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The Parabolic Equation with $q(x)-$ Triharmonic Equation: Blow up and Growth

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ABSTRACT: In this work, we investigate the $q(x)-$triharmonic equation with initial-boundary value conditions on a bounded domain. Firstly, we prove the blow up of solutions. Later, we prove the exponential growth of solutions.

Key Words: Blow up, growth, $q(x)-$ triharmonic equation, variable exponent.

Contents

1. Introduction

In this work, we consider the following parabolic-type $q(x)$-triharmonic equation with variable exponent

$$\begin{cases} \left(1+a\left|u\right|^{p(x)-2}\right)u_t-\Delta^3_{q(x)}u=b\left|u\right|^{r(x)-2}u, & Q=\Omega\times(0,T), \\ u\left(x,t\right)=\Delta u\left(x,t\right)=\Delta^2 u\left(x,t\right)=0, & \partial Q=\partial\Omega\times[0,T), \\ u\left(x,0\right)=u_0\left(x\right), & \Omega, \end{cases} \tag{1.1}$$

where $\Omega\subset R^n$ is a bounded domain in R^n, $n\geq 1$ with smooth boundary $\partial\Omega$ and $b>0$, $a\geq 0$ are constants. Also $\Delta^3_{q(x)}u$ is called the $q(x)$-triharmonic operator and is defined by

$$\Delta^3_{q(x)}u=\operatorname{div}\left(\Delta\left(\left|\nabla\Delta u\right|^{q(x)-2}\nabla\Delta u\right)\right).$$

The functions $p\left(.\right),q\left(.\right)$ and $r\left(.\right)$ are measurable and given on the set $\overline{\Omega}$ such that

$$2\leq p_1\leq p\left(x\right)\leq p_2<q_1\leq q\left(x\right)\leq q_2<r_1\leq r\left(x\right)<r_2<q^*\left(x\right), \tag{1.2}$$

with

$$q^*\left(x\right)=\begin{cases} \frac{nq(x)}{(n-q(x))_2} & \text{if } q_2<n, \\ +\infty & \text{if } q_2\geq n. \end{cases}$$

Additionally, we make the assumption that

$$\left|q\left(x\right)-q\left(y\right)\right|\leq\frac{A}{\ln\left(\frac{1}{|x-y|}\right)}, \text{ for all } x,\ y\ \in\Omega \text{ with } \left|x-y\right|<\delta, \tag{1.3}$$

and $A>0$, $0<\delta<1$

$$ess\inf\left(q^*\left(x\right)-r\left(x\right)\right)>0. \tag{1.4}$$

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The following problem was considered by Alaoui et al. [2]

$$u_t - \operatorname{div}\left(|\nabla u|^{m(x)-2}\nabla u\right) = |u|^{p(x)-2}u. \tag{1.5}$$

The authors proved the blow up of solutions. Later, Rahmoune [15] proved an upper bound for blow up time of solutions equation (1.5).

Di et al. [8] considered the following pseudo-parabolic equation with variable exponent

$$u_t - \Delta u_t - \operatorname{div}\left(|\nabla u|^{m(x)-2}\nabla u\right) = |u|^{p(x)-2}u. \tag{1.6}$$

They established both an upper and lower bound for the blow-up time. Subsequently, other researchers investigated the blow-up phenomena of solutions to the equation (1.6), (see [10,20]).

Liu [11] studied the $p(x)$-biharmonic heat equation

$$u_t + \Delta^2_{p(x)}u = |u|^{q(x)-2}u.$$

The author proved the local existence and blow up of solutions.

Belakdar et al. [3] investigated the $p(x)$-triharmonic equation

$$-\Delta^3_p(x)\,u = \lambda V_1(x)\,|u|^{q(x)-2}u.$$

They achieved the determination of both the existence and nonexistence of eigenvalues for the $p(x)$-triharmonic equation, subject to Navier boundary conditions, within a bounded domain in R^n.

In recent years, mush attention has been paid to the study of mathematical models of electrorheological fuids. This models include hyperbolic, parapolic or elliptic equations which are nonlinear with respect to the gradient of the thought solution with variable exponents of nonlinearity (see [16,17,7,12,13,4,5,18,19]).

