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The impact of time-fractional Cahn–Hilliard equation that arises during the process of digital picture reconstruction

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ABSTRACT: In this article, we examine the approximate solution of fourth and sixth order time-fractional Cahn-Hilliard equation by employing Shehu Adomian decomposition method. We employed Caputo, Caputo-Fabrizio, and Atangana-Baleanu in the Caputo sense fractional differential operators. When inpainting digital photographs, this equation is utilized to repair damaged or absent portions of deteriorated text and high luminance images. The acquired results are provided in the form of series. Numerical simulations were carried out and compared with the homotopy perturbation method, new iterative method and q-homotopy analysis method to ensure that the current technique is accurate. The aquired results are shown both numerically and visually to ensure the applicability and validity of the proposed technique. The numerical findings are coherent with previous findings. In the current investigation, uniqueness and convergence analysis are also mentioned.

Key Words: Caputo-Fabrizio, Caputo, Atangana-Baleanu in Caputo sense, time fractional Cahn-Hilliard equation, Shehu Adomian decomposition method.

Contents

1. Introduction

The technique of filling in an image's missing areas so that the final product resembles the original image is known as image interpolation or image inpainting. Its applications include photograph scratch removal and the restoration of antique paintings by museum artists. In order to recover images, painters have long used manual inpainting. Using the structural knowledge of the surrounding areas of the missing pieces, they were able to recreate the damaged or missing elements of the photos. As digital image processing advanced, there was an increased demand for unsupervised picture restoration, which gave rise to digital image inpainting. In digital image inpainting, the nonlinear Cahn–Hilliard equation is a stiff parabolic partial differential equation that is employed for extreme contrast images exceptionally high resolution and quick inpainting of damaged text. A natural relationship between the contours across missing regions is provided by the Cahn–Hilliard equation's smoothing feature [1,2].

In comparison to classical calculus, fractional calculus (FC) is effective at modelling real-world issues. FC theory provides an excellent and logical interpretation of instinctual reality. It has recently attracted the attention of numerous investigators due to its efficiency in providing a comprehensive explanation for non-linear complex systems [3,4,5,6,7]. The benefit of using fractional models of differential equations in any models is their nonlocal behaviour. Fractional-order derivative is nonlocal, but integer-order

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derivative is local in nature. The theory of fractional-order calculus has been associated to real-world applications, and it has been used to examine and investigate a wide range of phenomena, such as noisy environments [8], human diseases [9], financial models [10], chaos theory [11], nonlinear optics [12], and so on.

In fractional analysis, the inclusion of fractional derivative definitions allows the user to choose the most relevant one for the problem to get the best solution. In the last few decades, several fractional derivative definitions have emerged. Grunwald-Letnikov, Caputo-Fabrizio (CF), Riemann-Liouville (R-L), Caputo (C), and Atangana-Baleanu in the Caputo manner (ABC) fractional differential operators are some of the prominent definitions presented in literature [13,14]. The R-L and C fractional differential operators have a singular kernel. The R-L derivative of a constant is not equal to zero. Researchers have employed new fractional differential operators featuring a nonsingular kernel to address these types of problems. One is CF with an exponential kernel [15], and the other one is ABC with a kernel in the form of Mittag-Lefler function [16].

The Cahn-Hilliard equation was first proposed in the year 1958 as a model for phase separation of a binary alloy at critical temperature [17]. This equation is crucial to comprehending a number of exciting physical phenomena, such as spinodal decomposition [18] and phase ordering dynamics. It also covers important qualitative characteristics of two-phase systems that are related to phase separation mechanisms [19]. It has become increasingly significant in material sciences [20,21]. Researchers have looked into the mathematical and numerical solutions to this equation as a result of its real-world applicability in the many domains mentioned above [22,23,24,25,26,27,28,29].

In this study, we consider the fourth and sixth order time-fractional Cahn-Hilliard equation (TFCHE) [30]:

$$D_\tau^\mu \Phi = c\Phi_\chi + (-\Phi_{\chi\chi} - \Phi + \Phi^3)_{\chi\chi}, \tag{1.1}$$

and

$$D_\tau^\mu \Phi = c\Phi\Phi_\chi + (\Phi_{\chi\chi} + \Phi - \Phi^3)_{\chi\chi\chi\chi}, \tag{1.2}$$

with the initial condition (I.C)

$$\Phi(\chi, 0) = \tanh\left(\frac{\chi}{\sqrt{2}}\right). \tag{1.3}$$

Here, $0 < \mu \leq 1$ represents the order of fractional derivative and $c \geq 0$ is parameter. Various approaches are presented in literature to solve fourth and sixth order TFCHE namely, Akinyemi et al. solved both the orders using a new iterative method (NIM) and q-homotopy analysis method (q-HAM) with two different I.Cs [30], Bouhassoun and Cherif solved fourth order TFCHE using HPM [31], Arafa and Elmahdy solved fourth order TFCHE using RPSM [32], Prakasha et al. solved fourth order TFCHE using NTDM and q-HATM [33], Shah and Patel solved sixth order TFCHE using a new integrated project differential transform process [34]. Recently, Al-Maskari and Karaa used finite element method to study the solutions of TFCHE [35].

The Shehu Adomian decomposition method (SADM) is a decomposition method that combines the Shehu transform and the Adomian decomposition method. SADM has been proposed by Rashid et al. [37] in the year 2021. In the HPM solving the functional equation during each iteration is challenging and time-intensive. The VIM method possesses inherent precision in the determination of the Lagrange multiplier, corrective function, and stationary conditions for fractional order problems. Unlike the usual Adomian process, the proposed solution does not include the computation of the fractional derivative or fractional integrals in the recursive mechanism, making series term evaluation easier. There is no linearization, predefined assumptions, perturbation, or discretization in SADM, and there are no round-off errors. SADM is employed to solve various time fractional differential equations, such as the Zakharov-Kuznetsov equation [36], the KdV equations [37], the third-order dispersive PDEs [38], Belousov-Zhabotinsky System [39] and so on.

The main objective of this study is to find approximate solutions for fourth and sixth order TFCHE using SADM with singular and non singular fractional differential operators such as C, CF and ABC. The following is the outline of the paper: Section 2 covers the fractional derivative definitions and its Shehu transform (ST). We introduced the SADM to various fractional derivatives, namely ABC, C and

CF in Section 3. The convergence and uniqueness of the solutions were investigated in Section 4. Section 5 includes two test examples of the TFCHE. The results and discussions are presented in Section 6. The conclusions are presented in Section 7.

2. Basic Definitions

This section will highlight the fundamental FC preliminaries.

Definition 2.1 *[40] The fractional differential operator Caputo of order μ is given as*

$$D^{\mu}\Phi(\chi) = I^{m-\mu}D^{m}\Phi(\chi) = \frac{1}{\Gamma(m-\mu)}\int_0^{\chi}(\chi-\varrho)^{m-\mu-1}\Phi^{m}(\varrho)d\varrho,\ \chi>0,$$

where $m-1<\mu\leq m,\ m\in N$.

Definition 2.2 *[15] Let $\Phi\in\mathbb{H}^1(c,d),\ d>c$, then the CF of order $\mu\in(0,1)$ is given as*

$${}_{c}^{CF}D_{\chi}^{\mu}(\Phi(\chi)) = \frac{B(\mu)}{1-\mu}\int_c^{\chi} exp\Big(-\mu\frac{(\chi-\varrho)}{1-\mu}\Big)\Phi'(\varrho)d\varrho.$$

Definition 2.3 *[16] Let $\Phi\in\mathbb{H}^1(c,d),\ d>c$, then the ABC of order $\mu\in(0,1)$ is given as*

$${}_{c}^{ABC}D_{\chi}^{\mu}(\Phi(\chi)) = \frac{B(\mu)}{1-\mu}\int_c^{\tau} E_{\mu}\Big(-\mu\frac{(\chi-\varrho)^{\mu}}{1-\mu}\Big)\Phi'(\varrho)d\varrho.$$

Where E_{μ} indicates Mittag-Leffler function [41] and $B(\mu)$ represents a normalization function.

Definition 2.4 *[42] The ST of $\Phi(\chi)\ \forall\ \chi\geq 0$ is defined by*

$$\mathbb{S}[\Phi(\chi)] = \int_0^{\infty} e^{-\frac{s\chi}{\vartheta}}\Phi(\chi)\ \ d\chi \qquad \vartheta>0,\ s>0.$$

Where, $\mathbb{S}$ represents ST operator.

Definition 2.5 *[30] The ST of Caputo derivative is defined as*

$$\mathbb{S}[D_{\tau}^{\mu}\Phi(\chi,\tau)] = \frac{s^{\mu}}{\vartheta^{\mu}}\Big(\mathbb{S}[\Phi(\chi,\tau)] - \frac{\vartheta}{s}\Phi(\chi,0)\Big). \tag{2.1}$$

Definition 2.6 *[43] The ST of CF derivative is given as*

$$\mathbb{S}[D_{\tau}^{\mu}\Phi(\chi,\tau)] = \frac{1}{\mu\frac{\vartheta}{s}+1-\mu}\Big(\mathbb{S}[\Phi(\chi,\tau)] - \frac{\vartheta}{s}\Phi(\chi,0)\Big). \tag{2.2}$$

Definition 2.7 *[44] The ST of ABC derivative is given as*

$$\mathbb{S}[D_{\tau}^{\mu}\Phi(\chi,\tau)] = \frac{B(\mu)}{\mu\frac{\vartheta^{\mu}}{s^{\mu}}+1-\mu}\Big(\mathbb{S}[\Phi(\chi,\tau)] - \frac{\vartheta}{s}\Phi(\chi,0)\Big). \tag{2.3}$$

3. The Procedure of SADM

In this section, we discuss a SADM approach that we impose to the underlying non linear fractional partial differential equation (NFPDE)

$$D_{\tau}^{\mu}\Phi(\chi,\tau) = N(\Phi(\chi,\tau)) + E(\Phi(\chi,\tau)) + \zeta(\chi,\tau), \tag{3.1}$$

with the initial condition

$$\Phi(\chi,0) = \psi(\chi).$$

Where N, E, ζ represents the non linear, linear and known function respectively.
SADM$_C$: Employing ST on Eq. (3.1) and using Eq. (2.1), we derive

$$\mathbb{S}(\Phi(\chi,\tau)) = \frac{\vartheta}{s}\psi(\chi) + \frac{\vartheta^{\mu}}{s^{\mu}}\mathbb{S}[E(\Phi(\chi,\tau)) + N(\Phi(\chi,\tau)) + \zeta(\chi,\tau)]. \tag{3.2}$$

By taking inverse ST on Eq. (3.2), we get

$$\Phi(\chi,\tau) = Q(\chi,\tau) + \mathbb{S}^{-1}\Big[\frac{\vartheta^{\mu}}{s^{\mu}}\mathbb{S}[N(\Phi(\chi,\tau)) + E(\Phi(\chi,\tau))]\Big]. \tag{3.3}$$

Here $Q(\chi,\tau)$ indicates the term acquired from the known function $\zeta(\chi,\tau)$ and I.C. through the use of inverse ST.

