



A Study on Pairwise Neutrosophic Supra α -Open Set

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ABSTRACT: This paper aims to establish a foundation for the concepts of pairwise neutrosophic supra α -open set and pairwise neutrosophic supra α -continuous function in neutrosophic supra bitopological space. Next, we examine a few of their characteristics. In addition, we develop a few intriguing conclusions—theorems, propositions, etc. based on them.

Key Words: Neutrosophic Set; Neutrosophic Supra Bitopology; Pairwise Neutrosophic Supra α -Open Set.

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1. Introduction

The concept of Fuzzy Set (briefly FS) theory was first introduced by L.A. Zadeh [43] in 1965. Each element has a truth membership value in a FS. However, there are many issues in real life that we face on a daily basis for which there is no truth membership value. K. Atanassov [4] introduced the concept of Intuitionistic FS (briefly IFS) theory to understand these kinds of situations. Every element in an IFS has two memberships: truth membership and false membership. Subsequently, as a generalisation of FS and IFS, Smarandache [38] introduced the notion of Neutrosophic Set (briefly NS). Later on, Smarandache [39] further studied the notion of NS in 2005. Till now, the notion of NS and its extensions has been applied in both theoretical area ([5,16,19,31] as well as practical area [17,18,20,30,34]. Later, in 2012, Salama and Alblowi [35] established the concept of neutrosophic topological space (briefly NTS) by extending the notion of fuzzy topological space [6] and intuitionistic fuzzy topological space [7]. The generalised NS and generalised NTS were also examined by Salama and Alblowi [36]. In 2014, Salama et al. [37] studied the notion of neutrosophic continuous function and investigated on neutrosophic closed set via NTS. Arokiarani et al. [3] introduced the concept of neutrosophic α -open set via NTS for the first time. Additionally, they provided some context for the idea of neutrosophic semi-open functions and established a connection between them via NTS. In later work, Rao and Srinivasa [33] examined the concept of neutrosophic pre-open set and neutrosophic pre-closed set in NTS. The concepts of neutrosophic semi-continuous function and neutrosophic semi-open set were later grounded by Iswaraya and Bageerathi [28]. Ebenanjar et al. [27] introduced the notion of neutrosophic b -open set and neutrosophic b -continuous mapping via NTS in 2018. Subsequently, generalised neutrosophic b -open sets in NTS were introduced by Das and Pramanik [14] by extending the idea of b -open set [1]. The notions of neutrosophic Φ -open sets and neutrosophic Φ -continuous functions were also examined by Das and Pramanik [15]. Afterwards, Das and Tripathy [21] established the idea of neutrosophic simply b -open set in NTS. In 2022, Das et al. [12] presented the concept of neutrosophic separation axioms via NTS. The notion of neutrosophic b -locally open set via NTS was presented by Das and Tripathy [24].

The notion of neutrosophic supra topology (briefly NST) was first grounded by Dhavaseelan et al [25]

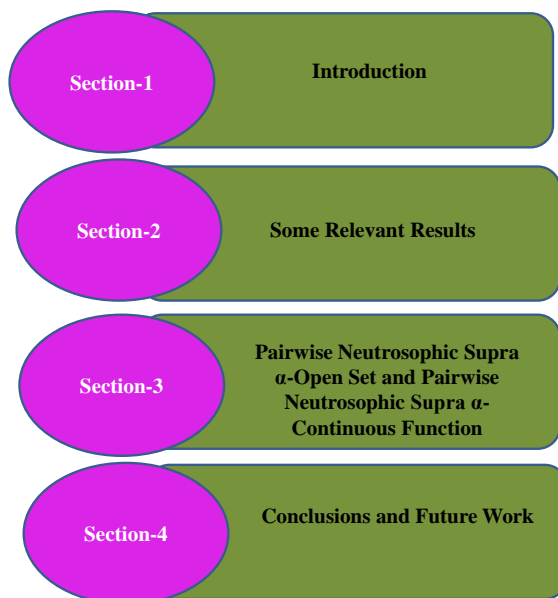
in 2017. They also established the notion of neutrosophic α -open set and neutrosophic α -continuous function, and studied several properties of them. Later on, Dhavaseelan et al. [26] introduced the notion of neutrosophic semi-supra open set via neutrosophic supra topological space (briefly NSTS). In 2021, Das [8] grounded the notion of neutrosophic supra simply open set in NSTS.

The concept of bitopological space (briefly BTS) was first introduced by Kelly [29] in 1963. By using BTS, Tripathy and Sarma [41] investigated the concept of pairwise b -locally closed function and pairwise b -locally open function. Eventually, in 2013, Tripathy and Sarma [42] also conducted research on weakly b -continuous mapping via BTS. In 2019, Ozturk and Ozkan [32] established the notion of neutrosophic BTS (briefly NBTS). The notion of Pairwise neutrosophic b -open set via NBTS were first proposed in 2020 by Das and Tripathy [20], who also examined their various characteristics. The concept of pairwise neutrosophic b -continuous function via NBTS was then presented by Tripathy and Das [40] in 2021. Later on, Das and Tripathy [23] established the notion of pairwise neutrosophic b -locally open set through NBTS. The notion of neutrosophic pre- I -open set via neutrosophic ideal bitopological space was studied by Das et al. [13]. The idea of neutrosophic Supra bi-topological space (briefly NSBTS) was recently defined by Das et al. [10], who also defined pairwise Neutrosophic Supra Pre-continuous function and pairwise Neutrosophic Supra Pre-open set. The notion of Neutrosophic n-Valued Refined Sets and Topologies defined by Arar [2] in the the year 2023. Das et al. [9,11] studied on "Topology on Quadripartitioned Neutrosophic Sets" and Single Valued Bipolar Pentapartitioned Neutrosophic Set and Its Application in MADM Strategy. Pentapartitioned Neutrosophic Topological Space defined by Das and Tripathy [22] in the year 2021. Also the notion of Bitopological Spaces was studied by Kelly [29] in the year 1963. In the year 2017 Rao and Srinivasa [33] worked on Neutrosophic pre-open sets and pre-closed sets in Neutrosophic topology. Tripathy and Das [40] worked on Pairwise neutrosophic b -continuous mapping in neutrosophic bitopological spaces in the year 2021.

Research gap: Till now, no investigation on pairwise neutrosophic supra α -open set and pairwise neutrosophic supra α -continuous function via NSBTS has been reported in the recent literature.

Motivation: To reduce the research gap, we grounded the notion of pairwise neutrosophic supra α -open set and pairwise neutrosophic supra α -continuous function via NSBTS in the context of NS.

