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Correction to "Blow-up directions at space infinity for solutions of semilinear heat equations" BSPM 23 (2005), 9–28.

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ABSTRACT: This work presents corrections to "Blow-up directions at space infinity for solutions of semilinear heat equations" published in BSPM 23(2005), 9-28.

Key Words: nonlinear heat equation, blow-up at space infinity, blow-up direction.

Contents

1. Introduction

In [2] we consider the equation

$$\begin{cases} u_t = \Delta u + f(u), & x \in \mathbf{R}^n, t > 0, \\ u(x,0) = u_0(x), & x \in \mathbf{R}^n, \end{cases} \tag{1}$$

and we had some results for the solution blowing up at space infinity. However, the assumptions of f and u_0 in [2, (1),(2), (5) and (6)] are too weak to achieve to the goal. Moreover, the statement of Theorem 3 (ii) is not precise. We shall correct these flaws.

We sent the revised version to the journal; however unfortunately the first version has been published. Moreover, the galley proof was not sent to the authors. We do not know the reason. It seems that there is a problem of e-mails.

First, we should change the condition of the nonlinear term f of (1) from

(A) "The nonlinear term f is assumed to be locally Lipschitz in $\mathbf{R}$ with the properly that

$$\liminf_{s\to\infty} \frac{f(s)}{s^p} > 0 \quad \text{for some } p > 1, \quad f' \geq 0."$$

to a stronger condition:

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(B) " The nonlinear term f is assumed to be a nondecreasing function and locally Lipschitz in $\mathbf{R}$ with the properly that

$$f(\delta b) \leq \delta^p f(b)$$

for all $b \geq b_0$ and for all $\delta \in (\delta_0, 1)$ with some $b_0 > 0$, some $\delta_0 \in (0,1)$ and some $p > 1$."

(The condition **(B)** is stronger than **(A)**; it is easy to construct an example of f a step-like function satisfies **(A)** but does not satisfy **(B)**.) If this condition is fulfilled, f satisfies **(A)** so that

$$\int^{\infty} \frac{ds}{f(s)} < \infty,$$

(see Appendix), and a spatially constant solution of (1) blows up in finite time.

Secondly, we have to change the part of the assumptions of initial data u_0. It should be changed from

(C) "We assume that

$$\operatorname{essinf}_{x \in B_m} (u_0(x) - M_m) \geq 0 \quad \text{for} \quad m = 1, 2, \ldots,$$

where

$$B_m = B_{r_m}(x_m)$$

with a sequence $\{r_m\}$ and a sequence of constants M_m satisfying

$$\lim_{m\to\infty} r_m = \infty, \quad \lim_{m\to\infty} |M - M_m| = 0,$$

and $\{x_m\}_{m=1}^{\infty}$ is some sequence of vectors."

to

(D) "We assume that

$$\operatorname{essinf}_{x \in \tilde{B}_m} (u_0(x) - M_m(x - x_m)) \geq 0 \quad \text{for} \quad m = 1, 2, \ldots,$$

where

$$\tilde{B}_m = B_{r_m}(x_m)$$

with a sequence $\{r_m\}$, a sequence of vectors $\{x_m\}_{m=1}^{\infty}$ and a sequence of functions $\{M_m(x)\}$ satisfying

$$\lim_{m\to\infty} r_m = \infty, \quad M_m(x) \leq M_{m+1}(x) \quad \text{for } m \geq 1$$
$$\lim_{m\to\infty} \inf_{s \in [1, r_m]} \frac{1}{|B_s|} \int_{B_s(0)} M_m(x)dx = M."$$

The condition **(C)** is not convenient to show Theorem 3.

Finally, we should correct Theorem 3 (ii) as follows;

(ii) If for any sequence $\{y_m\}_{m=1}^\infty$ satisfying $\lim_{m\to\infty} y_m/|y_m| = \psi$, there exists a constant $c \in (1/(M+N), \infty)$ such that

$$\limsup_{m\to\infty} \inf_{s\in(1,c)} A_m(s) \le M - \frac{1}{c},$$

then ψ is not a blow-up direction.

We should correct the proof of Theorem 3. In fact we have to change the text from line 4 from below starting from "Finally, we must ..." in page 25 as follows.