The nonlinear term $N(\Phi(\chi,\tau))$ and $\Phi(\chi,\tau)$ can be written as

$$N(\Phi(\chi,\tau)) = \sum_{\varUpsilon=0}^{\infty}\mathbb{C}_{\varUpsilon}, \tag{3.4}$$

where $\mathbb{C}_{\varUpsilon}$ indicates the Adomian polynomial [45].

$$\Phi(\chi,\tau) = \sum_{\varUpsilon=0}^{\infty}\Phi_{\varUpsilon}(\chi,\tau). \tag{3.5}$$

Now, put Eq. (3.4) and (3.5) into Eq.(3.3) to get

$$\sum_{\varUpsilon=0}^{\infty}\Phi_{\varUpsilon}(\chi,\tau) = Q(\chi,\tau) + \mathbb{S}^{-1}\Big[\mathbb{S}\Big[E\Big(\sum_{\varUpsilon=0}^{\infty}\Phi_{\varUpsilon}(\chi,\tau)\Big) + \sum_{\varUpsilon=0}^{\infty}\mathbb{C}_{\varUpsilon}\Big]\frac{\vartheta^{\mu}}{s^{\mu}}\Big]. \tag{3.6}$$

From Eq. (3.6), we obtain

$$\begin{aligned}
\Phi_0^C(\chi,\tau) =& Q(\chi,\tau),\\
\Phi_1^C(\chi,\tau) =& \mathbb{S}^{-1}\Big[\mathbb{S}\Big[E(\Phi_0)(\chi,\tau) + \mathbb{C}_0\Big]\frac{\vartheta^{\mu}}{s^{\mu}}\Big],\\
\Phi_2^C(\chi,\tau) =& \mathbb{S}^{-1}\Big[\mathbb{S}\Big[E(\Phi_1)(\chi,\tau) + \mathbb{C}_1\Big]\frac{\vartheta^{\mu}}{s^{\mu}}\Big],\\
&\vdots\\
\Phi_{\varUpsilon+1}^C(\chi,\tau) =& \mathbb{S}^{-1}\Big[\mathbb{S}\Big[E(\Phi_{\varUpsilon})(\chi,\tau) + \mathbb{C}_{\varUpsilon}\Big]\frac{\vartheta^{\mu}}{s^{\mu}}\Big], \varUpsilon \geq 0.
\end{aligned} \tag{3.7}$$

By putting Eq.(3.7) into Eq. (3.5), we get the solution of (3.1) *i.e*

$$\Phi^C = \Phi_0^C + \Phi_1^C + \Phi_2^C + \dots\ . \tag{3.8}$$

SADM$_{CF}$: Employing ST on Eq. (3.1) and using Eq. (2.2), we get

$$\mathbb{S}(\Phi(\chi,\tau)) = \frac{\vartheta}{s}\psi(\chi) + (\mu\frac{\vartheta}{s} - \mu + 1)\mathbb{S}[E(\Phi(\chi,\tau)) + N(\Phi(\chi,\tau)) + \zeta(\chi,\tau)]. \tag{3.9}$$

Consider $\sigma(\mu,\vartheta,s) = \mu\frac{\vartheta}{s} - \mu + 1$
By taking inverse ST on Eq. (3.2), we derive

$$\Phi(\chi,\tau) = Q(\chi,\tau) + \mathbb{S}^{-1}\Big[\sigma(\mu,\vartheta,s)\mathbb{S}[E(\Phi(\chi,\tau)) + N(\Phi(\chi,\tau))]\Big]. \tag{3.10}$$

Here $Q(\chi,\tau)$ indicates the term acquired from the known function $\zeta(\chi,\tau)$ and I.C. through the use of inverse ST.

Now, put Eq.(3.4) and (3.5) into (3.10) to get

$$\sum_{\Upsilon=0}^{\infty}\Phi_{\Upsilon}(\chi,\tau)=Q(\chi,\tau)+\mathbb{S}^{-1}\Big[\sigma(\mu,\vartheta,s)\mathbb{S}\Big[E\Big(\sum_{\Upsilon=0}^{\infty}\Phi_{\Upsilon}(\chi,\tau)\Big)+\sum_{\Upsilon=0}^{\infty}\mathbb{C}_{\Upsilon}\Big]\Big]. \tag{3.11}$$

From Eq. (3.11) we obtain

$$\begin{aligned}
\Phi_0^{CF}(\chi,\tau)=&Q(\chi,\tau),\\
\Phi_1^{CF}(\chi,\tau)=&\mathbb{S}^{-1}\Big[\sigma(\mu,\vartheta,s)\mathbb{S}\Big[E(\Phi_0)+\mathbb{C}_0\Big]\Big],\\
\Phi_2^{CF}(\chi,\tau)=&\mathbb{S}^{-1}\Big[\sigma(\mu,\vartheta,s)\mathbb{S}\Big[E(\Phi_1)+\mathbb{C}_1\Big]\Big],\\
&\vdots\\
\Phi_{\Upsilon+1}^{CF}(\chi,\tau)=&\mathbb{S}^{-1}\Big[\sigma(\mu,\vartheta,s)\mathbb{S}\Big[E(\Phi_{\Upsilon})+\mathbb{C}_{\Upsilon}\Big]\Big],\Upsilon\geq 0.
\end{aligned} \tag{3.12}$$

By putting Eq. (3.12) into (3.5), we get the solution of (3.1) *i.e*

$$\Phi^{CF}=\Phi_0^{CF}+\Phi_1^{CF}+\Phi_2^{CF}+\dots\ . \tag{3.13}$$

SADM$_{ABC}$**:** Taking ST to Eq. (3.1) and using Eq. (2.3), we derive

$$\mathbb{S}(\Phi(\chi,\tau))=\frac{\vartheta}{s}\psi(\chi)+\frac{(1-\mu+\mu\frac{\vartheta^{\mu}}{s^{\mu}})}{B(\mu)}\mathbb{S}[E(\Phi(\chi,\tau))+N(\Phi(\chi,\tau))+\zeta(\chi,\tau)]. \tag{3.14}$$

Consider $\omega(\mu,\vartheta,s)=\frac{1-\mu+\mu\frac{\vartheta^{\mu}}{s^{\mu}}}{B(\mu)}$
By taking inverse ST on Eq. (3.14), we get

$$\Phi(\chi,\tau)=Q(\chi,\tau)+\mathbb{S}^{-1}\Big[\omega(\mu,\vartheta,s)\mathbb{S}[E(\Phi(\chi,\tau))+N(\Phi(\chi,\tau))]\Big]. \tag{3.15}$$

Here $Q(\chi,\tau)$ indicates the term acquired from the known function $\zeta(\chi,\tau)$ and I.C. through the use of inverse ST.

Now, put Eq.(3.4) and (3.5) into (3.15) to get

$$\sum_{\Upsilon=0}^{\infty}\Phi_{\Upsilon}(\chi,\tau)=Q(\chi,\tau)+\mathbb{S}^{-1}\Big[\omega(\mu,\vartheta,s)\mathbb{S}\Big[E\Big(\sum_{\Upsilon=0}^{\infty}\Phi_{\Upsilon}(\chi,\tau)\Big)+\sum_{\Upsilon=0}^{\infty}\mathbb{C}_{\Upsilon}\Big]\Big]. \tag{3.16}$$

From Eq. (3.16) we obtain

$$\begin{aligned}
\Phi_0^{ABC}(\chi,\tau)=&Q(\chi,\tau),\\
\Phi_1^{ABC}(\chi,\tau)=&\mathbb{S}^{-1}\Big[\omega(\mu,\vartheta,s)\mathbb{S}\Big[E(\Phi_0)+\mathbb{C}_0\Big]\Big],\\
\Phi_2^{ABC}(\chi,\tau)=&\mathbb{S}^{-1}\Big[\omega(\mu,\vartheta,s)\mathbb{S}\Big[E(\Phi_1)+\mathbb{C}_1\Big]\Big],\\
&\vdots\\
\Phi_{\Upsilon+1}^{ABC}(\chi,\tau)=&\mathbb{S}^{-1}\Big[\omega(\mu,\vartheta,s)\mathbb{S}\Big[E(\Phi_{\Upsilon})+\mathbb{C}_{\Upsilon}\Big]\Big],\Upsilon\geq 0.
\end{aligned} \tag{3.17}$$

By putting Eq. (3.17) into (3.5), we get the solution of (3.1) *i.e*

$$\Phi^{ABC}=\Phi_0^{ABC}+\Phi_1^{ABC}+\Phi_2^{ABC}+\dots\ . \tag{3.18}$$

4. Convergence Analysis

In this section, we have presented the SADM$_C$, SADM$_{CF}$, and SADM$_{ABC}$ uniqueness and convergence.