The structure of this paper is organized as follows: In part 2, we state some results about the variable exponent Lebesgue spaces $L^{p(x)}(\Omega)$ and Sobolev spaces $W^{m,p(x)}(\Omega)$. In part 3, we provide the proofs for the blow-up results. In part 4, we establish demonstrating exponential growth when $a > 0$, along with negative initial energy in both cases.

2. Preliminaries

We recall some well-known results about the Lebesgue spaces and Sobolev spaces with variable exponents (see [9,14]).

Let $q : \Omega \to [1,\infty]$ be a measurable function, where Ω is a bounded domain of R^n. We define the Lebesgue space with variable exponent $q(.)$ by

$$L^{q(x)}(\Omega) = \left\{u : \Omega \to R,\ u \text{ is measurable and } \rho_{q(.)}(\lambda u) \ < \infty,\ \text{for some } \lambda > 0\right\},$$

where

$$\rho_{q(.)}(u) = \int_\Omega |u|^{q(x)}\,dx.$$

Also endowed with the Luxemburg-type norm

$$\|u\|_{q(x)} = \inf\left\{\lambda > 0 : \int_\Omega \left|\frac{u}{\lambda}\right|^{q(x)} dx \le 1\right\},$$

$L^{q(x)}(\Omega)$ is a Banach space.

The variable exponent Sobolev space $W^{m,q(x)}(\Omega)$ is defined by

$$W^{m,q(x)}(\Omega) = \left\{u \in L^{q(x)}(\Omega) : D^\alpha u \in L^{q(x)}(\Omega),\ |\alpha| \le m\right\}.$$

The variable exponent Sobolev space forms a Banach space when equipped with the following norm:

$$\|u\|_{2,q(x)} = \|u\|_{q(x)} + \|\nabla u\|_{q(x)} + \|\Delta u\|_{q(x)}.$$

Lemma 1 *(i) Assuming that (1.4) is satisfied, we have the inequality* $\|u\|_{q(.)} \leq C \|\nabla u\|_{q(.)}$ *for all* $u \in W_0^{1,q(.)}(\Omega)$, *within the context of a bounded* Ω. *Notably, the norm for the space* $W_0^{1,q(.)}(\Omega)$ *is given by* $\|u\|_{1,q(.)} = \|\nabla u\|_{q(.)}$ *for every* $u \in W_0^{1,q(.)}(\Omega)$.

(ii) If $q \in C(\overline{\Omega})$, $r : \Omega \to [1, \infty)$ *is a measurable function, the following conditions apply*

$$ess \inf (q^*(x) - r(x)) > 0,$$

with $q^*(x) = \frac{nq(x)}{(n-q(x))_2}$ *then*

$$W_0^{1,q(.)}(\Omega) \hookrightarrow\hookrightarrow L^{r(.)}(\Omega).$$

3. Blow up

In this part, we establish the finite-time blow-up of the solution to equation (1.1) when $a = 0$. We initiate our analysis with a local existence theorem for the problem (1.1), which is a direct result of the existence theorem by presented Agaki and Otani [1].

Theorem 2 *For all* $u_0 \in W_0^{3,q(.)}(\Omega)$, *there exists a value* $T_0 \in (0, T]$ *such that the equation (1.1) has a strong solution* u *over the interval* $[0, T_0]$ *meeting the following conditions:*

$$u \in C_w\left([0, T_0]; W_0^{3,q(.)}(\Omega)\right) \cap C\left([0, T_0], L^{r(.)}(\Omega)\right) \cap W^{1,2}\left(0, T_0; L^2(\Omega)\right).$$

Lemma 3 *We introduce the energy functional related with the equation*

$$E(t) = \int_\Omega \left(\frac{1}{q(x)} |\nabla \Delta u|^{q(x)} - \frac{b}{r(x)} |u|^{r(x)}\right) dx, \tag{3.1}$$

is a nonincreasing function for $t \geq 0$ *and*

$$E'(t) \leq 0.$$

Proof: By multiplying both sides of equation u_t with respect to (1.1), and subsequently integrating by parts, we arrive at the following expression:

$$\int_\Omega u_t^2 dx - \int_\Omega \Delta_{q(x)}^3 u u_t dx = \frac{d}{dt} \int_\Omega \frac{b}{r(x)} |u|^{r(x)} dx.$$

Subsequently, we define the energy using the following expression:

$$E(t) = \int_\Omega \left(\frac{1}{q(x)} |\nabla \Delta u|^{q(x)} - \frac{b}{r(x)} |u|^{r(x)}\right) dx,$$

and

$$E'(t) = -\|u_t\|_2^2,$$

then

$$E(t) \leq E(0) < 0. \tag{3.2}$$

□

Theorem 4 *Assuming that the conditions described in Lemma 3 are hold, and further supposing that*

$$E(0) < 0. \tag{3.3}$$

Then the solution of the problem (1.1), blow up finite time.