The layout of this article is given below:



2. Some Relevant Results:

We provide some pertinent definitions in this section that are crucial for understanding the paper's main findings.

Definition 2.1 [38] An NS R over a fixed set \tilde{X} is defined as follows:

$R = \{(\tilde{\alpha}, T_R(\tilde{\alpha}), I_R(\tilde{\alpha}), F_R(\tilde{\alpha})) : \tilde{\alpha} \in \tilde{X}\}$, where $T_R(\tilde{\alpha})$, $I_R(\tilde{\alpha})$ and $F_R(\tilde{\alpha})$ ($\in [0, 1]$) denotes the truth, indeterminacy and false membership values of each $\tilde{\alpha} \in \tilde{X}$. So, $0 \leq T_R(\tilde{\alpha}) + I_R(\tilde{\alpha}) + F_R(\tilde{\alpha}) \leq 3$, for each $\tilde{\alpha} \in \tilde{X}$.

EXAMPLE 2.1. Let $\tilde{X} = \{\alpha_1, \alpha_2\}$ is a non-empty set. Then, $P = \{(p, 0.4, 0.6, 0.3), (q, 0.8, 0.2, 0.7)\}$ is an NS over \tilde{X} , but $Q = \{(p, 0.9, -0.3, 0.2), (q, -0.5, 0.2, -0.3)\}$ is not an NS over \tilde{X} .

Definition 2.2 [25] Let τ be a family of NSs over \tilde{X} . Then, τ is referred to as an NST on \tilde{X} if the following conditions hold:

- (i) $0_N, 1_N \in \tau$;
- (ii) $\cup W_i \in \tau$ whenever $\{W_i : i \in \Delta\} \subseteq \tau$.

Then, (\tilde{X}, τ) is called an NSTS. If $W \in \tau$, then W is called an neutrosophic supra open set (NSOS) and its complement i.e., W^c is called an neutrosophic supra closed set (NSCS) in (\tilde{X}, τ) .

EXAMPLE 2.2. Let $\tilde{X} = \{\alpha_1, \alpha_2\}$ and let $G = \{(\alpha_1, 0.5, 0.4, 0.2), (\alpha_2, 0.5, 0.6, 0.2) : \alpha_1, \alpha_2 \in \tilde{X}\}$, $H = \{(\alpha_1, 0.4, 0.5, 0.7), (\alpha_2, 0.3, 0.8, 0.7) : \alpha_1, \alpha_2 \in \tilde{X}\}$, $K = \{(\alpha_1, 0.5, 0.5, 0.2), (\alpha_2, 0.3, 0.7, 0.5) : \alpha_1, \alpha_2 \in \tilde{X}\}$ be three NSs defined over \tilde{X} . Then, clearly the family $\tau = \{0_N, 1_N, G, H, K\}$ is a NST on \tilde{X} .

Definition 2.3 [25] Let (\tilde{X}, τ) be an NSTS. Assume that \tilde{Z} be an NS over \tilde{X} . Then, the neutrosophic supra interior (NS_{int}) and neutrosophic supra closure (NS_{cl}) of \tilde{Z} are defined as follows:

- (i) $NS_{int}(\tilde{Z}) = \cup \{E : E \text{ is an NSOS in } \tilde{X} \text{ and } E \subseteq \tilde{Z}\}$;
- (ii) $NS_{cl}(\tilde{Z}) = \cap \{F : F \text{ is an NSCS in } \tilde{X} \text{ and } \tilde{Z} \subseteq F\}$.

Definition 2.4 [25] Assume that (\tilde{X}, τ) be an NSTS. Let A be an NS defined over \tilde{X} . Then, A is called an neutrosophic supra α -open set in (\tilde{X}, τ) if and only if $A \subseteq NS_{int}(NS_{cl}(NS_{int}(A)))$. The complement of an neutrosophic supra α -open set is called an neutrosophic supra α -closed set in (\tilde{X}, τ) .

Definition 2.5 [25] A bijective mapping $\xi : (\tilde{X}, \tau) \rightarrow (\tilde{Y}, \sigma)$ is called an neutrosophic supra α -continuous mapping if the inverse image of every NSOS in \tilde{Y} is an neutrosophic supra α -open set in \tilde{X} .

Definition 2.6 [10] Let τ_1 and τ_2 be two different NSTs defined on the same set \tilde{X} . Then, the triplet $(\tilde{X}, \tau_1, \tau_2)$ is referred to as an NSBTS.

EXAMPLE 2.3. Let $\tilde{X} = \{a, b\}$ be a fixed set. Let $P = \{(a, 0.5, 0.4, 0.5), (b, 0.6, 0.4, 0.5) : a, b \in \tilde{X}\}$, $Q = \{(a, 0.3, 0.6, 0.7), (b, 0.2, 0.9, 0.8) : a, b \in \tilde{X}\}$, $R = \{(a, 0.2, 0.7, 0.8), (b, 0.1, 0.9, 0.9) : a, b \in \tilde{X}\}$, $X = \{(a, 0.3, 0.5, 0.7), (b, 0.6, 0.7, 0.6) : a, b \in \tilde{X}\}$, $Y = \{(a, 0.4, 0.4, 0.6), (b, 0.7, 0.4, 0.5) : a, b \in \tilde{X}\}$ and $Z = \{(a, 0.5, 0.3, 0.1), (b, 0.9, 0.1, 0.2) : a, b \in \tilde{X}\}$ are six NSs over \tilde{X} . Then, $\tau_1 = \{0_N, 1_N, P, Q, R\}$ and $\tau_2 = \{0_N, 1_N, X, Y, Z\}$ are two different NSTs on \tilde{X} . Hence, $(\tilde{X}, \tau_1, \tau_2)$ is an NSBTS.

Remark 2.1 [10] Assume that $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Let W be an NS defined over \tilde{X} . Then, $NS_{int}^i(W)$ and $NS_{cl}^i(W)$, respectively indicates the neutrosophic supra interior and neutrosophic supra closure of W with respect to the NSTS (\tilde{X}, τ_i) ($i=1, 2$).

Definition 2.7 [10] In an NSBTS $(\tilde{X}, \tau_1, \tau_2)$, an NS Q is called a pairwise neutrosophic supra open set (pairwise NSOS) if $Q = W \cup M$, where W is an NSOS in (\tilde{X}, τ_1) and M is an NSOS in (\tilde{X}, τ_2) .

Lemma 2.1 [10] Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Then, every NSOS in an NSTS (\tilde{X}, τ_i) is a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$.

