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# Inverse Planar Domination and Independent Planar Domination in Graphs

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ABSTRACT: Suppose that $G$ is a simple, undirected, finite graph with no isolated vertices. A dominating set $D$ of $G$ is said to be planar dominating set if $\langle D\rangle$ is a planar graph, such that all vertices in $D$ have at least two adjacent vertices in $G$. The planar domination number of $G$ denoted by $\gamma_{pl}$, is the cardinality of the minimum planar dominating set in $G$. Inverse and independent planar domination, the new models of domination are studied in this paper with several results and properties. The inverse planar domination number is evaluated and proved for some known graphs. Also, some possible applications of the planar domination are presented.

Key Words: Domination set, inverse planar domination set, inverse planar domination number, independent planar domination.

## Contents



## 1. Introduction

Let $G=(V,E)$ be a graph with size $m=|E|$, and order $n=|V|$ without isolated vertex. The degree of any vertex $v$ in $G$ ( $deg(v)$ ) is the number of edges that connected to $v$, $\delta(G)$ and $\Delta(G)$ are the minimum and maximum degree of $G$. An isolated vertex has degree zero. A vertex that has a degree one is called pendent, it is located next to a support vertex. The graph $\overline{G}$ is the complement graph of $G$ has the same vertices of $G$ but the $E\left(\overline{G}\right)=\left\{e: e\in\ \overline{G}\ \leftrightarrow \nexists\ e\in\ G\right\}$. For specifics on graph theoretic terminology, see [14]. The study of dominant set has a wide scope in graph theory. Refer to [15,16] for a thorough study of domination. A set $D\subseteq V$ is said to be a dominating set, if $v\in V-D$ is adjacent to at least one vertex in $D$. The domination number of a graph $G$: is $\gamma(G)=\min\{|\ D\ |: D\subseteq V(G)\ and\ D \text{ is a dominating set}\}$ The terms dominating set and domination number were first used by Ore [27,13]. There are several forms of dominance due to their significant uses. Certain definitions place restrictions on $G[D]$, as seen in [5,6,10,22,26]. For instance, see [2,4,8,19,24,25,29,30] for others imposing a constraint on $G[V-D]$. Constraint on $G[D]$ and $G[V-D]$ were incorporated in certain definitions (see, for instance, [3,17,18,23]. Additional to fuzzy domination [9,11,12,20,30,31,32] and fuzzy domination [7,21,28]. In [1], they presented the planar domination in a graph. Let $G=(V,E)$ be a graph without isolated vertex, a set $D\subseteq\ V(G)$ is a planar dominating set if induced subgraph $\langle D\rangle$ is planar graph and dominating set with for all $v\ \in\ D$, $\deg(v)\ \geq\ 2$ . The cardinality of a minimal planar dominating set in $G$ is represented by $\gamma_{pl}$ , which is the planar domination number of $G$. They looked at a graph with planar domination, including its order and certain constraints. They used domination of this kind on a few popular graphs. A new dominating kind known as "inverse planar domination" is presented here. A minimal inverse planar dominant set's cardinality is denoted by $\gamma_{pl}^{-1}$. This article examines the inverse planar domination number for the following graphs: path $P_n$ , cycle $C_n$ , complete $K_n$ , complete bipartite $K_{n,m}$, and wheel $W_n$. Also, input the condition of independence set on the

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planar domination to introduced an independent planar domination in a graph. Important question: the planar domination where can we use it? discuss this in this paper.