At this point, we consider

$$H(t) = -E(t) = -\int_\Omega \frac{1}{q(x)} |\nabla \Delta u|^{q(x)} + b\int_\Omega \frac{1}{r(x)} |u|^{r(x)} \leq b\int_\Omega \frac{1}{r(x)} |u|^{r(x)}, \tag{3.4}$$

and

$$F(t) = \frac{1}{2}\int_\Omega u^2 dx. \tag{3.5}$$

To demonstrate our result, we initially establish several lemmas.

Lemma 5 *Suppose that (1.2) and (1.3), hold and $E(0) < 0$. We get*

$$T_{q(.)}(\nabla \Delta u) = \int_\Omega |\nabla \Delta u|^{q(x)} dx, \tag{3.6}$$

where

$$T_{q(.)}(\nabla \Delta u) < \frac{bp_2}{r_1} T_{r(.)}(u), \tag{3.7}$$

and

$$\frac{r_1}{b} H(0) < T_{r(.)}(u). \tag{3.8}$$

Theorem 6 *Assume that $u_0 \in W_0^{3,m(.)}(\Omega)$ such that $\int_\Omega u_0^2 dx > 0$ and*

$$\int_\Omega \left(\frac{1}{q(x)} |\nabla \Delta u_0|^{q(x)} - \frac{b}{r(x)} |u_0|^{r(x)} \right) dx \geq 0.$$

Then

$$T_{r(.)}(u) \geq k \|u\|_{r_1}^{r_1}, \tag{3.9}$$

and

$$F(t) = \frac{1}{2}\int_\Omega u^2 dx, \tag{3.10}$$

blows up in finite time $t^* < +\infty$.

Proof. Upon differentiating F with respect to time t, we get

$$\begin{aligned}
F'(t) &= \int_\Omega u u_t dx \\
&= \int_\Omega u \left(-\Delta^3_{q(x)} u + b|u|^{r(x)-2} u \right) dx \\
&= \int_\Omega \left(b|u|^{r(x)} - |\nabla \Delta u|^{q(x)} \right) dx \\
&= \int_\Omega r(x) \left(\frac{b|u|^{r(x)}}{r(x)} - \frac{|\nabla \Delta u|^{q(x)}}{q(x)} \right) dx + \int_\Omega r(x) \left(\frac{1}{q(x)} - \frac{1}{r(x)} \right) |\nabla \Delta u|^{q(x)} dx \\
&= -T_{q(.)}(|\nabla \Delta u|) + bT_{r(.)}(|u|).
\end{aligned} \tag{3.11}$$

By combining the information from (3.7), (3.9) and (3.11), we can deduce that

$$F'(t) \geq kb\left(1 - \frac{p_2}{r_1}\right) \|u\|_{r_1}^{r_1}. \tag{3.12}$$

Next, we estimate $F(t)^{\frac{r_1}{2}}(t)$, using the embedding of $F^{r_1}(\Omega) \hookrightarrow L^2(\Omega)$, which yields

$$F(t)^{\frac{r_1}{2}}(t) \leq \left(\frac{1}{2}\|u\|_{r_1}^2\right)^{\frac{r_1}{2}} \leq k\|u\|_{r_1}^{r_1}. \tag{3.13}$$

By combining equations (3.12) and (3.13), we derive

$$F^{'}(t) \geq \chi F(t)^{\frac{r_1}{2}}(t). \tag{3.14}$$

Integrating equation (3.14), directly gives us

$$F(t)^{\frac{r_1}{2}-1}(t) \geq \frac{1}{\left(F(0)^{1-\frac{r_1}{2}} - \chi t\right)}.$$

Consequently, F blows up in a time

$$t^* \leq \frac{1}{F(t)^{\frac{r_1}{2}-1}(0)}. \tag{3.15}$$

4. Exponential growth

In this part, we establish the exponential growth of the solution to equation (1.1) when $a > 0$.