Theorem 4.1 *[37] The SADM$_C$ solution of NFPDE is unique when $0 < (\kappa_1+\kappa_2)\frac{\tau^\mu}{\Gamma(1+\mu)} < 1$.*

Proof: Let $P = (C[J], \|.\|)$ be the Banach space and $\|\varphi(\tau)\| = \max_{\tau\in J} \mid \varphi(\tau) \mid$, $\forall$ continuous functions on J. Let $H : P \to P$ is a nonlinear mapping, where

$$\Phi^C_{\Upsilon+1}(\chi,\tau) = \Phi^C_0 + \mathbb{S}^{-1}\Big[\mathbb{S}[E(\Phi_\Upsilon(\chi,\tau))\frac{\vartheta^\mu}{s^\mu}]\Big] + \mathbb{S}^{-1}\Big[\mathbb{S}[N(\Phi_\Upsilon(\chi,\tau))]\frac{\vartheta^\mu}{s^\mu}\Big],\ \Upsilon \geq 0.$$

Suppose that $\mid E(\Phi) - E(\Phi^*) \mid < \kappa_1 \mid \Phi - \Phi^* \mid$ and $\mid N(\Phi) - N(\Phi^*) \mid < \kappa_2 \mid \Phi - \Phi^* \mid$, where κ_1 and κ_2 are Lipschitz constants, Φ and Φ^* are two distinct functions and satisfies Eqn. (3.3) then

$$\begin{aligned}
\|H(\Phi) - H(\Phi^*)\| =& \max_{\tau\in J}\Big|\mathbb{S}^{-1}\Big[\mathbb{S}[E(\Phi)+N(\Phi)]\frac{\vartheta^\mu}{s^\mu}\Big] - \mathbb{S}^{-1}\Big[\mathbb{S}[E(\Phi^*)+N(\Phi^*)]\frac{\vartheta^\mu}{s^\mu}\Big]\Big|\\
\leq& \max_{\tau\in J}\Big|\mathbb{S}^{-1}\Big[\mathbb{S}[E(\Phi)-E(\Phi^*)]\frac{\vartheta^\mu}{s^\mu} + \mathbb{S}[N(\Phi)-N(\Phi^*)]\frac{\vartheta^\mu}{s^\mu}\Big]\Big|\\
\leq& \max_{\tau\in J}\Big[\kappa_1\mathbb{S}^{-1}\Big[\mathbb{S}|\Phi-\Phi^*|\frac{\vartheta^\mu}{s^\mu}\Big] + \kappa_2\mathbb{S}^{-1}\Big[\mathbb{S}|\Phi-\Phi^*|\frac{\vartheta^\mu}{s^\mu}\Big]\Big]\\
\leq& \max_{\tau\in J}(\kappa_1+\kappa_2)\Big[\mathbb{S}^{-1}\Big[\mathbb{S}|\Phi-\Phi^*|\Big]\frac{\vartheta^\mu}{s^\mu}\Big]\\
\leq&(\kappa_1+\kappa_2)\Big[\mathbb{S}^{-1}\Big[\mathbb{S}\left\|\Phi-\Phi^*\right\|\Big]\frac{\vartheta^\mu}{s^\mu}\Big]\\
=&(\kappa_2+\kappa_1)\frac{\tau^\mu}{\Gamma(1+\mu)}\left\|\Phi-\Phi^*\right\|.
\end{aligned}$$

H is contraction as $0 < (\kappa_2+\kappa_1)\frac{\tau^\mu}{\Gamma(1+\mu)} < 1$.Therefore the solution of (3.1) is unique. Similarly the solution of (3.1) using SADM$_{CF}$ and SADM$_{ABC}$ is unique when $0 < (\kappa_1+\kappa_2)(1-\mu+\mu\tau) < 1$ and $0 < (\kappa_1+\kappa_2)(\tau^\mu\frac{\mu}{\Gamma(\mu+1)} + 1 - \mu) < 1$ respectively. □

Theorem 4.2 *[37] The solution of (3.1) using SADM$_C$ is convergent.*

Proof: Let $\Phi_m = \sum_{\Upsilon=0}^{m}\Phi_\Upsilon(\chi,\tau)$. We prove that Φ_Υ is a Cauchy sequence in F i.e Banach Space. Consider,

$$\begin{aligned}
\|\Phi_\varsigma - \Phi_\Upsilon\| =& \max_{\tau\in J}|\Phi_\varsigma - \Phi_\Upsilon|\\
=& \max_{\tau\in J}\Big|\sum_{q=\Upsilon+1}^{\varsigma}\Phi_q\Big|,\ q = 1,2,3,\ \dots\ .\\
\leq& \max_{\tau\in J}\Big|\mathbb{S}^{-1}\Big[\mathbb{S}\Big[\sum_{q=\Upsilon}^{\varsigma-1}E(\Phi_q)+N(\Phi_q)\Big]\frac{\vartheta^\mu}{s^\mu}\Big]\Big|\\
\leq& \max_{\tau\in J}\Big|\mathbb{S}^{-1}\Big[\mathbb{S}[E(\Phi_{\varsigma-1})-E(\Phi_{\Upsilon-1})]\frac{\vartheta^\mu}{s^\mu}\Big]\Big| + \max_{\tau\in J}\Big|\mathbb{S}^{-1}\Big[\mathbb{S}[N(\Phi_{\varsigma-1})-N(\Phi_{\Upsilon-1})]\frac{\vartheta^\mu}{s^\mu}\Big]\Big|\\
\leq&\kappa_1\max_{\tau\in J}\Big|\mathbb{S}^{-1}\Big[\mathbb{S}[E(\Phi_{\varsigma-1})-E(\Phi_{\Upsilon-1})]\frac{\vartheta^\mu}{s^\mu}\Big]\Big| + \kappa_2\max_{\tau\in J}\Big|\mathbb{S}^{-1}\Big[\mathbb{S}[N(\Phi_{\varsigma-1})-N(\Phi_{\Upsilon-1})]\frac{\vartheta^\mu}{s^\mu}\Big]\Big|\\
=&(\kappa_1+\kappa_2)\frac{\tau^\mu}{\Gamma(\mu+1)}\left\|\Phi_{\varsigma-1}-\Phi_{\Upsilon-1}\right\|.
\end{aligned}$$

Let $\varsigma = \Upsilon + 1$, then

$$\begin{aligned}\|\Phi_{\Upsilon+1} - \Phi_{\Upsilon}\| &\leq \kappa \|\Phi_{\Upsilon} - \Phi_{\Upsilon-1}\| \\ &\leq \kappa^2 \|\Phi_{\Upsilon-1} - \Phi_{\Upsilon-2}\| \\ &\vdots \\ &\leq \kappa^{\Upsilon} \|\Phi_1 - \Phi_0\|,\end{aligned}$$

where $\kappa = (\kappa_1 + \kappa_2)\frac{\tau^{\mu}}{\Gamma(\mu+1)}$. Similarly, we have

$$\begin{aligned}\|\Phi_{\varsigma} - \Phi_{\Upsilon}\| &\leq \|\Phi_{\Upsilon+1} - \Phi_{\Upsilon}\| + \|\Phi_{\Upsilon+2} - \Phi_{\Upsilon+1}\| + \cdots + \|\Phi_{\varsigma} - \Phi_{\varsigma-1}\| \\ &\leq (\kappa^n + \kappa^{\Upsilon+1} + \cdots + \kappa^{\varsigma-1}) \|\Phi_1 - \Phi_0\| \\ &\leq \kappa^{\Upsilon}\left(\frac{1-\kappa^{m-n}}{1-\kappa}\right) \|\Phi_1\|.\end{aligned}$$

As $0 < \kappa < 1$, we get $1 - \kappa^{m-\Upsilon} < 1$. Therefore,

$$\|\Phi_{\varsigma} - \Phi_{\Upsilon}\| \leq \frac{\kappa^{\Upsilon}}{1-\kappa} \max_{\tau \in J} \|\Phi_1\|.$$

Since $\|\Phi_1\| < \infty$, as a result $n \to \infty$ then $\|\Phi_{\varsigma} - \Phi_{\Upsilon}\| \to 0$. Hence, Φ_{ς} is Cauchy sequence in F; therefore, the series Φ_{ς} is convergent.

Similarly we prove that the solution of (3.1) is convergent using SADM$_{CF}$ and SADM$_{ABC}$. □

5. Numerical Examples

In this section by employing fractional differential operators C, CF, and ABC derivatives, we derive the following solutions to Eq. (1.1) and (1.2) with SADM.

Example 1 Consider the general form of (1.1) as

$$D_{\tau}^{\mu}\Phi = c\Phi_{\chi} + 6\Phi\Phi_{\chi}^2 + 3\Phi^2\Phi_{\chi\chi} - \Phi_{\chi\chi} - \Phi_{\chi\chi\chi\chi}, \quad \text{where} \ \ 0 < \mu \leq 1, \tag{5.1}$$

with the initial condition $\Phi(\chi, 0) = \tanh\left(\frac{\chi}{\sqrt{2}}\right)$.

SADM$_C$: By employing the SADM$_C$, we get the subsequent results,

$$\begin{aligned}\Phi_0^C(\chi,\tau) &= \tanh\left(\frac{\chi}{\sqrt{2}}\right), \ \Phi_1^C(\chi,\tau) = \frac{c}{\sqrt{2}}\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\frac{\tau^{\mu}}{\Gamma(1+\mu)}, \\ \Phi_2^C(\chi,\tau) &= -c^2\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\frac{\tau^{2\mu}}{\Gamma(1+2\mu)}, \\ \Phi_3^C(\chi,\tau) &= \frac{3}{2}c^2\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\Bigg(30\tanh^5\left(\frac{\chi}{\sqrt{2}}\right) + \sqrt{2}c\tanh^2\left(\frac{\chi}{\sqrt{2}}\right) - 44\tanh^3\left(\frac{\chi}{\sqrt{2}}\right) \\ &\quad - \frac{c\sqrt{2}}{3} + 14\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\frac{\tau^{3\mu}}{\Gamma(1+3\mu)} - \frac{3}{2}c^2\Bigg(-15\tanh^5\left(\frac{\chi}{\sqrt{2}}\right) \\ &\quad + 22\tanh^3\left(\frac{\chi}{\sqrt{2}}\right) - 7\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\frac{\tau^{3\mu}}{(\Gamma(1+\mu))^2}\frac{\Gamma(1+2\mu)}{\Gamma(1+3\mu)}, \\ &\vdots\end{aligned} \tag{5.2}$$

by substituting (5.2) in (3.8), we get

$$\begin{aligned}\Phi^{C}(\chi,\tau)=&\tanh\left(\frac{\chi}{\sqrt{2}}\right)+\frac{c}{\sqrt{2}}\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\frac{\tau^{\mu}}{\Gamma(\mu+1)}-c^2\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\frac{\tau^{2\mu}}{\Gamma(2\mu+1)}\\&\frac{3}{2}c^2\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\Bigg(30\tanh^5\left(\frac{\chi}{\sqrt{2}}\right)+\sqrt{2}c\tanh^2\left(\frac{\chi}{\sqrt{2}}\right)-44\tanh^3\left(\frac{\chi}{\sqrt{2}}\right)-\frac{c\sqrt{2}}{3}\\&+14\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\frac{\tau^{3\mu}}{\Gamma(3\mu+1)}-\frac{3}{2}c^2\Bigg(-15\tanh^5\left(\frac{\chi}{\sqrt{2}}\right)+22\tanh^3\left(\frac{\chi}{\sqrt{2}}\right)\\&-7\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\frac{\tau^{3\mu}}{(\Gamma(1+\mu))^2}\frac{\Gamma(1+2\mu)}{\Gamma(3\mu+1)}+\dots\,.\end{aligned}$$

