Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2026 (44)** : 1–11.
ISSN-0037-8712
doi:10.5269/bspm.70943

Saigo Fractional q-Integral Operator for Product of General Class of q-Polynomial and Generalized q-Mittag-Leffler Function

Mulugeta Dawud Ali, Daya Lal Suthar and Sunil Dutt Purohit

ABSTRACT: We establish new image formulas for the Saigo fractional q-integral operator acting on a product of a general class of q-polynomials and a generalized q-analogue of the Mittag-Leffler function. The argument of the function involves the expression $x^{\rho}\left(xq^{-\lambda}+\xi\right)^{-\sigma}$. The findings are comprehensive and unifying, naturally specializing to yield corresponding results for the Weyl q-integral operator, Kober q-integral operator, and Riemann–Liouville q-integral operator.

Keywords: Saigo fractional q-integral operators, q-Mittag-Leffler function, general class of q-polynomial, q-binomial function, q-hypergeometric function.

Contents

1 Introduction and Preliminaries 1
2 Saigo's Fractional q-Integrals and q-Derivatives 3
3 Main Results 5
4 Special Cases and Concluding Observations 8

1. Introduction and Preliminaries

The computations of fractional order q-derivatives and q-integrals of special functions of single or several variables is significant since the results can be used to assess q-integrals and solve q-differential and q-integral equations. Physical sciences, Engineering, and Applied mathematics-such as Computational complexity, Lie theory, Quantum field theory, Partition theory, Number theory, and so forth-all utilize a familiarity with fractional q-hypergeometric functions and q-calculus of one or several variables. Quantum difference operators play a crucial role in mathematics since they are used in various fields, such in basic hypergeometric functions, orthogonal polynomials, calculus of variations, combinatorics, mechanics theory and fundamental relativity [6]. Numerous essential ideas in quantum calculus are explored in Kac and Cheung's book [9]. Recent decades have seen a great deal of research on q-calculus, and some new findings are presented in [3,18,25] and other sources listed therein.

Fractional calculus has shown to be a useful tool for interpreting various phenomena in kinematics, biology, chemistry, economics, and other fields over the past 200 years (see [11]). Further confirmation that q-calculus is a useful tool for solving discrete variants of continuous scientific problems can be found in [9].

For a real or complex number a and $|q|<1$, the q-shifted factorial in q-series theory (see [7]) is defined as follows:

$$(a;q)_0=1,\qquad (a;q)_n=\prod_{k=0}^{\infty}\left(1-aq^k\right),\qquad (n\in\mathbb{N}). \tag{1.1}$$

Equivalently

$$(a;q)_n=\frac{\Gamma_q(a+n)}{\Gamma_q(a)}\,(1-q)^n. \tag{1.2}$$

Defined by Gasper and Rahman [7], the q-gamma function is

$$\Gamma_q\left(\delta\right)=\frac{(q;q)_\infty}{(q^\delta;q)_\infty}\,(1-q)^{1-\delta}=\frac{(q;q)_{\delta-1}}{(1-q)^{\delta-1}},\quad (\delta\neq 0,-1,-2,...). \tag{1.3}$$

2020 *Mathematics Subject Classification:* Primary 26A33, 33E12; secondary 33D15, 33E20.
Submitted January 10, 2024. Published March 19, 2026

Moreover the power function $(y{\pm}z)^m$ has a q-analogue which is defined (see [18]) by

$$\begin{aligned}(y \pm z)^{(m)} &= (y \pm z)_m = y^n\left(\mp z/y;q\right)_m \\ &= y^m \textstyle\sum_{k=0}^{m} \begin{bmatrix} m \\ k \end{bmatrix}_q q^{k(k-1)/2}\left(\pm z/y\right)^k \quad , \quad (m \in \mathbb{N}),\end{aligned} \tag{1.4}$$

where the definition of the q-binomial function is

$$\begin{bmatrix} m \\ k \end{bmatrix}_q = \frac{(q^{-m};q)_k}{(q;q)_k}\left(-q^m\right)^k q^{-k(k-1)/2}. \tag{1.5}$$

As stated by Gasper and Rahman [7], the generalized basic hypergeometric series is given by

$${}_r\varphi_s\left[\begin{array}{c} a_1,...,a_r \\ b_1,...,b_s \end{array};q,x\right] = \sum_{n=0}^{\infty}\frac{(a_1,...,a_r;q)_n}{(q,b_1,...,b_s;q)_n}x^n\left\{(-1)^n q^{(n(n-1)/2)}\right\}^{(1+s-r)}, \tag{1.6}$$

where for convergence condition, we have $|q|<1$ and $|x|<1$ if $r=s+1$, and for any x if $r \leq s$. ${}_2\phi_1\,(.)$ is the abnormal type of generalized basic hypergeometric series and it is defined as

$${}_r\phi_s\left[\begin{array}{c} a_1,...,a_r;\; q,x \\ b_1,...,b_s;\; q^{\lambda} \end{array}\right] = \sum_{n=0}^{\infty}\frac{(a_1,...,a_r;q)_n}{(q,b_1,...,b_s;q)_n}x^n\left\{q^{(\lambda n(n+1)/2)}\right\} \tag{1.7}$$

where $\lambda>0$ and $|q|<1$.

The q-analogue of the Mittag-Leffler function has been studied extensively in the literature for its applications in many areas such as physics, engineering, and finance. For instance, in physics, the q-Mittag-Leffler function can be utilized to model the anomalous diffusion of particles in a medium with a fractal structure. In engineering, the q-Mittag-Leffler function can be utilized to model the viscoelastic behavior of materials. In finance, the q-Mittag-Leffler function can be utilized to model the volatility of stock prices.

In the area of q-theory, some scholars are exploring the q-analogue of Mittag-Leffler function. Rajkovic et al. [19], Purohit and Kalla [15], Sharma and Jain [20] are among the researchers who are pursuing this line of research. Nadeem et al. [13] presented a distinct method of extending q-analogue of the Mittag-Leffler function and examined its features. The q-analogue of the four-parameter generalized q-Mittag-Leffler function was additionally introduced, numerous of its properties were identified, and it was studied by Bairwa et al. [4]. An image formula for the q-analogue of the generalized Mittag-Leffler function was recently established by Ali and Suthar [1] under the Riemann–Liouville fractional q-calculus operators. This function maintains solutions to many physical problems in the form of integral and differential equations as well as fractional order differences.

