



## **$(-1, 1)$ ring of degree five satisfying the identity $(x, yz, w) = y(x, z, w)$**

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**ABSTRACT:** In this paper we describe  $(-1, 1)$  ring of degree-5. We derive the condition for associativity of a third power associative  $(-1, 1)$  ring of degree five satisfying the identity  $(x, yz, w) = y(x, z, w)$ . The ring is also associative even when we induce the condition of the semiprimeness.

**Key Words:** Nonassociative ring, semiprime ring, associative of degree five, antiflexible ring.

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### **1. Introduction**

A nonassociative ring  $R$  is called antiflexible if  $(x, y, xz) = x(x, y, z)$  holds for all  $x, y, z$  in  $R$ , where the associator is defined by  $(a, b, c) = (ab)c - a(bc)$ . A number of properties of antiflexible rings are given by Kosier in [6], by Rodabugh in [7], simple antiflexible rings are characterized to some extent by Anderson and Outcalt in [1]. In [3] Celik has proved that a prime antiflexible ring is either associative or the center is equal to the nucleus. In this paper, we study third power associative, antiflexible rings satisfying the identity  $(x, y, xz) = x(x, y, z)$  to show that the ring is associative of degree five.

### **2. Main Results**

A ring  $R$  is said to be  $(-1, 1)$  if satisfies the following two identities.

$$(x, y, z) + (x, z, y) = 0. \quad (2.1)$$

and

$$(x, y, z) + (y, z, x) + (z, x, y) = 0. \quad (2.2)$$

We first derive the identities which are useful to prove the results. We assume that  $R$  satisfies the following conditions

$$(x, [y, z], w) = 0. \quad (2.3)$$

$$(x, yz, w) = y(x, z, w). \quad (2.4)$$

$$(x, yz, w) + (x, wz, y) = (x, z, w)y + (x, z, y)w. \quad (2.5)$$

$$[x, (y, z, x)] + [x, (z, y, x)] = 0. \quad (2.6)$$

$$(x, yz, w) = y(x, z, w). \quad (2.7)$$

and

$$(x, y^2, z) = (x, y, yz + zy). \quad (2.8)$$

Identity (1.6) on rotation yields  $[x, (y, z, x)] = [x, (z, x, y)]$  and  $[x, (x, y, z)] = [x, (y, z, x)]$ . We commute (1.2) with  $x$  on the left side to get  $[x, (y, z, x)] + [x, (z, x, y)] + [x, (x, y, z)] = 0$  and use the above identities to obtain  $3[x, (z, x, y)] = 0$ . Since  $R$  is 3-torsion free, we have

$$[x, (x, z, y)] = 0. \quad (2.9)$$

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From (1.1) and (1.3), we get  $-(x, w, [y, z]) = 0$ . By plugging  $w = -x$ , we obtain

$$(x, x, [y, z]) = 0. \quad (2.10)$$

The Teichmuller identity holds in any nonassociative ring

$$(xy, z, w) - (x, yz, w) + (x, y, zw) = x(y, z, w) + (x, y, z)w. \quad (2.11)$$

From (1.11), we have  $(xx, x, y) - (x, xx, y) + (x, x, xy) = x(x, x, y)$ . To this we apply (1.4) and (1.7) to obtain

$$(xx, x, y) = (x, xx, y). \quad (2.12)$$

Now by multiplying  $x$  to (1.2) and with  $z = x$  we get,  $0 = x(x, y, x) + x(y, x, x) + x(x, x, y)$ . Using (1.1), (1.7), we get  $0 = (x, xy, x) + (x, xx, y)$ . using (1.3) and (1.7), we get  $0 = y(x, x, x) + (x, xx, y)$ . Thus,

$$(x, xx, y) = 0. \quad (2.13)$$

Put  $w = z = x$ , in (1.5), and applying (1.12) we see that  $(x, x, yx) = (x, y, x)x$  that is,  $-(x, yx, x) = -(x, x, y)x$ , from (1.1). Therefore, (1.3) and (1.1) together imply  $(x, xy, x) = (x, x, y)x = -(x, xy, x)$ . Thus,  $2(x, x, xy) = 0$  and hence,  $(x, x, xy) = 0$ . Using (1.4) and (1.9), we get

