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A Fractional Calculus Model for Worm Propagation in Computer Network

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ABSTRACT: Fractional Calculus emerges as a new field with wide applications in the fields of science and engineering. There is an increasing trend to find fractional calculus applications in various real-life non-linear and non-local problems, to develop new models for existing problems. Various results reported by the researchers, and many more are on the way to be discovered. Among all these problems is a computer science problem, worm propagation over various networks. This paper will provide an overview of existing worm propagation models and their applications in analyzing and combating computer virus threats using fractional calculus. We specifically look at various classical and fractional models described in the literature for mobile and computer networks. This work makes an important addition by developing a fractional SEIR model that uses the Caputo derivative to represent worm spread in a network context. This model's equilibrium points and asymptotic stability are systematically examined to better understand its dynamical behavior.This paper aims to present some short summaries of the work by distinguished researchers in modeling virus and worm propagation problems using fractional calculus. We believe this incomplete, but important, information will guide many researchers and help them to see some of the main real-world applications and gain an understanding of this powerful mathematical tool. We expect this collection will also benefit our community. Along with this a fractional SEIR model of virus propagation with the help of Caputo derivative is taken in this work. Its equilibrium point and asymptotic stability are discussed.

Key Words: Worm Propagation Model, Caputo Fractional Derivative, Reproduction Number, Adam-Bashforth's Algorithm, Lagrange's Interpolation.

Contents

1. Introduction

Fractional Calculus is now an emerging field, with its application in many real-life non-linear problems that are dynamic in nature. One such result is the problem of worm propagation. The problem of worm propagation is seen at places where multiple systems are connected through Wi-Fi, internet, Bluetooth, or other means. Fractional Calculus has become a wonderful tool for modeling these problems. Moreover, researchers find that fractional calculus is not universal but it has its place in application, hence successful application of fractional calculus guides us on its application in the future. Fractional Calculus emerges as a rapidly evolving field with several applications in science and engineering. In recent years, there has been a growing trend of using fractional calculus to solve real-world nonlinear and non-local issues, such as modeling the spread of worms in computer networks [19,20,21]. Worm propagation has become a serious cybersecurity concern, as network worms use weaknesses to spread across networked systems, causing significant harm and disruption. In addition, numerical simulations utilizing the Adam-Bashforth-Moulton

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method are used to validate the theoretical conclusions. The importance of fractional-order modeling in capturing memory effects and non-local interactions is also highlighted. Comparisons to classical models and alternative numerical methods show our approach's benefits and prospective uses in cybersecurity and network security assessments. The findings of this study help to promote fractional calculus in mathematical modeling and provide useful tools for developing more effective worm propagation tactics [18].

As the epidemic spreading model and worm propagation models share similar features, therefore most of the worm propagation models are borrowed from epidemic models. Organization of this paper starts with; the first section provides a summary of worm propagation over mobile networks [1,3,4]. The Second section is useful for worm Propagation over computer networks [2,6,7,8,5,9]. In section three fractional SEIR model is discussed. Section four discusses the equilibrium and stability of this model. Finally, in section five, some numerical schemes are discussed.

The contribution of renowned authors is collected and presented in this paper. So that, one gets a better understanding of Fractional Calculus along with its application in worm propagation problems. The first two sections contain contributions of some eminent researchers. References are also provided to help readers gain a better understanding of debated topics [13]. In this paper, the contribution of the collected ten articles is presented in such a manner that helps researchers in further development of the Fractional Calculus concept in worm propagation in the future [14,15,16,17].

2. Preliminaries

2.1. Definitions

Definition 2.1 *([11]) For an integrable function h, the Caputo derivative of fractional order $\nu \in (0,1)$ is given by*

$$^{C}D^{\nu}h\left(t\right)=\frac{1}{\Gamma(m-\nu)}\int_{0}^{t}\frac{h^{m}(\tau)}{\left(t-\tau\right)^{\nu-m+1}}d\tau, \quad m=\lceil\nu\rceil+1. \tag{2.1}$$

Also, the corresponding fractional integral of order ν with $Re\left(\nu\right)>0$ is given by

$$^{C}I^{\nu}h\left(t\right)=\frac{1}{\Gamma(\nu)}\int_{0}^{t}\left(t-\tau\right)^{\nu-1}h\left(\tau\right)d\tau. \tag{2.2}$$

