zimbabwe's matches are removed, the results of the teams are shown in Table 4.3.

Table 3: Summary of team performances over 369 test matches excluding matches involving Bangladesh and Zimbabwe

Team	**Matches**	**Wins**	**Losses**	**Draws**	**Win (%)**	**Loss (%)**	**Draw (%)**
Australia	111	57	37	17	51.35	33.33	15.32
England	128	57	46	25	44.53	35.94	19.53
India	102	50	31	21	49.02	30.39	20.59
New Zealand	75	25	32	18	33.33	42.67	24.00
Pakistan	79	27	37	15	34.18	46.84	18.99
South Africa	87	41	28	18	47.13	32.18	20.69
Sri Lanka	85	24	39	22	28.24	45.88	25.88
West Indies	71	12	43	16	16.90	60.56	22.54

7. Ranking of Test Match Teams

The International Cricket Council's official ranking can be replaced by a new ranking system based on a player's abilities. A comparison of the two ranking systems is thought to be quite interesting. ICC presently ranks the teams with David Kendix's methodology. While the Bradley-Terry framework is used to rank test match teams in order to estimate the model parameters, data for the time period from 3 January 2010 to 25 August 2020 is employed. In Table 3, the model's rankings and the ICC rankings are displayed. For the four test match teams, both rankings—the model rankings and the ICC rankings—are identical. Other teams almost share the same ranking with only a few exceptions, such as South Africa and New Zealand, whose ICC rankings are 6 and 2, respectively, while their rankings in our model are 2 and 5, respectively. The same difference in ranking can be seen when the home factor is taken into account; four teams have identical ranks, several have little differences, and South Africa and New Zealand have a big gap in rating.

Table 4: Comparison of ranking from ICC and Bradley-Terry model. Using data from 3rd January 2010 and 25th August 2020

Teams	Estimate	Std. Error	Model Ranking	ICC Ranking
India	1.6898	0.3726	1	3
South Africa	1.6076	0.3901	2	6
Australia	1.6000	0.3672	3	1
England	1.3837	0.3510	4	4
New Zealand	0.8812	0.3786	5	2
Pakistan	0.7423	0.3749	6	7
Sri Lanka	0.7057	0.3857	7	5
West Indies	0.0000	0.0000	8	8
Bangladesh	-1.1588	0.4963	9	9
Zimbabwe	-1.9203	0.6804	10	10

These inconsistencies result from the fact that the ICC ranking system ranks teams based on how frequently they have competed in matches rather than their overall performance. The algorithm utilised in this study, however, ranks teams based on historical performance rather than on the number of matches a club has played.