According to Nadeem et al. [13] q-analogue of the generalized Mittag-Leffler function was recently presented for $\alpha,\tau,\gamma \in \mathbb{C}$, $\Re\left(l\right)>\Re\left(\gamma\right)>0$ and $|q|<1$, by:

$$E_{\alpha,\tau}^{(\gamma,l)}(x;q) = \sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)}{\mathrm{B}_q(\gamma,l-\gamma)}\frac{(q^l;q)_n}{(q;q)_n}\frac{x^n}{\Gamma_q(\alpha n+\tau)} \tag{1.8}$$

where $B_q(.)$ is the q-analogue of beta function. The specific cases of q-analogue of the generalized Mittag-Leffler function are as follows:

1. If we set $l=1$ in (1.8), we obtain

$$E_{\alpha,\tau}^{(\gamma,1)}(x;q) = \sum_{n=0}^{\infty}\frac{(q^{\gamma};q)_n}{(q;q)_n}\frac{x^n}{\Gamma_q(\alpha n+\tau)} = E_{\alpha,\tau}^{\gamma}(x;q), \tag{1.9}$$

which was presented by Sharma and Jain [20].

2. Similarly, letting $\gamma = 1$ in (1.8), we have

$$E_{\alpha,\tau}^{(1,l)}(x;q) = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma_q(\alpha n + \tau)} = e_{\alpha,\tau}(x;q). \tag{1.10}$$

Monsoor [12] presented the of the generalized Mittag-Leffler function that has q-analogue, which is $e_{\alpha,\tau}(x;q)$.

3. Applying $\gamma = \alpha = \tau = 1$ using (1.8), we arrive at

$$E_{1,1}^{(1,l)}(x;q) = \sum_{n=0}^{\infty} \frac{(q^l;q)_n}{(q;q)_n} x^n = {}_1\phi_0(q^l;-;q,x), \tag{1.11}$$

where q-binomial function can be express in function ${}_1\phi_0(q^l;-;q,x) = (1-x)^{-l}$.

According to [24], the set of general class of q-polynomial $f_{r,N}(x;q)$ is defined as follows in terms of a bounded complex series $\{S_{r,q}\}_{r=0}^{\infty}$.

$$f_{r,N}(x;q) = \sum_{j=0}^{[r/N]} \begin{bmatrix} r \\ Nj \end{bmatrix}_q S_{r,q}\, x^j, \qquad (r = 0, 1, 2, ...), \tag{1.12}$$

where N is a positive integer. After suitably specialization the coefficient $S_{r,q}$, $(x;q)$ yields a variety that utilizes recognized q-polynomials as its specific examples.
The theory of particular functions of one or more variables has been investigated by numerous researchers using fractional q-calculus operators, inspired by the potential applications of these functions. We derive the image formula for the Saigo fractional q-integral formulation to the product of general class of polynomial and generalized q-Mittag-Leffler function.

2. Saigo's Fractional q-Integrals and q-Derivatives

Purohit and Yadav [17] recently defined Saigo fractional q-integral operators with the constraint that the parameter η is positive integer or zero. With that constraint, it was difficult to give a fractional derivative definitions. Further, Garg and Chanchalani [8] (see also [21,22,23]) provide the formulations of the Saigo fractional q-integral operators defined in the following manner to avoid these issue.

For real or complex ϑ and η and $\Re(\delta) > 0$, the generalized fractional q-integral operators $I_q^{\delta,\vartheta,\eta}(.)$ and $K_q^{\delta,\vartheta,\eta}(.)$ is given by:

$$I_q^{\delta,\vartheta,\eta} f(x) = \frac{x^{-\vartheta-1}}{\Gamma_q(\delta)} \int_0^x (tq/x;q)_{\delta-1} \sum_{m=0}^{\infty} \frac{(q^{\delta+\vartheta};q)_m\,(q^{-\eta};q)_m}{(q^{\delta};q)_m\,(q;q)_m} (-1)^m q^{(\eta-\vartheta)m} q^{-\binom{m}{2}} \times \left(\frac{t}{x}-1\right)_q^m f(t)\, d_q t, \tag{2.1}$$

and

$$K_q^{\delta,\vartheta,\eta} f(x) = \frac{q^{-\delta(\delta+1)/2-\vartheta}}{\Gamma_q(\delta)} \int_x^{\infty} (x/t;q)_{\delta-1}\, t^{-\vartheta-1} \sum_{m=0}^{\infty} \frac{(q^{\delta+\vartheta};q)_m\,(q^{-\eta};q)_m}{(q^{\delta};q)_m\,(q;q)_m} (-1)^m q^{(\eta-\vartheta)m} \times q^{-\binom{m}{2}} \left(\frac{x}{qt}-1\right)_q^m f\left(tq^{1-\delta}\right) d_q t, \tag{2.2}$$

where $f(x)$ is a real function on $(0,\infty)$ and $0 < |q| < 1$.

The q-integrals of a function $f(t)$ are defined in [7] as follows

$$\int_0^c f(t)\, d_q t = c(1-q)\sum_{k=0}^{\infty} q^k f\left(cq^k\right), \tag{2.3}$$

and

$$\int_c^{\infty} f(t)\, d_q t = c(1-q)\sum_{k=1}^{\infty} q^{-k} f\left(cq^{-k}\right). \tag{2.4}$$

In view of (2.3) and (2.4), the definitions given by (2.1) and (2.2), can be expressed as

$$\begin{aligned} I_q^{\delta,\vartheta,\eta} f(x) = x^{-\vartheta}(1-q)^{\delta} \sum_{m=0}^{\infty} & \frac{\left(q^{\delta+\vartheta};q\right)_m \left(q^{-\eta};q\right)_m}{(q;q)_m} q^{(\eta-\vartheta+1)m} \\ & \times \sum_{k=0}^{\infty} q^k \frac{\left(q^{\delta+m};q\right)_k}{(q;q)_k} f\left(xq^{k+m}\right), \end{aligned} \tag{2.5}$$

and

$$\begin{aligned} K_q^{\delta,\vartheta,\eta} f(x) = x^{-\vartheta} q^{-\delta(\delta+1)/2}(1-q)^{\delta} \sum_{m=0}^{\infty} & \frac{\left(q^{\delta+\vartheta};q\right)_m \left(q^{-\eta};q\right)_m}{(q;q)_m} q^{\eta m} \\ & \times \sum_{k=0}^{\infty} q^{\vartheta k} \frac{\left(q^{\delta+m};q\right)_k}{(q;q)_k} f\left(xq^{-\delta-k-m}\right). \end{aligned} \tag{2.6}$$

Let $\delta \in \mathbb{C}, (n-1 < \Re(\delta) \leq n)$, $n \in \mathbb{N}$ and for a real function $f(x)$ defined on $(0,\infty)$, the q-fractional derivatives of Saigo [17] of order δ is given by;

$$D_q^{\delta,\vartheta,\eta} f(x) = D_q^n I_q^{-\delta+m,-\vartheta-n,\delta+\eta-n} f(x) \tag{2.7}$$

and

$$P_q^{\delta,\vartheta,\eta} f(x) = q^{\delta(\delta+\vartheta)}\left(-q^{-(\delta+\vartheta)} D_q\right)^n K_q^{-\delta+n,-\vartheta-n,\delta+\eta} f(x), \tag{2.8}$$

where η belongs either real or complex and $0 < |q| < 1$; ϑ, $I_q^{\delta,\vartheta,\eta}$ and $K_q^{\delta,\vartheta,\eta}$ are defined in (2.1) and (2.2) respectively.