$$x(x, x, y) = 0. \quad (2.14)$$

Linearization of (1.14) results in  $x(x, y, z) + x(z, y, x) + z(x, y, x) + z(z, y, x) + z(x, y, z) + x(z, y, z) = 0$ . Since this is a nonhomogeneous identity, we have  $x(x, y, z) + x(z, y, x) + z(x, y, x) = 0$ . Using (1) and (7), we get  $x(x, y, z) - (z, x^2, y) + z(x, y, x) = 0$ . Now by applying (1.8), (1.3) and (1.1), we get  $x(x, y, z) + 2(z, yx, x) + z(x, y, x) = 0$ . Using (1.7) and (1.1), we get

$$x(x, y, z) = z(x, x, y). \quad (2.15)$$

From identities (1.14) and (1.7), we get  $(x, x, xy) = 0$ . Linearization of this implies  $(z, x, xy) + (x, z, xy) + (x, x, zy) + (x, z, zy) + (z, x, zy) + (z, z, xy) = 0$ . Since this is nonhomogeneous identity, we have  $(z, x, xy) + (x, z, xy) + (x, x, zy) = 0$ . Using (1.1), (1.3) and (1.7), we get  $(x, z, xy) + (x, x, zy) = 0$ . which implies that  $(x, z, xy) = -(x, x, zy)$ . Therefore from (1.1) and (1.3), we get  $(x, xy, z) = (x, x, yz)$ . Again from (1.3) and (1.10) we see that

$$(x, xy, z) = (x, x, yz) = (x, x, zy). \quad (2.16)$$

From (1.11), we have  $(xx, y, x) - (x, xy, x) + (x, x, yx) = x(x, y, x) + (x, x, y)x$ . Applying (1.1) we get  $(xx, y, x) + (x, x, xy) + (x, x, yx) = x(x, y, x) + (x, x, y)x$ . That is,  $(xx, y, x) + (x, x, y)x + (x, x, yx) = x(x, y, x) + (x, x, y)x$ , from (1.4), which implies that,  $-(xx, x, y) + (x, x, yx) = x(x, y, x)$ . applying (1.1), (1.12) and (1.13)  $(x, xx, y) = x(x, y, x)$ . using (1.7) and (1.1)  $x(x, x, y) = -x(x, x, y)$ . Thus, from (1.3) we obtain  $(x, x, y)x = -(x, x, y)x$ . Hence  $2[(x, x, y)x] = 0$  and thereby resulting in

$$(x, x, y)x = 0. \quad (2.17)$$

Now from (1.1) and (1.17) we have  $(x, y, x)x = 0$ . Linearization of this implies  $(z, y, x)x + (x, y, z)x + (x, y, x)z + (x, y, z)z + (z, y, z)x + (z, y, x)z = 0$ . Since this is nonhomogeneous identity we have  $(z, y, x)x + (x, y, z)x + (x, y, x)z = 0$ . Using (1.9), (1.7), we get  $y(z, x, x) + (x, y, z)x + (x, y, x)z = 0$ . Using (1.3) and (1.7) we obtain  $y(z, x, x) + (x, y, z)x + (x, y, x)z = 0$ . And thus, from (1.1) we have  $(x, y, x)z = -(x, y, z)x$ . using (1.1) and (1.9), we get  $x(x, z, y) = 0$ . Applying (1.7) and (1.3), we obtain  $(x, zx, y) = 0$ . Applying (1.7)  $z(x, x, y) = 0$ . From (1.1) we see that  $z(x, x, y) = -(x, x, y)z$ . Hence  $2z(x, x, y) = 0$  and therefore, from (1.9) we get

$$(x, x, y)z = 0. \quad (2.18)$$

We begin with the following Lemmas of identities:

**Lemma 2.1** *The identity  $(x, zw, y) = (z, xy, w)$  holds in  $R$ .*

Proof: From (1.15), we have  $x(x, y, w) = w(x, x, y)$ . using (1.9), we get  $(x, xy, w) = (x, wx, y)$ . Linearization of this implies  $(x, zy, w) + (z, xy, w) = (x, wz, y) + (z, wx, y)$ . That is,  $z(x, y, w) + (z, xy, w) = (x, wz, y) + (z, wx, y)$  using (1.7) and (1.3), we get  $(x, zy, w) + (z, xy, w) = (x, zw, y) + (z, xw, y)$ . using (1.7) and (1.1), we obtain  $2z(x, y, w) + (z, xy, w) = -x(z, y, w) = -(z, xy, w)$  from (1.7). That is,  $2z(x, y, w) = 2(z, xy, w)$ . To the left hand side we apply (1.1) to get  $2z(x, y, w) = 2(z, xy, w)$ . which is equivalent to  $(x, zw, y) = (z, xy, w)$ . This finishes the proof of the Lemma 2. 1.