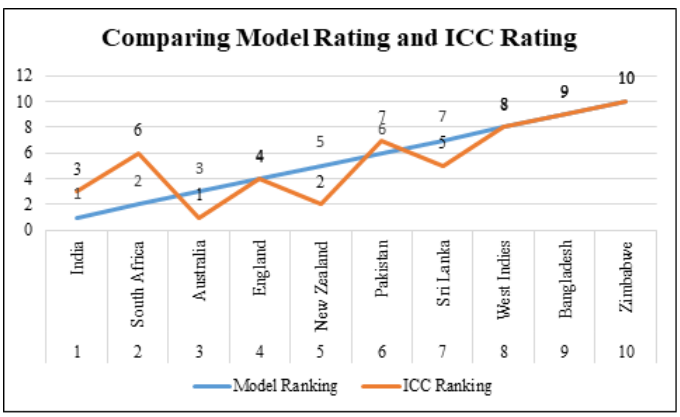

Figure 1

Table 5: Probabilities of victory for teams

	SA	NZ	WI	B	E	SL	A	I	Z	P
SA		0.674	0.833	0.941	0.556	0.711	0.502	0.480	0.972	0.704
NZ	0.326		0.707	0.885	0.377	0.544	0.328	0.308	0.943	0.535
WI	0.167	0.293		0.761	0.200	0.331	0.168	0.156	0.872	0.323
B	0.059	0.115	0.239		0.073	0.134	0.060	0.055	0.682	0.130
E	0.444	0.623	0.800	0.927		0.663	0.446	0.424	0.965	0.655
SL	0.289	0.456	0.670	0.866	0.337		0.290	0.272	0.933	0.491
A	0.498	0.672	0.832	0.940	0.554	0.710		0.478	0.971	0.702
I	0.521	0.692	0.844	0.945	0.576	0.728	0.522		0.974	0.721
Z	0.029	0.057	0.128	0.318	0.035	0.068	0.029	0.026		0.065
P	0.296	0.465	0.678	0.870	0.345	0.509	0.298	0.279	0.935	

In Table 6 model fit is shown using data from 3 January 2010 to 25 August 2020, with the West Indies used as a reference category and the home factor taken into account. Log-probability ratios are used to

present estimates.

Table 6: Bradley-Terry model fit including Home Factor

Variable	**Estimate**	**Std. Error**	**p**
India	2.1854	0.5118	< 0.0001
Australia	1.9335	0.4385	< 0.0343
South Africa	1.523	0.3982	< 0.0001
England	1.5089	0.3624	< 0.0428
Pakistan	1.2407	0.5117	< 0.0000
Sri Lanka	0.9404	0.4208	< 0.0254
New Zealand	0.8242	0.3893	<0.0000
Bangladesh	-1.029	0.508	< 0.0474
Zimbabwe	-1.4665	0.7398	< 0.0153
Home	0.3759	0.2557	0.1416

Table 7: Comparison of ranking from ICC and Bradley-Terry model after including home factor

Teams	**Estimate**	**Std. Error**	**Bradley-Terry Ranking**	**ICC Ranking**
India	2.1854	0.5118	1	3
Australia	1.9335	0.4385	2	1
South Africa	1.5230	0.3982	3	6
England	1.5089	0.3624	4	4
Pakistan	1.2407	0.5117	5	7
Sri Lanka	0.9404	0.4208	6	5
New Zealand	0.8242	0.3893	7	2
West Indies	0.0000	0.0000	8	8
Bangladesh	-1.0290	0.5080	9	9
Zimbabwe	-1.4665	0.7398	10	10

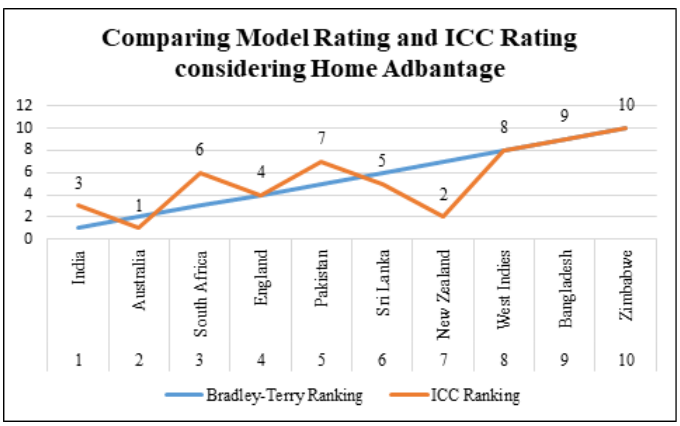

Figure 2

Improved model fit for comparison of ranking from ICC and Bradley-Terry model is provide in Table 4.7

after including home factor as potential covariate. Model is fitted using data from 3^{rd} January 2010 and 25^{th} August 2020.

