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Strong Forms of Weakly e-continuity *

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ABSTRACT: The main purpose of this study is to introduce and study two new classes of continuity called eR-continuous functions and weakly eR-continuous functions via e-regular sets. Both forms of continuous functions we have described are stronger than the weakly e-continuity. Furthermore, we obtain various characterizations of weakly eR-continuous functions. In addition, we examine not only the relations of these functions with some other forms of existing continuous functions, but also some of their fundamental properties.

Key Words: e-regular set, e-connectedness, eR-continuity, weakly eR-continuity.

Contents

1. Introduction

There is no doubt that one of the fundamental concepts of general topology is different forms of the open sets. The discussion about e-open set types, one of the generalized open set concepts, is still a rich field to study in terms of general topology. Some forms of this concept such as $ge\Lambda$-closed sets [2], generalized e-closed sets [3], πge-closed sets [4], Λ_e-sets and V_e-sets [20] have been investigated in recent years. Apart from these, some forms of e-continuity and e-openness of functions have been studied in [21,22] as well.

In recent years, many authors have studied on generalizations of strong continuity such as strongly θ-continuous functions [16], strongly θ-precontinuous functions [17], strongly θ-semi continuous functions [12], strongly θ-b-continuous functions [24], strongly θ-e-continuous functions [18]. On the other hand, many researchers have introduced and investigated some properties of the weakly clopen functions [25] and weakly e-continuous functions [19].

In this paper, we investigate different classes of continuity called eR-continuous functions, weakly eR-continuous functions and strongly-θ-e-continuous functions and study some of their fundamental properties. So, it turns out that weakly eR-continuous functions are weaker than strongly θ-e-continuous functions, weakly clopen functions and eR-continuous functions and also stronger than weakly e-continuous functions.

2. Preliminaries

Throughout this paper, X and Y represent topological spaces. For a subset A of a space X, $cl(A)$ and $int(A)$ denote the closure of A and the interior of A, respectively. The family of every closed (resp. open, clopen) sets of X is denoted by $C(X)$ (resp. $O(X)$, $CO(X)$). A subset A is called regular open [26] (resp. regular closed [26]) if $A = int(cl(A))$ (resp. $A = cl(int(A))$). A point $x \in X$ is called δ-cluster point [28] of A if $int(cl(U)) \cap A \neq \emptyset$ for every open neighborhood U of x. The set of all δ-cluster points of

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A is called the δ-closure [28] of A and is denoted by $cl_\delta(A)$. If $A = cl_\delta(A)$, then A is called δ-closed [28] and the complement of a δ-closed set is called δ-open [28]. The set $\{x|(\exists U \in O(X,x))(int(cl(U)) \subseteq A)\}$ is called the δ-interior of A and is denoted by $int_\delta(A)$.

A subset A of a space X is called semiopen [14] (resp. preopen [15], b-open [1], e-open [11], a-open [10]) if $A \subseteq cl(int(A))$ (resp. $A \subseteq int(cl(A))$, $A \subseteq cl(int(A)) \cup int(cl(A))$, $A \subseteq cl(int_\delta(A)) \cup int(cl_\delta(A))$, $A \subseteq int(cl(int_\delta(A)))$. The complement of a semiopen (resp. preopen, b-open, e-open, a-open) set is called semiclosed [14] (resp. preclosed [15], b-closed [1], e-closed [11], a-closed [10]). The intersection of all semiclosed (resp. preclosed, b-closed, e-closed, a-closed) sets of X containing A is called the semi-closure [14] (resp. pre-closure [15], b-closure [1], e-closure [11], a-closure [10]) of A and is denoted by $scl(A)$ (resp. $pcl(A)$, $bcl(A)$, e-$cl(A)$, a-$cl(A)$). The union of every semiopen (resp. preopen, b-open, e-open, a-open) sets of X contained in A is called the semi-interior [14] (resp. pre-interior [15], b-interior [1], e-interior [11], a-interior [10]) of A and is denoted by $sint(A)$ (resp. $pint(A)$, $bint(A)$, e-$int(A)$, a-$int(A)$).

A point x of X is said to be θ-cluster (e-θ-cluster) point of A if $cl(U) \cap A \neq \emptyset$ (e-$cl(U) \cap A \neq \emptyset$) for all open ($e$-open) set U containing x. The set of all θ-cluster (e-θ-cluster) points of A is called the θ-closure [28] (e-θ-closure [18]) of A and is denoted by $cl_\theta(A)$ (e-$cl_\theta(A)$). A subset A is called to be θ-closed (e-θ-closed) if $A = cl_\theta(A)$ ($A = e$-$cl_\theta(A)$). The complement of a θ-closed (e-θ-closed) set is called a θ-open [28] (e-θ-open [18]). A point x of X called to be a θ-interior [28] (e-θ-interior [18]) point of a subset A, denoted by $int_\theta(A)$ (e-$int_\theta(A)$), if there exists an open (e-open) set U of X containing x such that $cl(U) \subseteq A$ (e-$cl(U) \subseteq A$).