Definition 2.8 [10] Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. An NS P over \tilde{X} is called a

- (i) τ_{ij} neutrosophic supra semi-open set if and only if $P \subseteq NS_{cl}^i(NS_{int}^j(P))$;
- (ii) τ_{ij} neutrosophic supra pre-open set if and only if $P \subseteq NS_{int}^j(NS_{cl}^i(P))$.

EXAMPLE 2.4. Let us consider the NSBTS $(\tilde{X}, \tau_1, \tau_2)$ as shown in Example 2.3. Then, Clearly, the NS $\tilde{B} = \{(a, 0.4, 0.5, 0.6), (b, 0.5, 0.5, 0.6) : a, b \in \tilde{X}\}$ is a τ_{12} neutrosophic supra semi-open set, and the NS $X = \{(a, 0.3, 0.5, 0.7), (b, 0.6, 0.7, 0.6) : a, b \in \tilde{X}\}$ is a τ_{12} neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$.

Definition 2.9 [10] An NS L is referred to as a pairwise neutrosophic supra pre-open set (resp. pairwise neutrosophic supra semi-open set) in an NSBTS $(\tilde{X}, \tau_1, \tau_2)$ if $L = K \cup M$, where K is a τ_{ij} neutrosophic supra pre-open set (resp. τ_{ij} neutrosophic supra semi-open set) and M is a τ_{ji} neutrosophic supra pre-open set (resp. τ_{ji} neutrosophic supra semi-open set) in $(\tilde{X}, \tau_1, \tau_2)$.

Definition 2.10 [10] In an NSBTS $(\tilde{X}, \tau_1, \tau_2)$, an NS W is referred to as a τ_{ij} neutrosophic supra b-open set if and only if $W \subseteq NS_{cl}^i NS_{int}^j(W) \cup NS_{int}^j NS_{cl}^i(W)$. An NS M over \tilde{X} is said to be a τ_{ij} neutrosophic supra b-closed set if and only if M^c is a τ_{ij} neutrosophic supra b-open set in $(\tilde{X}, \tau_1, \tau_2)$.

Definition 2.11 [10] An NS L is referred to as a pairwise neutrosophic supra b-open set in an NSBTS $(\tilde{X}, \tau_1, \tau_2)$ if $L = K \cup M$, where K is a τ_{ij} neutrosophic supra b-open set and M is a τ_{ji} neutrosophic supra b-open set in $(\tilde{X}, \tau_1, \tau_2)$. An NS L is said to be a pairwise neutrosophic supra b-closed in $(\tilde{X}, \tau_1, \tau_2)$ if and only if L^c is a pairwise neutrosophic supra b-open set in $(\tilde{X}, \tau_1, \tau_2)$.

3. Pairwise Neutrosophic Supra α -Open Set and Pairwise Neutrosophic Supra α -Continuous Function

The concepts of pairwise neutrosophic supra α -open set, pairwise neutrosophic supra α -continuous function and pairwise neutrosophic supra α -open function via NSBTS are introduced in this section. Furthermore, we derive a number of intriguing conclusions from them.

Definition 3.1 Assume that $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Let P be an NS defined over \tilde{X} . Then, P is called a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$ iff $P \subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(P)))$. If P is a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$, then the complement of P i.e., P^c is called a τ_{ij} neutrosophic supra α -closed set in $(\tilde{X}, \tau_1, \tau_2)$.

EXAMPLE 3.1. Let us consider a NSBTS as shown in Example 2.3. Then, the NS $\tilde{B} = \{(a, 0.4, 0.5, 0.6), (b, 0.5, 0.5, 0.6) : a, b \in \tilde{X}\}$ is a τ_{12} neutrosophic supra α -open set, and $\tilde{B}^c = \{(a, 0.6, 0.5, 0.4), (b, 0.5, 0.5, 0.4) : a, b \in \tilde{X}\}$ is a τ_{12} neutrosophic supra α -closed set.

Similarly, it can be shown that, the NS $\tilde{C} = \{(a, 0.6, 0.3, 0.4), (b, 0.7, 0.3, 0.4) : a, b \in \tilde{X}\}$ is a τ_{21} neutrosophic supra α -open set, and $\tilde{C}^c = \{(a, 0.4, 0.7, 0.6), (b, 0.3, 0.4, 0.6) : a, b \in \tilde{X}\}$ is a τ_{21} neutrosophic supra α -closed set.

Theorem 3.1 Assume that $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Then, every NSOS in NSTS (\tilde{X}, τ_j) is a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$ ($i, j = 1, 2$).

Proof. Assume that $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS, and Q be an NSOS in (\tilde{X}, τ_j) . Therefore, $NS_{int}^j(Q) = Q$. It is known that, $NS_{int}^j(Q) \subseteq NS_{cl}^i(NS_{int}^j(Q))$.

$$\begin{aligned} \text{Now, } NS_{int}^j(Q) &\subseteq NS_{cl}^i(NS_{int}^j(Q)) \\ \implies Q = NS_{int}^j(Q) &\subseteq NS_{cl}^i(NS_{int}^j(Q)) \\ \implies Q &\subseteq NS_{cl}^i(NS_{int}^j(Q)) \\ \implies NS_{int}^j(Q) &\subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(Q))) \\ \implies Q = NS_{int}^j(Q) &\subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(Q))) \\ \implies Q &\subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(Q))) \end{aligned}$$

Therefore, Q is a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$.

Lemma 3.1 Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Then, every NSCS in NSTS (\tilde{X}, τ_j) is a τ_{ij} neutrosophic supra α -closed set in $(\tilde{X}, \tau_1, \tau_2)$ ($i, j = 1, 2$).

Theorem 3.2 In an NSBTS $(\tilde{X}, \tau_1, \tau_2)$,

- (i) every τ_{ij} neutrosophic supra α -open set is a τ_{ij} neutrosophic supra semi-open set;
- (ii) every τ_{ij} neutrosophic supra α -open set is a τ_{ij} neutrosophic supra pre-open set.

Proof.(i) Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Let Q be a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. So, $Q \subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(Q)))$. It is known that, $NS_{int}^j(NS_{cl}^i(NS_{int}^j(Q))) \subseteq NS_{cl}^i(NS_{int}^j(Q))$. Therefore, $Q \subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(Q))) \subseteq NS_{cl}^i(NS_{int}^j(Q))$. This implies, $Q \subseteq NS_{cl}^i(NS_{int}^j(Q))$. Hence, Q is a τ_{ij} neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$. Therefore, every τ_{ij} neutrosophic supra α -open set is a τ_{ij} neutrosophic supra semi-open set.