If ϑ and η are in $\mathbb{C}$ or $\mathbb{R}$, $\Re(\delta) > 0$ and$0 < |q| < 1$, then the fractional q-integrals $I_q^{\delta,\vartheta,\eta}$ and $K_q^{\delta,\vartheta,\eta}$ of the power function $\left(x^{\beta}\right)$ is defined as follows [17]

$$I_q^{\delta,\vartheta,\eta}\left(x^{\beta}\right) = \frac{\Gamma_q(\beta+1)\,\Gamma_q(\beta-\vartheta+\eta+1)}{\Gamma_q(\beta-\vartheta+1)\,\Gamma_q(\beta+\delta+\eta+1)} x^{\beta-\vartheta}, \tag{2.9}$$

with $1+\Re(\beta) > 0$ and $\Re(1+\beta-\vartheta+\eta) > 0$.
and

$$K_q^{\delta,\vartheta,\eta}\left(x^{\beta}\right) = \frac{\Gamma_q(\vartheta-\beta)\,\Gamma_q(\eta-\beta)}{\Gamma_q(-\beta)\,\Gamma_q(\vartheta+\delta-\beta+\eta)} x^{\beta-\vartheta} q^{-\delta\beta-\delta(\delta+1)/2}, \tag{2.10}$$

provided $\Re(\vartheta-\beta) > 0$ and $\Re(\eta-\beta) > 0$.

If ϑ , β and η are in $\mathbb{C}$ or $\mathbb{R}$, $m-1 < \Re(\delta) < m$ and $0 < |q| < 1$, then the fractional q-derivatives $D_q^{\delta,\vartheta,\eta}$ and $P_q^{\delta,\vartheta,\eta}$ of the power function $\left(x^{\beta}\right)$ is defined as follows [17]

$$D_q^{\delta,\vartheta,\eta}\left(x^{\beta}\right) = \frac{\Gamma_q(\beta+1)\,\Gamma_q(\beta+\delta+\vartheta+\eta+1)}{\Gamma_q(\beta+\vartheta+1)\,\Gamma_q(\beta+\eta+1)} x^{\beta+\vartheta}, \tag{2.11}$$

provided $\Re\left(\beta+1\right)>0$and $\Re\left(\beta+\delta+\vartheta+\eta+1\right)>0$.

$$P_q^{\delta,\vartheta,\eta}\left(x^\beta\right)=\frac{\Gamma_q\left(-\vartheta-\beta\right)\Gamma_q\left(\delta+\eta-\beta\right)}{\Gamma_q\left(-\beta\right)\Gamma_q\left(-\vartheta+\eta-\beta\right)}x^{\beta-\vartheta}q^{\delta(\vartheta+\beta)+\delta(\delta-1)/2}, \tag{2.12}$$

provided $\Re\left(-\vartheta-\beta\right)>0$ and $\Re\left(\delta+\eta-\beta\right)>0$.

Furthermore, the defined operator (2.1) and (2.2) can be taken as the generalizations of the fractional q-integral operators of Kober, Weyl and Riemann-Liouville according to the following relations:

$$I_q^{\delta,0,\eta}f(x)=I_q^{\eta,\delta}f(x), \tag{2.13}$$
$$I_q^{\delta,-\delta,\eta}f(x)=I_q^{\delta}f(x), \tag{2.14}$$
$$K_q^{\delta,0,\eta}f(x)=q^{-\delta(\delta+1)/2}K_q^{\eta,\delta}f(x), \tag{2.15}$$
$$K_q^{\delta,-\delta,\eta}f(x)=K_q^{\delta}f(x). \tag{2.16}$$

3. Main Results

In this section, we will assess the following fractional q-integrals involving the generalized q-analogue of Mittag-Leffler function and general class of q-polynomials $f_{r,N}\left(x;q\right)$. The main theorems are as under:

Theorem 3.1 *If $\Re(k+\rho n+1)>0$, $\Re(k+\rho n+1-\vartheta+\eta)>0$, $\left|xq^{-\lambda-\sigma\,n}/\xi\right|<1$, $\min(k,\lambda,\sigma,\rho)>0$, , a x^k-weighted basic binomial function, the generalized q-analog of Mittag-Leffler function (1.8) and general class of q- $f_{r,N}\left(x;q\right)$, then generalized fractional q-integral of Saigo type $I_q^{\delta,\vartheta,\eta}(.)$ is given by*

$$\begin{aligned}
&I_q^{\delta,\vartheta,\eta}\left\{x^k\left(x+\xi\right)^{(-\lambda)}f_{r,N}\left(x;q\right)E_{\alpha,\tau}^{(\gamma,l)}\left(x^\rho\left(xq^{-\lambda}+\xi\right)^{(-\sigma)};q\right)\right\}\\
&=x^{k-\vartheta}\,\xi^{-\lambda}\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r\\ Nj\end{array}\right]_q S_{r,q}\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}\frac{x^{\rho n+j}}{\xi^\sigma}\\
&\times\frac{\Gamma_q\left(k+\rho n+j+1\right)\ \Gamma_q\left(k+\rho n+j+1-\vartheta+\eta\right)}{\Gamma_q\left(k+\rho n+j+1-\vartheta\right)\ \Gamma_q\left(k+\rho n+j+1+\eta+\delta\right)}\\
&\times{}_3\varphi_2\left[\begin{array}{c} q^{\lambda+\sigma n},q^{1+k+\rho n+j},q^{1+k-\vartheta+\eta+\rho n+j};\\ q^{1+k-\vartheta+\rho\,n+j},q^{1+k+\eta+\delta+\rho\,n+j};\end{array}q,-\left(xq^{-\lambda-\sigma\,n}/\xi\right)\right].
\end{aligned}\tag{3.1}$$

where N is a positive integer and $r=0,1,2,\ldots$.