SADM$_{CF}$:By employing the SADM$_{CF}$, we get the subsequent results,

$$\begin{aligned}\Phi_0^{CF}(\chi,\tau)=&\tanh\left(\frac{\chi}{\sqrt{2}}\right),\ \Phi_1^{CF}(\chi,\tau)=\frac{c}{\sqrt{2}}\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)(\tau\mu+1-\mu),\\\Phi_2^{CF}(\chi,\tau)=&-c^2\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\left(\tau^2\frac{\mu^2}{\Gamma(3)}-2(\tau\mu^2-\tau\mu)+(\mu-1)^2\right),\\\Phi_3^{CF}(\chi,\tau)=&\frac{3}{2}c^2\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\Bigg(30\tanh^5\left(\frac{\chi}{\sqrt{2}}\right)+\sqrt{2}c\tanh^2\left(\frac{\chi}{\sqrt{2}}\right)-44\tanh^3\left(\frac{\chi}{\sqrt{2}}\right)\\&-\frac{c\sqrt{2}}{3}+14\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\left(\tau^3\frac{\mu^3}{\Gamma(4)}+\tau^2(\mu-1)\frac{3\mu^2}{\Gamma(3)}-(\mu-1)^3\right.\\&\left.+3\tau(\mu-1)^2\mu\right)-\frac{3}{2}\Bigg(-15\tanh^5\left(\frac{\chi}{\sqrt{2}}\right)+22\tanh^3\left(\frac{\chi}{\sqrt{2}}\right)-7\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\\&\left(\tau^3\frac{2\mu^3}{\Gamma(4)}-2\tau^2(\mu^3-\mu^2)-(\mu-1)^3+3(\mu-1)^2\mu\tau\right),\\&\vdots\end{aligned}\tag{5.3}$$

by substituting (5.3) in (3.13), we get

$$\begin{aligned}\Phi^{CF}(\chi,\tau)=&\tanh\left(\frac{\chi}{\sqrt{2}}\right)+\frac{c}{\sqrt{2}}\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)(\tau\mu+1-\mu)-c^2\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\\&\left(\tau^2\frac{\mu^2}{\Gamma(3)}-2(\tau\mu^2-\tau\mu)+(\mu-1)^2\right)+\frac{3}{2}c^2\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\Bigg(30\tanh^5\left(\frac{\chi}{\sqrt{2}}\right)\\&+\sqrt{2}c\tanh^2\left(\frac{\chi}{\sqrt{2}}\right)-44\tanh^3\left(\frac{\chi}{\sqrt{2}}\right)-\frac{c\sqrt{2}}{3}+14\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\\&\left(\tau^3\frac{\mu^3}{\Gamma(4)}+\tau^2(\mu-1)\frac{3\mu^2}{\Gamma(3)}-(\mu-1)^3+3\tau(\mu-1)^2\mu\right)\\&-\frac{3}{2}\Bigg(-15\tanh^5\left(\frac{\chi}{\sqrt{2}}\right)+22\tanh^3\left(\frac{\chi}{\sqrt{2}}\right)-7\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\\&\left(\tau^3\frac{2\mu^3}{\Gamma(4)}-2\tau^2(\mu^3-\mu^2)-(\mu-1)^3+3(\mu-1)^2\mu\tau\right)+\dots\,.\end{aligned}$$

SADM$_{ABC}$: By employing the SADM$_{ABC}$, we get the subsequent results,

$$\begin{aligned}
\Phi_0^{ABC}(\chi,\tau) =& \tanh\left(\frac{\chi}{\sqrt{2}}\right),\ \Phi_1^{ABC}(\chi,\tau) = \frac{c}{\sqrt{2}}\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\left(\frac{\mu}{\Gamma(1+\mu)}\tau^{\mu}+1-\mu\right),\\
\Phi_2^{ABC}(\chi,\tau) =& -c^2\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\left(\frac{\mu^2}{\Gamma(1+2\mu)}\tau^{2\mu}+(\mu-1)^2-\frac{2\mu^2-\mu}{\Gamma(1+\mu)}\tau^{\mu}\right),\\
\Phi_3^{ABC}(\chi,\tau) =& \frac{3}{2}c^2\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\Bigg(30\tanh^5\left(\frac{\chi}{\sqrt{2}}\right)+\sqrt{2}c\tanh^2\left(\frac{\chi}{\sqrt{2}}\right)-44\tanh^3\left(\frac{\chi}{\sqrt{2}}\right)\\
&-\frac{c\sqrt{2}}{3}+14\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\left(\frac{\mu^3}{\Gamma(1+3\mu)}\tau^{3\mu}-\frac{3\mu-3}{\Gamma(1+2\mu)}\mu^2\tau^{2\mu}-(\mu-1)^3\right.\\
&\left.+3\tau^{\mu}\frac{(\mu-1)^2\mu}{\Gamma(1+\mu)}\right)-\frac{3}{2}\Bigg(-15\tanh^5\left(\frac{\chi}{\sqrt{2}}\right)+22\tanh^3\left(\frac{\chi}{\sqrt{2}}\right)-7\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\\
&\left(\frac{\mu^3\Gamma(1+2\mu)}{\Gamma(1+3\mu)}\frac{\tau^{3\mu}}{(\Gamma+1)^2}-\frac{(3\mu-3)\mu^2}{\Gamma(1+2\mu)}\tau^{2\mu}-(\mu-1)^3-\frac{3(\mu-1)^2\mu}{\Gamma(1+\mu)}\tau^{\mu}\right),\\
&\vdots
\end{aligned} \tag{5.4}$$

by substituting (5.4) in (3.18), we get

$$\begin{aligned}
\Phi^{ABC}(\chi,\tau) =& \tanh\left(\frac{\chi}{\sqrt{2}}\right)+\frac{c}{\sqrt{2}}\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\left(\frac{\mu}{\Gamma(1+\mu)}\tau^{\mu}+1-\mu\right)\\
&-c^2\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\left(\frac{\mu^2}{\Gamma(1+2\mu)}\tau^{2\mu}+(\mu-1)^2-\frac{2\mu^2-\mu}{\Gamma(1+\mu)}\tau^{\mu}\right),\\
&+\frac{3}{2}c^2\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\Bigg(30\tanh^5\left(\frac{\chi}{\sqrt{2}}\right)+\sqrt{2}c\tanh^2\left(\frac{\chi}{\sqrt{2}}\right)-44\tanh^3\left(\frac{\chi}{\sqrt{2}}\right)\\
&-\frac{c\sqrt{2}}{3}+14\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\left(\frac{\mu^3}{\Gamma(1+3\mu)}\tau^{3\mu}-\frac{3\mu-3}{\Gamma(1+2\mu)}\mu^2\tau^{2\mu}-(\mu-1)^3\right.\\
&\left.+3\tau^{\mu}\frac{(\mu-1)^2\mu}{\Gamma(1+\mu)}\right)-\frac{3}{2}\Bigg(-15\tanh^5\left(\frac{\chi}{\sqrt{2}}\right)+22\tanh^3\left(\frac{\chi}{\sqrt{2}}\right)-7\tanh\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\\
&\left(\frac{\mu^3\Gamma(1+2\mu)}{\Gamma(1+3\mu)}\frac{\tau^{3\mu}}{(\Gamma+1)^2}-\frac{(3\mu-3)\mu^2}{\Gamma(1+2\mu)}\tau^{2\mu}-(\mu-1)^3-\frac{3(\mu-1)^2\mu}{\Gamma(1+\mu)}\tau^{\mu}\right)+\dots\,.
\end{aligned}$$

When $\mu = 1$ and $c = 1$, the exact solution of Eq. (5.1) is $\Phi(\chi,\tau) = \tanh\left(\frac{\chi+\tau}{\sqrt{2}}\right)$.

Example 2 Consider the general form of (1.2) as

$$D_{\tau}^{\mu}\Phi = c\Phi\Phi_{\chi}-18\Phi\Phi_{\chi\chi}^2-36\Phi_{\chi}^2\Phi_{\chi\chi}-24\Phi\Phi_{\chi}\Phi_{\chi\chi\chi}-3\Phi^2\Phi_{\chi\chi\chi\chi}+\Phi_{\chi\chi\chi\chi}+\Phi_{\chi\chi\chi\chi\chi\chi},\quad \text{where}\quad 0<\mu\le 1, \tag{5.5}$$

with the initial condition $\Phi(\chi,0) = \tanh\left(\frac{\chi}{\sqrt{2}}\right)$.

SADM$_C$: By employing the $SADM_C$, we get the subsequent results,

$$\begin{aligned}
\Phi_0^{C}(\chi,\tau) =& \tanh\left(\frac{\chi}{\sqrt{2}}\right),\ \Phi_1^{C}(\chi,\tau) = \frac{c}{\sqrt{2}}\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\frac{\tau^{\mu}}{\Gamma(\mu+1)},\\
\Phi_2^{C}(\chi,\tau) =& \frac{c}{\sqrt{2}}\Bigg(-282-1260\tanh^8\left(\frac{\chi}{\sqrt{2}}\right)+3870\tanh^6\left(\frac{\chi}{\sqrt{2}}\right)+c\sqrt{2}-3\sqrt{2}\tanh^2\left(\frac{\chi}{\sqrt{2}}\right)\\
&(-319\sqrt{2}+c)+2(-2121+c\sqrt{2})\tanh^4\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\tanh\left(\frac{\chi}{\sqrt{2}}\right)\frac{\tau^{2\mu}}{\Gamma(2\mu+1)},\\
&\vdots
\end{aligned} \tag{5.6}$$

by substituting (5.6) in (3.8), we get

$$\Phi^{C}(\chi,\tau)=\tanh\left(\frac{\chi}{\sqrt{2}}\right)+\frac{c}{\sqrt{2}}\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\frac{\tau^{\mu}}{\Gamma(\mu+1)}+\frac{c}{\sqrt{2}}\Bigg(-282-1260\tanh^8\left(\frac{\chi}{\sqrt{2}}\right) + 3870\tanh^6\left(\frac{\chi}{\sqrt{2}}\right)+c\sqrt{2}-3\sqrt{2}\tanh^2\left(\frac{\chi}{\sqrt{2}}\right)(-319\sqrt{2}+c)+2(-2121+c\sqrt{2}) \tanh^4\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\tanh\left(\frac{\chi}{\sqrt{2}}\right)\frac{\tau^{2\mu}}{\Gamma(2\mu+1)}+\dots .$$