Lemma 7 *Assume that (1.2) holds and $E(0) < 0$. Then,*

$$\int_\Omega |u|^{p(x)}\,dx \leq k\left(\|u\|_{r_1}^{r_1} + H(t)\right). \tag{4.1}$$

Proof: We have

$$\int_\Omega |u|^{p(x)}\,dx = \int_{\Omega_1} |u|^{p(x)}\,dx + \int_{\Omega_2} |u|^{p(x)}\,dx,$$

then

$$\Omega_2 = \{x \in \Omega : |u(x,t)| \geq 1\} \text{ and } \Omega_1 = \{x \in \Omega : |u(x,t)| < 1\}.$$

Thus, we obtain

$$\begin{aligned}\int_\Omega |u|^{p(x)}\,dx &\leq k\left[\left(\int_{\Omega_1} |u|^{r_1}\,dx\right)^{\frac{p_1}{r_1}} + \left(\int_{\Omega_2} |u|^{r_1}\,dx\right)^{\frac{p_2}{r_1}}\right] \\ &\leq k\left(\|u\|_{r_1}^{p_1} + \|u\|_{r_1}^{p_2}\right).\end{aligned}$$

Utilizing the algebraic inequality

$$x^y \leq (x+1) \leq \left(1 + \frac{1}{a}\right)(x+a),\ \forall x > 0,\ 0 < y \leq 1,\ a \geq 0,$$

we get

$$\begin{aligned}\|u\|_{r_1}^{p_1} &\leq k\left(\|u\|_{r_1}^{r_1}\right)^{\frac{p_1}{r_1}} \leq k\left(1 + \frac{1}{H(0)}\right)\left(\|u\|_{r_1}^{r_1} + H(0)\right) \\ &\leq k\left(\|u\|_{r_1}^{r_1} + H(0)\right).\end{aligned}$$

Likewise,

$$\begin{aligned} \|u\|_{r_1}^{p_2} &\leq k\left(\|u\|_{r_1}^{r_1}\right)^{\frac{p_2}{r_1}} \leq k\left(1+\frac{1}{H(0)}\right)\left(\|u\|_{r_1}^{r_1}+H(0)\right) \\ &\leq k\left(\|u\|_{r_1}^{r_1}+H(0)\right), \end{aligned}$$

this results in

$$\int_{\Omega} |u|^{p(x)}\,dx \leq k\left(\|u\|_{r_1}^{r_1}+H(0)\right).$$

□

Theorem 8 *Assuming that the conditions of Lemma 3, are satisfied and that (3.3) holds, we can conclude that the solution of the problem (1.1), exponential growth.*

Proof: Applying the same procedure used in the proof of Lemma 5, we obtain

$$E'(t) = -\|u_t\|_2^2 - a\int_{\Omega} |u|^{p(x)-2} u_t^2 dx \leq 0. \tag{4.2}$$

Subsequently, we get

$$H'(t) = \|u_t\|_2^2 + a\int_{\Omega} |u|^{p(x)-2} u_t^2 dx \geq 0. \tag{4.3}$$

We define the following

$$G(t) = H(t) + \gamma F(t), \tag{4.4}$$

for a small value of γ to be chosen later. Taking the time derivative of equation (4.4), we get

$$G'(t) = H'(t) + \gamma\int_{\Omega} uu_t dx.$$

Utilizing equation (1.1), we obtain

$$G'(t) = H'(t) - \gamma T_{q(.)}(|\nabla\Delta u|) + \gamma b T_{r(.)}(u) - \gamma a\int_{\Omega} |u|^{p(x)-2} u_t u dx. \tag{4.5}$$

To bound the last term on the right hand side of equation (4.5), we can make use of Young's inequality:

$$AB \leq \delta A^2 + \delta^{-1}B^2, \quad A, B \geq 0,\ \delta > 0.$$

$$\begin{aligned} \int_{\Omega} |u|^{p(x)-2} u_t u dx &= \int_{\Omega} |u|^{\frac{p(x)-2}{2}} u_t |u|^{\frac{p(x)-2}{2}} u dx \\ &\leq \delta\int_{\Omega} |u|^{p(x)-2} u_t^2 dx + \delta^{-1}\int_{\Omega} |u|^{p(x)} dx. \end{aligned}$$