(ii) Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Suppose that Q be a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. So, $Q \subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(Q)))$. It is known that, $NS_{int}^j(Q) \subseteq Q$.

$$\begin{aligned} \text{Now, } NS_{int}^j(Q) &\subseteq Q \\ \implies NS_{cl}^i(NS_{int}^j(Q)) &\subseteq NS_{cl}^i(Q) \\ \implies NS_{int}^j(NS_{cl}^i(NS_{int}^j(Q))) &\subseteq NS_{int}^j(NS_{cl}^i(Q)) \\ \implies Q &\subseteq NS_{int}^j(NS_{cl}^i(Q)) \text{ [Since, } Q \subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(Q)))] \end{aligned}$$

Hence, Q is a τ_{ij} neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$. Therefore, every τ_{ij} neutrosophic supra α -open set $(\tilde{X}, \tau_1, \tau_2)$ is a τ_{ij} neutrosophic supra pre-open set.

Theorem 3.3 In an NSBTS $(\tilde{X}, \tau_1, \tau_2)$, every τ_{ij} neutrosophic supra α -open set is a τ_{ij} neutrosophic supra b -open set.

Proof. Suppose that $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Let Q be a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Since, every τ_{ij} neutrosophic supra α -open set is a τ_{ij} neutrosophic supra semi-open set, and every τ_{ij} neutrosophic supra semi-open set is a τ_{ij} neutrosophic supra b -open set in $(\tilde{X}, \tau_1, \tau_2)$, so Q is a τ_{ij} neutrosophic supra b -open set in $(\tilde{X}, \tau_1, \tau_2)$. Hence, every τ_{ij} neutrosophic supra α -open set $(\tilde{X}, \tau_1, \tau_2)$ is a τ_{ij} neutrosophic supra b -open set.

Theorem 3.4 Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. If G is both an NSOS in (\tilde{X}, τ_j) and τ_{ij} neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$, then G is a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$.

Proof. Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Assume that G be both NSOS in (\tilde{X}, τ_j) and τ_{ij} neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$. Since, G is an NSOS in (\tilde{X}, τ_j) , so $NS_{int}^j(G) = G$. Further, since G is a τ_{ij} neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$, so $G \subseteq NS_{cl}^i(NS_{int}^j(G))$.

$$\text{Now, } G \subseteq NS_{cl}^i(NS_{int}^j(G))$$

$$\begin{aligned}
&\Rightarrow NS_{int}^j(G) \subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(G))) \\
&\Rightarrow G = NS_{int}^j(G) \subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(G))) \\
&\Rightarrow G \subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(G)))
\end{aligned}$$

Therefore, G is a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$.

Theorem 3.5 *Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. If G is an NSOS in (\tilde{X}, τ_j) and τ_{ij} neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$, then G is a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$.*

Proof. Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Assume that G be both NSOS in (\tilde{X}, τ_j) and τ_{ij} neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$. Since, G is an NSOS in (\tilde{X}, τ_j) , so $NS_{int}^j(G) = G$. Further, since G is a τ_{ij} neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$, so $G \subseteq NS_{int}^j(NS_{cl}^i(G))$.

$$\begin{aligned}
&\text{Now, } G \subseteq NS_{int}^j(NS_{cl}^i(G)) \\
&\Rightarrow G \subseteq NS_{int}^j(NS_{cl}^i(NS_{int}^j(G))) \text{ [since } G = NS_{int}^j(G)\text{]}
\end{aligned}$$

Therefore, G is a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$.

Definition 3.2 *Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Then A , an NS over \tilde{X} is called a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$ if there exists a τ_{ij} neutrosophic supra α -open set G and a τ_{ji} neutrosophic supra α -open set H in $(\tilde{X}, \tau_1, \tau_2)$ such that $A = G \cup H$.*

EXAMPLE 3.2. Let us consider a NSBTS as shown in Example 2.3. Then, the NS $\tilde{B} = \{(a, 0.4, 0.5, 0.6), (b, 0.5, 0.5, 0.6)\}$: $a, b \in \tilde{X}$ is a τ_{12} neutrosophic supra α -open set, and the NS $\tilde{C} = \{(a, 0.6, 0.3, 0.4), (b, 0.7, 0.3, 0.4)\}$: $a, b \in \tilde{X}$ is a τ_{21} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$, which is shown in Example 3.1. Clearly, the NS $\{(a, 0.6, 0.3, 0.4), (b, 0.7, 0.3, 0.4)\}$: $a, b \in \tilde{X}$ is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$, because it can be expressed as the union of \tilde{B} and \tilde{C} .

Theorem 3.6 *Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Then, every pairwise neutrosophic supra open set in $(\tilde{X}, \tau_1, \tau_2)$ is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$.*

Proof. Suppose that G be a pairwise neutrosophic supra open set in the NSBTS $(\tilde{X}, \tau_1, \tau_2)$. Therefore, there exists an NSOS Y in (\tilde{X}, τ_i) and an NSOS R in (\tilde{X}, τ_j) such that $G = Y \cup R$. Since, every NSOS in (\tilde{X}, τ_i) is a τ_{ji} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$, so Y is a τ_{ji} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Further, since every NSOS in (\tilde{X}, τ_j) is a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$, so R is a τ_{ij} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Therefore, G can be expressed as the union of a τ_{ij} neutrosophic supra α -open set R and a τ_{ji} neutrosophic supra α -open set Y . Hence, G is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$.

Theorem 3.7 *Let $(\tilde{X}, \tau_1, \tau_2)$ be an NSBTS. Then,*

- (i) *every pairwise neutrosophic supra α -open set is a pairwise neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$;*
- (ii) *every pairwise neutrosophic supra α -open set is a pairwise neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$;*
- (iii) *every pairwise neutrosophic supra α -open set is a pairwise neutrosophic supra b-open set in $(\tilde{X}, \tau_1, \tau_2)$.*

Proof. (i) Let G be a pairwise neutrosophic supra α -open set in the NSBTS $(\tilde{X}, \tau_1, \tau_2)$. Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic supra α -open set and Y is a τ_{ji} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Since, every τ_{ij} neutrosophic supra α -open set is a τ_{ij} neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$, so R is a τ_{ij} neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$. Further, since every τ_{ji} neutrosophic supra α -open set is a τ_{ji} neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$, so Y is a

τ_{ji} neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$. Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic supra semi-open set and Y is a τ_{ji} neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$. Hence, G is a pairwise neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$.