A subset A is called e-regular [18] if it is e-open and e-closed. Also, it is noted in [13] that

$$e\text{-regular} \Rightarrow e\text{-}\theta\text{-open} \Rightarrow e\text{-open}.$$

The family of every e-θ-open (resp. e-θ-closed, e-regular, regular open, regular closed, δ-open, δ-closed, θ-open, θ-closed, semiopen, semiclosed, preopen, preclosed, b-open, b-closed, e-open, e-closed, a-open, a-closed) subsets of X is denoted by $e\theta O(X)$ (resp. $e\theta C(X)$, $eR(X)$, $RO(X)$, $RC(X)$, $\delta O(X)$, $\delta C(X)$, $\theta O(X)$, $\theta C(X)$, $SO(X)$, $SC(X)$, $PO(X)$, $PC(X)$, $BO(X)$, $BC(X)$, $eO(X)$, $eC(X)$, $aO(X)$, $aC(X)$). The family of every open (resp. closed, e-θ-open, e-θ-closed, e-regular, regular open, regular closed, δ-open, δ-closed, θ-open, θ-closed, semiopen, semiclosed, preopen, preclosed, b-open, b-closed, e-open, e-closed, a-open, a-closed) sets of X containing a point x of X is denoted by $O(X,x)$ (resp. $C(X,x)$, $e\theta O(X,x)$, $e\theta C(X,x)$, $eR(X,x)$, $RO(X,x)$, $RC(X,x)$, $\delta O(X,x)$, $\delta C(X,x)$, $\theta O(X,x)$, $\theta C(X,x)$, $SO(X,x)$, $SC(X,x)$, $PO(X,x)$, $PC(X,x)$, $BO(X,x)$, $BC(X,x)$, $eO(X,x)$, $eC(X,x)$, $aO(X,x)$, $aC(X,x)$).

We shall use the well-known accepted language almost in the whole of the proofs of the theorems in this article. The following fundamental properties of e-θ-closure are useful in the sequel:

Lemma 2.1 *[18]; [13] Let A and B be subsets of a space X. Then the followings are hold:*
(1) $A \subseteq e\text{-}cl(A) \subseteq e\text{-}cl_\theta(A)$,
(2) $e\text{-}cl_\theta(A)$ is e-θ-closed,
(3) If A is e-θ-closed, then $A = e\text{-}cl_\theta(A)$,
(4) If $A \subseteq B$, then $e\text{-}cl_\theta(A) \subseteq e\text{-}cl_\theta(B)$,
(5) $e\text{-}cl_\theta(e\text{-}cl_\theta(A)) = e\text{-}cl_\theta(A)$,
(6) $e\text{-}cl_\theta(X \setminus A) = X \setminus e\text{-}int_\theta(A)$,
(7) $x \in e\text{-}cl_\theta(A)$ iff $A \cap U \neq \emptyset$ for all $eR(X,x)$,
(8) $e\text{-}cl_\theta(A) = \bigcap\{V|(A \subseteq V)(V \in eR(X))\} = \bigcap\{V|(A \subseteq V)(V \in e\theta C(X))\}$,
(9) $A \in e\theta O(X)$ iff for all $x \in A$ there exists $U \in eR(X,x)$ such that $U \subseteq A$,
(10) Any intersection (union) of e-θ-closed (e-θ-open) sets is e-θ-closed (e-θ-open).

Lemma 2.2 *[6] Let A be a subset of a space X. If A is an open set in X, then $cl(A) = cl_\theta(A)$.*

Definition 2.1 *A function $f : X \to Y$ is called to be:*
(a) e-continuous (briefly e.c.) [11] if the inverse image of each open set in Y is e-open in X.
(b) strongly θ-continuous (briefly, st.θ.c.) [16] if for all $x \in X$ and all open set V of Y containing $f(x)$, there exists $U \in O(X,x)$ such that $f[cl(U)] \subseteq V$.
(c) strongly θ-semicontinuous (briefly, st.θ.s.c.) [12] if for all $x \in X$ and all open set V of Y containing

$f(x)$, there exists $U \in SO(X,x)$ such that $f[scl(U)] \subseteq V$.
(d) strongly θ-precontinuous (briefly, st.θ.p.c.) [17] if for all $x \in X$ and all open set V of Y containing $f(x)$, there exists $U \in PO(X,x)$ such that $f[pcl(U)] \subseteq V$.
(e) strongly θ-b-continuous (briefly, st.θ.b.c.) [24] if for all $x \in X$ and all open set V of Y containing $f(x)$, there exists $U \in BO(X,x)$ such that $f[bcl(U)] \subseteq V$.
(f) strongly θ-e-continuous (briefly, st.θ.e.c.) [18] if for all $x \in X$ and all open set V of Y containing $f(x)$, there exists $U \in eO(X,x)$ such that $f[e\text{-}cl(U)] \subseteq V$.
(g) weakly clopen (briefly, w.co.) [25] if for all $x \in X$ and all open set V of Y containing $f(x)$, there exists $U \in CO(X,x)$ such that $f[U] \subseteq cl(V)$.
(h) weakly b-continuous (briefly w.b.c.) [27] if for all $x \in X$ and all open set V of Y containing $f(x)$, there exists $U \in BO(X,x)$ such that $f[U] \subseteq cl(V)$.
(i) weakly a-continuous (briefly w.a.c.) [5] if for all $x \in X$ and all open set V of Y containing $f(x)$, there exists $U \in aO(X,x)$ such that $f[U] \subseteq cl(V)$.
(j) weakly e-continuous (briefly w.e.c.) [19] if for all $x \in X$ and all open set V of Y containing $f(x)$, there exists $U \in eO(X,x)$ such that $f[U] \subseteq cl(V)$.
(k) weakly BR-continuous (briefly, w.BR.c.) [8] if for all $x \in X$ and all open set V of Y containing $f(x)$, there exists $U \in BR(X,x)$ such that $f[U] \subseteq cl(V)$.
(l) BR-continuous (briefly, BR.c.) [8] if $f^{-1}[V]$ is b-regular in X for all open set V of Y.