Proof: To proof the generalized fractional q-integral formula (3.1), we expressing the generalized q-Mittag-Leffler functions $E_{\alpha,\tau}^{(\gamma,l)}(.)$ and general class of q-polynomial $f_{r,N}\left(x;q\right)$ occurring its left hand side (say L) in the series form given by equation (1.8) and (1.12), then it yields to

$$\begin{aligned}
L=&I_q^{\delta,\vartheta,\eta}\left\{x^k\left(x+\xi\right)^{(-\lambda)}\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}x^{\rho\,n}\xi^{-\sigma\,n}\left(\frac{-xq^{-\lambda}}{\xi};q\right)_{(-\sigma\,n)}\right.\\
&\left.\times\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r\\ Nj\end{array}\right]_q S_{r,q}\,x^j\right\}.
\end{aligned}$$

By using the q-binomial theorem given by (1.4), we get

$$\begin{aligned}
L=&\xi^{-\lambda}\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}\,\xi^{-\sigma\,n}\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r\\ Nj\end{array}\right]_q S_{r,q}\sum_{m=0}^{\infty}\frac{\left(q^{\lambda+\sigma n};q\right)_m}{(q;q)_m}\\
&\times\left(\frac{-q^{-\lambda-\sigma j}}{\xi}\right)^m I_q^{\delta,\vartheta,\eta}\left\{x^{k+m+\rho n+j}\right\}.
\end{aligned}\tag{3.2}$$

Using (2.9), we have

$$L = \xi^{-\lambda} \sum_{n=0}^{\infty} \frac{\mathrm{B}_q(\gamma+n, l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma, l-\gamma)(q;q)_n \Gamma_q(\alpha n+\tau)} \xi^{-\sigma n} \sum_{j=0}^{[r/N]} \left[\begin{array}{c} r \\ Nj \end{array} \right]_q S_{r,q} \sum_{m=0}^{\infty} \frac{\left(q^{\lambda+\sigma n};q\right)_m}{(q;q)_m}$$
$$\times \left(\frac{-q^{-\lambda-\sigma n}}{\xi}\right)^m \frac{\Gamma_q\left(k+\rho n+m+j+1\right)\Gamma_q\left(k+\rho n-\vartheta+\eta+m+j+1\right)}{\Gamma_q\left(k+\rho n-\vartheta+m+j+1\right)\Gamma_q\left(k+\rho n+\delta+\eta+m+j+1\right)}$$
$$\times x^{k+m+j+\rho n-\vartheta}. \tag{3.3}$$

Rearrangement of parameters and simple calculation, we obtain

$$L = \xi^{-\lambda} x^{k-\vartheta} \sum_{n=0}^{\infty} \frac{\mathrm{B}_q(\gamma+n, l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma, l-\gamma)(q;q)_n \Gamma_q(\alpha n+\tau)} \left(\frac{x^{\rho}}{\xi^{\sigma}}\right)^n \sum_{j=0}^{[r/N]} \left[\begin{array}{c} r \\ Nj \end{array} \right]_q S_{r,q} x^j$$
$$\times \sum_{m=0}^{\infty} \frac{\left(q^{\lambda+\sigma n};q\right)_m}{(q;q)_m} \left(\frac{-q^{-\lambda-\sigma n}}{\xi}\right)^m$$
$$\times \frac{\Gamma_q\left(k+\rho n+j+1\right)\,\Gamma_q\left(k+\rho n-\vartheta+\eta+j+1\right)\left(q^{k+\rho n+j+1},q\right)_m \left(q^{k+\rho n-\vartheta+\eta+j+1},q\right)_m}{\Gamma_q\left(k+\rho n-\vartheta+j+1\right)\Gamma_q\left(k+\rho n+\delta+\eta+j+1\right)\left(q^{k+\rho n-\vartheta+j+1},q\right)_m \left(q^{k+\rho n+\delta+\eta+j+1},q\right)_m}. \tag{3.4}$$

Further using the series expansion of ${}_3\phi_2(.)$ during the course of analysis, we arrive at the required result (3.1). □

If we set $\vartheta = 0$ and $\vartheta = -\delta$, in Theorem 3.1, then we have the following special cases stated as corollaries:

Corollary 3.1 *Consider x^k-weighted basic binomial function. the q-Mittag-Leffler function and general class of q-polynomial $f_{r,N}(x;q)$ defined in (1.8) and (1.12) respectively, then the Kober fractional q-integral $I_q^{\eta,\delta}(.)$ is given by*

$$I_q^{\eta,\delta}\left\{x^k\,(x+\xi)^{(-\lambda)}\,f_{r,N}(x;q)\;E_{\alpha,\tau}^{(\gamma,l)}\left(x^{\rho}\left(xq^{-\lambda}+\xi\right)^{(-\sigma)};q\right)\right\}$$
$$= x^k\,\xi^{-\lambda} \sum_{j=0}^{[r/N]} \left[\begin{array}{c} r \\ Nj \end{array} \right]_q S_{r,q} \sum_{n=0}^{\infty} \frac{\mathrm{B}_q(\gamma+n, l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma, l-\gamma)(q;q)_n \Gamma_q(\alpha n+\tau)} \frac{x^{\rho n+j}}{\xi^{\sigma n}}$$
$$\times \frac{\Gamma_q\left(k+\rho n+j+1+\eta\right)}{\Gamma_q\left(k+\rho n+j+1+\eta+\delta\right)}\,{}_2\varphi_1\left[\begin{array}{c} q^{\lambda+\sigma n}, q^{1+k+\eta+\rho n+j}\,; \\ q^{1+k+\eta+\delta+\rho\,n+j}\,; \end{array} \quad q, -\left(xq^{-\lambda-\sigma\,n}/\xi\right)\right], \tag{3.5}$$

where $\Re(k+\rho n+j+1+\eta) > 0,\ \left|xq^{-\lambda-\sigma\,n}/\xi\right| < 1,\ \min(k,\lambda,\sigma,\rho) > 0.$

Corollary 3.2 *Consider a x^k-weighted basic binomial function, the q-Mittag-Leffler function and and general class of q-polynomial $f_{r,N}\,(x;q)$ defined (1.8) and (1.12), then the fractional q-integral of Riemann-Liouville type $I_q^{\delta}(.)$ is given by*

$$I_q^{\delta}\left\{x^k\,(x+\xi)^{(-\lambda)}\,f_{r,N}\,(x;q)\,E_{\alpha,\tau}^{(\gamma,l)}\left(x^{\rho}\left(xq^{-\lambda}+\xi\right)^{(-\sigma)};q\right)\right\}$$
$$= x^{k+\delta}\,\xi^{-\lambda} \sum_{j=0}^{[r/N]} \left[\begin{array}{c} r \\ Nj \end{array} \right]_q S_{r,q} \sum_{n=0}^{\infty} \frac{\mathrm{B}_q(\gamma+n, l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma, l-\gamma)(q;q)_n \Gamma_q(\alpha n+\tau)} \left(\frac{x^{\rho n+j}}{\xi^{\sigma n}}\right)$$
$$\times \frac{\Gamma_q\left(k+\rho n+j+1\right)}{\Gamma_q\left(k+\rho n+j+1+\delta\right)}\,{}_2\varphi_1\left[\begin{array}{c} q^{\lambda+\sigma n}, q^{1+k+\rho n+j}\,; \\ q^{1+k-\vartheta+\rho\,n+j}\,; \end{array} \quad q, -\left(xq^{-\lambda-\sigma\,n}/\xi\right)\right], \tag{3.6}$$

where $\Re(k+\rho n+j+1) > 0,\ \left|xq^{-\lambda-\sigma\,n}/\xi\right| < 1,\ \min(k,\lambda,\sigma,\rho) > 0.$