(ii) Let G be a pairwise neutrosophic supra α -open set in the NSBTS $(\tilde{X}, \tau_1, \tau_2)$. Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic supra α -open set and Y is a τ_{ji} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Since, every τ_{ij} neutrosophic supra α -open set is a τ_{ij} neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$, so R is a τ_{ij} neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$. Further, since every τ_{ji} neutrosophic supra α -open set is a τ_{ji} neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$, so Y is a τ_{ji} neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$. Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic supra pre-open set and Y is a τ_{ji} neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$. Hence, G is a pairwise neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$.

(iii) Let G be a pairwise neutrosophic supra α -open set in the NSBTS $(\tilde{X}, \tau_1, \tau_2)$. Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic supra α -open set and Y is a τ_{ji} neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Since, every τ_{ij} neutrosophic supra α -open set is a τ_{ij} neutrosophic supra b -open set in $(\tilde{X}, \tau_1, \tau_2)$, so R is a τ_{ij} neutrosophic supra b -open set in $(\tilde{X}, \tau_1, \tau_2)$. Further, since every τ_{ji} neutrosophic supra α -open set is a τ_{ji} neutrosophic supra b -open set in $(\tilde{X}, \tau_1, \tau_2)$, so Y is a τ_{ji} neutrosophic supra b -open set in $(\tilde{X}, \tau_1, \tau_2)$. Therefore, G can be expressed as $G = R \cup Y$, where R is a τ_{ij} neutrosophic supra b -open set and Y is a τ_{ji} neutrosophic supra b -open set in $(\tilde{X}, \tau_1, \tau_2)$. Hence, G is a pairwise neutrosophic supra b -open set in $(\tilde{X}, \tau_1, \tau_2)$.

Definition 3.3 A bijective mapping $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ is called a pairwise neutrosophic supra α -continuous mapping iff $f^{-1}(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$ whenever D is a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$.

Theorem 3.8 (i) Every pairwise neutrosophic supra continuous mapping is also a pairwise neutrosophic supra α -continuous mapping;

(ii) Every pairwise neutrosophic supra α -continuous mapping is a pairwise neutrosophic supra semi-continuous mapping;

(iii) Every pairwise neutrosophic supra α -continuous mapping is a pairwise neutrosophic supra pre-continuous mapping;

(iv) Every pairwise neutrosophic supra α -continuous mapping is a pairwise neutrosophic supra b -continuous mapping.

Proof. (i) Let $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra continuous mapping. Assume that D be a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$. By hypothesis, $f^{-1}(D)$ is a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$. Since, every pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$ is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$, so $f^{-1}(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Therefore, $f^{-1}(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$ whenever D is a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$. Hence, the mapping $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra α -continuous mapping.

(ii) Let $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra α -continuous mapping. Assume that D be a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$. By hypothesis, $f^{-1}(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Since, every pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$ is a pairwise neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$, so $f^{-1}(D)$ is a pairwise neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$. Therefore, $f^{-1}(D)$ is a pairwise neutrosophic supra semi-open set in $(\tilde{X}, \tau_1, \tau_2)$ whenever D is

a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$. Hence, the mapping $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra semi-continuous mapping.

(iii) Let $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra α -continuous mapping. Suppose that D be a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$. By hypothesis, $f^{-1}(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Since, every pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$ is a pairwise neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$, so $f^{-1}(D)$ is a pairwise neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$. Therefore, $f^{-1}(D)$ is a pairwise neutrosophic supra pre-open set in $(\tilde{X}, \tau_1, \tau_2)$ whenever D is a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$. Hence, the mapping $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra pre-continuous mapping.

(iv) Let $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra α -continuous mapping. Assume that D be a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$. By hypothesis, $f^{-1}(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Since, every pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$ is a pairwise neutrosophic supra b -open set in $(\tilde{X}, \tau_1, \tau_2)$, so $f^{-1}(D)$ is a pairwise neutrosophic supra b -open set in $(\tilde{X}, \tau_1, \tau_2)$. Therefore, $f^{-1}(D)$ is a pairwise neutrosophic supra b -open set in $(\tilde{X}, \tau_1, \tau_2)$ whenever D is a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$. Hence, the mapping $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra b -continuous mapping.

Theorem 3.9 *Let $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra α -continuous mapping, and $g : (\tilde{Y}, \sigma_1, \sigma_2) \rightarrow (\tilde{Z}, \theta_1, \theta_2)$ be a pairwise neutrosophic supra continuous mapping, then the composition mapping $g \circ f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Z}, \theta_1, \theta_2)$ is a pairwise neutrosophic supra α -continuous mapping.*

Proof. Let $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra α -continuous mapping, and $g : (\tilde{Y}, \sigma_1, \sigma_2) \rightarrow (\tilde{Z}, \theta_1, \theta_2)$ be a pairwise neutrosophic supra continuous mapping. Assume that D be a pairwise NSOS in $(\tilde{Z}, \theta_1, \theta_2)$. Since $g : (\tilde{Y}, \sigma_1, \sigma_2) \rightarrow (\tilde{Z}, \theta_1, \theta_2)$ is a pairwise neutrosophic supra continuous mapping, so $g^{-1}(D)$ is a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$. Further, since $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra α -continuous mapping, so $f^{-1}(g^{-1}(D)) = (g \circ f)^{-1}(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Therefore, $(g \circ f)^{-1}(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$ whenever D is a pairwise NSOS in $(\tilde{Z}, \theta_1, \theta_2)$. Hence, the composition mapping $g \circ f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Z}, \theta_1, \theta_2)$ is a pairwise neutrosophic supra α -continuous mapping.

Definition 3.4 *A mapping $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ is referred to as a pairwise neutrosophic supra α -open mapping iff $f(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$ whenever D is a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$.*

Theorem 3.10 (i) *Every pairwise neutrosophic supra open mapping is also a pairwise neutrosophic supra α -open mapping;*

(ii) *Every pairwise neutrosophic supra α -open mapping is a pairwise neutrosophic supra semi-open mapping;*

(iii) *Every pairwise neutrosophic supra α -open mapping is a pairwise neutrosophic supra pre-open mapping;*

(iv) *Every pairwise neutrosophic supra α -open mapping is a pairwise neutrosophic supra b -open mapping.*

Proof. (i) Let $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra open mapping. Assume that D be a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$. By hypothesis, $f(D)$ is a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$. Since, every pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra α -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$, so $f(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$. Therefore, $f(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$ whenever D is a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$. Hence, the mapping $f : (\tilde{X}, \tau_1, \tau_2)$

$\rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra α -open mapping.