3. eR-continuity and Weakly eR-continuity

Definition 3.1 *A function $f : X \to Y$ is called weakly eR-continuous (briefly w.eR.c.) at $x \in X$ if for each open set V containing $f(x)$, there exists an e-regular set U in X containing x such that $f[U] \subseteq cl(V)$. The function f is w.eR.c. if and only if f is w.eR.c. for all $x \in X$.*

Definition 3.2 *A function $f : X \to Y$ is called eR-continuous (briefly eR.c.) if $f^{-1}(V)$ is e-regular in X for every open set V of Y.*

Theorem 3.1 *Let $f : X \to Y$ be a function. If f is eR-continuous, then f is weakly eR-continuous.*

Proof: Let $x \in X$ and $V \in O(Y, f(x))$.

$$\left.\begin{array}{r}(x \in X)(V \in O(Y,f(x)))\\ f \text{ is } eR.\text{c.}\end{array}\right\} \Rightarrow \left.\begin{array}{r} f^{-1}[V] \in eR(X,x)\\ U := f^{-1}[V]\end{array}\right\} \Rightarrow (U \in eR(X,x))(f[U] \subseteq V \subseteq cl(V)).$$ □

Theorem 3.2 *Let $f : X \to Y$ be a function. If f is eR-continuous, then f is e-continuous.*

Proof: It is obvious from Definitions 2.1(a) and 3.2. □

Remark 3.1 *From Definitions 2.1, 3.1 and 3.2, we have the following diagram. The converses of the below implications are not true in general, as shown in the related articles and following examples.*

$$\begin{array}{ccccccccc}
 & & st.\theta.s.c. & & & & BR.c. & \longrightarrow & w.b.c.\\
 & \nearrow & & \searrow & & & \downarrow & \nearrow & \uparrow\\
st.\theta.c. & & & & st.\theta.b.c. & \longrightarrow & w.BR.c. & \longleftarrow & w.co.\\
 & \searrow & & \nearrow & & & & \swarrow & \downarrow\\
 & & st.\theta.p.c. & \longrightarrow & st.\theta.e.c. & \longrightarrow & w.eR.c. & \longrightarrow & w.e.c.\\
 & & & & & & \uparrow & \nearrow & \uparrow\\
 & & & & & & eR.c. & \longrightarrow & e.c.
\end{array}$$

Example 3.1 Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, X\}$ and $\sigma = \{\emptyset, X, \{b, c, d\}\}$. Then the function $f : (X, \tau) \to (X, \sigma)$ by $f = \{(a, e), (b, b), (c, c), (d, d), (e, a)\}$ is weakly eR-continuous but it is not strongly θ-e-continuous.

Example 3.2 Let $X = \{a, b, c, d, e\}$ and $\tau = \{\emptyset, \{a\}, \{c\}, \{a, c\}, \{c, d\}, \{a, c, d\}, X\}$ and $\sigma = \{\emptyset, X, \{b, c, d\}\}$. Then the function $f : (X, \tau) \to (X, \sigma)$ by $f = \{(a, b), (b, a), (c, c), (d, d), (e, e)\}$ is weakly eR-continuous but it is not eR-continuous.

Example 3.3 Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ and $\sigma = \{\emptyset, X, \{a\}, \{b, c\}, \{a, b, c\}\}$. Then the function $f : (X, \tau) \to (X, \sigma)$ by $f = \{(a, a), (b, c), (c, b), (d, d)\}$ is both e-continuous and weakly e-continuous but it is not eR-continuous.

Question: Is there any weakly e-continuous function which is not weakly eR-continuous?

Theorem 3.3 *For a function $f : X \to Y$, the followings are equivalent:*
(1) f is weakly eR-continuous;
(2) for all $x \in X$ and all open set V of Y containing $f(x)$, there exists an e-θ-open set U in X containing x such that $f[U] \subseteq cl(V)$;
(3) e-$cl_\theta(f^{-1}[U]) \subseteq f^{-1}[cl(U)]$ for all preopen set U of Y;
(4) $f^{-1}[U] \subseteq e$-$int_\theta(f^{-1}[cl(U)])$ for all preopen set U of Y;
(5) e-$cl_\theta(f^{-1}[int(cl(B))]) \subseteq f^{-1}[cl(B)]$ for all subset B of Y;
(6) e-$cl_\theta(f^{-1}[int(F)]) \subseteq f^{-1}[F]$ for all regular closed set F of Y;
(7) e-$cl_\theta(f^{-1}[U]) \subseteq f^{-1}[cl(U)]$ for all open subset U of Y;
(8) $f^{-1}[U] \subseteq e$-$int_\theta(f^{-1}[cl(U)])$ for all open subset U of Y;
(9) $f[e$-$cl_\theta(A)] \subseteq cl_\theta(f[A])$ for all subset A of X;
(10) e-$cl_\theta(f^{-1}[B]) \subseteq f^{-1}[cl_\theta(B)]$ for all subset B of Y;
(11) e-$cl_\theta(f^{-1}[int(cl_\theta(B))]) \subseteq f^{-1}[cl_\theta(B)]$ for all subset B of Y.

Proof: $(1) \Rightarrow (2)$: Let $x \in X$ and $V \in O(Y, f(x))$.

$$\left.\begin{array}{r}(x \in X)(V \in O(Y, f(x))) \\ \text{Hypothesis}\end{array}\right\} \Rightarrow \left.\begin{array}{r}(\exists U \in eR(X, x))(f[U] \subseteq cl(V)) \\ eR(X) \subseteq e\theta O(X)\end{array}\right\} \Rightarrow$$
$$\Rightarrow (\exists U \in e\theta O(X, x))(f[U] \subseteq cl(V)).$$

$(2) \Rightarrow (3)$: Let $x \in X \setminus f^{-1}[cl(G)]$ and $G \in PO(Y)$.