Definition 2.2 *([12]) Let y(t) be fractally differentiable with order β and continuous on (x,y), then RL (Riemann-Liouville) fractal fractional derivative of y(t) of order α with power law kernel is:*

$$^{FFP}D^{\nu,\beta}\left(y\left(t\right)\right)=\frac{1}{\Gamma\left(m-\nu\right)}\frac{d}{dt^{\beta}}\int_{0}^{t}\frac{h\left(\tau\right)}{\left(t-\tau\right)^{\nu-m+1}}d\tau. \tag{2.3}$$

Where,

$$m-1<\nu,\beta\leq m\in\mathbb{N} \quad and \quad \frac{dy(s)}{dt^{\beta}}=\lim_{t\to s}\frac{y(t)-y(s)}{t^{\beta}-s^{\beta}}\ . \tag{2.4}$$

Definition 2.3 *([12]) Let y(t) be fractal differentiable with order β and continuous on (x,y), then fractal fractional derivative in RL (Riemann-Liouville) of y(t) of order α with exponentially decay kernel is:*

$$^{FFE}D^{\nu,\beta}\left(y\left(t\right)\right)=\frac{M(\nu)}{\Gamma\left(1-\nu\right)}\frac{d}{dt^{\beta}}\int_{0}^{t}\frac{h\left(\tau\right)}{\left(t-\tau\right)^{\nu-m+1}}d\tau \tag{2.5}$$

3. Worm Propagation Models

3.1. Worm propagation over mobile network

(a) Classical Models for this problem: Models like susceptible-infected (SI), susceptible-infected-susceptible (SIS), susceptible-infected-recovered (SIR), susceptible-infected-immunized (SII), susceptible-affected-infected-suspended (SAID), Gao *et al.* (VEIQS) (2020) [3] model and many more models exist in the literature for the problem of worm propagation. The classical Model given by Xaio *et al.* [4] in 2017, susceptible-affected-infected-suspended-recovered (SAIDR) model is one of the most effective models to model worm propagation problems over mobile networks and also over the social network. In this model, the whole population denoted by N of mobile phones is divided into five classes, given by susceptible-affected-infected-suspended-recovered (SAIDR).

$$N \ = \ S\left(t\right)+A\left(t\right)+I\left(t\right)+D\left(t\right)+R\left(t\right).$$

This model has local and global stability, which is predicted using the reproduction number R_0. Numerical experiments are performed in order to validate theoretical findings.

(b) Fractional Models for this problem: In a sequence of the above models by Xaio *et al.*, Ucar *et al.*(2021) proposed a fractional SAIDR model with the help of AB derivative, for modeling the SMS-based worm propagation problem. Providing existing and unique solutions to this problem, by using various analytical and numerical methods. For more understanding of this model, one can go through [1].

3.2. Worm Propagation over Computer Network

(a) Classical Models for this problem: Hua Yuan (2008) [2] discussed the spread of computer network viruses by using the e-SEIR model to eliminate its ill effects. Some safety and security measures are discussed based on the stability conditions of the e-SEIR model. B. K. Mishra [6] gives a dynamic SEIS-V model for worm propagation in computer networks. Stability analysis of the SEIS-V model is done by using the reproduction number R0. Based on this, some antivirus software was also suggested. The malware propagation SIRS model by Feng [7] is one in this series of worm propagation models. Fangfang Yang [8] provides a Hopf bifurcation of a VEIQS worm propagation model in mobile networks with two delays. The classical SEIR model for computer virus propagation in a total population of N systems by Bonyah *et al.* [10] is given as

$$N = S\left(t\right)+E\left(t\right)+I\left(t\right)+R\left(t\right)$$

$$\frac{dS}{dt} = \left(1-p\right)N-\beta_1 SI-\beta_2 SE-(p+\mu)S. \tag{3.1}$$

$$\frac{dE}{dt} = \beta_1 SI+\beta_2 SE-kE-\sigma E-\mu E. \tag{3.2}$$

$$\frac{dI}{dt} = \sigma E-dI-\mu I. \tag{3.3}$$

$$\frac{dR}{dt} = pS+kE+dI. \tag{3.4}$$

where S(t), E(t), I(t), and R(t) are susceptible, exposed, infectious, and recovered computers, respectively. at the time t.
Last equation among the above four equations is independent of the above three, therefore (2.1) can be written as