Theorem 3.2 *Let $\Re(\vartheta-k-\rho n)>0$, $\Re(\eta-k-\rho n)>0$, $\left|xq^{-\delta-\lambda-\sigma n}/\xi\right|<1$, $\min(k,\lambda,\sigma,\rho)>0$, a x^k-weighted basic binomial function, q-Mittag-Leffler function (1.8) and $f_{r,N}(x;q)$ (1.12), and then general fractional q-integrals of Saigo $K_q^{\delta,\vartheta,\eta}(.)$ is given by*

$$\begin{aligned}
&K_q^{\delta,\vartheta,\eta}\left\{x^k(x+\xi)^{(-\lambda)} f_{r,N}(x;q)\, E_{\alpha,\tau}^{(\gamma,l)}\left(x^\rho\left(xq^{-\lambda}+\xi\right)^{(-\sigma)};q\right)\right\}\\
&=x^{k-\vartheta}q^{-\delta k-\delta(\delta+1)/2}\,\xi^{-\lambda}\sum_{j=0}^{[r/N]}\begin{bmatrix} r\\ Nj\end{bmatrix}_q S_{r,q}\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}x^{\rho n+j}\xi^{-\sigma n}\\
&\times\frac{\Gamma_q(\vartheta-k-\rho n-j)\,\Gamma_q(\eta-k-\rho n-j)}{\Gamma_q(-k-\rho n-j)\,\Gamma_q(\vartheta+\delta-k+\eta-\rho n-j)}\left(\frac{x^{\rho n+j}02}{q^\delta}\right)\\
&\times\ {}_3\varphi_2\left[\begin{matrix} q^{\lambda+\sigma n},\ q^{1+k+\rho n+j},\ q^{1-\vartheta-\delta+k-\eta+\rho n+j}; \\ q^{1-\vartheta+k+\rho n+j},\ q^{1-\eta+k+\rho n+j}\ q;\end{matrix}\quad q,-\left(xq^{-\delta-\lambda-\sigma n}/\xi\right)\right],
\end{aligned}\tag{3.7}$$

Proof: To proof the generalized fractional q-integral formula (3.7), we expressing the general class of q-Mittag-Leffler function $E_{\alpha,\tau}^{(\gamma,l)}$ and the general class of q-polynomial $f_{r,N}(x;q)$ occurring its left hand side (say ℓ) in the series form given by equations (1.8) and (1.12), then it yields to

$$\begin{aligned}
\ell=K_q^{\delta,\vartheta,\eta}&\left\{x^k(x+\xi)^{(-\lambda)}\sum_{j=0}^{[r/N]}\begin{bmatrix} r\\ Nj\end{bmatrix}_q S_{r,q}\,x^j\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}\left(\frac{x^\rho}{\xi^\sigma}\right)^n\right.\\
&\left.\times\left(\frac{-xq^{-\lambda}}{\xi};q\right)_{(-\sigma n)}\right\}.
\end{aligned}$$

By applying the q-binomial theorem given by (1.4), we get

$$\begin{aligned}
\ell=&\xi^{-\lambda}\sum_{j=0}^{[r/N]}\begin{bmatrix} r\\ Nj\end{bmatrix}_q S_{r,q}\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}\ \xi^{-\sigma n}\sum_{m=0}^{\infty}\frac{\left(q^{\lambda+\sigma n};q\right)_m}{(q;q)_m}\\
&\times\left(\frac{-q^{-\lambda-\sigma n}}{\xi}\right)^m\ K_q^{\delta,\vartheta,\eta}\left\{x^{k+m+\rho n+j}\right\}.
\end{aligned}\tag{3.8}$$

Using (2.10), we obtain

$$\begin{aligned}
\ell=&\ \xi^{-\lambda}\sum_{j=0}^{[r/N]}\begin{bmatrix} r\\ Nj\end{bmatrix}_q S_{r,q}\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}\ \xi^{-\sigma n}\\
&\times\sum_{m=0}^{\infty}\frac{\left(q^{\lambda+\sigma n};q\right)_m}{(q;q)_m}\left(\frac{-q^{-\lambda-\sigma n}}{\xi}\right)^m\frac{\Gamma_q(\vartheta-k-\rho n-j-m)\,\Gamma_q(\eta-k-\rho n-j-m)}{\Gamma_q(-k-\rho n-j-m)\,\Gamma_q(\vartheta+\delta-k-\rho n-j+\eta-m)}\\
&\times x^{k+\rho n-j-\vartheta+m}q^{-\delta(k+\rho n+j+m)-\delta(\delta+1)/2}.
\end{aligned}\tag{3.9}$$

Applying the formula $\left(q^{\eta-k-\rho n},q\right)_{-m}=\frac{\Gamma_q(\eta-k-m-\rho n)}{\Gamma_q(\eta-k-\rho n)}(1-q)^{-m}$, we get

$$\begin{aligned}
\ell=&\xi^{-\lambda}\sum_{j=0}^{[r/N]}\begin{bmatrix} r\\ Nj\end{bmatrix}_q S_{r,q}\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}\xi^{-\sigma n}\sum_{m=0}^{\infty}\frac{\left(q^{\lambda+\sigma n};q\right)_m}{(q;q)_m}\left(\frac{-q^{-\lambda-\sigma n}}{\xi}\right)^m\\
&\times\frac{\Gamma_q(\vartheta-k-\rho n-j)\,\Gamma_q(\eta-k-\rho n-j)\,\left(q^{\vartheta-k-\rho n-j},q\right)_{-m}\left(q^{\eta-k-\rho n-j},q\right)_{-m}}{\Gamma_q(-k-\rho n-j)\,\Gamma_q(\vartheta+\delta-k+\eta-\rho n-j)\left(q^{-k-\rho n-j},q\right)_{-m}\left(q^{\vartheta+\delta-k+\eta-\rho n-j},q\right)_{-m}}\\
&\times\ x^{k+m+\rho n+j-\vartheta}q^{\delta(-k-m-\rho n-j)-\delta(\delta+1)/2}.
\end{aligned}\tag{3.10}$$

Further using the series expansion of ${}_3\phi_2(.)$ during the course of analysis, we arrive at the required result (3.7). □

If we set $\vartheta = 0$ and $\vartheta = -\delta$, in Theorem 3.2, then we obtain the following special case stated as corollaries:

Corollary 3.3 *Consider a x^k-weighted basic binomial function, q-Mittag-Leffler function and the general class of q-polynomial defined by (1.8) and (1.12) respectively, and then fractional q-integral of generalized Weyl type $K_q^{\eta,\delta}(.)$ is given by*

$$K_q^{\eta,\delta}\left\{x^k\,(x+\xi)^{(-\lambda)}\,f_{r,N}\,(x;q)\,E_{\alpha,\tau}^{(\gamma,l)}\left(x^\rho\left(xq^{-\lambda}+\xi\right)^{(-\sigma)};q\right)\right\}$$
$$= \xi^{-\lambda}\left(\frac{x}{q^\delta}\right)^k\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r\\ Nj\end{array}\right]_q S_{r,q}\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}\,\xi^{-\sigma n}\left(\frac{x}{q^\delta}\right)^{\rho n}$$
$$\times\frac{\Gamma_q\left(\eta-k-\rho n-j\right)}{\Gamma_q\left(\delta+\eta-k-\rho n-j\right)}{}_2\varphi_1\left[\begin{array}{c} q^{\lambda+\sigma n},q^{1-\eta-\delta+k+\rho n+j};\\ q^{1-\eta+k+\rho n+j}\,;\end{array}\;q,-\left(xq^{-\delta-\lambda-\sigma n}/\xi\right)\right], \tag{3.11}$$

where $\Re\left(\eta-k-\rho n-j\right)>0,\ \left|xq^{-\delta-\lambda-\sigma\,n}/\xi\right|<1,$ *and* $\min(k,\lambda,\sigma,\rho)>0.$

Corollary 3.4 *Consider a x^k-weighted basic binomial function, q-Mittag-Leffler function and the general class of q-polynomial given by (1.8) and (1.12), and then the Weyl fractional q-integral $K_q^\delta(.)$ is given by*

$$K_q^{\delta}\left\{x^k\,(x+\xi)^{(-\lambda)}\,f_{r,N}\,(x;q)\,E_{\alpha,\tau}^{(\gamma,l)}\left(x^\rho\left(xq^{-\lambda}+\xi\right)^{(-\sigma)};q\right)\right\}$$
$$= x^{k+\delta}q^{\delta(-k)-\delta(\delta+1)/2}\,\xi^{-\lambda}\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r\\ Nj\end{array}\right]_q S_{r,q}\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}\,\xi^{-\sigma n}\left(\frac{x}{q^\delta}\right)^{\rho n+j}$$
$$\times\frac{\Gamma_q\left(-\delta-k-\rho n-j\right)}{\Gamma_q\left(-k-\rho n-j\right)}{}_2\varphi_1\left[\begin{array}{c} q^{\lambda+\sigma n},\;q^{1+k+\rho n+j};\\ q^{1+\delta+k+\rho n+j}\,;\end{array}\;q,-\left(xq^{-\delta-\lambda-\sigma n}/\xi\right)\right], \tag{3.12}$$

where $\Re\left(-\delta-k-\rho n-j\right)>0,\ \left|xq^{-\delta-\lambda-\sigma\,n}/\xi\right|<1,\ \min(k,\lambda,\sigma,\rho)>0\ .$

4. Special Cases and Concluding Observations

Here, we take further interesting special cases of Theorems.
If we set $l=1$ in Theorem 3.1 and Theorem 3.2, we obtain the following results respectively:

$$I_q^{\delta,\vartheta,\eta}\left\{x^k\,(x+\xi)^{-\lambda}\,f_{r,N}\,(x;q)\,E_{\alpha,\tau}^{\gamma}\left(x^\rho\left(xq^{-\lambda}+\xi\right)^{-\sigma};q\right)\right\}=x^{k-\vartheta}\,\xi^{-\lambda}\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r\\ Nj\end{array}\right]_q S_{r,q}$$
$$\times\sum_{n=0}^{\infty}\frac{(q^\gamma;q)_n}{\Gamma_q(\alpha n+\tau)}\left(\frac{x^{\rho n+j}}{\xi^{\sigma n}}\right)\frac{\Gamma_q\left(k+\rho n+j+1\right)\ \Gamma_q\left(k+\rho n+j+1-\vartheta+\eta\right)}{\Gamma_q\left(k+\rho n+j+1-\vartheta\right)\ \Gamma_q\left(k+\rho n+j+1+\eta+\delta\right)}$$
$$\times{}_3\varphi_2\left[\begin{array}{c} q^{\lambda+\sigma n},q^{1+k+\rho n+j},q^{1+k-\vartheta+\eta+\rho n+j};\\ q^{1+k-\vartheta+\rho\,n+j},q^{1+k+\eta+\delta+\rho\,n+j};\end{array}\;q,-\left(xq^{-\lambda-\sigma\,n}/\xi\right)\right], \tag{4.1}$$

and

$$K_q^{\delta,\vartheta,\eta}\left\{x^k\,(x+\xi)^{-\lambda}\,f_{r,N}\,(x;q)\,E_{\alpha,\tau}^{\gamma}\left(x^\rho\left(xq^{-\lambda}+\xi\right)^{-\sigma};q\right)\right\}$$
$$=x^{k-\vartheta}q^{-\delta k-\delta(\delta+1)/2}\,\xi^{-\lambda}\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r\\ Nj\end{array}\right]_q S_{r,q}\sum_{n=0}^{\infty}\frac{(q^\gamma;q)_n}{\Gamma_q(\alpha n+\tau)}x^{\rho n+j}\xi^{-\sigma n}$$
$$\times\frac{\Gamma_q\left(\vartheta-k-\rho n-j\right)\Gamma_q\left(\eta-k-\rho n-j\right)}{\Gamma_q\left(-k-\rho n-j\right)\Gamma_q\left(\vartheta+\delta-k+\eta-\rho n-j\right)}\left(\frac{x}{q^\delta}\right)^{\rho n+j}$$
$$\times\ {}_3\varphi_2\left[\begin{array}{c} q^{\lambda+\sigma n},\;q^{1+k+\rho n+j},\;q^{1-\vartheta-\delta+k-\eta+\rho n+j};\\ q^{1-\vartheta+k+\rho n+j},\;q^{1-\eta+k+\rho n+j}\;q;\end{array}\;q,-\left(xq^{-\delta-\lambda-\sigma n}/\xi\right)\right]. \tag{4.2}$$