Table 8: Estimated Abilities of Top 10 ranking teams by considering Home Factor

Teams	**Abilities**	**S.E**
South Africa	1.5230	0.3982
New Zealand	0.8242	0.3893
West Indies	0.0000	0.0000
Bangladesh	-1.0290	0.5080
England	1.5089	0.3624
Sri Lanka	0.9404	0.4208
Australia	1.9335	0.4385
India	2.1854	0.5118
Zimbabwe	-1.4665	0.7398
Pakistan	1.2407	0.5117

Table 9: Probabilities of victory for the teams including home factor

	SA	**NZ**	**WI**	**B**	**E**	**SL**	**A**	**I**	**Z**	**P**
SA		0.668	0.821	0.928	0.504	0.642	0.399	0.340	0.952	0.570
NZ	0.332		0.695	0.865	0.335	0.471	0.248	0.204	0.908	0.397
WI	0.179	0.305		0.737	0.181	0.281	0.126	0.101	0.813	0.224
B	0.072	0.136	0.263		0.073	0.123	0.049	0.039	0.608	0.094
E	0.497	0.665	0.819	0.927		0.638	0.395	0.337	0.952	0.567
SL	0.358	0.529	0.719	0.878	0.362		0.270	0.224	0.917	0.426
A	0.601	0.752	0.874	0.951	0.605	0.730		0.437	0.968	0.667
I	0.660	0.796	0.899	0.961	0.663	0.776	0.563		0.975	0.720
Z	0.048	0.092	0.188	0.392	0.049	0.083	0.032	0.025		0.063
P	0.430	0.603	0.776	0.906	0.433	0.575	0.333	0.280	0.938	

Probabilities of victory of the teams in the left-hand column against teams on top row. NZ: New Zealand, WI: West Indies, B: Bangladesh, E: England, SL: Sri Lanka, A: Australia, I: India, Z: Zimbabwe, P: Pakistan. (Including home factor

Three measures of performance are presented in Table 4.11 below, as discussed in section 3.4. According to the results of all performance measures used, for all data under study model gives good fit.

Table 10: Measures of predictive performance for fitted model

	Model	
	Away Factor	Home Factor
Classification Accuracy m_1	67.42%	68.61%
Average Probability m_2	0.5955	0.5969
Average Log Probability m_3	-0.5871	-0.5850

8. Conclusion

In this study, the Bradley-Terry model has been introduced to develop an alternate method of predicting rating that is used by the ICC rating system (which may be kept confidential) of underlying teams. Our objective was to improve the rating system that is adopted and applied by ICC for teams and to propose a new rating system that will resolve all issues faced in this system. Rating in general is very beneficial in all aspects of life. Which can be used to evaluate more relevant factors related to team match. In

this thesis, Bradley-Terry Model was mainly used to develop an alternate team rating system for Top 10 ranking teams that is compared with standard international ICC rating system. By applying Spearman Rank Order (rho) Correlation between proposed model ranking and ICC ranking, interdependence has been checked between them, which is strongly correlated.
In chapter IV after fitting the model, through different criterions and graphical representation of under consideration data set gathered from ESPNcricinfo, the model is being utilized for further analysis and it was observed that our obtained results were close to predetermined odds. Each team may get rewards in the form of ranking for their performance according to some suitable criterions that are part of test match outcome. To apply Bradley Terry approach and achieve the objectives of research historical data of test matches for the period 3^{rd} January 2010 to 25^{th} August 2020, was used. Using coefficients of applied model, ratings of teams were obtained for home factor and also without considering home factor. Wining probabilities of players were then computed using the obtained ratings. Model adequacy was checked using three measures of performance. Using measures of performance it was concluded that model has shown favourable results for that data in which home factor is considered.
In this study Bradley Terry model has been applied on historical match results to obtain forecasts, and prediction using historical match results is considered more accurate because official ranking doesn't contain the complete information which we need for forecasting the outcomes accurately.
Only 444 test matches from 3^{rd} January 2010 to 25^{th} August 2020 have been used in the current study for estimating the parameters and fitting the model. To improve the accuracy of model predictions; data set can be increased by considering more years, data of T20 and One Day International can also be added in the fitting the model and estimation of parameters.
Although our model has considered multiple parameters and also home effect, some more factors can also be included in the analysis to further validate the results and to enhance accuracy, i.e., bat first, bat last, winning toss, players' individual performance, punters, bowling rate, batting rate, playing strategy and pitch smoothness etc.

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