$$\frac{dS}{dt} = \left(1-p\right)N-\beta_1 SI-\beta_2 SE-(p+\mu)S. \tag{3.5}$$

$$\frac{dE}{dt} = \beta_1 SI+\beta_2 SE-kE-\sigma E-\mu E. \tag{3.6}$$

$$\frac{dI}{dt} = \sigma E-(d+\mu)I. \tag{3.7}$$

where the meaning of notations is given in Table 1
'N' stands for the rate at which external computers are connected to the network.
'p' stands for the recovery rate of susceptible computers.
'k' stands for the recovery rate of exposed computers.
$'\beta_1'$ stands for the rate at which one susceptible computer can turn into exposed, having a connection to one infected computer.
'β_2' stands for the rate at which one susceptible computer can turn into exposed, having a connection to one exposed computer.
$'\sigma'$ stands for the rate of the exposed computers.
'D' stands for the recovery rate of infected computers.

N	Addition rate of external computers.
P	The recovery rate of susceptible computers.
K	Rate of recovery of exposed computers.
$\boldsymbol{\beta}_1$	The rate at which one susceptible computer can turn into exposed, having a connection to one infected computer.
$\boldsymbol{\beta}_2$	The rate at which one susceptible computer can turn into exposed, having a connection to one exposed computer.
$\boldsymbol{\sigma}$	Rate of the exposed computers.
D	The recovery rate of infected computers.
$\boldsymbol{\mu}$	The rate at which one computer is removed from the network.

Table 1: Descriptions of the parameters used in the model.

'μ' stands for the rate at which one computer is removed from the network.
as $N \geq 0$ so $\frac{dS}{dt}, \frac{dE}{dt}, \frac{dI}{dt} \geq 0$ as well as initial conditions are taken as $S(0) = S_0,\ E(0) = E_0, I(0) = I_0$.
Basic reproduction number for integer order model i. e. for $\nu = 1$ is given as

$$R_0 = \frac{N(1-p)(\beta_1 \nu + \beta_2(r+\mu))}{(p+\mu)(r+\mu)(k+\mu+\nu)}$$

(b) Fractional Models for this problem: Pinto *et al.* [5] propose a fractional model for computer virus propagation. His model includes the interaction between computers and removable devices. Dong *et al.* [9] have given a fuzzy fractional SIQR model to describe the dynamics of virus propagation with quarantine in the network.
The fractional Model in Caputo Sense for a system of equations in (3.5), (3.6), and (3.7) is given by

$$^{C}D^{\nu} S = (1-p)N - \beta_1 SI - \beta_2 SE - (p+\mu)S. \tag{3.8}$$

$$^{C}D^{\nu} E = \beta_1 SI + \beta_2 SE - kE - \sigma E - \mu E. \tag{3.9}$$

$$^{C}D^{\nu} I = \sigma E - (d+\mu)I. \tag{3.10}$$

4. Equilibrium Ponits and Stability

We have an initial value problem with $0 < \alpha \leq 1$ to estimate the equilibrium points
let, $^{C}D^{\alpha} S = 0,\ ^{C}D^{\alpha} E = 0,\ ^{C}D^{\alpha} I = 0$
Then the equilibrium points are $E_0 = (1, 0, 0)$ and $E_0 = (S^*, E^*, I^*)$, where

$$S^* = \frac{A}{aR_0}$$

$$E^* = \frac{A(R_0 - 1)}{bR_0}$$

$$I^* = \frac{A\alpha(R_0 - 1)}{bcR_0}$$

The Jacobian Matrix $J(E_0)$ computed for disease-free equilibrium is given by

$$J(E_0) = \begin{pmatrix} -(p+\mu) & -\beta_2 & -\beta_1 \\ 0 & \beta_2 - (k+\alpha+\mu) & \beta_1 \\ 0 & \alpha & -(r+\mu) \end{pmatrix}$$

This disease-free equilibrium of equation (2.3) is asymptotically stable if

$$\frac{N(1-p)(\beta_1 \alpha + \beta_2(r+\mu))}{(p+\mu)(r+\mu)(k+\mu+\alpha)} < 1$$