If we put $\gamma = 1$ in Theorem 3.1 and Theorem 3.2, we get the subsequent results:

$$I_q^{\delta,\vartheta,\eta}\left\{x^k\left(x+\xi\right)^{-\lambda} f_{r,N}\left(x;q\right)e_{\alpha,\tau}\left(x^\rho\left(xq^{-\lambda}+\xi\right)^{-\sigma};q\right)\right\} = x^{k-\vartheta}\,\xi^{-\lambda}\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r \\ Nj \end{array}\right]_q S_{r,q}$$
$$\times\sum_{n=0}^{\infty}\frac{\Gamma_q\left(k+\rho n+j+1\right)\,\Gamma_q\left(k+\rho n+j+1-\vartheta+\eta\right)}{\Gamma_q\left(k+\rho n+j+1-\vartheta\right)\,\Gamma_q\left(k+\rho n+j+1+\eta+\delta\right)\Gamma_q(\alpha n+\tau)}\left(\frac{x^{\rho n+j}}{\xi^{\sigma n}}\right)$$
$$\times\,{}_3\varphi_2\left[\begin{array}{c} q^{\lambda+\sigma n}, q^{1+k+\rho n+j}, q^{1+k-\vartheta+\eta+\rho n+j}; \\ q^{1+k-\vartheta+\rho n+j}, q^{1+k+\eta+\delta+\rho n+j}; \end{array} q,-\left(xq^{-\lambda-\sigma n}/\xi\right)\right], \tag{4.3}$$

and

$$K_q^{\delta,\vartheta,\eta}\left\{x^k\left(x+\xi\right)^{-\lambda} f_{r,N}\left(x;q\right)e_{\alpha,\tau}\left(x^\rho\left(xq^{-\lambda}+\xi\right)^{-\sigma};q\right)\right\}$$
$$= x^{k-\vartheta}q^{-\delta k-\delta(\delta+1)/2}\,\xi^{-\lambda}$$
$$\times\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r \\ Nj \end{array}\right]_q S_{r,q}\sum_{n=0}^{\infty}\frac{\Gamma_q\left(\vartheta-k-\rho n-j\right)\,\Gamma_q\left(\eta-k-\rho n-j\right)}{\Gamma_q\left(-k-\rho n-j\right)\,\Gamma_q\left(\vartheta+\delta-k+\eta-\rho n-j\right)\Gamma_q(\alpha n+\tau)}x^{\rho n+j}\xi^{-\sigma n}$$
$$\times\left(\frac{x}{q^\delta}\right)^{\rho n+j}{}_3\varphi_2\left[\begin{array}{c} q^{\lambda+\sigma n},\ q^{1+k+\rho n+j},\ q^{1-\vartheta-\delta+k-\eta+\rho n+j}; \\ q^{1-\vartheta+k+\rho n+j},\ q^{1-\eta+k+\rho n+j}\ q; \end{array} q,-\left(xq^{-\delta-\lambda-\sigma n}/\xi\right)\right], \tag{4.4}$$

Furthermore, if we take $\sigma = 0$ in the results (3.1),and (3.7) , we deduce the following results:

$$I_q^{\delta,\vartheta,\eta}\left\{x^k\left(x+\xi\right)^{-\lambda} f_{r,N}\left(x;q\right)E_{\alpha,\tau}^{(\gamma,l)}\left(x^\rho;q\right)\right\} = x^{k-\vartheta}\,\xi^{-\lambda}\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r \\ Nj \end{array}\right]_q S_{r,q}$$
$$\times\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}\frac{\Gamma_q\left(k+\rho n+j+1\right)\Gamma_q\left(k-\vartheta+\eta+\rho n+j+1\right)}{\Gamma_q\left(k-\vartheta+\rho n+j+1\right)\Gamma_q\left(k+\eta+\delta+\rho n+j+1\right)}x^{\rho n+j}$$
$$\times\,{}_3\varphi_2\left[\begin{array}{c} q^{\lambda}, q^{1+k+\rho n+j}, q^{1+k-\vartheta+\eta+\rho n+j}; \\ q^{1+k-\vartheta+\rho n+j}, q^{1+k+\eta+\delta+\rho n+j}; \end{array} q,-\left(xq^{-\lambda}/\xi\right)\right], \tag{4.5}$$

and

$$K_q^{\delta,\vartheta,\eta}\left\{x^k\left(x+\xi\right)^{-\lambda} f_{r,N}\left(x;q\right)E_{\alpha,\tau}^{(\gamma,l)}\left(x^\rho;q\right)\right\} = x^{k-\vartheta}q^{-\delta k-\delta(\delta+1)/2}\xi^{-\lambda}\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r \\ Nj \end{array}\right]_q S_{r,q}$$
$$\times\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}\left(\frac{x}{q^\delta}\right)^{\rho n}\frac{\Gamma_q\left(\vartheta-k-\rho n-j\right)\,\Gamma_q\left(\eta-k-\rho n-j\right)}{\Gamma_q\left(-k-\rho n-j\right)\,\Gamma_q\left(\vartheta+\delta-k+\eta-\rho n-j\right)}$$
$$\times\,{}_3\varphi_2\left[\begin{array}{c} q^{\lambda}, q^{1+k+\rho n+j}, q^{1-\vartheta-\delta+k-\eta+\rho n+j}; \\ q^{1-\vartheta+k+\rho n+j}, q^{1-\eta+k+\rho n+j}; \end{array} q,-\left(xq^{-\delta-\lambda}/\xi\right)\right]. \tag{4.6}$$

Again, if we take $\lambda = 0$ in the above results (4.5) and, (4.6), these formulae reduces to

$$I_q^{\delta,\vartheta,\eta}\left\{x^k f_{r,N}\left(x;q\right)E_{\alpha,\tau}^{(\gamma,l)}\left(x^\rho;q\right)\right\} = x^{k-\vartheta}\sum_{j=0}^{[r/N]}\left[\begin{array}{c} r \\ Nj \end{array}\right]_q S_{r,q}\sum_{n=0}^{\infty}\frac{\mathrm{B}_q(\gamma+n,l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma,l-\gamma)(q;q)_n\Gamma_q(\alpha n+\tau)}$$
$$\times\frac{\Gamma_q\left(k+\rho n+j+1\right)\Gamma_q\left(k+\rho n+j+1-\vartheta+\eta\right)}{\Gamma_q\left(k+\rho n+j+1-\vartheta\right)\Gamma_q\left(k+\rho n+j+1+\eta+\delta\right)}\,x^{\rho n+j}, \tag{4.7}$$

and

$$K_q^{\delta,\vartheta,\eta}\left\{x^k f_{r,N}(x;q)\, E_{\alpha,\tau}^{(\gamma,l)}(x^\rho;q)\right\} = x^{k-\vartheta} q^{-\delta k-\delta(\delta+1)/2} \sum_{j=0}^{[r/N]} \begin{bmatrix} r \\ Nj \end{bmatrix}_q S_{r,q}$$
$$\times \sum_{n=0}^{\infty} \frac{\mathrm{B}_q(\gamma+n, l-\gamma)(q^l;q)_n}{\mathrm{B}_q(\gamma, l-\gamma)(q;q)_n \Gamma_q(\alpha n+\tau)} \frac{\Gamma_q(\vartheta-k-\rho n-j)\,\Gamma_q(\eta-k-\rho n-j)}{\Gamma_q(-k-\rho n-j)\,\Gamma_q(\vartheta+\delta-k+\eta-\rho n-j)} \left(\frac{x}{q^\delta}\right)^{\rho n+j}. \tag{4.8}$$

Moreover, for $l=1$, $\sigma=\lambda=0$, $k=\tau-1$, $\alpha=\rho$ and using the identity

$$\lim_{q\to 1}\Gamma_q(x)=\Gamma(x) \quad \textit{and} \quad \lim_{q\to 1}\frac{(q^x;q)_n}{(1-q)^n}=(x)_n$$

Corollary 3.1 leads to the result found in the publications of Chaurasia and Pandey [[5], Theorem 1, p.116].