SADM$_{CF}$:By employing the $SADM_{CF}$, we get the subsequent results,

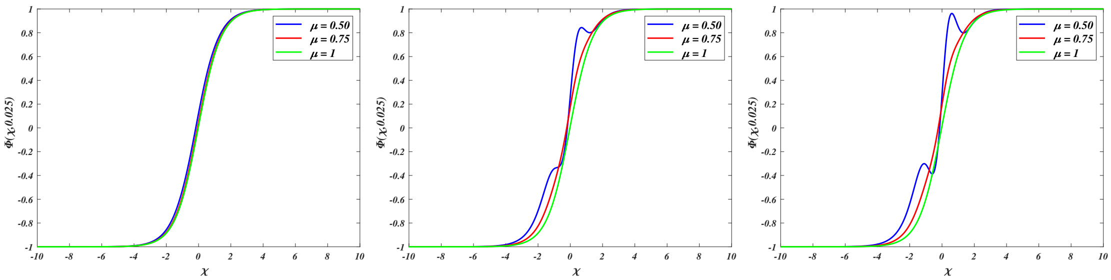
$$\begin{aligned}
&\Phi_0^{CF}(\chi,\tau)=\tanh\left(\frac{\chi}{\sqrt{2}}\right),\ \Phi_1^{CF}(\chi,\tau)=\frac{c}{\sqrt{2}}\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)(\tau\mu+1-\mu),\\
&\Phi_2^{CF}(\chi,\tau)=\frac{c}{\sqrt{2}}\Bigg(-282-1260\tanh^8\left(\frac{\chi}{\sqrt{2}}\right)+3870\tanh^6\left(\frac{\chi}{\sqrt{2}}\right)+c\sqrt{2}-3\sqrt{2}\tanh^2\left(\frac{\chi}{\sqrt{2}}\right)\\
&\qquad(-319\sqrt{2}+c)+2(-2121+c\sqrt{2})\tanh^4\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\tanh\left(\frac{\chi}{\sqrt{2}}\right)\\
&\qquad\left(\tau^2\frac{\mu^2}{\Gamma(3)}-2(\tau\mu^2-\tau\mu)+(\mu-1)^2\right),\\
&\qquad\vdots
\end{aligned}\tag{5.7}$$

by substituting (5.7) in (3.13), we get

$$\Phi^{CF}(\chi,\tau)=\tanh\left(\frac{\chi}{\sqrt{2}}\right)+\frac{c}{\sqrt{2}}\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)(\tau\mu+1-\mu)+\frac{c}{\sqrt{2}}\Bigg(-282-1260\tanh^8\left(\frac{\chi}{\sqrt{2}}\right)+3870\tanh^6\left(\frac{\chi}{\sqrt{2}}\right)+c\sqrt{2}-3\sqrt{2}\tanh^2\left(\frac{\chi}{\sqrt{2}}\right)(-319\sqrt{2}+c)+2(-2121+c\sqrt{2})\tanh^4\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\tanh\left(\frac{\chi}{\sqrt{2}}\right)\left(\tau^2\frac{\mu^2}{\Gamma(3)}-2(\tau\mu^2-\tau\mu)+(\mu-1)^2\right)+\dots .$$

SADM$_{ABC}$: By employing the $SADM_{ABC}$, we get the subsequent results,

$$\begin{aligned}
&\Phi_0^{ABC}(\chi,\tau)=\tanh\left(\frac{\chi}{\sqrt{2}}\right),\ \Phi_1^{ABC}(\chi,\tau)=\frac{c}{\sqrt{2}}\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\left(\frac{\mu}{\Gamma(1+\mu)}\tau^{\mu}+1-\mu\right),\\
&\Phi_2^{ABC}(\chi,\tau)=\frac{c}{\sqrt{2}}\Bigg(-282-1260\tanh^8\left(\frac{\chi}{\sqrt{2}}\right)+3870\tanh^6\left(\frac{\chi}{\sqrt{2}}\right)+c\sqrt{2}\\
&\qquad-3\sqrt{2}\tanh^2\left(\frac{\chi}{\sqrt{2}}\right)(-319\sqrt{2}+c)+2(-2121+c\sqrt{2})\\
&\qquad\tanh^4\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\tanh\left(\frac{\chi}{\sqrt{2}}\right)\left(\frac{\mu^2}{\Gamma(1+2\mu)}\tau^{2\mu}+(\mu-1)^2-\frac{2\mu^2-\mu}{\Gamma(1+\mu)}\tau^{\mu}\right),\\
&\qquad\vdots
\end{aligned}\tag{5.8}$$



(a) SADM_C (b) SADM_{CF} (c) SADM_{ABC}

Figure 1: SADM_C, SADM_{CF} and SADM_{ABC} solutions of Example 1 for different μ values with c=1.

by substituting (5.8) in (3.18), we get

$$\Phi^{ABC}(\chi,\tau) = \tanh\left(\frac{\chi}{\sqrt{2}}\right) + \frac{c}{\sqrt{2}}\tanh\left(\frac{\chi}{\sqrt{2}}\right)\operatorname{sech}^2\left(\frac{\chi}{\sqrt{2}}\right)\left(\frac{\mu}{\Gamma(1+\mu)}\tau^{\mu} + 1 - \mu\right) + \frac{c}{\sqrt{2}}\tanh\left(\frac{\chi}{\sqrt{2}}\right)\times$$
$$\Bigg(-282 - 1260\tanh^8\left(\frac{\chi}{\sqrt{2}}\right) + 3870\tanh^6\left(\frac{\chi}{\sqrt{2}}\right) + c\sqrt{2} - 3\sqrt{2}\tanh^2\left(\frac{\chi}{\sqrt{2}}\right)$$
$$(-319\sqrt{2}+c) + 2(-2121 + c\sqrt{2})\tanh^4\left(\frac{\chi}{\sqrt{2}}\right)\Bigg)\left(\frac{\mu^2}{\Gamma(1+2\mu)}\tau^{2\mu} + (\mu-1)^2 - \frac{2\mu^2-\mu}{\Gamma(1+\mu)}\tau^{\mu}\right) + \dots\ .$$

6. Results and Discussion

This section discusses the numerical simulations of the fourth and sixth order TFCHE under the impact of the described technique SADM using C, CF, and ABC. Table 1 exhibits the absolute error of Example 1 for $\mu = 1$ at various c values and compares results with q-HAM, NIM and HPM. Table 2 exhibits the approximate solution of Example 1 at various μ values with distinct c values. Table 3 exhibits the absolute error of Example 2 for various μ values at $c = 0.01$ and compares results with q-HAM and NIM. It can be observed from Tables 1 and 3 that the proposed method solutions align closely with the existing methods in the literature.

Figure 1 shows $2D$ solitons with respect to SADM_C, SADM_{CF} and SADM_{ABC} of Example 1 with different μ values and at $c = 1$. Figure 2 shows the $3D$ solitons with respect to SADM_C, SADM_{CF} and SADM_{ABC} of Example 1 with different μ values at $c = 1$. Figure 3 shows the $2D$ solitons with respect to SADM_C, SADM_{CF} and SADM_{ABC} of Example 1 with different μ values at $c = 0.1$. Figure 4 shows the $3D$ solitons with respect to SADM_C, SADM_{CF} and SADM_{ABC} of Example 1 with different μ values at $c = 0.1$. Figure 5 shows $2D$ solitons with respect to SADM_C, SADM_{CF} and SADM_{ABC} of Example 2 with different μ values and at $c = 0.01$. Figure 6 shows the $3D$ solitons with respect to SADM_C, SADM_{CF} and SADM_{ABC} of Example 2 with different μ values at $c = 0.01$. From the figures, it is clear that the suggested method solution behaviour is similar for all three fractional derivatives. Furthermore, as the fractional order approaches 1, we observe that the approximate solutions converge to the exact solution. The outcome of this work shows the accuracy of the proposed approach.

7. Conclusion

The time-fractional Cahn–Hilliard equation's approximate solutions are obtained by using the SADM. To showcase the effectiveness of the method, we analysed two test cases of the TFCHE. We investigated the considered model within the framework of the Caputo, CF, and ABC fractional derivatives. According to the tables and graphs, the approximate solution of the differential equation converges to the exact solution when the fractional order approaches 1. The results obtained using SADM have shown good agreement with the existing methods in the literature, namely, HPM [22], q-HAM, and NIM [30]. The presented findings show that the scheme is extremely accurate, adaptable, effective, and simple to use.

Table 1: Comparison of SADM absolute error at $c = 1,\ 0.1$ and $\mu = 1$ of Example 1