Comparing with Related Works: The following current methodologies are contrasted with our suggested fractional SEIR model and numerical techniques: i. Computer Model by Ali Akgül [14]: In order to account for memory effects and long-term dependencies in worm spread, our model goes one

step further by introducing fractional derivatives.
ii. Pinto *et al.* [17] fractional Model: By permitting non-integer differentiation, the model includes the interaction between computers and removable devices. Simulate numerically the model for distinct values of the order of the fractional derivative and for two sets of initial conditions adopted in the literature.
iii Nguyen Phuong Dong *et al.* [15]: This work is devoted to studying the uncertain attacking behavior of computer viruses in wireless sensor networks involving fuzzy fractional derivatives with non-local Mittag-Leffler function kernel. Based on epidemic theory and fractional calculus, we propose a fuzzy fractional Susceptible – Infectious – Quarantine – Recovered (SIQR) model to describe the dynamics of virus propagation with quarantine in the network.
iv. The Fuzzy Fractional SIQR Model by Dong *et al.* [16]: Here concentrate on deterministic fractional modeling with thorough stability analysis, even if their approach uses fuzzy logic. By contrasting these approaches, we show how fractional-order models can improve prediction accuracy and offer a more profound understanding of the dynamics of worm spread.
Numerical methods are most effective in solving Fractional differential equations. For the numerical solution of (3.5), (3.6), and (3.7) Adams-Bashforth-Moulton Method is applied
for an approximate solution we consider following non-linear fractional differential equation

$$D^{\nu} y\left(t\right) = f\left(t, y\left(t\right)\right), \; 0 \leq t \leq T$$

$$y^{k}\left(0\right) = y_{0}^{k} \quad k = 0, 1, 2, \ldots, \; m-1$$

This equation corresponds to the Volterra Integral Equation

$$y\left(t\right) = \sum_{k=0}^{m-1} y_{0}^{k} \frac{t^{k}}{k!} + \frac{1}{\Gamma\nu.} \int_{0}^{t} \left(t-s\right)^{\nu-1} f\left(s, y\left(s\right)\right) ds. \tag{4.1}$$

Diethelm *et al.* employed the predictor-correctors scheme depending on the Adams-Bashforth-Moulton, algorithm to integrate Eq. (4.1). By employing this scheme to the fractional-order model for computer viruses and putting $h = \frac{T}{N}$ $t_n = nh$, $n = 0, 1, 2, \ldots, N \in \mathbb{Z}^{+}$, equation (4.1) becomes

$$\begin{aligned} S_{n+1} = S_0 + \frac{h^{\nu}}{\Gamma(\nu+2)}\Big((1-p)N - \beta_1 S_{n+1}^{p} I_{n+1}^{p} - \beta_2 S_{n+1}^{p} E_{n+1}^{p} - (p+\mu) S_{n+1}^{p}\Big) \\ + \frac{h^{\nu}}{\Gamma(\nu+2)} \sum_{j=0}^{n} a_{j,n+1}\Big((1-p)N - \beta_1 S_j I_j - \beta_2 S_j E_j - (p+\mu) S_j\Big) \end{aligned} \tag{4.2}$$

$$\begin{aligned} E_{n+1} = E_0 + \frac{h^{\nu}}{\Gamma(\nu+2)}\Big(\beta_1 S_{n+1}^{p} I_{n+1}^{p} + \beta_2 S_{n+1}^{p} E_{n+1}^{p} - k E_{n+1}^{p} - \sigma E_{n+1}^{p} - \mu E_{n+1}^{p}\Big) \\ + \frac{h^{\nu}}{\Gamma(\nu+2)} \sum_{j=0}^{n} a_{j,n+1}\Big(\beta_1 S_j I_j + \beta_2 S_j E_j - k E_j - \sigma E_j - \mu E_j\Big) \end{aligned} \tag{4.3}$$

$$I_{n+1} = I_0 + \frac{h^{\nu}}{\Gamma\left(\nu+2\right)}\left(\sigma E_{n+1}^{p} - \left(d+\mu\right) I_{n+1}^{p}\right) + \frac{h^{\nu}}{\mathit{\Gamma}\left(\nu+2\right)} \sum_{j=0}^{n} a_{j,n+1}(\sigma E_j - (d+\mu) I_j). \tag{4.4}$$