$$\left.\begin{array}{r}x \in X \setminus f^{-1}[cl(G)] \Rightarrow f(x) \in Y \setminus cl(G) \Rightarrow (\exists V \in O(Y, f(x)))(V \cap G = \emptyset) \\ \text{Hypothesis}\end{array}\right\} \Rightarrow$$
$$\Rightarrow (\exists U \in e\theta O(X, x))(f[U] \subseteq cl(V))(V \cap G = \emptyset)$$
$$\Rightarrow (\exists U \in e\theta O(X, x))(f[U] \cap G = \emptyset)$$
$$\Rightarrow (\exists U \in e\theta O(X, x))(U \cap f^{-1}[G] = \emptyset)$$
$$\Rightarrow x \notin e\text{-}cl_\theta(f^{-1}[G])$$
$$\Rightarrow x \in X \setminus e\text{-}cl_\theta(f^{-1}[G]).$$

$(3) \Rightarrow (4)$: Let $G \in PO(Y)$.

$$\left.\begin{array}{r}G \in PO(Y) \Rightarrow Y \setminus cl(G) \in O(Y) \subseteq PO(Y) \\ \text{Hypothesis}\end{array}\right\} \Rightarrow e\text{-}cl_\theta(f^{-1}[Y \setminus cl(G)]) \subseteq f^{-1}[cl(Y \setminus cl(G))]$$
$$\Rightarrow X \setminus e\text{-}int_\theta(f^{-1}[cl(G)]) \subseteq X \setminus f^{-1}[int(cl(G))] \subseteq X \setminus f^{-1}[G]$$
$$\Rightarrow f^{-1}[G] \subseteq e\text{-}int_\theta(f^{-1}[cl(G)]).$$

$(4) \Rightarrow (5)$: Let $B \subseteq Y$.

$$\left.\begin{array}{r}B \subseteq Y \Rightarrow Y \setminus cl(B) \in O(Y) \subseteq PO(Y) \\ \text{Hypothesis}\end{array}\right\} \Rightarrow f^{-1}[Y \setminus cl(B)] \subseteq e\text{-}int_\theta(f^{-1}[cl(Y \setminus cl(B))])$$
$$\Rightarrow X \setminus f^{-1}[cl(B)] \subseteq X \setminus e\text{-}cl_\theta(f^{-1}[int(cl(B))])$$
$$\Rightarrow e\text{-}cl_\theta(f^{-1}[int(cl(B))]) \subseteq f^{-1}[cl(B)].$$

$(5) \Rightarrow (6)$: Let $F \in RC(Y)$.

$$\left.\begin{array}{r}F \in RC(Y) \Rightarrow int(F) \subseteq Y \\ \text{Hypothesis}\end{array}\right\} \Rightarrow$$

$\Rightarrow e\text{-}cl_\theta(f^{-1}[int(F)]) = e\text{-}cl_\theta(f^{-1}[int(cl(int(F)))]) \subseteq f^{-1}[cl(int(F))] = f^{-1}[F]$.

(6) $\Rightarrow$ (7) : Let $U \in O(Y)$.

$$\left.\begin{array}{r} U \in O(Y) \Rightarrow cl(U) \in RC(Y) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow e\text{-}cl_\theta(f^{-1}[U]) \subseteq e\text{-}cl_\theta(f^{-1}[int(cl(U))]) \subseteq f^{-1}[cl(U)].$$

(7) $\Rightarrow$ (8) : Let $U \in O(Y)$.

$$\left.\begin{array}{r} U \in O(Y) \Rightarrow Y \setminus cl(U) \in O(Y) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow e\text{-}cl_\theta(f^{-1}[Y \setminus cl(U)]) \subseteq f^{-1}[cl(Y \setminus cl(U))]$$

$\Rightarrow X \setminus e\text{-}int_\theta(f^{-1}[cl(U)]) \subseteq X \setminus f^{-1}[int(cl(U))] \subseteq X \setminus f^{-1}[U]$
$\Rightarrow f^{-1}[U] \subseteq f^{-1}[int(cl(U))] \subseteq e\text{-}int_\theta(f^{-1}[cl(U)])$.

(8) $\Rightarrow$ (9) : Let $A \subseteq X$.

$$\left.\begin{array}{r} A \subseteq X \Rightarrow int(Y \setminus f[A]) \in O(Y) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow f^{-1}[int(Y \setminus f[A])] \subseteq e\text{-}int_\theta(f^{-1}[cl(int(Y \setminus f[A]))])$$

$\Rightarrow X \setminus f^{-1}[cl(f[A])] \subseteq X \setminus e\text{-}cl_\theta(f^{-1}[int(cl(f[A]))])$
$\Rightarrow f[e\text{-}cl_\theta(f^{-1}[int(cl(f[A]))])] \subseteq f[f^{-1}[cl(f[A])]]$
$\Rightarrow f[e\text{-}cl_\theta(A)] \subseteq cl_\theta(f[A])$.

(9) $\Rightarrow$ (10) : Let $B \subseteq Y$.

$$\left.\begin{array}{r} B \subseteq Y \Rightarrow f^{-1}[B] \subseteq X \\ \text{Hypothesis} \end{array}\right\} \Rightarrow f[e\text{-}cl_\theta(f^{-1}[B])] \subseteq cl_\theta(f[f^{-1}[B]]) \subseteq cl_\theta(B)$$

$\Rightarrow e\text{-}cl_\theta(f^{-1}[B]) \subseteq f^{-1}[f[e\text{-}cl_\theta(f^{-1}[B])]] \subseteq f^{-1}[cl_\theta(f[f^{-1}[B]])] \subseteq f^{-1}[cl_\theta(B)]$.