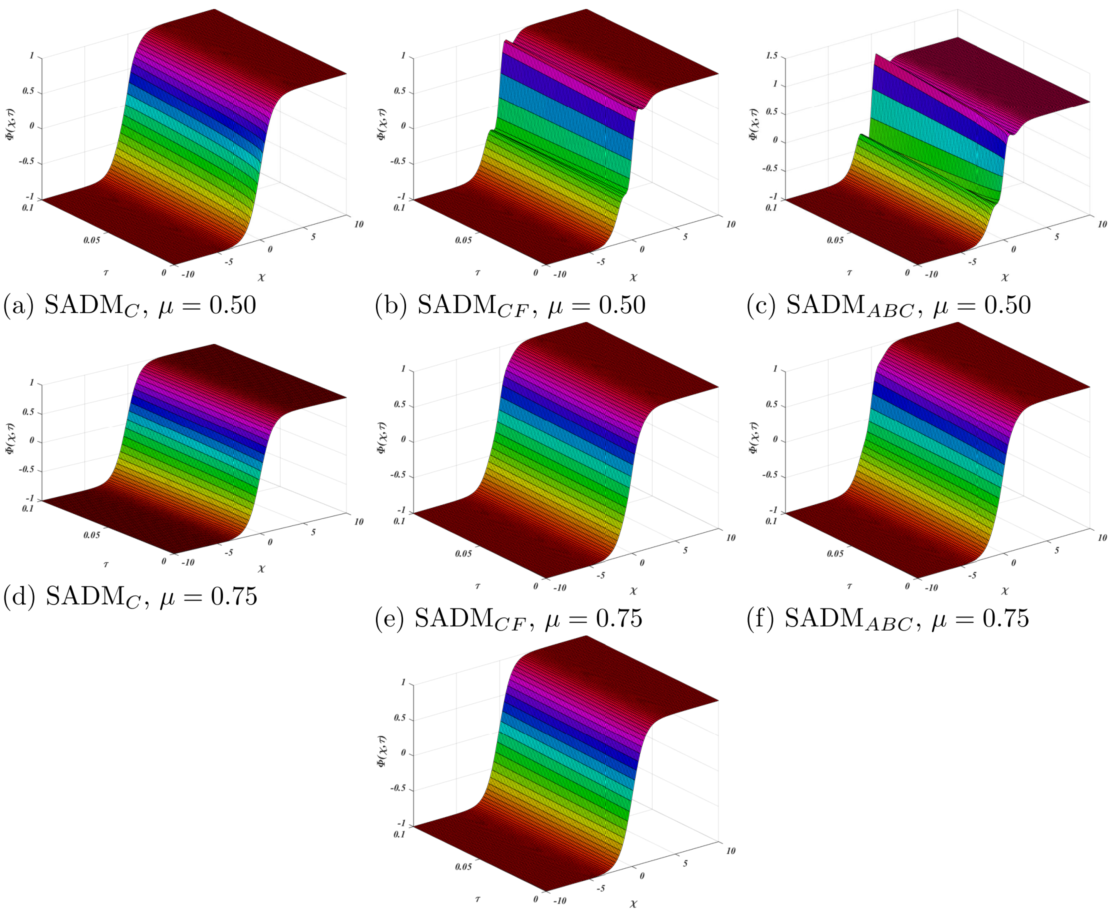
$c = 1$							
τ	χ	$SADM_C$	$SADM_{CF}$	$SADM_{ABC}$	NIM [30]	q-HAM [30]	HPM [22]
0.01	0	2.356974E-12	2.356974E-12	2.356974E-12	1.151971E-07	2.356975E-12	2.356974E-12
	1	2.823764E-10	2.823764E-10	2.823764E-10	1.810671E-07	2.823765E-10	2.823764E-10
	2	5.749510E-11	5.749510E-11	5.749510E-11	6.167394E-08	5.749512E-11	5.749510E-11
	3	3.757251E-11	3.757251E-11	3.757251E-11	1.165205E-09	3.757261E-11	3.757251E-11
0.05	0	7.361971E-09	7.361971E-09	7.361971E-09	4.940148E-05	7.713501E-08	7.361971E-09
	1	1.736922E-07	1.736922E-07	1.736922E-07	8.990891E-05	1.124520E-06	1.736922E-07
	2	3.622408E-08	3.622408E-08	3.622408E-08	3.218897E-05	2.387229E-07	3.622408E-08
	3	2.328496E-08	2.328496E-08	2.328496E-08	4.548965E-07	1.516340E-07	2.328496E-08
0.08	0	7.713501E-08	7.713501E-08	7.713501E-08	1.306675E-05	7.361971E-09	7.713501E-08
	1	1.124520E-06	1.124520E-06	1.124520E-06	2.224480E-05	1.736922E-07	1.124520E-06
	2	2.387229E-07	2.387229E-07	2.387229E-07	7.794449E-06	3.622408E-08	2.387229E-07
	3	1.516340E-07	1.516340E-07	1.516340E-07	1.257660E-07	2.328496E-08	1.516340E-07
0.10	0	2.352262E-07	2.352262E-07	2.352262E-07	9.109940E-05	2.352262E-07	2.352262E-07
	1	2.722916E-06	2.722916E-06	2.722916E-06	1.740220E-04	2.722916E-06	2.722916E-06
	2	5.848640E-07	5.848640E-07	5.848640E-07	6.321236E-05	5.848640E-07	5.848640E-07
	3	3.686350E-07	3.686350E-07	3.686350E-07	8.108096E-07	3.686350E-07	3.686350E-07
$c = 0.1$							
0.01	0	6.363843E-03	6.363843E-03	6.363843E-03	6.363843E-03	6.363843E-03	6.363843E-03
	1	3.985821E-03	3.985821E-03	3.985821E-03	3.985823E-03	3.985821E-03	3.985821E-03
	2	1.332104E-03	1.332104E-03	1.332104E-03	1.332103E-03	1.332104E-03	1.332104E-03
	3	3.528252E-04	3.528252E-04	3.528252E-04	3.528251E-04	3.528252E-04	3.528252E-04
0.05	0	3.180510E-02	3.180510E-02	3.180510E-02	3.180508E-02	3.180510E-02	3.180510E-02
	1	1.955096E-02	1.955096E-02	1.955096E-02	1.955118E-02	1.955096E-02	1.955096E-02
	2	6.479184E-03	6.479184E-03	6.479184E-03	6.479069E-03	6.479184E-03	6.479184E-03
	3	1.711813E-03	1.711813E-03	1.711813E-03	1.711801E-03	1.711813E-03	1.711813E-03
0.08	0	5.085149E-02	5.085149E-02	5.085149E-02	5.085144E-02	5.085149E-02	5.085149E-02
	1	3.082979E-02	3.082979E-02	3.082979E-02	3.083068E-02	3.082979E-02	3.082979E-02
	2	1.015465E-02	1.015465E-02	1.015465E-02	1.015418E-02	1.015465E-02	1.015465E-02
	3	2.678064E-03	2.678064E-03	2.678064E-03	2.678014E-03	2.678064E-03	2.678064E-03
0.10	0	6.352211E-02	6.352211E-02	6.352211E-02	6.352202E-02	6.352211E-02	6.352211E-02
	1	3.816223E-02	3.816223E-02	3.816223E-02	3.816396E-02	3.816223E-02	3.816223E-02
	2	1.251988E-02	1.251988E-02	1.251988E-02	1.251895E-02	1.251988E-02	1.251988E-02
	3	3.298003E-03	3.298003E-03	3.298003E-03	3.297906E-03	3.298003E-03	3.298003E-03

Table 2: Approximate solution of Example 1 at distinct μ values with $c = 1$ and 0.1.

$c = 1$							
τ	χ	$\mu = 0.50$			$\mu = 0.75$		
		SADM$_C$	SADM$_{CF}$	SADM$_{ABC}$	SADM$_C$	SADM$_{CF}$	SADM$_{ABC}$
0.01	0	0.079257	0.266036	0.272420	0.024321	0.170022	0.180326
	1	0.655560	0.809319	0.835538	0.623885	0.707770	0.714358
	2	0.903327	0.905266	0.899438	0.893374	0.912946	0.913594
	3	0.975619	0.982900	0.983704	0.972987	0.978921	0.979276
0.05	0	0.172465	0.269251	0.273465	0.081024	0.186894	0.212655
	1	0.705577	0.819176	0.874111	0.656957	0.718524	0.736076
	2	0.916591	0.903213	0.889729	0.903995	0.914026	0.915128
	3	0.979330	0.983208	0.984901	0.975783	0.979505	0.980401
0.08	0	0.213640	0.271380	0.270657	0.114789	0.199220	0.230777
	1	0.727512	0.826625	0.893948	0.675530	0.726540	0.749172
	2	0.920898	0.901630	0.884337	0.909701	0.914735	0.915606
	3	0.980761	0.983445	0.985531	0.977287	0.979935	0.981050
0.10	0	0.235492	0.272663	0.268085	0.135257	0.207276	0.241347
	1	0.739509	0.831618	0.905540	0.686442	0.731860	0.757216
	2	0.922738	0.900555	0.881087	0.912949	0.915161	0.915737
	3	0.981493	0.983606	0.985906	0.978149	0.980219	0.981439
$c = 0.1$							
0.01	0	0.007978	0.035618	0.039224	0.002433	0.018196	0.019488
	1	0.613845	0.631030	0.633333	0.610388	0.620153	0.620950
	2	0.890047	0.895092	0.895646	0.888897	0.892054	0.892299
	3	0.972108	0.973492	0.973655	0.971803	0.972645	0.972711
0.05	0	0.017835	0.037021	0.044107	0.008135	0.020313	0.023754
	1	0.619927	0.631923	0.636481	0.613948	0.621459	0.623583
	2	0.892037	0.895311	0.896359	0.890084	0.892456	0.893099
	3	0.972636	0.973556	0.973870	0.972118	0.972753	0.972928
0.08	0	0.022556	0.038073	0.046443	0.011572	0.021900	0.026325
	1	0.622820	0.632593	0.638001	0.616080	0.622438	0.625170
	2	0.892960	0.895475	0.896684	0.890792	0.892755	0.893573
	3	0.972882	0.973604	0.973971	0.972305	0.972834	0.973057
0.10	0	0.025215	0.038775	0.047759	0.013680	0.022958	0.027901
	1	0.624445	0.633039	0.638861	0.617383	0.623091	0.626142
	2	0.893472	0.895583	0.896863	0.891222	0.892954	0.893860
	3	0.973019	0.973635	0.974027	0.972419	0.972888	0.973135

Table 3: Approximate solution of Example 2 at $c = 0.01$ and $\mu = 0.5,\ 0.75,\ 1$.

τ	χ	$SADM_C$	$SADM_{CF}$	$SADM_{ABC}$	NIM [30]	q-HAM [30]
$\mu = 0.5$						
0.01	0	0	0	0	0	0
	1	0.609938	0.629926	0.634230	0.609937	0.609938
	2	0.888531	0.888954	0.889001	0.888531	0.888531
	3	0.971683	0.971147	0.971015	0.971683	0.971683
0.05	0	0	0	0	0	0
	1	0.613405	0.631548	0.640578	0.613405	0.613405
	2	0.888700	0.888972	0.889062	0.888700	0.888700
	3	0.971625	0.971097	0.970818	0.971625	0.971625
0.08	0	0	0	0	0	0
	1	0.615904	0.632785	0.643829	0.615903	0.615904
	2	0.888777	0.888986	0.889090	0.888777	0.888777
	3	0.971566	0.971060	0.970716	0.971566	0.971566
0.10	0	0	0	0	0	0
	1	0.617551	0.633620	0.645722	0.617549	0.617551
	2	0.888819	0.888995	0.889105	0.888819	0.888819
	3	0.971524	0.971034	0.970657	0.971524	0.971524
$\mu = 0.75$						
0.01	0	0	0	0	0	0
	1	0.609011	0.614677	0.615464	0.609011	0.609011
	2	0.888431	0.888700	0.888721	0.888431	0.888431
	3	0.971679	0.971581	0.971561	0.971679	0.971679
0.05	0	0	0	0	0	0
	1	0.609821	0.615969	0.618296	0.609821	0.609821
	2	0.888535	0.888734	0.888787	0.888535	0.888535
	3	0.971688	0.971548	0.971487	0.971688	0.971688
0.08	0	0	0	0	0	0
	1	0.610618	0.616984	0.620178	0.610618	0.610618
	2	0.888596	0.888759	0.888827	0.888596	0.888596
	3	0.971683	0.971522	0.971437	0.971683	0.971683
0.10	0	0	0	0	0	0
	1	0.611221	0.617682	0.621396	0.611221	0.611221
	2	0.888632	0.888775	0.888850	0.888632	0.888632
	3	0.971675	0.971504	0.971403	0.971675	0.971675
$\mu = 1$						
0.01	0	0	0	0	0	0
	1	0.608890	0.608890	0.608890	0.608890	0.608890
	2	0.888399	0.888399	0.888399	0.888399	0.888399
	3	0.971672	0.971672	0.971672	0.971672	0.971672
0.05	0	0	0	0	0	0
	1	0.609091	0.609091	0.609091	0.609091	0.609091
	2	0.888451	0.888451	0.888451	0.888451	0.888451
	3	0.971684	0.971684	0.971684	0.971684	0.971684
0.08	0	0	0	0	0	0
	1	0.609323	0.609323	0.609323	0.609323	0.609323
	2	0.888490	0.888490	0.888490	0.888490	0.888490
	3	0.971690	0.971690	0.971690	0.971690	0.971690
0.10	0	0	0	0	0	0
	1	0.609517	0.609517	0.609517	0.609517	0.609517
	2	0.888516	0.888516	0.888516	0.888516	0.888516
	3	0.971692	0.971692	0.971692	0.971692	0.971692