where,

$$S_{n+1} = S_0 + \frac{1}{\Gamma\nu} \sum_{j=0}^{n} b_{j,n+1}((1-p)\,N - \beta_1 S_j I_j - \beta_2 S_j E_j - (p+\mu) S_j) \tag{4.5}$$

$$E_{n+1} = E_0 + \frac{1}{\mathit{\Gamma}\nu} \sum_{j=0}^{n} b_{j,n+1}(\beta_1 S_j I_j + \beta_2 S_j E_j - k E_j - \sigma E_j - \mu E_j) \tag{4.6}$$

$$I_{n+1} = I_0 + \frac{1}{\mathit{\Gamma}\nu} \sum_{j=0}^{n} b_{j,n+1}(\sigma E_j - (d+\mu) I_j) \tag{4.7}$$

$$a_{j,n+1} = \begin{cases} n^{\nu+1} - (n-\nu)(n-j) & j-0 \\ (n-j+2)^{\nu+1} + (n-j)^{\nu+1} - 2(n-j+1)^{\nu+1} & 1 \le j \le n \\ 1 & n+1 \end{cases}$$

$$b_{j,n+1} = \frac{h^{\nu}}{\nu}\left((n-j+1)^{\nu} - (n-j)^{\nu}\right), \qquad 0 \le j \le n$$

The other numerical technique created for Caputo-Fractional derivative operators is:

$$^{C}D^{\nu}S = (1-p)\,N - \beta_1 SI - \beta_2 SE - (p+\mu)\,S. \tag{4.8}$$

$$^{C}D^{\nu}E = \beta_1 SI + \beta_2 SE - kE - \sigma E - \mu E. \tag{4.9}$$

$$^{C}D^{\nu}I = \sigma E - (d+\mu)I. \tag{4.10}$$

Caputo-fractal -fractional derivative differential operator in the Caputo sense [12] is

$$^{C}D^{\nu}h\,(t) = \frac{1}{\Gamma(1-\nu)}\frac{d}{dt}\int_0^t (t-\tau)^{\nu} S^{\tau-1} f(\tau)\frac{1}{\tau t^{\tau-1}}d\tau$$

such that the system becomes

$$^{RL}D^{\nu}S = \tau t^{\tau-1}[(1-p)\,N - \beta_1 SI - \beta_2 SE - (p+\mu)\,S]. \tag{4.11}$$

$$^{RL}D^{\nu}E = \tau t^{\tau-1}[\beta_1 SI + \beta_2 SE - kE - \sigma E - \mu E]. \tag{4.12}$$

$$^{RL}D^{\nu}I = \tau t^{\tau-1}[\sigma E - (d+\mu)\,I]. \tag{4.13}$$

Caputo derivative is replaced by Riemann-Liouville (RL). On applying the RL fractional integral on both sides of the above equations, we get:

$$S(t) \;=\; S(0) \;+ \frac{\tau}{\Gamma\nu}\int_0^t \lambda^{\tau-1}(t-\lambda)^{\nu-1} f(S,E,I,\lambda)d\lambda. \tag{4.14}$$

$$E\,(t) = \;E\,(0) + \frac{\tau}{\Gamma\nu}\int_0^t \lambda^{\tau-1}(t-\lambda)^{\nu-1} g\,(S,E,I,\lambda)\,d\lambda. \tag{4.15}$$

$$I(t) \;=\; I(0) \;+ \frac{\tau}{\Gamma\nu}\int_0^t \lambda^{\tau-1}(t-\lambda)^{\nu-1} h(S,E,I,\lambda)d\lambda. \tag{4.16}$$

where,

$$f\,(S,E,I,\lambda) = (1-p)\,N - \beta_1 SI - \beta_2 SE - (p+\mu)S.$$
$$g\,(S,E,I,\lambda) = \beta_1 SI + \beta_2 SE - kE - \sigma E - \mu E.$$
$$(S,E,I,\lambda) = \sigma E - (d+\mu)I.$$