Acknowledgments

The authors would like to thank referee sincerely for very helpful comments improving the paper. The authors declare that there is no conflict of interest regarding the publication of this paper.

References

1. Ali, M. D., and Suthar, D.L., *On the Riemann–Liouville fractional q-calculus operator involving q-Mittag–Leffler function*, Res. Math. 11(1), 2292549, 1-7, (2024).
2. Ali, M. D., and Suthar, D.L., *Saigo fractional q-integral operator involving generalized q-Mittag-Leffler function.* Advances in Mathematical Sciences and Applications 34(1), 87–100, (2025).
3. Annaby, M. H., and Mansour, Z. S., *q-Fractional Calculus and Equations. Lecture Notes in Mathematics*, vol. 2056. Springer, Heidelberg, (2012).
4. Bairwa, R. K., Kumar, A., and Kumar, D., *Certain properties of generalized q-Mittag–Leffler type function and its application in fractional q-kinetic equation*, Int. J. Appl. Comput. Math. 8:219, 1-13, (2022).
5. Chaurasia, V. B. L., and Pandey, S. C., *On the fractional calculus of generalized Mittag-Leffler function*, J. Math. Sci. 20 113–122, (2010).
6. Ernst, T., *The History of q-Calculus and a New Method*, Department of Mathematics, Uppsala University, Sweden, (2000).
7. Gasper, G., and Rahman, M., *Basic Hypergeometric Series*, Cambridge University Press, Cambridge, (1990).
8. Garg, M., and Chanchlani, L., *q-analogues of Saigo's fractional calculus operators*, Bull. Math. Anal. Appl. 3(4), 169-179, (2011).
9. Kac, V., and Cheung, P., *Quantum Calculus*, Springer-Verlag, New York, (2002).
10. Kumar, D. Ayant, F., Nisar, K. S., and Suthar, D. L., *On fractional q-integral operators involving the basic analogue of multivariable Aleph-function*, Proc. Natl. Acad. Sci. India Sect. A Phys. Sci. 2, 211-218, (2022). https://doi.org/10.1007/s40010-022-00796-7.
11. Miller, K. S. and Ross, B., *An Introduction to the Fractional Calculus and Fractional Differential Equations*, John Wiley and Sons Inc., New York, (1993).
12. Mansour, Z. S. I., *Linear sequential q-difference equations of fractional order*, Fract. Calc. Appl. Anal. 12(2), 159-178, (2009).
13. Nadeem, R., Usman, T., Nisar, K. S. and Abdeljawad, T., *A new generalization of Mittag-Leffler function via q-calculus*, Adv. Differ. Equ., 695-105, (2020). doi.org/10.1186/s13662-02 0-03157-z.
14. Al-Omari, S., Suthar, D. L. and Araci, S., *A fractional q-integral operator associated with a certain class of q-Bessel functions and q-generating series*, Adv. Differ, 441-454, (2021). https://doi.org/10.1186/s13662-021-03594-4.
15. Purohit, S. D., and Kalla, S.L., *On q-analogue of Mittag-Leffler function and its properties*, J. Comput. Appl. 296, 1-11, (2016).
16. Purohit, S. D., Murugusundaramoorthy, G., Kaliappan, V., Suthar, D. L. and Jangid, K., *A unified class of spiral-like functions including Kober fractional operators in quantum calculus* Palest. J. Math. 2, 487–498, (2022).

17. Purohit, S. D., and Yadav, R. K., *On generalized fractional q-integral operators involving the q-Gauss hypergeometric function*, Bull. Math. Anal. Appl. 2(4), 35-44, (2010).

18. Rajković, P. M., Marinkovrć, S. D., Stanković, M. S., *Fractional integrals and derivatives in q-calculus*, Appl. Anal. Discret. 1, 311-323, (2007).

19. Rajkovic, P. M., Marinkovrc, S. D., Stankovic, M. S., *On q-analogues of Caputo derivative and Mittag-Leffler function*, Fract. Calc. Appl. Anal. 10(4), 359-374, (2007).

20. Sharma, S. K., and Jain, R., *On some properties of generalized q-Mittag Leffler function*, Math. Aeterna. 4(6), 613-619, (2014).

21. Shimelis, B., and Suthar, D. L., *Saigo fractional q-integral involving the generalized q-hypergeometric series and a general class of q-polynomials*, Arab Journal of Basic and Applied Sciences 30(1), 691-701, (2023).

22. Shimelis, B., and Suthar, D. L., *On Saigo fractional q-calculus of a general class of q-Polynomials*, Sahand Communications in Mathematical Analysis 21(2) 147-166, (2024). DOI: 10.22130/scma.2023.2001797.1317

23. Shimelis, B., and Suthar, D. L., *Image formulae for the Saigo fractional q-derivative operator with basic hypergeometric series*. Nonlinear Analysis and Computational Techniques: Proceedings of the ICNACT-2024 Conference Held During 8-10 August, 2024, edited by Hemant Kumar Nashine, Ranis Ibragimov and Hemanta Kalita, De Gruyter, 55-72, (2025). https://doi.org/10.1515/9783111724638-004

24. Srivastava, H., and Agarwal, A., *Generating functions for a class of q-polynomials*, Annali di Matematica Pura ed Applicata 154, 99–109, (1989).

25. Younus, A., Asif, M., and Farhad, K., *Interval-valued fractional q-calculus and applications*, Inform. Sci. 569, 241-263, (2021).

Mulugeta Dawud Ali,
Department of Mathematics,
Wollo University, P.O. Box: 1145, Dessie,
Ethiopia.
E-mail address: mulugeta.dawud@wu.edu.et

and

Daya Lal Suthar,
Department of Mathematics,
Wollo University, P.O. Box: 1145, Dessie,
Ethiopia.
E-mail address: dlsuthar@gmail.com

and

Sunil Dutt Purohit,
Department of HEAS (Mathematics),
Rajasthan Technical University, Kota,
India.

E-mail address: sunil_a_purohit@yahoo.com