(10) $\Rightarrow$ (11) : Let $B \subseteq Y$.

$$\left.\begin{array}{r} B \subseteq Y \Rightarrow int(cl_\theta(B)) \subseteq Y \\ \text{Hypothesis} \end{array}\right\} \Rightarrow e\text{-}cl_\theta(f^{-1}[int(cl_\theta(B))]) \subseteq f^{-1}[cl_\theta(int(cl_\theta(B)))] \subseteq f^{-1}[cl_\theta(B)].$$

(11) $\Rightarrow$ (1) : Let $x \in X$ and $U \in O(Y, f(x))$.

$$\left.\begin{array}{r} (x \in X)(U \in O(Y, f(x))) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow \left.\begin{array}{r} e\text{-}cl_\theta(f^{-1}[int(cl_\theta(U))]) \subseteq f^{-1}[cl_\theta(U)] \\ U \subseteq int(cl(U)) = int(cl_\theta(U)) \end{array}\right\} \Rightarrow$$

$$\Rightarrow e\text{-}cl_\theta(f^{-1}[U]) \subseteq e\text{-}cl_\theta(f^{-1}[int(cl_\theta(U))]) \subseteq f^{-1}[cl_\theta(U)] \overset{\text{Lemma 2.2}}{=} f^{-1}[cl(U)]$$

It follows by (1) $\Leftrightarrow$ (7) that f is weakly eR continuous. □

Definition 3.3 *[13] A function $f : X \to Y$ is called to be contra $e\theta$-continuous (briefly c.$e\theta$.c.) if $f^{-1}[V]$ is e-θ-closed in X for every open set V of Y.*

Theorem 3.4 *If $f : X \to Y$ is contra $e\theta$-continuous, then f is weakly eR-continuous.*

Proof: Let $V \in O(Y)$.

$$\left.\begin{array}{r} V \in O(Y) \\ f \text{ is c.}e\theta\text{.c.} \end{array}\right\} \Rightarrow f^{-1}[V] \in e\theta C(X) \Rightarrow e\text{-}cl_\theta(f^{-1}[V]) = f^{-1}[V] \subseteq f^{-1}[cl(V)]$$

Then by Theorem 3.3(7) f is weakly eR-continuous. □

Theorem 3.5 *For a function $f : X \to Y$, the followings are equivalent:*
(1) f is weakly eR-continuous at $x \in X$;
(2) $x \in e\text{-}int_\theta(f^{-1}[cl(V)])$ for each neighborhood V of $f(x)$.

Proof: (1) $\Rightarrow$ (2) : Let $V \in O(Y, f(x))$.

$$\left.\begin{array}{r} V \in O(Y, f(x)) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow (\exists U \in eR(X, x))(f[U] \subseteq cl(V)) \Rightarrow (\exists U \in eR(X, x))(U \subseteq f^{-1}[cl(V)])$$

$\Rightarrow (\exists U \in eR(X,x))(U \subseteq e\text{-}int_\theta(U) \subseteq e\text{-}int_\theta(f^{-1}[cl(V)]))$
$\Rightarrow x \in e\text{-}int_\theta(f^{-1}[cl(V)])$.

(2) $\Rightarrow$ (1) : Let $V \in O(Y, f(x))$.

$$\left.\begin{array}{r} V \in O(Y,f(x)) \\ \text{Hypothesis} \end{array}\right\} \Rightarrow x \in e\text{-}int_\theta(f^{-1}[cl(V)]) \Rightarrow (\exists U \in eR(X,x))(f[U] \subseteq cl(V))$$

Thus, f is weakly eR-continuous at $x \in X$. □

Lemma 3.1 *[23] Let X be a topological space and $A, B \subseteq X$. If $A \in aO(X)$ and $B \in eO(X)$, then $A \cap B \in eO(X)$.*

Theorem 3.6 *If $f : X \to Y$ is weakly eR-continuous at $x \in X$, then there exists a nonempty e-open set $U \subseteq H$ such that $U \subseteq e\text{-}cl_\theta(f^{-1}[cl(V)])$ for every neighborhood V of $f(x)$ and every a-open neighborhood H of x.*

Proof: Let $H \in aO(X,x)$ ve $V \in O(Y, f(x))$.

$$\left.\begin{array}{r} (H \in aO(X,x))(V \in O(Y,f(x))) \\ f \text{ is w.}eR\text{.c. at } x \in X \end{array}\right\} \overset{\text{Theorem 3.5}}{\Rightarrow} x \in e\text{-}int_\theta(f^{-1}[cl(V)])$$

$$\overset{\text{Lemma 2.1(9)}}{\Rightarrow} \left.\begin{array}{r} (\exists G \in eR(X,x))(G \subseteq e\text{-}int_\theta(f^{-1}[cl(V)])) \\ U := G \cap H \end{array}\right\} \overset{\text{Lemma 3.1}}{\Rightarrow}$$

$\Rightarrow (U \in eO(X,x))(U \subseteq H)(U \subseteq G \subseteq e\text{-}int_\theta(f^{-1}[cl(V)]) \subseteq e\text{-}cl_\theta(f^{-1}[cl(V)]))$. □

Theorem 3.7 *Let $f : X \to Y$ be a function. If $f^{-1}[cl_\theta(U)]$ is e-θ-closed in X for every subset U of Y, then f is weakly eR-continuous.*

Proof: Let $f^{-1}[cl_\theta(U)]$ is e-θ-closed in X for every subset U of Y.
$(U \subseteq Y)(f^{-1}[cl_\theta(U)]) \in e\theta C(X)) \Rightarrow e\text{-}cl_\theta(f^{-1}[U]) \subseteq e\text{-}cl_\theta(f^{-1}[cl_\theta(U)]) = f^{-1}[cl_\theta(U)]$
Then by Theorem 3.3(10), f is weakly eR-continuous. □

Theorem 3.8 *Let $f : X \to Y$ be a weakly eR-continuous function. Then the followings hold:*
(1) $f^{-1}[U]$ is e-θ-open in X for each θ-open set U of Y,
(2) $f^{-1}[V]$ is e-θ-closed in X for each θ-closed set V of Y.