(ii) Let $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra α -open mapping. Assume that D be a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$. By hypothesis, $f(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$. Since, every pairwise neutrosophic supra α -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra semi-open set in $(\tilde{Y}, \sigma_1, \sigma_2)$, so $f(D)$ is a pairwise neutrosophic supra semi-open set in $(\tilde{Y}, \sigma_1, \sigma_2)$. Therefore, $f(D)$ is a pairwise neutrosophic supra semi-open set in $(\tilde{Y}, \sigma_1, \sigma_2)$ whenever D is a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$. Hence, the mapping $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra semi-open mapping.

(iii) Let $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra α -open mapping. Assume that D be a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$. By hypothesis, $f(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$. Since, every pairwise neutrosophic supra α -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra pre-open set in $(\tilde{Y}, \sigma_1, \sigma_2)$, so $f(D)$ is a pairwise neutrosophic supra pre-open set in $(\tilde{Y}, \sigma_1, \sigma_2)$. Therefore, $f(D)$ is a pairwise neutrosophic supra pre-open set in $(\tilde{Y}, \sigma_1, \sigma_2)$ whenever D is a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$. Hence, the mapping $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra pre-open mapping.

(iv) Let $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra α -open mapping. Assume that D be a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$. By hypothesis, $f(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$. Since, every pairwise neutrosophic supra α -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra b -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$, so $f(D)$ is a pairwise neutrosophic supra b -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$. Therefore, $f(D)$ is a pairwise neutrosophic supra b -open set in $(\tilde{Y}, \sigma_1, \sigma_2)$ whenever D is a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$. Hence, the mapping $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra b -open mapping.

Theorem 3.11 Let $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra open mapping, and $g : (\tilde{Y}, \sigma_1, \sigma_2) \rightarrow (\tilde{Z}, \theta_1, \theta_2)$ be a pairwise neutrosophic supra α -open mapping, then the composition mapping $g \circ f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Z}, \theta_1, \theta_2)$ is a pairwise neutrosophic supra α -open mapping.

Proof. Suppose that $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ be a pairwise neutrosophic supra open mapping, and $g : (\tilde{Y}, \sigma_1, \sigma_2) \rightarrow (\tilde{Z}, \theta_1, \theta_2)$ be a pairwise neutrosophic supra α -open mapping. Let D be a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$. Since $f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \sigma_1, \sigma_2)$ is a pairwise neutrosophic supra open mapping, so $f(D)$ is a pairwise NSOS in $(\tilde{Y}, \sigma_1, \sigma_2)$. Further, since $g : (\tilde{Y}, \sigma_1, \sigma_2) \rightarrow (\tilde{Z}, \theta_1, \theta_2)$ is a pairwise neutrosophic supra α -open mapping, so $g(f(D)) = (g \circ f)(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{Z}, \theta_1, \theta_2)$. Therefore, $(g \circ f)(D)$ is a pairwise neutrosophic supra α -open set in $(\tilde{Z}, \theta_1, \theta_2)$ whenever D is a pairwise NSOS in $(\tilde{X}, \tau_1, \tau_2)$. Hence, the composition mapping $g \circ f : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Z}, \theta_1, \theta_2)$ is a pairwise neutrosophic supra α -open mapping.

Definition 3.5 A family $\{A_i : i \in \Delta\}$, where Δ is an index set and A_i is a pairwise neutrosophic supra open set in an NSBTS $(\tilde{X}, \tau_1, \tau_2)$, for each $i \in \Delta$, is called a pairwise neutrosophic supra open cover of an NS A if $A \subseteq \cup_{i \in \Delta} A_i$.

Definition 3.6 An NSBTS $(\tilde{X}, \tau_1, \tau_2)$ is called a pairwise neutrosophic supra compact space if each pairwise neutrosophic supra open cover of 1_N has a finite sub-cover.

Definition 3.7 A neutrosophic sub-set B of an NSBTS $(\tilde{X}, \tau_1, \tau_2)$ is called a pairwise neutrosophic supra compact set relative to \tilde{X} if every pairwise neutrosophic supra open cover of B has a finite pairwise neutrosophic supra open sub-cover.

Theorem 3.12 *Every pairwise neutrosophic supra closed sub-set of a pairwise neutrosophic supra compact space $(\tilde{X}, \tau_1, \tau_2)$ is a pairwise neutrosophic supra compact relative to \tilde{X} .*

Proof. Assume that $(\tilde{X}, \tau_1, \tau_2)$ be a pairwise neutrosophic supra compact space. Suppose that K be a pairwise neutrosophic supra closed sub-set of $(\tilde{X}, \tau_1, \tau_2)$. Therefore, K^c is a pairwise neutrosophic supra open set in $(\tilde{X}, \tau_1, \tau_2)$. Suppose that $U = \{H_i : i \in \Delta \text{ and } H_i \text{ is a pairwise neutrosophic supra open set in } \tilde{X}\}$ be a pairwise neutrosophic supra open cover of K . Then, $H = \{K^c\} \cup U$ is a pairwise neutrosophic supra open cover of 1_N . Since $(\tilde{X}, \tau_1, \tau_2)$ is a pairwise neutrosophic supra compact space, so it has a finite sub-cover say $\{H_1, H_2, H_3, \dots, H_n, K^c\}$. This implies, $\{H_1, H_2, H_3, \dots, H_n\}$ is a finite pairwise neutrosophic supra open cover of K . Hence, K is a pairwise neutrosophic supra compact set relative to \tilde{X} .

Theorem 3.13 *If $\xi : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \delta_1, \delta_2)$ is a pairwise neutrosophic supra continuous mapping, then for each pairwise neutrosophic supra compact set Q relative to \tilde{X} , $\xi(Q)$ is a pairwise neutrosophic supra compact set relative to \tilde{Y} .*

Proof. Let $\xi : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \delta_1, \delta_2)$ be a pairwise neutrosophic supra continuous mapping. Let Q be a pairwise neutrosophic supra compact set relative to \tilde{X} . Suppose that $H = \{H_i : i \in \Delta, \text{ and } H_i \text{ is a pairwise neutrosophic supra open set in } \tilde{X}\}$ be a pairwise neutrosophic supra open cover of $\xi(Q)$. By hypothesis, $\xi^{-1}(H) = \{\xi^{-1}(H_i) : i \in \Delta \text{ and } \xi^{-1}(H_i) \text{ is a pairwise neutrosophic supra open set in } \tilde{X}\}$ is a pairwise neutrosophic supra open cover of $\xi^{-1}(\xi(Q)) = Q$. Since Q is a pairwise neutrosophic supra compact set relative to \tilde{X} , so there exists a finite pairwise neutrosophic supra open sub-cover of Q say $\{\xi^{-1}(H_1), \xi^{-1}(H_2), \xi^{-1}(H_3), \dots, \xi^{-1}(H_n)\}$ such that $Q \subseteq \cup_i \{\xi^{-1}(H_i) : i = 1, 2, \dots, n\}$.