Finally, we must show that the conditions of ψ in (i) and (ii) cover all of S^{n-1} exclusively. Let $\{s_m\}_{m=1}^\infty$ be a sequence satisfying $\lim_{m\to\infty} s_m = \infty$. We set $D = (1,\infty) \cap [1/(M+N), \infty)$ and the set of sequence

$$S(\psi) = \left\{ \{y_m\}_{m=1}^\infty \,\middle|\, \lim_{m\to\infty} \frac{y_m}{|y_m|} = \psi, \lim_{m\to\infty} |y_m| = \infty \right\}.$$

Let $\Psi^* = \Psi^*(u_0)$ and $\Psi_* = \Psi_*(u_0)$ be the sets of directions of the form

$$\Psi^* = \left\{ \psi \in S^{n-1} \,\middle|\, \exists \{y_m\}_{m=1}^\infty \in S(\psi), \limsup_{m\to\infty} \inf_{s\in(1,s_m)} A_m(s) = M \right\},$$
$$\Psi_* = \left\{ \psi \in S^{n-1} \,\middle|\, \forall \{y_m\}_{m=1}^\infty \in S(\psi), \exists c \in D, \limsup_{m\to\infty} \inf_{s\in(1,c)} A_m(s) \le M - \frac{1}{c} \right\}.$$

Here, Ψ^* and Ψ_* are the sets of all $\psi \in S^{n-1}$ satisfying, respectively, (i) and (ii) of Theorem 3.

Define two other sets $\Psi^\sharp = \Psi^\sharp(u_0)$ and $\Psi_\sharp = \Psi_\sharp(u_0)$ as follows:

$$\Psi^\sharp = \left\{ \psi \in S^{n-1} \,\middle|\, \exists \{y_m\}_{m=1}^\infty \in S(\psi), \forall c \in D, \limsup_{m\to\infty} \inf_{s\in(1,c)} A_m(s) = M \right\},$$
$$\Psi_\sharp = \left\{ \psi \in S^{n-1} \,\middle|\, \forall \{y_m\}_{m=1}^\infty \in S(\psi), \exists c \in D, \limsup_{m\to\infty} \inf_{s\in(1,c)} A_m(s) < M \right\}.$$

It is clear that $\Psi^\sharp = (\Psi_\sharp)^c$. We shall show that $\Psi^* = \Psi^\sharp$ and $\Psi_* = \Psi_\sharp$.

First we show $\Psi^* = \Psi^\sharp$. It is clear that $\Psi^* \subset \Psi^\sharp$. We shall show $\Psi^* \supset \Psi^\sharp$. Take a sequence $\{c_l\}_{l=1}^\infty \subset \mathbf{R}$ such that $c_l < c_{l+1}$ for $l \ge 1$ and $\lim_{l\to\infty} c_l = \infty$. From the condition of $\Psi^\sharp$ we have

$$\lim_{m\to\infty} \inf_{s\in(1,c_l)} A_m(s) = M$$

for any $l \ge 1$. For $k \in \mathbf{N}$, take the subsequences $\{m_k\} \subset \{m\}$ and $\{l_k\} \subset \{l\}$ satisfying $m_k < m_{k+1}$, $l_k < l_{k+1}$, $\lim_{k\to\infty} = \infty$ and $\lim_{k\to\infty} l_k = \infty$. We set $\tilde{c}_k = c_{l_k}$ to get

$$\lim_{k\to\infty} \inf_{s\in(1,\tilde{c}_k)} A_{m_k}(s) = M.$$

Thus we have $\Psi^* \supset \Psi^\sharp$ and $\Psi^* = \Psi^\sharp$.

Next, we show $\Psi_* = \Psi_\sharp$. It is clear that $\Psi_* \subset \Psi_\sharp$. We shall prove that $\Psi_* \supset \Psi_\sharp$. By the condition of $\Psi_\sharp$ we have

$$\exists c \in D \text{ and } \exists \epsilon > 0, \lim_{m\to\infty} \inf_{s\in(1,c)} A_m(s) \leq M - \epsilon.$$

Take $c' = \max\{c, 1/\epsilon\}$. Then we have

$$\lim_{m\to\infty} \inf_{s\in(1,c')} A_m(s) \leq M - \frac{1}{c'},$$

so we have $\Psi_* \supset \Psi_\sharp$ and $\Psi_* = \Psi_\sharp$

Since $\Psi^* = \Psi^\sharp$, $\Psi_* = \Psi_\sharp$ and $\Psi^\sharp = (\Psi_\sharp)^c$, we obtain $\Psi^* = (\Psi_*)^c$, and the proof is now complete.

We also mention that the proof of Lemma 3.7 of [2] is incomplete. A complete proof is given in [1, Lemma 4. 2. 1]. □

2. Appendix

We shall give a proof of that the condition **(B)** implies that the ODE $v_t = f(v)$ blows up in finite time for sufficiently large initial data.

For the nonlinear term of the first equation of (1), we have one proposition.

Proposition A. *Let f be a continuous function. If f satisfies* **(B)**, *then* **(A)** *holds. In particular*

$$\int_a^\infty \frac{ds}{f(s)} < \infty$$

for any $a \in \boldsymbol{R}$ satisfying $f(r) > 0$ for $r \geq a$.