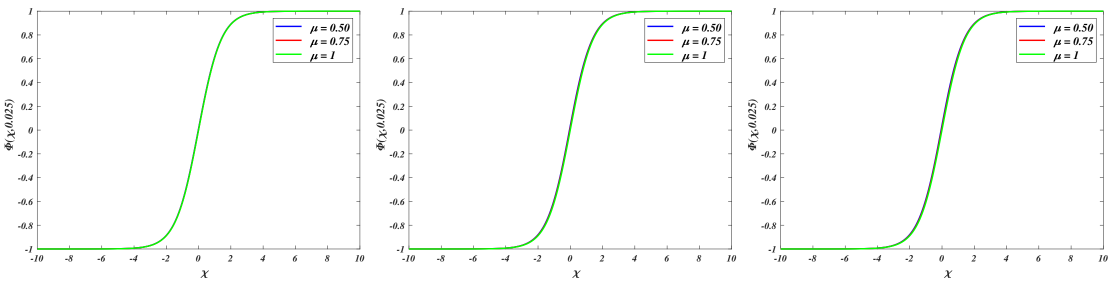


(a) SADM_C, $\mu = 0.50$ (b) SADM_{CF}, $\mu = 0.50$ (c) SADM_{ABC}, $\mu = 0.50$

(d) SADM_C, $\mu = 0.75$ (e) SADM_{CF}, $\mu = 0.75$ (f) SADM_{ABC}, $\mu = 0.75$

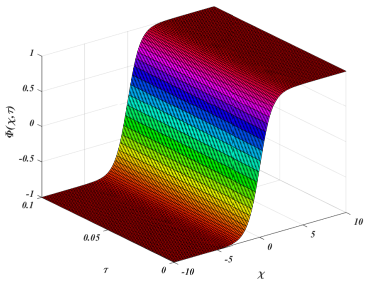
(g) SADM_{Exact}

Figure 2: SADM_C, SADM_{CF} and SADM_{ABC} solutions of Example 1 for different μ values with c=1.

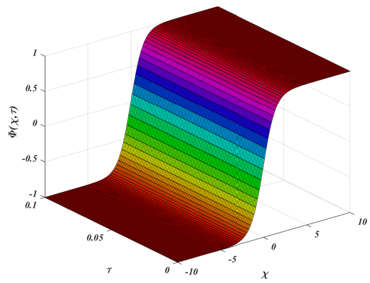


(a) SADM_C (b) SADM_{CF} (c) SADM_{ABC}

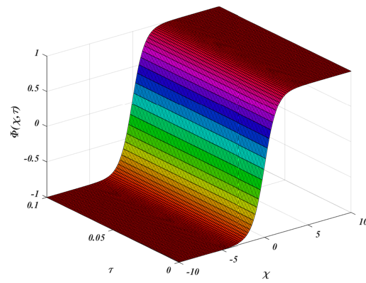
Figure 3: SADM_C, SADM_{CF} and SADM_{ABC} solutions of Example 1 for different μ values with c=0.1.



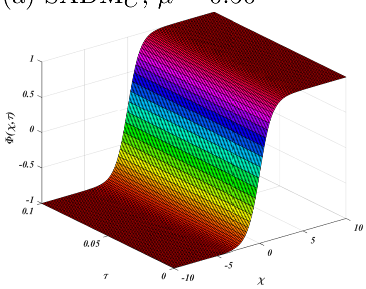
(a) SADM_C, $\mu = 0.50$



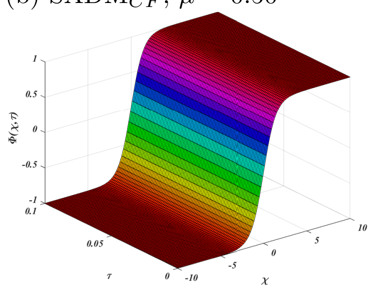
(b) SADM_{CF}, $\mu = 0.50$



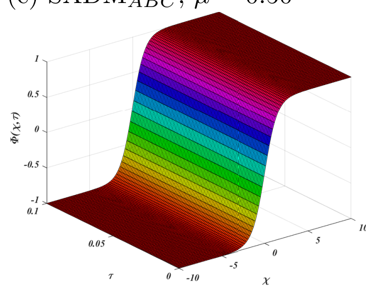
(c) SADM_{ABC}, $\mu = 0.50$



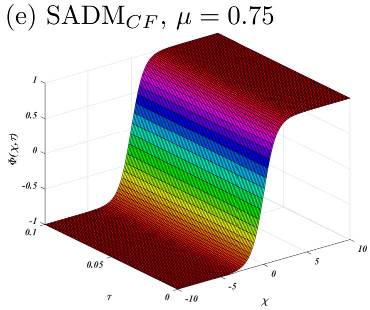
(d) SADM_C, $\mu = 0.75$



(e) SADM_{CF}, $\mu = 0.75$

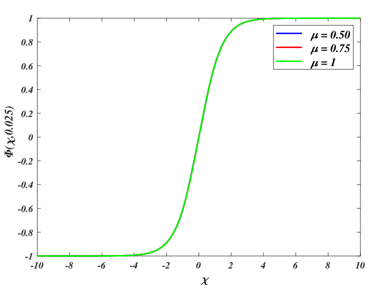


(f) SADM_{ABC}, $\mu = 0.75$

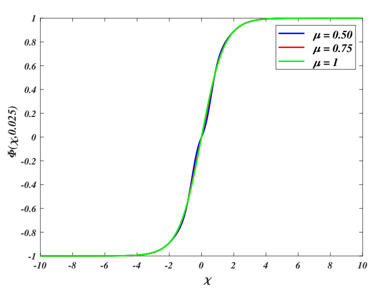


(g) SADM_{Exact}

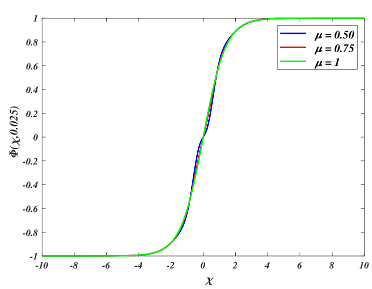
Figure 4: SADM_C, SADM_{CF} and SADM_{ABC} solutions of Example 1 for different μ values with $c = 0.1$.



(a) SADM_C

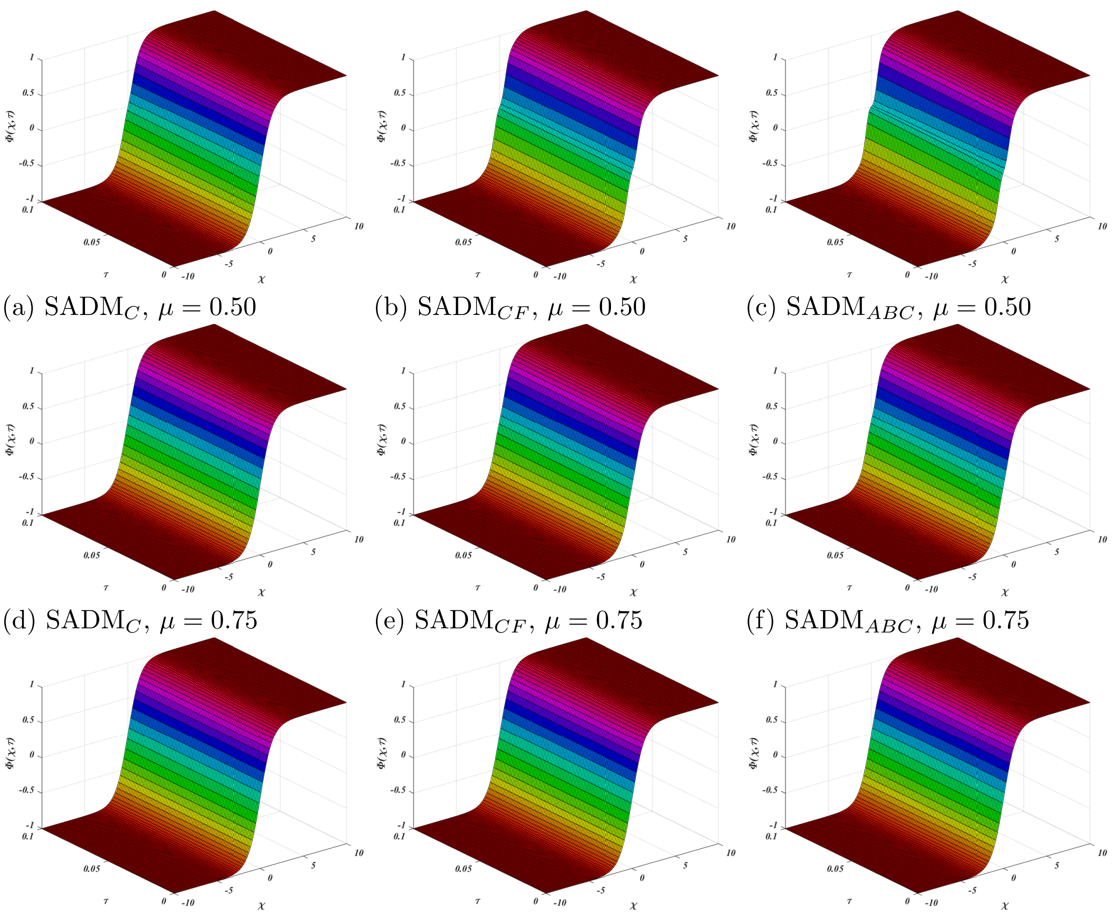


(b) SADM_{CF}



(c) SADM_{ABC}

Figure 5: SADM_C, SADM_{CF} and SADM_{ABC} solutions of Example 2 for different μ values with $c = 0.01$.