Now $Q \subseteq \cup_i \{\xi^{-1}(H_i) : i = 1, 2, \dots, n\}$

$\implies \xi(Q) \subseteq \cup_i \{\xi(\xi^{-1}(H_i)) : i = 1, 2, \dots, n\}$

$\implies \xi(Q) \subseteq \cup_i \{H_i : i = 1, 2, \dots, n\}$

Therefore, there exist a finite pairwise neutrosophic supra open sub-cover $\{H_1, H_2, H_3, \dots, H_n\}$ of $\xi(Q)$ such that $\xi(Q) \subseteq \cup_i \{H_i : i = 1, 2, \dots, n\}$. Hence, $\xi(Q)$ is a pairwise neutrosophic supra compact set relative to \tilde{Y} .

Theorem 3.14 *If $\xi : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \delta_1, \delta_2)$ is a pairwise neutrosophic supra open function, and B is a pairwise neutrosophic supra compact set relative to $(\tilde{Y}, \delta_1, \delta_2)$, then $\xi^{-1}(B)$ is also a pairwise neutrosophic supra compact set relative to $(\tilde{X}, \tau_1, \tau_2)$.*

Proof. Assume that $\xi : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \delta_1, \delta_2)$ be a pairwise neutrosophic supra open function. Let B is a pairwise neutrosophic supra compact set relative to $(\tilde{Y}, \delta_1, \delta_2)$. Suppose that $H = \{H_i : i \in \Delta \text{ and } H_i \text{ is a pairwise neutrosophic supra open set in } \tilde{X}\}$ be a pairwise neutrosophic supra open cover of $\xi^{-1}(B)$. By hypothesis, $\xi(H) = \{\xi(H_i) : i \in \Delta \text{ and } \xi(H_i) \text{ is a pairwise neutrosophic supra open set in } \tilde{Y}\}$ is a pairwise neutrosophic supra open cover of $\xi(\xi^{-1}(B)) = B$. Since, B is a pairwise neutrosophic supra compact set relative to $(\tilde{Y}, \delta_1, \delta_2)$, so there exists a finite sub-cover say $\{\xi(H_1), \xi(H_2), \dots, \xi(H_n)\}$ such that $B \subseteq \cup \{\xi(H_i) : i = 1, 2, \dots, n\}$.

Now, $B \subseteq \cup \{\xi(H_i) : i = 1, 2, \dots, n\}$

$\implies \xi^{-1}(B) \subseteq \cup \{\xi^{-1}(\xi(H_i)) : i = 1, 2, \dots, n\}$

$\implies \xi^{-1}(B) \subseteq \cup \{H_i : i = 1, 2, \dots, n\}$

This implies, $\{H_1, H_2, \dots, H_n\}$ is a finite sub-cover for $\xi^{-1}(B)$. Hence, $\xi^{-1}(B)$ is a pairwise neutrosophic supra compact set relative to $(\tilde{X}, \tau_1, \tau_2)$.

Definition 3.8 *A family $\{A_i : i \in \Delta\}$, where Δ is an index set and A_i is a pairwise neutrosophic supra α -open set in an NSBTS $(\tilde{X}, \tau_1, \tau_2)$, for each $i \in \Delta$, is called a pairwise neutrosophic supra α -open cover of an NS A if $A \subseteq \cup_{i \in \Delta} A_i$.*

Definition 3.9 An NSBTS $(\tilde{X}, \tau_1, \tau_2)$ is called a pairwise neutrosophic supra α -compact space if each pairwise neutrosophic supra α -open cover of 1_N has a finite sub-cover.

Definition 3.10 A neutrosophic sub-set B of an NSBTS $(\tilde{X}, \tau_1, \tau_2)$ is called a pairwise neutrosophic supra α -compact relative to \tilde{X} if every pairwise neutrosophic supra α -open cover of B has a finite pairwise neutrosophic supra α -open sub-cover.

Theorem 3.15 Every pairwise neutrosophic supra α -closed sub-set of a pairwise neutrosophic supra α -compact space $(\tilde{X}, \tau_1, \tau_2)$ is a pairwise neutrosophic supra α -compact set relative to \tilde{X} .

Proof. Let $(\tilde{X}, \tau_1, \tau_2)$ be a pairwise neutrosophic supra α -compact space. Let K be a pairwise neutrosophic supra α -closed sub-set of $(\tilde{X}, \tau_1, \tau_2)$. Therefore, K^c is a pairwise neutrosophic supra α -open set in $(\tilde{X}, \tau_1, \tau_2)$. Suppose that $U = \{H_i : i \in \Delta \text{ and } H_i \text{ is a pairwise neutrosophic supra } \alpha\text{-open set in } \tilde{X}\}$ be a pairwise neutrosophic supra α -open cover of K . Then, $H = \{K^c\} \cup U$ is a pairwise neutrosophic supra α -open cover of 1_N . Since $(\tilde{X}, \tau_1, \tau_2)$ is a pairwise neutrosophic supra α -compact space, so it has a finite sub-cover say $\{H_1, H_2, H_3, \dots, H_n, K^c\}$. This implies, $\{H_1, H_2, H_3, \dots, H_n\}$ is a finite pairwise neutrosophic supra α -open cover of K . Hence, K is a pairwise neutrosophic supra α -compact set relative to \tilde{X} .

Theorem 3.16 If $\xi : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \delta_1, \delta_2)$ is a pairwise neutrosophic supra α -continuous mapping, then for each pairwise neutrosophic supra α -compact set Q relative to \tilde{X} , $\xi(Q)$ is a pairwise neutrosophic supra α -compact set relative to \tilde{Y} .