Proof: It follows from Theorem 3.3. □

Theorem 3.9 *[18] A function $f : X \to Y$ strongly θ-e-continuous if and only if for all $x \in X$ and all open set V of Y containing $f(x)$, there exists $U \in eR(X,x)$ such that $f(U) \subseteq V$.*

Theorem 3.10 *Let $f : X \to Y$ be a function. If Y is regular, then the followings are equivalent:*
(1) f is weakly eR-continuous;
(2) f is strongly θ-e-continuous.

Proof: (1) $\Rightarrow$ (2) : Let $x \in X$ and $V \in O(Y, f(x))$.

$$\left.\begin{array}{r} (x \in X)(V \in O(Y,f(x))) \\ Y \text{ is regular} \end{array}\right\} \Rightarrow \left.\begin{array}{r} (\exists H \in O(Y,f(x)))(cl(H) \subseteq V) \\ f \text{ is w.}eR\text{.c.} \end{array}\right\} \Rightarrow$$

$\Rightarrow (\exists U \in eR(X,x))(f[U] \subseteq cl(H) \subseteq V)$
Then by Theorem 3.9 f is strongly θ-e-continuous.

(2) $\Rightarrow$ (1) : It is obvious from Definitions 2.1(f) and 3.1. □

4. Some Basic Properties of Weakly eR-continuity

Definition 4.1 *A space X is called to be e-connected [9] if X is not the union of two disjoint nonempty e-open sets.*

Theorem 4.1 *If $f : X \to Y$ is a weakly eR-continuous surjection and X is e-connected, then Y is connected.*

Proof: Suppose that Y is not connected.

$$Y \text{ is not connected} \Rightarrow (\exists A, B \in O(Y) \setminus \{\emptyset\})(A \cap B = \emptyset)(A \cup B = Y)$$
$$\left.\begin{array}{r} \Rightarrow (A, B \in CO(Y) \setminus \{\emptyset\})(A \cap B = \emptyset)(A \cup B = Y) \\ f \text{ is w.}eR\text{.c.} \end{array}\right\} \overset{\text{Theorem 3.3(8)}}{\Rightarrow}$$
$$\Rightarrow f^{-1}[A] \subseteq e\text{-}int_\theta(f^{-1}[cl(A)]) = e\text{-}int_\theta(f^{-1}[A])$$
$$\left.\begin{array}{r} (f^{-1}[B] \subseteq e\text{-}int_\theta(f^{-1}[cl(B)]) = e\text{-}int_\theta(f^{-1}[B]))(f^{-1}[A \cap B] = \emptyset)(f^{-1}[A \cup B] = X) \\ f \text{ is surjection} \end{array}\right\} \Rightarrow$$
$$\Rightarrow (f^{-1}[A], f^{-1}[B] \in eO(X) \setminus \{\emptyset\})(f^{-1}[A] \cap f^{-1}[B] = \emptyset)(f^{-1}[A] \cup f^{-1}[B] = X)$$

This is a contradiction to the fact that X is e-connected. Thus, Y is connected. □

Definition 4.2 *A space X is called:*
(1) Urysohn [29] if for all distinct two points x and y in X, there exist open sets U and V such that $x \in U$, $y \in V$ and $cl(U) \cap cl(V) = \emptyset$.
(2) Clopen T_2 [7] if for all distinct two points x and y in X, there exist disjoint clopen sets U and V of X such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.
(3) eR-T_1 if for all distinct two points x and y in X, there exist e-regular sets U and V of X containing x and y, respectively, such that $y \notin U$ and $x \notin V$.
(4) eR-T_2 if for all distinct two points x and y in X, there exist disjoint e-regular sets U and V of X such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$.

Remark 4.1 *Every clopen T_2 space is eR-T_2. This implication is not reversible as shown in the following example.*

Example 4.1 Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$. It is not difficult to see $CO(X) = \{\emptyset, X\}$ and $eR(X) = 2^X \setminus \{\{c\}, \{d\}, \{c, d\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$. Then (X, τ) is eR-T_2 but it is not clopen T_2.

Theorem 4.2 *Let $f : X \to Y$ is a weakly eR-continuous function and $g : X \to Y$ is a weakly a-continuous function. If Y is Urysohn, then $A = \{x | f(x) = g(x)\} \in eC(X)$.*

Proof: Let $x \notin A$.

$$\left.\begin{array}{r} x \notin A \Rightarrow f(x) \neq g(x) \\ Y \text{ is Urysohn} \end{array}\right\} \Rightarrow$$
$$\left.\begin{array}{r} \Rightarrow (\exists V_1 \in O(Y, f(x)))(\exists V_2 \in O(Y, g(x)))(cl(V_1) \cap cl(V_2) = \emptyset) \\ (f \text{ is w.}eR\text{.c.})(g \text{ is w.}a\text{.c.}) \end{array}\right\} \Rightarrow$$
$$\left.\begin{array}{r} \Rightarrow (\exists U \in eR(X, x))(\exists G \in aO(X, x))(f[U] \subseteq cl(V_1))(g[G] \subseteq cl(V_2)) \\ W := U \cap G \end{array}\right\} \overset{\text{Lemma 3.1}}{\Rightarrow}$$
$$\Rightarrow (\exists W \in eO(X, x))(f[W] \cap g[W] \subseteq f[U] \cap g[G] \subseteq cl(V_1) \cap cl(V_2) = \emptyset)$$
$$\Rightarrow (\exists W \in eO(X, x))(W \cap A = \emptyset)$$
$$\Rightarrow x \notin e\text{-}cl(A).$$
□