In conclusion, we have

$$\begin{aligned} G'(t) &\geq (1-\gamma\delta)\int_{\Omega} |u|^{p(x)-2} u_t^2 dx + \|u_t\|_2^2 - \gamma T_{q(.)}(|\nabla\Delta u|) \\ &\quad + \gamma b T_{q(.)}(u) - \gamma a\delta^{-1}\int_{\Omega} |u|^{p(x)} dx. \end{aligned} \tag{4.6}$$

Consequently

$$\begin{aligned} G^{'}(t) &\geq (1-\gamma\delta)\int\limits_{\Omega}|u|^{p(x)-2}u_t^2dx+\|u_t\|_2^2-\gamma a\delta^{-1}\int\limits_{\Omega}|u|^{p(x)}dx \\ &+\gamma(1-\mu)r_1H(t)+\gamma b\mu T_{r(.)}(u)+\gamma\left((1-\mu)\frac{r_1}{q_2}-1\right)T_{q(.)}(|\nabla\Delta u|), \end{aligned}$$

where μ is a constant satisfying

$$0<\mu\leq 1-\frac{q_2}{r_1}.$$

Additionally, by utilizing (3.7) we deduce

$$\begin{aligned} G^{'}(t) &\geq (1-\gamma\delta)\int\limits_{\Omega}|u|^{p(x)-2}u_t^2dx+\|u_t\|_2^2-\gamma a\delta^{-1}\int\limits_{\Omega}|u|^{p(x)}dx \\ &+\gamma(1-\mu)r_1H(t)+\gamma\left(b\mu+1-\mu-\frac{q_2}{r_1}\right)T_{r(.)}(u). \end{aligned} \tag{4.7}$$

Subsequently, in accordance with Lemma 7 and (3.9), the expression (4.7) transforms

$$\begin{aligned} G^{'}(t) &\geq (1-\gamma\delta)\int\limits_{\Omega}|u|^{p(x)-2}u_t^2dx+\|u_t\|_2^2-\gamma ka\delta^{-1}\left(\|u\|_{r_1}^{r_1}+H(t)\right) \\ &+\gamma(1-\mu)r_1H(t)+\gamma k\left(b\mu+1-\mu-\frac{q_2}{r_1}\right)\|u\|_{r_1}^{r_1}. \end{aligned} \tag{4.8}$$

As a result,

$$\begin{aligned} G^{'}(t) &\geq (1-\gamma\delta)\int\limits_{\Omega}|u|^{p(x)-2}u_t^2dx+\|u_t\|_2^2+\gamma\left((1-\mu)r_1-ka\delta^{-1}\right)H(t) \\ &+\gamma\left(k\left(b\mu+1-\mu-\frac{q_2}{r_1}\right)-ka\delta^{-1}\right)\|u\|_{r_1}^{r_1}. \end{aligned} \tag{4.9}$$

Thus, by selecting a sufficiently large value for δ and a small enough value for γ we can find positive constants $\lambda_1,\lambda_2>0$, such that

$$G^{'}(t)\geq\lambda_1H(t)+\lambda_2\|u\|_{r_1}^{r_1}\geq K_1\left(\|u\|_{r_1}^{r_1}+H(t)\right), \tag{4.10}$$

and

$$G(0)=H(0)+\gamma L(0).$$

Likewise, in equation (4.1), we obtain

$$\|u\|_2^2\leq k\left(\|u\|_{r_1}^{r_1}+H(t)\right), \tag{4.11}$$

on the contrary, utilizing (4.11), we arrive at

$$G(t)\leq K_2\left(\|u\|_{r_1}^{r_1}+H(t)\right), \tag{4.12}$$

when combining the expressions (4.12) and (4.10), we get

$$G^{'}(t)\geq\psi G(t). \tag{4.13}$$

In conclusion, a simple integration of (4.13) we obtain

$$G(t)\geq G(0)e^{\psi t},\quad\forall t\geq 0. \tag{4.14}$$

This concludes the proof. □

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