**Lemma 2.2** *The identity  $x(z, y, w) = y(z, x, w)$  holds in  $R$ .*

Proof : From (1.11), we have

$$(z, xy, z) = (zx, y, z) + (z, x, yz) - z(x, y, z) - (z, x, y)z \quad (2.19)$$

$$(z, yz, x) = (zy, z, x) + (z, y, zx) - z(y, z, x) - (z, y, z)x \quad (2.20)$$

and

$$(z, zx, y) = (zz, x, y) + (z, z, xy) - z(z, x, y) - (z, z, x)y. \quad (2.21)$$

Adding (1.19), (1.20) and (1.21) and then using (1.2), we get

$(z, xy, z) + (z, yz, x) + (z, zx, y) = (zx, y, z) + (zy, z, x) + (zz, x, y) + (z, x, yz) + (z, y, zx) + (z, z, xy) - [z(x, y, z) + z(y, z, x) + z(z, x, y)] - (z, x, y)z - (z, y, z)x - (z, z, x)y$ . From (1.1) this is equivalent to  $(z, xy, z) + (z, zy, x) + (z, zx, y) = (zx, y, z) + (zy, z, x) + (zz, x, y) + (z, x, yz) + (z, y, zx) + (z, z, xy) - (z, x, y)z - (z, y, z)x - (z, z, x)y$ . Using (1.16) and (1.14), we get  $(z, z, xy) = (zx, y, z) + (z, x, yz) - (z, x, y)z + (zy, z, x) + (z, y, zx) - (z, y, z)x + (zz, x, y) + (z, z, xy) - (z, z, x)y$ . From (1.14), we derive  $(zx, y, z) + (zy, z, x) + (zz, x, y) = 0$ . By applying (1.2), we obtain  $-(y, z, zx) - (z, zx, y) - (z, x, zy) - (x, zy, z) - (x, y, zz) - (y, zz, x) = 0$ . Using (1.9), (1.4) and (1.1), we obtain  $z(y, x, z) - (z, zx, y) - (z, x, zy) - y(x, z, z) - (x, y, zz) - (y, zz, x) = 0$ . Using (1.7) and (1.3), we have  $x(y, z, z) - (z, zx, y) - (z, x, zy) - y(x, z, z) - (x, y, zz) - (y, zz, x) = 0$ .

Using (1.7),  $-z(z, x, y) - y(z, x, z) - z(x, y, z) - z(y, z, x) = 0$ . Applying (1.2), we get  $-y(z, x, z) = 0$ . That is,  $y(z, x, z) = 0$ . Linearization of this results in  $y(z, w, x) + y(w, z, x) = (z, yw, x) + (w, zy, x) = 0$ , by using (1.7). Thus, we have  $(z, yw, x) = -(w, zy, x)$ . Using (Lemma 2.1), we get  $-(z, wx, y)$ . Now by applying (1.3), we get  $-(z, xw, y) = 0$ . Applying (1.7) and (1.1) to the above expression we get  $y(z, x, w) = -x(z, w, y) = x(z, y, w)$ . This finishes the proof of Lemma 2.2.

**Lemma 2.3** *The identity  $(x, y, wz) = (y, x, zw)$  holds in  $R$ .*

Proof: From identity (1.11), we have  $(x^2, y, w) - (x, xy, w) + (x, x, yw) = x(x, y, w) + (x, x, y)w$ . Using (1.16), we get  $(x^2, y, w) - (x, x, yw) + (x, x, yw) = x(x, y, w) + (x, x, y)w$  which implies that  $(x^2, y, w) = x(x, y, w) + (x, x, y)w$ , this can be written as  $x(x, y, w) + (x, x, y)w = (x^2, y, w) = (xx, y, w)$ . Applying (1.2) to the right hand side, we get  $x(x, y, w) + (x, x, y)w = -(y, w, xx) - (w, xx, y)$ . Applying (1.1) and (1.8) we get  $-x(y, w, x) + (w, xy, x) = 0$ . Now by applying (1.8) and (1.3), we have  $-(y, wx, x) + y(w, x, x) = 0$ . Applying (1.3), we get  $-w(y, x, x) + y(w, x, x) = 0$ . Now applying (1.8) and (1.9), we get  $(x, xy, w) + w(x, x, y) = 0$ . Using (1.7) and (1.3) we get  $(x, xy, w) + (x, xw, y) = 0$ . Applying Lemma 1.1 and (7), we have  $(x, xy, w) + x(x, y, w) = 0$ . Linearization of this implies  $x(z, y, w) + z(x, y, w) + (z, y, w)x + (x, y, w)z = 0$ . Hence  $x(z, y, w) + z(x, y, w) = 0$ . That is  $z(x, y, w) = -x(z, y, w)$ . Applying (1.7) and Lemma 2.1, we get  $z(x, y, w) = -(y, zw, x)$ . Hence,  $-z(x, w, y) = (y, x, zw)$  from (1.3), (1.7), and (1.1) implies  $(y, x, zw) = (x, y, wz)$ .