Proof: We take $\delta_1 \in (\delta_0, 1)$ and $v_1 > b_0$. Since $\delta_1^{-\alpha_0} v_1 > b_0$ for $\alpha_0 \in [0,1]$, we have

$$f(v_1) \leq \delta_1^{\alpha_0 p} f(\delta_1^{-\alpha_0} v_1)$$

by (2). Next, since $\delta_1^{-\alpha_0-1} v_1 > b_0$, we obtain

$$f(v_1) \leq \delta_1^{(\alpha_0+1)p} f(\delta_1^{-\alpha_0-1} v_1)$$

by the same argument. By induction, we have

$$f(v_1) \leq \delta_1^{(\alpha_0+N)p} f(\delta_1^{-\alpha_0-N} v_1)$$

for α_0 and $n \in \mathbf{N}$. Then, we obtain **(A)**. It is clear that

$$f(v_1) \leq \delta_1^{\alpha p} f(\delta_1^{-\alpha} v_1)$$

for each $\alpha \geq 0$. Take $s = \delta_1^{-\alpha} v_1$ to get

$$f(v_1) \leq t^{-p} f(t v_1)$$

for $s > v_1 > v_0$. Thus we obtain

$$\int_a^\infty \frac{ds}{f(s)} < \infty.$$

The proof is now complete. □

3. List of typographical errors

There are other typographical errors we should correct.

1. P. 9, line 2 of ABSTRACT, "$\liminf f(u)/u^p > 0$" ⇒ "$f(\delta b) \le \delta^p f(b)$ for all $b \ge b_0$ and all $\delta \in (0.\delta_0)$ with some $b_0 > 0$, some $\delta_0 \in (0,1)$ and some $p > 1$, " .
2. P. 13, line 1 from bottom and P.14, Line 2, "$G_R(x,y,t)$" ⇒ "$\tilde{G}_R(x,y,t)$".
3. P. 14, line 7, line 19 and line 5 from bottom, "M_m" ⇒ "$M_m(x - x_m)$".
4. P. 14, line 17, "$X_m \le X_{m+1}$" ⇒ " $X_m(x + x_m, t) \le X_{m+1}(x + x_{m+1}, t)$". From P. 14, line 19 to P. 15, line 12, all "G_m" ⇒ "$\hat{G}_m$". ".
5. P. 14, line 9 from bottom, "$G_m(x,y,t)$ be the ... domain B_{R_m}" ⇒ "$G_m = \tilde{G}_{R_m}$".
6. P. 14, line 4 from bottom and P. 15, line 2, after "$\int_{\mathbf{R}^n}$", insert "$G_m(x,y,t-s)$".
7. P. 15, line 3, remove "{" between "...[" and "(...".
8. P, 15, line 6, "M_m" ⇒ "$M_m(y - x_m)$".
9. P. 15, line 10 from bottom, insert "$G_m(x,y,t-s)$" between "$\int_0^t$" and "C".
10. P. 19, line 3, "$(M = 2,3,...l)$" ⇒ "$(m = 2,3,...,l)$"
11. P. 22, line 8, "...$\Delta w_m = \phi_m(f(u))$..." ⇒ "$\Delta w_m + \phi_m(f(u))$...".
12. P. 22, line 11, "$g(x,s)$" ⇒ "$g_m(x,s)$".
13. P. 22, line 14, "$e^{t\Delta}$" ⇒ "$e^{(t-s)\Delta}$".
14. P. 22, lines 18 and 19, "$\|w(\cdot,s)\|_{L^\infty(B_{\eta,m})}$ " ⇒ "$\|w_m(\cdot,s)\|_{L^\infty(B_{\eta,m})}$".
15. P. 22, line 4 from bottom, remove "ϵ" between "C" and "$\int_{\tau - r_0^2}^t$".
16. P. 23, line 12 from bottom, "$\in (0, 1/2 - \epsilon^{q-1}/(q-1)$" ⇒ "$\in [\epsilon^{q-1}/(q-1) - 1/2, 0)$".
17. P, 23, line 11 from bottom, "$m \in (2/(q-1-2\epsilon^{q-1}), 2/(q-1-2\epsilon^{q-1}) + 1]$" ⇒ "$m \in [(3 + 2\epsilon^{q-1} - q)/(q-1-2\epsilon^{q-1}), 2/(q-1-2\epsilon^{q-1}))$ ".

18. P. 24, line 5, insert “for any $r \in (0, 1)$” after “then”.

19. P. 24, line 7, remove “$r \in (0, 1)$ and” between “with” and “D”.

20. P. 25, line 2, insert “in” between “as” and “proof”.

21. P. 25, line 12 from bottom and line 10 from bottom, “+” $\Rightarrow$ “−”.

22. P. 25, line 12 from bottom and line 10 from bottom, “c_n” $\Rightarrow$ “c_m”.

References

1. Y. Giga, Y. Seki, N. Umeda, *Blow-up at space infinity for nonlinear heat equation*, EPrint series of Department of Mathematics, Hokkaido University ♯856 (2007), http://eprints.math.sci.hokudai.ac.jp/archive/00001709/

2. Y. Giga, N. Umeda, *Blow-up directions at space infinity for solutions of semilinear heat equations*, Bol. Soc. Parana. Mat. **23** (2005), 9–28.

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