Theorem 4.3 *Let $f : X \to Y$ be a weakly eR-continuous injection. If Y is Hausdorff, then X is eR-T_1.*

Proof: Let $x, y \in X$ and $x \neq y$.

$$\left.\begin{array}{r}(x,y\in X)(x\neq y)\overset{f\text{ is injective}}{\Rightarrow} f(x)\neq f(y)\\ Y\text{ is Hausdorff}\end{array}\right\}\Rightarrow(\exists U\in O(Y,f(x))(\exists V\in O(Y,f(y))(U\cap V=\emptyset)$$

$$\left.\begin{array}{r}\Rightarrow(\exists U\in O(Y,f(x))(\exists V\in O(Y,f(y))(f(x)\notin cl(V))(f(y)\notin cl(U))\\ f\text{ is w.}eR\text{.c.}\end{array}\right\}\Rightarrow$$

$$\Rightarrow(\exists A\in eR(X,x))(\exists B\in eR(X,y))(f[A]\subseteq cl(U))(f[B]\subseteq cl(V))$$

$$\Rightarrow(\exists A\in eR(X,x))(\exists B\in eR(X,y))(y\notin A)(x\notin B).$$

□

Theorem 4.4 *Let $f : X \to Y$ be a weakly eR-continuous injection. If Y is Urysohn, then X is eR-T_2.*

Proof: Let $x, y \in X$ and $x \neq y$.

$$\left.\begin{array}{r}(x,y\in X)(x\neq y)\overset{f\text{ is injective}}{\Rightarrow} f(x)\neq f(y)\\ Y\text{ is Urysohn}\end{array}\right\}\Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow(\exists U\in O(Y,f(x))(\exists V\in O(Y,f(y))(cl(U)\cap cl(V)=\emptyset)\\ f\text{ is w.}eR\text{.c.}\end{array}\right\}\Rightarrow$$

$$\Rightarrow(\exists A\in eR(X,x))(\exists B\in eR(X,y))(f[A]\subseteq cl(U))(f[B]\subseteq cl(V))$$

$$\Rightarrow(\exists A\in eR(X,x))(\exists B\in eR(X,y))(A\subseteq f^{-1}[cl(U)])(B\subseteq f^{-1}[cl(V)])$$

$$\Rightarrow(\exists A\in eR(X,x))(\exists B\in eR(X,y))(A\cap B\subseteq f^{-1}[cl(U)]\cap f^{-1}[cl(V)]=\emptyset).$$

□

Corollary 4.1 *Let $f : X \to Y$ be a weakly clopen injection. If Y is Urysohn, then X is eR- T_2.*

Let $\{X_\alpha|\alpha \in I\}$ and $\{Y_\alpha|\alpha \in I\}$ be any two families of topological spaces with the same index set I. The product space of $\{X_\alpha|\alpha \in I\}$ (resp. $\{Y_\alpha|\alpha \in I\}$) is simply denoted by ΠX_α (resp. ΠY_α). Let $f_\alpha : X_\alpha \to Y_\alpha$ be a function for all $\alpha \in I$. Let $f : \Pi X_\alpha \to \Pi Y_\alpha$ be the product function defined as follows: $f(\{x_\alpha\}) = \{f_\alpha(x_\alpha)\}$ for all $\{x_\alpha\} \in \Pi X_\alpha$.

Theorem 4.5 *Let $\{X_\alpha|\alpha \in I\}$ and $\{Y_\alpha|\alpha \in I\}$ be any two families of topological spaces. If $f_\alpha : X_\alpha \to Y_\alpha$ is weakly eR-continuous for each $\alpha \in I$, then the function $f : \Pi X_\alpha \to \Pi Y_\alpha$ is weakly eR-continuous.*

Proof: Let $x = \{x_\alpha\} \in \Pi X_\alpha$ and $V \in O(\Pi Y_\alpha, f(x))$.

$$V\in O(\Pi Y_\alpha,f(x))\Rightarrow(\exists J=\{\alpha_1,\alpha_2,\ldots,\alpha_n\}\subseteq I)$$

$$\left.\begin{array}{r}\left(W_\alpha:=\left\{\begin{array}{lcl}W_{\alpha_j}\in O(Y_{\alpha_j}) & , & \alpha\in J\\ Y_\alpha & , & \alpha\notin J\end{array}\right.\right)(\Pi W_\alpha\in O(\Pi Y_\alpha,f(x)))\,(\Pi W_\alpha\subseteq V)\\ (\forall\alpha\in I)(f_\alpha\text{ is w.}eR\text{.c.})\end{array}\right\}\Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow(\exists U_\alpha\in eR(X_\alpha,x_\alpha))\,(f_\alpha[U_\alpha]\subseteq cl(W_\alpha))\\ U:=\prod_{j=1}^{n}U_{\alpha_j}\times\prod_{\alpha\notin J}X_\alpha\end{array}\right\}\Rightarrow$$

$$\Rightarrow(U\in eR(\Pi X_\alpha,x))\left(f[U]\subseteq\prod_{j=1}^{n}f_\alpha[U_{\alpha_j}]\times\prod_{\alpha\notin J}Y_\alpha\subseteq\prod_{j=1}^{n}cl(W_{\alpha_j})\times\prod_{\alpha\notin J}Y_\alpha\subseteq cl(V)\right).$$

□

Recall that the graph of a function $f : X \to Y$ is the subset $\{(x, f(x))|x \in X\}$ of the product space $X \times Y$ and denoted by $G(f)$.