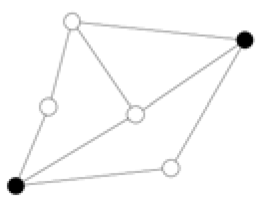
**Proposition 1.1** *[1] Let $G$ be any graph of order $n$ having a planar domination number, then we have:*

1. *The order of $G$ is $n \geq 3$.*
2. *$\delta(G) \geq 1$ and $\Delta(G) \geq 2$.*
3. *For each vertex $v \in D$, $deg(v) \geq 2$.*
4. *If $v$ is a support vertex then $v \in D$.*
5. *Every pendent vertex $v$, $v \in V - D$.*
6. *$\gamma_{pl}(G) \geq 1$.*

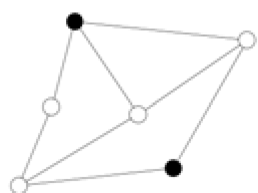
## 2. Inverse planar domination in graphs

The dominating set is the main set in any graph or network, and it actively contributes to issues in projects involving graphs or networks when there is a network failure in some of the dominating nodes. Maintaining the functionality of the system will be handled by the inverse dominating set. Focusing on the dominating set and inverse dominating set is helpful in this context. We shall thus introduce the concept of an "inverse planar dominating set" in this section.

**Definition 2.1** *Let $G = (V, E)$ be a graph without isolated vertex, a subset $D \subseteq V(G)$ is a* ***planar dominating set*** *if $D$ dominating set and induced subgraph $\langle D \rangle$ is planar graph such that $deg(v) \geq 2$ for all $v \in D$. A set is referred to be an inverse planar dominating set of $D$ in $G$ if $V - D$ includes a planar dominating set in $G$, which is represented by $D^{-1}$. (for example, see Fig. 1)*



(a) Minimum planar dominating



(b) Minimum inverse planar dominating

Figure 1: Minimum planar dominating set and minimum inverse planar dominating set in graph $G$

**Definition 2.2** *Let $G = (V, E)$ be a graph, a subset $D^{-1} \subseteq V(G) - D$ is called* ***a minimal inverse planar dominating set if*** *it has no proper planar dominating set. A minimum inverse planar dominating set is an inverse planar dominating set of smallest size in a graph $G$.*

**Definition 2.3** *The inverse planar domination number denotes $\Upsilon_{pl}^{-1}(G)$ is a minimum cardinality over all the inverse planar dominating sets in $G$.*

**Definition 2.4** *Let $G$ be a graph has planar domination, the minimum planar dominating set of $G$ is denoted by $\Upsilon_{pl}^{-1}-$ set.*

**Proposition 2.1** *If a vertex in a graph $G$ has degree1, then there isn't an inverse planar dominating set.*

**Proof:** Assume that $G$ contains a vertex $u$ such that $deg(u) = 1$. Let $D$ be any planar dominating set in $G$.By definition, every vertex in $D$ must have a degree at least 2 then $u \in v - D$ is adjacent to exactly one other vertex, say $v \in D$ . To form an inverse planar dominating set $D^{-1}$from the set $V - D$, the set must include $u$ (since no other vertex in $V - D$ dominates it), which represent a contradiction. Therefore, $G$ cannot have an inverse planar dominating set $D^{-1}$. □

**Remark 2.1**

1. *There is no inverse planar dominating set in the following graphs: path $P_n$, star $S_n$*
2. *For the complete graph $K_n$, $\forall n \geq 3$ the $\Upsilon_{pl}^{-1}(K_n) = 1$.*

**Theorem 2.1** *The cycle graph $C_n$ has an inverse planar dominating set and $\gamma_{pl}^{-1}(C_n) = \gamma_{pl}(C_n) = \left\lceil \frac{n}{3} \right\rceil$ , where $n \geq 3$.*

**Proof:** Since the cycle graph $C_n$ is a planar graph so, any minimum domination set is also planar with every vertex with degree two. thus $\gamma_{pl}^{-1}(C_n) = \gamma_{pl}(C_n) = \left\lceil \frac{n}{3} \right\rceil$. [ 15] □

**Theorem 2.2** *If $K_{n,m}$is a complete bipartite graph then it has an inverse planar dominating set and $\gamma_{pl}^{-1}(K_{m,n}) = 2$ where $n, m \geq 2$.*

**Proof:** Let $V_1$ and $V_2$ are the two sets of the vertices of $K_{n,m}$. Since $n, m \geq 2$, by starting with planar dominating set $D^{-1}$ contains two vertices, one from set $V_1$ and the other from set $V_2$, which are distinct from the two vertices utilized in the planar dominant set $D$ of $K_{m,n}$. (for example, see Fig. 2 ).