We apply new numerical techniques at t_{n+1}, resulting in equations given below:

$$S^{n+1} \;=\; S^0 \;+ \frac{\tau}{\Gamma\nu}\int_0^{t_{n+1}} \lambda^{\tau-1}(t_{n+1}-\lambda)^{\nu-1} f(S,E,I,\lambda)d\lambda.$$

$$E^{n+1} \;=\; E^0 \;+ \frac{\tau}{\Gamma\nu}\int_0^{t_{n+1}} \lambda^{\tau-1}(t_{n+1}-\lambda)^{\nu-1} g(S,E,I,\lambda)d\lambda.$$

$$I^{n+1} \;=\; I^0 \;+ \frac{\tau}{\Gamma\nu}\int_0^{t_{n+1}} \lambda^{\tau-1}(t_{n+1}-\lambda)^{\nu-1} h(S,E,I,\lambda)d\lambda.$$

this can be represented as,

$$S^{n+1} = S^0 + \frac{\tau}{\Gamma\nu}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} \lambda^{\tau-1}(t_{n+1}-\lambda)^{\nu-1} f(S,E,I,\lambda)d\lambda. \tag{4.17}$$

$$E^{n+1} = E^0 + \frac{\tau}{\Gamma\nu}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} \lambda^{\tau-1}(t_{n+1}-\lambda)^{\nu-1} g(S,E,I,\lambda)d\lambda. \tag{4.18}$$

$$I^{n+1} = I^0 + \frac{\tau}{\Gamma\nu}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} \lambda^{\tau-1}(t_{n+1}-\lambda)^{\nu-1} h(S,E,I,\lambda)d\lambda. \tag{4.19}$$

Now using Lagrange's piecewise interpolation, we approximate the function $\lambda^{\tau-1} f(S,E,I,\lambda)$ within the interval $[t_j,\ t_{j+1}]$ such that

$$\mathrm{U_j}(\lambda) = \frac{\lambda - t_{j-1}}{\mathrm{t_j} - t_{j-1}} \mathrm{t}_{\mathrm{j-1}}^{\tau-1} f\left(S^j, E^j, I^j, t_j\right) - \frac{\lambda - t_{j-1}}{t_j - t_{j-1}} \mathrm{t}_{\mathrm{j-1}}^{\tau-1} f\left(S^{j-1}, E^{j-1}, I^{j-1}, t_{j-1}\right). \tag{4.20}$$

$$\mathrm{V_j}(\lambda) = \frac{\lambda - t_{j-1}}{\mathrm{t_j} - t_{j-1}} \mathrm{t}_{\mathrm{j-1}}^{\tau-1} g\left(S^j, E^j, I^j, t_j\right) - \frac{\lambda - t_{j-1}}{t_j - t_{j-1}} \mathrm{t}_{\mathrm{j-1}}^{\tau-1} g\left(S^{j-1}, E^{j-1}, I^{j-1}, t_{j-1}\right). \tag{4.21}$$

$$\mathrm{W_j}(\lambda) = \frac{\lambda - t_{j-1}}{\mathrm{t_j} - t_{j-1}} \mathrm{t}_{\mathrm{j-1}}^{\tau-1} h\left(S^j, E^j, I^j, t_j\right) - \frac{\lambda - t_{j-1}}{t_j - t_{j-1}} \mathrm{t}_{\mathrm{j-1}}^{\tau-1} h\left(S^{j-1}, E^{j-1}, I^{j-1}, t_{j-1}\right). \tag{4.22}$$

Thus, we have

$$S^{n+1} = S^0 + \frac{\tau}{\Gamma\nu}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} \lambda^{\tau-1}(t_{n+1}-\lambda)^{\nu-1} U_j(\lambda)\, d\lambda. \tag{4.23}$$

$$E^{n+1} = E^0 + \frac{\tau}{\Gamma\nu}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} \lambda^{\tau-1}(t_{n+1}-\lambda)^{\nu-1} V_j(\lambda)\, d\lambda. \tag{4.24}$$

$$I^{n+1} = I^0 + \frac{\tau}{\Gamma\nu}\sum_{j=0}^{n}\int_{t_j}^{t_{j+1}} \lambda^{\tau-1}(t_{n+1}-\lambda)^{\nu-1} W_j(\lambda)\, d\lambda. \tag{4.25}$$