This completes the proof of Lemma 2.3.

**Lemma 2.4** *The following identities hold in  $R$*

(i)  $(x, y[z, w], v) = 0$ .

(ii)  $(x, [y, z]w, v) = 0$ .

Proof. (i) From Lemma (2.2), we have  $(x, y[z, w], v) = (y, xv, [z, w])$ .

Using (1.1), we obtain

$-(y, xv, [z, w]) = 0$ . Using (1.3), we get  $(x, y[z, w], v) = 0$ .

This completes the proof of Lemma 2.4 (i). (ii)  $(x, [y, z]w, v) = 0$ . From Lemma 2.4 (i) and (2.3), it follows that  $(x, [y, z]w, v) = 0$ . This completes the proof of Lemma 2.4 (ii).

**Lemma 2.5** *The identity  $(x, (y, z, w), v) = 0$  holds in  $R$ .*

Proof. From  $[xy, z] - [x, yz] - [x, z]y = (x, y, z) - (x, z, y) + (z, x, y)$ , with (1.1) we have  $(x, [yz, w], v) = (x, y[z, w], v) + (x, [y, w]z, v) - 2(x, (y, w, z), v) + (x, (w, y, z), v)$  by Lemma 2.4 (i), (ii) and (1.3), we get  $= -2(x, (y, z, w), v) + (x, (w, y, z), v) = 0$ . Using (1.2), we get  $-(x, (y, z, w), v) - [(x, (y, z, w), v)] + (x, (w, y, z), v) = 0$ .

Thus, on rotation we obtain  $(x, (y, z, w), v) = (x, (z, w, y), v) = (x, (w, y, z), v)$ .

Therefore, (1.2) and above relations together imply  $3(x, (w, y, z), v) = 0$ .

Since the ring is 3-torsion free we get  $(x, (w, y, z), v) = 0$ .

This completes the proof of Lemma 2.5.

**Lemma 2.6** *The following identities hold in  $R$ : (i)  $(R, R(RR), R) = (R, (RR)R, R) = 0$ .*

*(ii)  $(RR, RR, R) = (R, RR, RR) = 0$ .*

Proof. (i) From (1.13), we have  $(xx, wv, x) = 0$ . Using Lemma 2.3 we get  $(w, (xx), x, v) = 0$ .

Linearization of this implies  $(w, (xx)y + (xx)z + (yy)z + (yy)x + (zz)x + (zz)y + (xy)x + (yx)x + (yz)y + (zy)y + (zx)z + (xz)z + (xy)y + (yx)y + (yz)z + (zy)z + (zx)x + (xz)x + (xy)z + (yz)x + (zx)y + (yx)z + (xz)y + (zy)x, v) = 0$ .

Since this is a nonhomogeneous identity, we have  $(w, (xy)z + (yz)x + (zx)y + (yx)z + (xz)y + (zy)x, v) = 0$ .

Therefore, using Lemmas 2.3 and 2.5 and identity (1.1), we get

$6(w, (xy)z, v) = 0$ . Since  $R$  is a 2, 3-torsion free ring, we have  $(w, (xy)z, v) = 0$ . Using Lemma 2.5, we get  $(w, x(yz), v) = 0$ . This finishes the proof of Lemma 2.6 (i). From Lemma 2.1, we have  $(xy, zw, v) = (z, (xy)v, w)$ . Using Lemma 2.6 (i), we get  $(xy, zw, v) = 0$ . Similarly using Lemmas 2.1 and 2.6 (i), we obtain  $b(x, yz, wv) = 0$ . This finishes the proof of the Lemma 2.6.