(a) Minimum planar dominating set

(b) Minimum inverse planar dominating set

Figure 2: Minimum planar dominating set and minimum inverse planar dominating set in $K_{n,m}$

□

**Proposition 2.2** *Let $W_n$ be a wheel graph, then we have $\gamma_{pl}{}^{-1}(W_n) = \left\lceil \frac{n-1}{3} \right\rceil$, $n \geq 3$.*

**Proof:** Since $W_n = C_{n-1} + K_1$ . Let $D^{-1}$ be the inverse planar dominating set with respect to planar dominating set $D = \{v\}$. Since the wheel graph is planar, then $\gamma(W_n) = \gamma_{pl}(W_n) = 1$, $n \geq 3$ . In this instance, the center vertex $v$ in the wheel graph is contained in the planar dominating set. Thus, to find vertices in $V - D$ to be the planar dominant set, the induced subgraph of vertices must be taken from the cycle of order $n - 1$. According to Theorem2.4, we have $\gamma_{pl}{}^{-1}(W_n) = \left\lceil \frac{n-1}{3} \right\rceil$, $n \geq 3$. □

**Theorem 2.3** *For $K$- partite graph the inverse planar dominating set is*

$$\Upsilon_{pl}^{-1}(K_{n_1,n_2,\ldots,n_k}) = \begin{cases} 1 & if \quad n_i = 1 \quad \exists i = 1, \ldots, k, \ k \geq 2, \quad n_j \geq 2 \quad where\ j = k - i \\ 2 & if \quad n_i = 1, i = 1 \quad n_j \geq 2 \quad where\ j = k - i, \\ & \quad n_i \geq 2 \quad \forall i = 1, \ldots, k \end{cases}$$

**Proof:** Take the sets to $A_1$, $A_2$ , . . . , $A_k$ be the graph's partite of orders $n_1, n_2, \ldots, n_k$ respectively. next the following three instances are described. In this theorem there are three cases:

**Case 1** *If $n_i = 1$ for some $i = 1, \ldots, k$ , then $D = \{v\}$, $v \in A_i$ thus $\gamma_{pl}\left(K_{n_1,n_2,\ldots,n_k}\right) = 1$ . Now to find $D^{-1}$ , it is clear that any one vertex from disjoint partite $A_i$ of graph $\left(K_{n_1,n_2,\ldots,n_k}\right)$ are planar dominating set in it. then $\Upsilon_{pl}^{-1}\left(K_{n_1,n_2,\ldots,n_k}\right) = 1$.*

**Case 2** *If $n_i = 1$ for some $i = 1$ , then $D = \{v\}$, $v \in A_i$ thus $\gamma_{pl}\left(K_{n_1,n_2,\ldots,n_k}\right) = 1$ . Now to find $D^{-1}$ , it is clear that any two vertices from two disjoint partite $A_j$ of graph $\left(K_{n_1,n_2,\ldots,n_k}\right)$ are planar dominating set in it. then $\Upsilon_{pl}^{-1}\left(K_{n_1,n_2,\ldots,n_k}\right) = 2$.*

**Case 3** *If $n_i \geq 2 \quad \forall i = 1, \ldots, k$, then let the set $D$ has two vertices according, in this case, without losing generality, choose any new vertices from disjoint sets do not contain in $D$ and dominating other vertices in the graph $\left(K_{n_1,n_2,\ldots,n_k}\right)$.*

□

## 3. Independent planar domination

The independent planar dominion in in graphs is introduced in this section.