Proof. Let $\xi : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \delta_1, \delta_2)$ be a pairwise neutrosophic supra α -continuous mapping. Let Q be a pairwise neutrosophic supra α -compact set relative to \tilde{X} . Suppose that $H = \{H_i : i \in \Delta, \text{ and } H_i \text{ is a pairwise neutrosophic supra } \alpha\text{-open set in } \tilde{Y}\}$ be a pairwise neutrosophic supra α -open cover of $\xi(Q)$. By hypothesis, $\xi^{-1}(H) = \{\xi^{-1}(H_i) : i \in \Delta \text{ and } \xi^{-1}(H_i) \text{ is a pairwise neutrosophic supra } \alpha\text{-open set in } \tilde{X}\}$ is a pairwise neutrosophic supra α -open cover of $\xi^{-1}(\xi(Q)) = Q$. Since Q is a pairwise neutrosophic supra α -compact set relative to \tilde{X} , so there exists a finite pairwise neutrosophic supra α -open sub-cover of Q say $\{\xi^{-1}(H_1), \xi^{-1}(H_2), \xi^{-1}(H_3), \dots, \xi^{-1}(H_n)\}$ such that $Q \subseteq \cup_i \{\xi^{-1}(H_i) : i = 1, 2, \dots, n\}$.

Now $Q \subseteq \cup_i \{\xi^{-1}(H_i) : i = 1, 2, \dots, n\}$
 $\implies \xi(Q) \subseteq \cup_i \{\xi(\xi^{-1}(H_i)) : i = 1, 2, \dots, n\}$
 $\implies \xi(Q) \subseteq \cup_i \{H_i : i = 1, 2, \dots, n\}$

Therefore, there exist a finite pairwise neutrosophic supra α -open sub-cover $\{H_1, H_2, H_3, \dots, H_n\}$ of $\xi(Q)$ such that $\xi(Q) \subseteq \cup_i \{H_i : i = 1, 2, \dots, n\}$. Hence, $\xi(Q)$ is a pairwise neutrosophic supra α -compact set relative to \tilde{Y} .

Theorem 3.17 If $\xi : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \delta_1, \delta_2)$ is a pairwise neutrosophic supra α -open function, and B is a pairwise neutrosophic supra α -compact set relative to $(\tilde{Y}, \delta_1, \delta_2)$, then $\xi^{-1}(B)$ is also a pairwise neutrosophic supra α -compact set relative to $(\tilde{X}, \tau_1, \tau_2)$.

Proof. Let $\xi : (\tilde{X}, \tau_1, \tau_2) \rightarrow (\tilde{Y}, \delta_1, \delta_2)$ be a pairwise neutrosophic supra α -open function. Assume that B is a pairwise neutrosophic supra α -compact set relative to $(\tilde{Y}, \delta_1, \delta_2)$. Suppose that $H = \{H_i : i \in \Delta \text{ and } H_i \text{ is a pairwise neutrosophic supra } \alpha\text{-open set in } \tilde{X}\}$ be a pairwise neutrosophic supra α -open cover of $\xi^{-1}(B)$. By hypothesis, $\xi(H) = \{\xi(H_i) : i \in \Delta \text{ and } \xi(H_i) \text{ is a pairwise neutrosophic supra } \alpha\text{-open set in } \tilde{Y}\}$ is a pairwise neutrosophic supra α -open cover of $\xi(\xi^{-1}(B)) = B$. Since, B is a pairwise neutrosophic supra α -compact set relative to $(\tilde{Y}, \delta_1, \delta_2)$, so there exists a finite sub-cover say $\{\xi(H_1), \xi(H_2), \dots, \xi(H_n)\}$ such that $B \subseteq \cup \{\xi(H_i) : i = 1, 2, \dots, n\}$.

Now, $B \subseteq \cup \{\xi(H_i) : i = 1, 2, \dots, n\}$

$$\implies \xi^{-1}(B) \subseteq \cup \{\xi^{-1}(\xi(H_i)) : i = 1, 2, \dots, n\}$$

$$\implies \xi^{-1}(B) \subseteq \cup \{H_i : i = 1, 2, \dots, n\}$$

This implies, $\{H_1, H_2, \dots, H_n\}$ is a finite sub-cover for $\xi^{-1}(B)$. Hence, $\xi^{-1}(B)$ is a pairwise neutrosophic supra α -compact set relative to $(\tilde{\tilde{X}}, \tau_1, \tau_2)$.

Theorem 3.18 *Let $(\tilde{\tilde{X}}, \tau_1, \tau_2)$ be an NSBTS. Then, every pairwise neutrosophic supra α -compact set relative to $\tilde{\tilde{X}}$ is also a pairwise neutrosophic supra compact set relative to $\tilde{\tilde{X}}$.*

Proof. Let $(\tilde{\tilde{X}}, \tau_1, \tau_2)$ be an NSBTS. Assume that B be a pairwise neutrosophic supra α -compact set relative to $\tilde{\tilde{X}}$. Let $U = \{E_i : i \in \Delta \text{ and } E_i \text{ is a pairwise neutrosophic supra open set in } \tilde{\tilde{X}}\}$ be a pairwise neutrosophic supra open cover of B . Since every pairwise neutrosophic supra open set in $(\tilde{\tilde{X}}, \tau_1, \tau_2)$ is also a pairwise neutrosophic supra α -open set in $(\tilde{\tilde{X}}, \tau_1, \tau_2)$, so $U = \{E_i : i \in \Delta \text{ and } E_i \text{ is a pairwise neutrosophic supra } \alpha\text{-open set in } \tilde{\tilde{X}}\}$ is a pairwise neutrosophic supra α -open cover of B . Further, since B is a pairwise neutrosophic supra α -compact set relative to $\tilde{\tilde{X}}$, so there exist a finite pairwise neutrosophic supra α -open sub-cover say $\{E_1, E_2, E_3, \dots, E_n\}$ such that $B \subseteq \cup_{i=1}^n E_i$. Therefore, $\{E_1, E_2, E_3, \dots, E_n\}$ is a finite pairwise neutrosophic supra open sub-cover of B . This implies, B is a pairwise neutrosophic supra compact set relative to $\tilde{\tilde{X}}$. Hence, every pairwise neutrosophic supra α -compact set relative to $(\tilde{\tilde{X}}, \tau_1, \tau_2)$ is also a pairwise neutrosophic supra compact set relative to $(\tilde{\tilde{X}}, \tau_1, \tau_2)$.

4. Conclusions and Future Work

In this article, we have introduced the concept of pairwise neutrosophic supra α -open set and pairwise neutrosophic supra α -continuous function in neutrosophic supra bitopological spaces, and formulated several results related to them in the form of theorems, propositions, etc. in the context of neutrosophic set theory.

Further, it is hoped that, the notion of pairwise neutrosophic supra α -open set and pairwise neutrosophic supra α -continuous function can also be applied in the field of refined neutrosophic set [2], quadripartitioned neutrosophic topological space [9], pentapartitioned neutrosophic topological space [22], bipolar pentapartitioned neutrosophic set [11], etc.

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