Theorem 4.6 *Let $f : X \to Y$ be a function. If the graph function g is weakly eR-continuous, then f is weakly eR-continuous.*

Proof: Let $x \in X$ and $V \in O(Y, f(x))$.

$$\left.\begin{array}{r}(x\in X)(V\in O(Y,f(x)))\Rightarrow X\times V\in O(X\times Y,g(x))\\ g\text{ is w.}eR\text{.c.}\end{array}\right\}\Rightarrow$$

$$\Rightarrow(\exists U\in eR(X,x))(g[U]\subseteq cl(X\times V)=X\times cl(V))$$

$$\Rightarrow(\exists U\in eR(X,x))(f[U]\subseteq cl(V)).$$

□

Definition 4.3 *A function $f : X \to Y$ has an er-graph if for all $(x, y) \notin G(f)$, there exist $U \in eR(X, x)$ and $V \in O(Y, y)$ such that $(U \times cl(V)) \cap G(f) = \emptyset$.*

Lemma 4.1 *A function $f : X \to Y$ has an er-graph if and only if for all $(x, y) \notin G(f)$, there exist $U \in eR(X, x)$ and $V \in O(Y, y)$ such that $f[U] \cap cl(V) = \emptyset$.*

Proof: It is obvious from Definition 4.3. □

Theorem 4.7 *If $f : X \to Y$ is weakly eR-continuous and Y is a Urysohn space, then $G(f)$ is an er-graph.*

Proof: Let $(x, y) \notin G(f)$.

$$\left.\begin{array}{r}(x,y)\notin G(f)\Rightarrow y\neq f(x)\\ Y \text{ is Urysohn}\end{array}\right\}\Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow(\exists A\in O(X,f(x)))(\exists B\in O(Y,y))(cl(A)\cap cl(B)=\emptyset)\\ f\text{ is w.}eR\text{.c.}\end{array}\right\}\Rightarrow$$

$$\Rightarrow(\exists G\in eR(X,x))(\exists B\in O(Y,y))(f[G]\subseteq cl(A))$$
$$\Rightarrow(\exists G\in eR(X,x))(\exists B\in O(Y,y))(f[G]\cap cl(B)=\emptyset)$$
$$\Rightarrow(\exists G\in eR(X,x))(\exists B\in O(Y,y))((G\times cl(B))\cap G(f)=\emptyset).$$
□

Theorem 4.8 *If $f : X \to Y$ has an er-graph and a weakly eR-continuous injection, then X is eR-T_2.*

Proof: Let $x, y \in X$ and $x \neq y$.

$$\left.\begin{array}{r}(x,y\in X)(x\neq y)\overset{f\text{ is injective}}{\Rightarrow} f(x)\neq f(y)\Rightarrow(x,f(y))\notin G(f)\\ G(f)\text{ is }er\text{-graph}\end{array}\right\}\Rightarrow$$

$$\Rightarrow(\exists U\in eR(X,x))(\exists V\in O(Y,f(y)))((U\times cl(V))\cap G(f)=\emptyset)$$

$$\left.\begin{array}{r}\Rightarrow(\exists U\in eR(X,x))(\exists V\in O(Y,f(y)))(f[U]\cap cl(V)=\emptyset)\\ f\text{ is w.}eR\text{.c.}\end{array}\right\}\Rightarrow$$

$$\Rightarrow(\exists U\in eR(X,x))(\exists W\in eR(X,y))(f[W]\subseteq cl(V))$$
$$\Rightarrow(\exists U\in eR(X,x))(\exists W\in eR(X,y))(f[U]\cap f[W]=\emptyset)$$
$$\Rightarrow(\exists U\in eR(X,x))(\exists W\in eR(X,y))(U\cap V=\emptyset).$$
□

Definition 4.4 *A space X is called to be eR-compact if every cover of X by e-regular sets has a finite subcover.*

Theorem 4.9 *Let $f : X \to Y$ be a function having an er-graph $G(f)$, then $f[K]$ is θ-closed in Y for all eR-compact relative to X subset K.*

Proof: Let K be eR-compact relative to X and $y \notin f[K]$.

$$\left.\begin{array}{r}y\notin f[K]\Rightarrow(\forall x\in K)((x,y)\notin G(f))\\ G(f)\text{ is }er\text{-graph}\end{array}\right\}\overset{\text{Lemma 4.1}}{\Rightarrow}$$

$$\Rightarrow(\exists U_x\in eR(X,x))(\exists V_x\in O(Y,y))(f[U_x]\cap cl(V_x)=\emptyset)$$

$$\left.\begin{array}{r}\Rightarrow(\{U_x|x\in K\}\subseteq eR(X))(K\subseteq\bigcup\{U_x|x\in K\})\\ K\text{ is }eR\text{-compact relative to }X\end{array}\right\}\Rightarrow$$

$$\left.\begin{array}{r}\Rightarrow(\exists K^*\subseteq K)(|K^*|<\aleph_0)(K\subseteq\bigcup\{U_x|x\in K^*\})\\ V:=\bigcap_{x\in K^*}V_x\in O(Y,y)\end{array}\right\}\Rightarrow$$

$$\Rightarrow(V\in O(Y,y))\left(f[K]\cap V\subseteq\left(\bigcup_{x\in K^*}f[U_x]\right)\cap cl(V)\subseteq\bigcup_{x\in K^*}(f[U_x]\cap cl(V))=\emptyset\right)$$

$$\Rightarrow y\notin cl_\theta(f[K]).$$
□

5. Conclusion

This study is concerned with the concept of eR-continuity and weakly eR-continuity defined by utilizing the notion of e-regular sets. It turns out that both eR-continuous and weakly eR-continuous functions are stronger than weakly e-continuous functions, as will be seen in Remark 3.1. We believe that this paper will pave the way for future studies relevant to continuity and convergence etc. known from functional analysis.

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