**Definition 3.1** *Let $G = (V, E)$ be simple graph with no isolated vertex, a set $D \subseteq V(G)$ is an independent planar dominating set if the induced subgraph $\langle D \rangle$ is planar graph such that $deg(v) \geq 2$ for all $v \in D$, and $D$ is an independent set. The minimum cardinality of an independent planar dominating set is denoted by $\gamma_{pl}^{i}(G)$. (For example, see Fig. 3).*

(a) Minimum planar dominating set (b) Minimum independent planar dominating set

Figure 3: Minimum planar dominating set and minimum independent planar dominating set in $G$

**Proposition 3.1** *For a graph $G$ with $(n \geq 3)$ then we have:*

1. $\gamma_{pl}^{i} C_n = \gamma_{pl}(C_n) = \gamma(C_n) = \left\lceil \frac{n}{3} \right\rceil$.
2. $\gamma_{pl}^{i} K_n = \gamma_{pl}((K_n\ ) = 1$.

**Proposition 3.2** *If $G$ have an independent planar domination number $\gamma_{pl}^{i}$ then the size of $G$ is $(n - \gamma_{pl}^{i}) \leq m \leq \binom{n}{2} + \frac{1}{2}\left(-\gamma_{pl}^{2i} + \gamma_{pl}^{i}\right)$.*

**Proof:** Two instances are necessary for this proof. Assume the subset $D$ is a $\gamma_{pl}^{i}-$ set

**Case 4** *Now we start prove the lower bound $(n - \gamma_{pl}^{i}) \leq m$ . According to the Definition 3.1 of the independent planar dominating set, $G[D]$ is a null graph, every vertex in $V - D$ must be adjacent to at least one vertex in $D$ to be dominated and the degree of every vertex in $D$ is 2. Let $G\,[V - D]$ be another null graph. Next, we have $G$ have the fewest edges possible and $G\,[V - D]$ will not violate the definition of the independent planar domination. Thus, each vertex in $V - D$ contributes at least one edge to $m$. Therefore, $|V - D| = \left(n - \gamma_{pl}^{i}\right) \leq m$.*

**Case 5** *To prove the upper bound* $m \leq \binom{n}{2} + \frac{1}{2}\ (-\gamma_{pl}^{2i} + \gamma_{pl}^{i})$*. All vertices of* $G[D]$ *are isolated because set* $D$ *is independent. Since the vertices of set* $D$ *are unaffected by the number of* $G[V-D]$ *edges, then, then we can suppose* $G[V-D]$ *is completely induced subgraph. Assume that* $D$ *and* $V-D$ *edge counts are equal to* $m_1$ *and* $m_2$ *respectively, since* $D$ *is an independent planar dominating set then then we have*

$$m_1 = 0 \text{ and } m_2 = \frac{(|V-D|)(|V-D|-1)}{2} = \frac{(n-\gamma_{pl}^{i})(n-\gamma_{pl}^{i}-1)}{2}$$

*By definition of planar dominating set, any vertex in* $D$ *may be adjacent to all vertices in* $V-D$*. Then the number of edges from* $D$ *to* $V-D$ *is equal to*

$$m_3 = |D||V-D| = \left(\gamma_{pl}^{i}\right)\left(n-\gamma_{pl}^{i}\right) = (n\gamma_{pl}^{i} - \gamma_{pl}^{2i})$$

*Therefore, the number of edges of* $G$ *is*

$$\begin{aligned} m = m_1 + m_2 + m_3 &= 0 + \frac{1}{2}\ (n^2 - n\gamma_{pl}^{i} - n - n\gamma_{pl}^{i} + \gamma_{pl}^{2i} + \gamma_{pl}^{i}) + (n\gamma_{pl}^{i} - \gamma_{pl}^{2i}) \\ &= \frac{1}{2}\left(n^2 - n\gamma_{pl}^{i} - n - n\gamma_{pl}^{i} + \gamma_{pl}^{2i} + \gamma_{pl}^{i} + 2n\gamma_{pl}^{i} - 2\gamma_{pl}^{2i}\right) \\ &= \frac{(n^2-n)}{2} + \frac{1}{2}(-\gamma_{pl}^{2i} + \gamma_{pl}^{i}) = \binom{n}{2} + \frac{1}{2}(-\gamma_{pl}^{2i} + \gamma_{pl}^{i}). \end{aligned}$$