We obtain the following numerical scheme,

$$\begin{aligned} S^{n+1} = S^0 + \frac{\tau(\Delta t)}{\Gamma(\nu+2)}\sum_{j=0}^{n}\Bigg[\int_{t_j}^{t_{j+1}} & t_j^{\tau-1} f\left(S^j, E^j, I^j, t_j\right) \times ((n+1-j)^{\alpha}(n-j+2+\alpha) \\ & -(n-j)^{\alpha}(n-j+2+2\alpha)) - t_{j-1}^{\tau-1} f\left(S^{j-1}, E^{j-1}, I^{j-1}, t_{j-1}\right) \times \\ & \left((n+1-j)^{\alpha+1} - (n-j)^{\alpha}(n-j+1+\alpha)\right) d\lambda\Bigg]. \end{aligned} \tag{4.26}$$

$$\begin{aligned} E^{n+1} = E^0 + \frac{\tau(\Delta t)}{\Gamma(\nu+2)}\sum_{j=0}^{n}\Bigg[\int_{t_j}^{t_{j+1}} & t_j^{\tau-1} g\left(S^j, E^j, I^j, t_j\right) \times ((n+1-j)^{\alpha}(n-j+2+\alpha) \\ & -(n-j)^{\alpha}(n-j+2+2\alpha)) - t_{j-1}^{\tau-1} g\left(S^{j-1}, E^{j-1}, I^{j-1}, t_{j-1}\right) \times \\ & \left((n+1-j)^{\alpha+1} - (n-j)^{\alpha}(n-j+1+\alpha)\right) d\lambda\Bigg]. \end{aligned} \tag{4.27}$$

$$\begin{aligned} I^{n+1} = I^0 + \frac{\tau(\Delta t)}{\Gamma(\nu+2)}\sum_{j=0}^{n}\Bigg[\int_{t_j}^{t_{j+1}} & t_j^{\tau-1} h\left(S^j, E^j, I^j, t_j\right) \times ((n+1-j)^{\alpha}(n-j+2+\alpha) \\ & -(n-j)^{\alpha}(n-j+2+2\alpha)) - t_{j-1}^{\tau-1} h\left(S^{j-1}, E^{j-1}, I^{j-1}, t_{j-1}\right) \times \\ & \left((n+1-j)^{\alpha+1} - (n-j)^{\alpha}(n-j+1+\alpha)\right) d\lambda\Bigg]. \end{aligned} \tag{4.28}$$

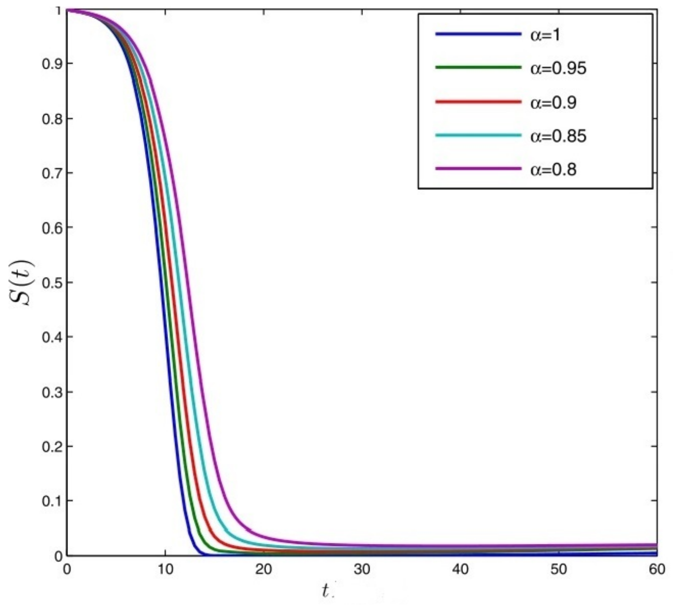

Figure 1: S(t) with Caputo derivative of order 1, 0.95, 0.9, 0.85, 0.8. The figure represents the function S(t) plotted against time t for different values of the fractional-order parameter α. The curves correspond to **Caputo fractional derivatives** with orders $\boldsymbol{\alpha = 1, 0.95, 0.9, 0.85, 0.8}$.