**Lemma 2.7** *The following identities hold in  $R$ : (i)  $(x, yz, w)v = 0$ .  $x(y, zw, v) = 0$ .  $(x, y, z(wv)) = z(x, y, wv)$ .*

Proof: Again (1.11), we have  $(x(yz), v, w) - (x, (yz)v, w) + (x, yz, vw) = x(yz, v, w) + (x, yz, v)w$ .

That is  $(x(yz), v, w) = (yz, v, w) + (x, yz, v)w$  from Lemma 1.6(i) which from (1.2), equivalent to

$-(v, w, x(yz)) - (w, x(yz), v) = x(yz, v, w) + (x, yz, v)w$ . Hence, from (1.1) and Lemma 2.6 (i) we see that  $x(yz, v, w) + (x, yz, v)w = 0$ . Thus,  $w(x, yz, v) + w(yz, v, x) = 0$ .

This from (1.2) implies that  $-w(v, x, yz) = 0$ . Using (1.7), we get  $-(v, xw, yz) = 0$ .

To this if we apply Lemma 2.1 we get  $-(x, v(yz), w) = 0$ . from (1.7) and (1.1), we have  $v(x, w, yz) = 0$ .

From (1.1) and (1.9) we get  $(x, yz, w)v = 0$ .

From the above equation we have  $(y, x(zw), v) = x(y, zw, v) + (y, zw, v)x$ . Using Lemmas 1.6(i) and 1.7(i), we get  $x(y, zw, v) = 0$ .

From (1.7), we have  $(x, zy, (wv)) = z(x, y, (wv))$ . Using (1.1), we get  $(x, y, z(wv)) = -(x, z(wv), y)$ .

Using (1.7) and (1.1), we obtain  $(x, y, z(wv)) = z(x, y, wv)$ . This finishes the proof of Lemma 2.7.

**Lemma 2.8** *The identity  $(x, y, z)w^2 = 0$  holds in  $R$ .*

Proof: Using (1.2), we have  $(w^2, x, y) + (x, y, w^2) + (y, w^2, x) = 0$ .

This implies  $z(w^2, x, y) + z(x, y, w^2) + z(y, w^2, x) = 0$ .

Using Lemma 2.7(ii), we get  $z(w^2, x, y) + z(x, y, w^2) = 0$ .

That is,  $z(w^2, x, y) = -z(x, y, w^2)$ .

Using (1.1), (1.7) and (4), we have  $z(w^2, x, y) = (x, w^2z, y)$ .

By Lemma 2.2,  $z(w^2, x, y) = (w^2, xy, z)$ . Using (7)  $z(w^2, x, y) = x(w^2, y, z)$ .

Using (1.1), (1.7), (1.3) and (1.7), we have  $z(w^2, x, y) = -z(w^2, x, y)$ . Therefore,  $2(z(w^2, x, y)) = 0$  and hence,  $z(w^2, x, y) = 0$ .

Using (1.7), (1.3) and Lemma 2.1, we obtain  $(x, w^2y, z) = 0$ . Applying (1.7), (1.1) and (1.7)  $-(x, w^2z, y) = 0$ . Using (1.3) (1.7) and (1.1), we obtain

$$z(x, y, w^2) = 0. \quad (2.22)$$

Using Lemma 2.7(iii) and (1.7), we obtain

$$(x, y, z(w^2)) = 0. \quad (2.23)$$

From (1.11), we have  $(xy, z, w^2) - (x, yz, w^2) + (x, y, zw^2) = x(y, z, w^2) + (x, y, z)w^2$ .  
Using Lemma 2.6, identities (1.22) and (1.23), we get  $(x, y, z)w^2 = 0$ .  
This finishes the proof of Lemma 2.8.

**Lemma 2.9**  $(R, R, R)$  is an ideal.

Proof. Using (1.8) and (1.15), we have  $(x, y, xz) = x(x, y, z)$ . using (1.1), (1.7) and (1.3), we have  $-(x, zx, y) = 0$ .