□

**Proposition 3.3** *The set* $D$ *is an independent planar dominating in graph* $G$ *if and only if* $D$ *is an independent dominating set in* $G$ *and* $v \geq 2 \quad \forall v \in D$ *.*

**Proof:** $\Longrightarrow$ It is trivial.
$\Longleftarrow$ If $D$ is an independent dominating set in $G$ and $v \geq 2 \quad \forall v \in D$ then $D$ is also a planar dominating set in $G$ . Therefore, $D$ is independent planar dominating in $G$. □

**Proposition 3.4** *If* $D$ *is an independent planar dominating set in graph* $G$ *then* $D$ *is a planar dominating set in graph* $G$ *but the converse is not true .*

**Proof:** $\Longrightarrow$ It is trivial. □

The example for converse $P_4$ has a planar dominating set but has no independent planar dominating set.

**Proposition 3.5** *Every* $T_n$ *with* $(n \geq 3)$ *has an independent planar domination number* $\gamma_{pl}^{i}$ *if the support vertices in the tree are not adjacent.*

**Proof:** Since $T$ is a connected planar graph has order $n$ $(n \geq 3)$ and let $D$ be the dominating set in $T$ with $deg(v) \geq 2$ for all $v \in D$ then $T$ has a planar dominating set $D$.
Now we need to proof $D$ is independent set in $T$. By hypothesis the set $D$ has $deg(v) \geq 2$ for all $v \in D$ then we have two following cases:

**Case 6** *If* $\deg(v) = 2$ *for all* $v \in D$ *then in such a way that each* $v$ *in* $D$ *forms a part of a path. Therefore,* $D$ *is independent set because no two vertices in* $D$ *can be adjacent due to the structure of the tree and the conditions are given.*

**Case 7** *If* $\deg(v) > 2$ *for all* $v \in D$ *must be connected to at least three other vertices. these three neighbors of* $v$ *must either be leaves or other vertices not in* $D$*. Thus* $D$ *remains an independent set in* $T$*. Given the both cases, the set* $D$ *in the tree* $T$ *is independent, and therefore,* $T$ *has an independent planar domination number* $\gamma_{pl}^{i}$

□

## 4. Applications of the planar domination in graphs

1. **Geographic Information Systems (GIS):** In GIS, important sites (such buildings or service points) that cover a region without overlapping and provide effective connectivity can be represented as a planar dominant set.

2. **Graph Drawing and Visualization:** Making sure the induced subgraph in graph visualization is planar, facilitates the creation of diagrams that are easy to read and comprehend. This is very helpful for network topology and circuit design.

3. **Robotics and Autonomous Systems:** Robots must cover an area without overlapping pathways, and for coverage issues in robotics, a planar dominant set guarantees effective path planning.

4. **Urban Planning:** A planar dominating set may be used to simulate the process of choosing strategic places for urban planning, such as hospitals and fire stations, that are effectively connected and cover the whole city.

**Example 4.1** *Consider a graph $G = (V, E)$ representing a city layout, where $V$ vertices represent intersections and $E$ edges represent roads. A planar dominating set $D$ could represent key intersections that ensure every road (edge) and intersection (vertex) is monitored by a traffic camera. The condition $deg(v) \geq 2$ ensures each key intersection has at least two roads passing through it, ensuring redundancy in monitoring. Therefore, by applying the concept of a planar dominating set, we can efficiently design and manage networks, systems, and layouts, ensuring optimal coverage and connectivity.*

## 5. Discussion and Conclusion

According to the previous discussion, any graphs—whether planar or non-planar, and whether connected or unconnected—can utilize a new type of inverse domination in graphs called inverse planar domination. For a large number of known graphs, the inverse planar dominance was calculated. Additionally, we connect planar and independent domination.

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