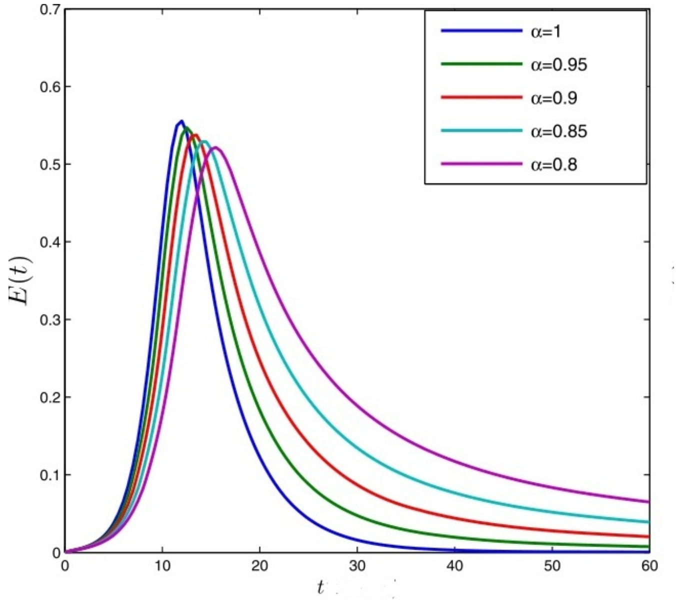

Figure 2: E(t) with Caputo derivative of order 1, 0.95, 0.9, 0.85, 0.8. The figure represents the function E(t) plotted against time $\boldsymbol{t}$, with different curves corresponding to different values of the fractional-order parameter $\boldsymbol{\alpha}$.

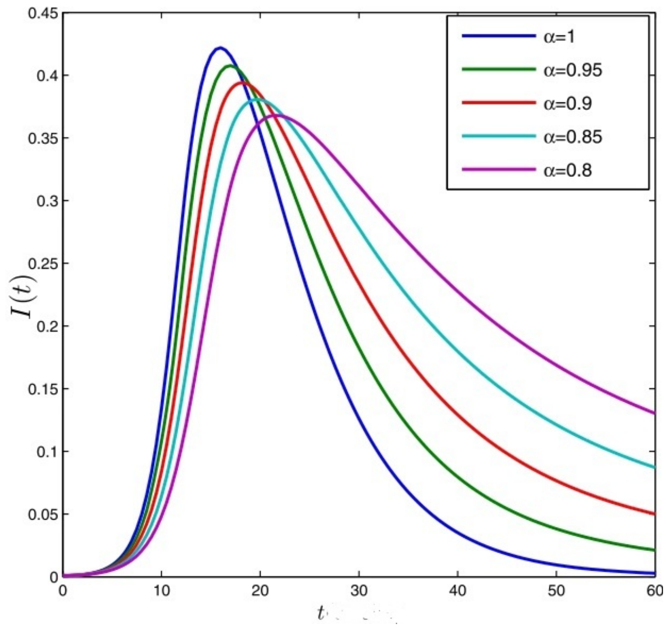

Figure 3: I(t) with Caputo derivative of order 1, 0.95, 0.9, 0.85, 0.8. The figure represents the function I(t) plotted against time t, with different curves corresponding to different values of the fractional-order parameter $\boldsymbol{\alpha}$. The parameter $\boldsymbol{\alpha}$ controls the order of the **Caputo fractional derivative** applied in the model.

5. Conclusions

Using fractional calculus, this paper provides a thorough examination of worm spread in computer networks. The intricacy of worm propagation is effectively captured by the suggested fractional SEIR model, which takes into account both local and global infection dynamics. Our simulation-backed numerical analysis confirms that the fractional approach is more effective than conventional models at describing network worm behavior. We have emphasized how fractional derivatives are a useful tool in cybersecurity because they can be used to incorporate memory effects and non-local interactions. Additionally, comparisons with current models demonstrate how our method improves stability analysis, equilibrium points, and numerical solutions.This work has taken into account the mathematical SEIR model that shows the potential propagation of computer viruses among computer systems with four classifications. These classes included the systems' populations that were susceptible, exposed, infectious, and recovered. The classical form was modified by using various fractional derivatives and integrals, they further capture additional complexity and various spread possibilities. Various numerical techniques are used to solve these models, due to their non-linearity Additionally, utilizing the derived numerical solutions, some numerical simulations were carried out. This work can be expanded in the future by incorporating more real-world characteristics, investigating hybrid models that blend machine learning with fractional calculus, and looking into the best control methods for worm containment. The study's conclusions lay the groundwork for creating cybersecurity frameworks that are more resilient and flexible in order to lessen the negative effects of malicious software on networked systems.

Compliance with ethical standards
Conflict of interest: The authors assert no conflict of interest.
Ethical Approval: None of the authors conducted any studies that involved human participants or animals for this article.

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