Therefore, (1.7) and (1.1), together imply  $(x, y, xz) = z(x, y, x)$ . Linearization of this yields  $(x, y, wz) + (w, y, xz) = z(w, y, x) + z(x, y, w)$ . Apply (1.7), Lemma 2.1, (1.3) and (1.1), we get  $(x, y, wz) + w, y, xz = z(w, y, x) - w(y, z, x)$ . now by applying (1.7) and (1.3)  $(x, y, wz) + (w, y, xz) = z(w, y, x) + z(y, x, w)$ . Thus, from (1.2) we see that  $(x, y, wz) + (w, y, xz) = -z(x, w, y)$ . Applying (1.1) to this, we obtain

$$(x, y, wz) + (w, y, xz) = z(x, y, w). \quad (2.24)$$

Thus,  $(R, R, R)$  is a left ideal. Therefore, from (1.11) it follows that  $(R, R, R)$  is a two-sided ideal.

**Lemma 2.10** The identity  $(x, y, (z, w, v)) = 0$  holds in  $R$ .

Proof. Linearization of identity (1.22) implies

$$(x, y, z(wv + vw)) = 0. \quad (2.25)$$

Using Lemma 3 and identity (1.23), we obtain

$$(x, y, (wv)z) = 0. \quad (2.26)$$

Linearization of (1.25) implies

$$(x, y, (wv + vw)z) = 0. \quad (2.27)$$

Now from (1.3), and using Lemma 2.6 (i), we obtain  $(x, y, (zw)v + v(zw)) + (y, (zw)v + v(zw), x) + ((zw)v + v(zw), x, y) = 0$ . This implies  $(x, (zw)v + v(zw), y) = 0$ . Using (1.1), we get  $(x, y, (z, w, v)) = (x, y, (zw)v - z(wv))$ . Using (1.26), we have

$$(x, y, (z, w, v)) = (x, y, -v(zw) + (vw)z).$$

Using (1.25) and (1.27), we get  $(x, y, (z, w, v)) = -(x, y, (v, w, z))$ . Using (1.1), we get

$$(x, y, (zw, v)) = (x, y, (v, z, w)). \text{ On rotation, we see that } (x, y, (v, z, w)) = (x, y, (w, v, z)).$$

Similarly,  $(x, y, (w, v, z)) = (x, y, (z, w, v))$ . Applying this to (1.2), we get

$$(x, y, (z, w, v)) + (x, y, (w, v, z)) + (x, y, (v, w, z)) = 0.$$

This implies,  $3(x, y, (z, v, w)) = 0$ .

Since  $R$  is 3-torsion free, we have  $(x, y, (z, v, w)) = 0$ .

This completes the proof of Lemma 2.10.

**Lemma 2.11** The identity  $(x, y, (zw)v) = 0$  holds in  $R$ .

Proof. Using (1.26), we have  $(x, y, (zw)v) = -(x, y, (wz)v)$ . Using Lemma 2.6, we have

$$(x, y, (zw)v) = -(x, y, w(zv)) = (x, y, w(vz)) = -(x, y, (vz)w) \text{ from (1.26) and (1.27). We now apply (1.28) thrice, to get}$$

$$(x, y, (zw)v) = (x, y, (vz)w) = (x, y, (wv)z) = -(x, y, (zw)v). \text{ Therefore, we obtain } 2(x, y, (zw)v) = 0.$$

Since  $R$  is a 2-torsion free ring, we have  $(x, y, (zw)v) = 0$ .

This completes the proof of the Lemma 2.11. We now prove the main theorem of this section.

**Theorem 2.1** Products of degree five are associative.

From (1.11), we have  $(x, y, z)w = (xy, z, w) - (x, yz, w) + (x, y, zw) - x(y, z, w)$ . Applying (1.7), we get

$$(x, y, z)w = (xy, z, w) - (x, yz, w) + (x, y, zw) - (y, z, wx) - (w, z, yx). \quad (2.28)$$

From (1.24) and (1.29), it follows that to prove that products of degree five are associative it is sufficient to prove the associator  $(1, 1, 3)$ ,  $(1, 3, 1)$ ,  $(3, 1, 1)$ ,  $(2, 2, 1)$ ,  $(2, 1, 2)$ ,  $(1, 2, 2)$  are zero where the number represents the degree of the terms.

We have already proved these associators are zero in Lemmas 2.6, 2.10, 2.11 and identity (1.1). This completes the proof of the Theorem.

**Corollary 2.1** *Let  $R$  be semiprime, then  $R$  is associative.*

Proof. In Lemma 2.9, we have already proved that  $(R, R, R)$  is an ideal. Now since any product of degree five (or more) containing an associator is zero, we have  $I^2 = 0$ . Since  $R$  is semiprime  $I = 0$ . This shows that  $R$  is associative.

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