(a) SADM$_C$, $\mu = 0.50$ (b) SADM$_{CF}$, $\mu = 0.50$ (c) SADM$_{ABC}$, $\mu = 0.50$

(d) SADM$_C$, $\mu = 0.75$ (e) SADM$_{CF}$, $\mu = 0.75$ (f) SADM$_{ABC}$, $\mu = 0.75$

(g) SADM$_C$, $\mu = 1$ (h) SADM$_{CF}$, $\mu = 1$ (i) SADM$_{ABC}$, $\mu = 1$

Figure 6: SADM$_C$, SADM$_{CF}$ and SADM$_{ABC}$ solutions of Example 2 for different μ values with $c = 0.01$.

According to the given results, the scheme is simple to use, flexible, highly accurate, and efficient. The results demonstrate how SADM can be applied to solve a variety of fractional linear and nonlinear models in science and engineering, including thermodynamics, ocean engineering, and epidemiology.

References

1. J. Bosch, D. Kay, M. Stoll and A. J. Wathen, Fast solvers for Cahn-Hilliard inpainting, SIAM Journal on Imaging Sciences. 7(1) (2014) 67-97.
2. A. L. Bertozzi, S. Esedoglu, and A. Gillette, Inpainting of binary images using the Cahn–Hilliard equation, IEEE Transactions on image processing. 16(1) (2006) 285-291.
3. K. Manikandan, D. Aravinthan, J. B. Sudharsan and S. R. R. Reddy, Soliton and rogue wave solutions of the space–time fractional nonlinear Schrodinger equation with PT-symmetric and time-dependent potentials, Optik. 266 (2022) 169594.
4. M. L. Rupa, and K. Aruna, Optical solitons of time fractional Kundu–Eckhaus equation and massive Thirring system arises in quantum field theory, Optical and Quantum Electronics. 56(3) (2024) 460.
5. M. X. Zhou, A. R. Kanth, K. Aruna, K. Raghavendar, H. Rezazadeh, M. Inc and A. A. Aly, Numerical Solutions of Time Fractional Zakharov-Kuznetsov Equation via Natural Transform Decomposition Method with Nonsingular Kernel Derivatives, Journal of Function Spaces. 2021(1) (2021) 9884027.
6. R. K. Adivi Sri Venkata, A. Kirubanandam, and R. Kondooru, Numerical solutions of time fractional Sawada Kotera Ito equation via natural transform decomposition method with singular and nonsingular kernel derivatives, Mathematical Methods in the Applied Sciences. 44(18) (2021) 14025-14040.
7. A. S. V. Ravi Kanth, K. Aruna, and K. Raghavendar, Natural transform decomposition method for the numerical treatment of the time fractional Burgers–Huxley equation, Numerical Methods for Partial Differential Equations. 39(3) (2023) 2690-2718.
8. D. Y. Liu, O. Gibaru, W. Perruquetti, and T. M. Laleg-Kirati, Fractional order differentiation by integration and error analysis in noisy environment, IEEE Transactions on Automatic Control. 60(11) (2015) 2945-2960.
9. D. G. Prakasha, P. Veeresha and H. M. Baskonus, Analysis of the dynamics of hepatitis E virus using the Atangana-Baleanu fractional derivative, The European Physical Journal Plus. 134(5) (2019) 1-11.
10. N. H. Sweilam, M. M. Abou Hasan and D. Baleanu, New studies for general fractional financial models of awareness and trial advertising decisions, Chaos, Solitons and Fractals. 104 (2017) 772-784.
11. D. Baleanu, G. C. Wu, and S. D. Zeng. Chaos analysis and asymptotic stability of generalized Caputo fractional differential equations, Chaos, Solitons and Fractals. 102 (2017) 99-105.
12. K. Pavani, K. Raghavendar, and K. Aruna, Soliton solutions of the time-fractional Sharma–Tasso–Olver equations arise in nonlinear optics. Optical and Quantum Electronics, 56 (5) (2024) 748.
13. D. Baleanu, K. Diethelm, E. Scalas, and J. J. Trujillo, Series on complexity, nonlinearity and chaos, Fractional Calculus Models and Numerical Methods. (2012).
14. A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, Theory and applications of fractional differential equations, North-Holland Mathematics Studies. (2006).
15. M. Caputo and M. Fabrizio, A new definition of fractional derivative without singular kernel, Progress in Fractional Differentiation and Applications. 1(2) (2015) 1-13.
16. A. Atangana, and D. Baleanu, New fractional derivatives with nonlocal and non-singular kernel: theory and application to heat transfer model, Thermal Science. 20(2) (2016) 763-769.
17. J. W. Cahn and J. E. Hilliard, Free energy of a nonuniform system. I. Interfacial free energy, The Journal of chemical physics. 28(2) (1958) 258-267.
18. J. W. Cahn, On spinodal decomposition, Acta metallurgica. 9(9) (1961) 795-801.
19. C. M. Elliott, The Cahn-Hilliard model for the kinetics of phase separation, In Mathematical models for phase change problems. (1989) 35-73.
20. M. E. Gurtin. Generalized Ginzburg-Landau and Cahn-Hilliard equations based on a microforce balance, Physica D: Nonlinear Phenomena. 92(3-4) (1996) 178-192.
21. S. M. Choo, S. K. Chung and Y. J. Lee, A conservative difference scheme for the viscous Cahn–Hilliard equation with a nonconstant gradient energy coefficient, Applied Numerical Mathematics. 51(2-3) (2004) 207-219.
22. Y. Ugurlu and D. Kaya, Solutions of the Cahn–Hilliard equation, Computers and Mathematics with Applications. 56(12) (2008) 3038-3045.
23. P. Rybka and K. H. Hoffnlann, Convergence of solutions to Cahn-Hilliard equation, Communications in partial differential equations. 24(5-6) (1999) 1055-1077.
24. M. S. Mohamed, K. S. Mekheimer and S. A. Taif, Analytical approximate solution for nonlinear space-time fractional Cahn-Hilliard equation, International Electronic Journal of Pure and Applied Mathematics. 7(4) (2014) 145-159.

25. J. Kim, A numerical method for the Cahn–Hilliard equation with a variable mobility, Communications in Nonlinear Science and Numerical Simulation. 12(8) (2007) 1560-1571.

26. M. I. M. Copetti and C. M. Elliott, Kinetics of phase decomposition processes: Numerical solutions to Cahn–Hilliard equation, Materials Science and Technology. 6(3) (1990) 273-284.

27. E. V. L. De Mello and O. T. Silveira Filho, Numerical study of the Cahn–Hilliard equation in one, two and three dimensions, Physica A: Statistical Mechanics and its Applications. 347 (2005) 429-443.

28. Z. Dahmani and M. Benbachir, Solutions of the Cahn-Hilliard equation with time-and space-fractional derivatives, International Journal of Nonlinear Science. 8(1) (2009) 19-26.

29. A. Novick-Cohen, The cahn–hilliard equation, Handbook of differential equations: evolutionary equations. 4 (2008) 201-228.

30. L. Akinyemi, O. S. Iyiola and U. Akpan, Iterative methods for solving fourth-and sixth-order time-fractional Cahn-Hillard equation, Mathematical Methods in the Applied Sciences. 43(7) (2020) 4050-4074.

31. A. Bouhassoun and M. Hamdi Cherif, Homotopy perturbation method for solving the fractional Cahn-Hilliard equation, Journal of Interdisciplinary Mathematics, 18(5) (2015) 513-524.

32. A. Arafa and G. Elmahdy, Application of residual power series method to fractional coupled physical equations arising in fluids flow, International Journal of Differential Equations. (2018).

33. D. G. Prakasha, P. Veeresha and H. M. Baskonus, Two novel computational techniques for fractional Gardner and Cahn-Hilliard equations, Computational and Mathematical Methods. 1(2) (2019) e1021.

34. K. Shah and H. Patel, A Novel Hybrid Approach to the Sixth-Order Cahn-Hillard Time-Fractional Equation, In Mathematical Modeling, Computational Intelligence Techniques and Renewable Energy. (2021) 65-77.

35. M. Al-Maskari and S. Karaa, Strong approximation of the time-fractional Cahn–Hilliard equation driven by a fractionally integrated additive noise. Computers & Mathematics with Applications. 180 (2025) 28-45.

36. M. Al-Qurashi, S. Rashid, F. Jarad, M. Tahir and A. M. Alsharif, New computations for the two-mode version of the fractional Zakharov-Kuznetsov model in plasma fluid by means of the Shehu decomposition method, AIMS Mathematics. 7(2) (2022) 2044-2060.

37. S. Rashid, A. Khalid, S. Sultana, Z. Hammouch, R. Shah and A. M. Alsharif, A novel analytical view of time-fractional Korteweg-De Vries equations via a new integral transform, Symmetry. 13(7) (2021) 1254.

38. Y. M. Chu, E. H. Bani Hani, E. R. El-Zahar, A. Ebaid and N. A. Shah. Combination of Shehu decomposition and variational iteration transform methods for solving fractional third order dispersive partial differential equations, Numerical Methods for Partial Differential Equations. (2021).

39. M. L. Rupa, K. Aruna, and K. Raghavendar, Insights into the time Fractional Belousov-Zhabotinsky System Arises in Thermodynamics, International Journal of Theoretical Physics. 63(9) (2024) 222.

40. I. Podlubny, Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications, Elsevier. (1998).

41. H. J. Haubold, A. M. Mathai and R. K. Saxena, Mittag-Leffler functions and their applications, Journal of applied mathematics. (2011) 2011.

42. S. Maitama and W. Zhao, New integral transform: Shehu transform a generalization of Sumudu and Laplace transform for solving differential equations, International Journal of Analysis and Applications. 17(2) (2019) 167-190.

43. M. L. Rupa and K. Aruna, Optical soliton solutions of nonlinear time fractional Biswas–Milovic equation, Optik. 270 (2022) 169921.

44. A. Bokhari, Application of Shehu transform to Atangana-Baleanu derivatives, Journal of Mathematics and Computer Science. 20 (2019) 101-107.

45. G. Adomian, A new approach to nonlinear partial differential equations, Journal of Mathematical Analysis and Applications. 102(2) (1984) 420-434.

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