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New class of cyclic contraction and fixed point results in b-metric spaces

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ABSTRACT: Notion of C-class cyclic contractive mapping is introduced in this communication. Further a new fixed point result in b-metric spaces is investigated and as applications of our main results we have deduce some corollaries. Examples are also given here to notify the importance and novelty of our main result.

Key Words: C-class function, cyclic contraction, b-metric spaces, fixed point.

2020 Mathematics Subject Classification: 47H10, 54H25.

Contents

1. Introduction and preliminaries

Bourbaki [1] and Bakhtin [2] works inspired Czerwik [3] to give an extension and wider-class of metric spaces. Czerwik [3]named and initiated the hypothesis of b -metric space and defined the following definition.

Definition 1.1 *[3] Let W be a non-empty set and $\Omega : W \times W \to \mathbb{R}^+$ be a functional satisfying following assertions:*

1. $\Omega(s,l) = 0$ *iff* $s = l$,
2. $\Omega(s,l) = \Omega(l,s)$,
3. $\Omega(s,m) \leq \lambda[\Omega(s,l) + \Omega(l,m)]$, *where* $\lambda \geq 1$ *is a fixed real*

for all $s,l,m \in W$*. Then* Ω *is b-metric and the pair* (W,Ω) *is known as b-metric space.*

If we let $\lambda = 1$, in general, we obtain the usual definition of metric spaces but conversely, every metric is not a b-metric. Aghajani et al. [4]) have justified this statement by presenting a suitable example. Here one of most significant point to note is that b-metric is discontinuous.
Boriceanu [5], in 2008, initiate the notion of closure, compactness, completeness, and convergences in a b-metric spaces.

Definition 1.2 *[5] Let the pair* (W,Ω) *be a b-metric space. Then sequence* $\{s_n\} \in W$ *is called*

1. *b-convergent if and only if* $\lim_{n\to\infty} \Omega(s_n, s) = 0$ *for all* $s \in W$
2. *b-Cauchy if and only if* $\lim_{n,m\to\infty} \Omega(s_n, s_m) \to 0$.

For some more results on fixed point theorems in b- metric and other spaces we refer to [11,12,13,14,15,16,17,21] and references there in. Aghajani et al. [4] derived a lemma (see Lemma [[4]]) to discuss the convergence in b-metric spaces. Han and Hieu [9] initiated the notions of generalized cyclic contractive mapping as follows.

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Definition 1.3 *[9] Let (W, Ω, λ) be a b-metric space, S and T be two nonempty subset of W. Let $Y = S \cup T$ and define a function $P : Y \to Y$. Then P is called a generalized cyclic contractive mapping if:*

(i) $Y = S \cup T$ is a cyclic representation of Y w.r.t. P, i.e., $P(S) \subset T$ and $P(T) \subset S$.
(ii) $\psi(\lambda^4 \Omega(Ps, Pv)) \leq \varphi(\psi(M(s, v))) + LN(s, v)$
for all $(s, v) \in S \times T$ or $(s, v) \in T \times S$, where ψ is control function, $\varphi \in \Phi_u$, $L \geq 0$ be a constant and

$$M(s,v) = \max \left\{ \begin{array}{c} \Omega(s,v), \Omega(s,Ps), \Omega(v,Pv), \\ \frac{\Omega(s,Pv)+\Omega(v,Ps)}{2s}, \\ \frac{\Omega(P^2s,s)+\Omega(P^2s,Pv)}{2s}, \\ \Omega(P^2s,Ps), \Omega(P^2s,v), \Omega(P^2s,Pv) \end{array} \right\}$$

and

$$N(s, v) = \min\{\Omega(s, Ps), \Omega(v, Ps), \Omega(P^2s, P^2v)\}.$$

Theorem 1.1 *[9] Let (W, Ω, λ) be a b-metric space, S and T be two nonempty closed subset of W. Let $Y = S \cup T$ and $P : Y \to Y$ be a generalized cyclic contractive mapping. Then P has a unique fixed point in $S \cap T$.*

Huang et al. [20] for contractive maps derived few results to get invariant points. They also discussed both (P property, T-stability of Picard's iteration) for such mappings. Additionally,with some initial conditions, they gave applications for two distinct classes of ODE and provided a concise mathematical expression of solutions to these ordinary differential equations. In continuity, Pant and Panicker [19] for admissible maps, obtained some fixed point results and displayed an application to a quadratic non linear integral equation for existence and uniqueness of solution under some assumptions is also given. Ansari [7,8] initiated a C -class notion as major generalization of Banach contraction principle. Ansari in [7] defined a family of ultra distance function and denoted by Φ_u.

Definition 1.4 *[7] Let $H : [0, \infty)^2 \to \mathbb{R}$ be a map. Then function H is known as C-class function, if it is continuous and satisfies:*

1. *$H(b_1, b_2) \leq b_1$;*
2. *$H(b_1, b_2) = b_1 \implies$ either $b_1 = 0$ or $b_2 = 0$; for all $b_1, b_2 \in [0, \infty)$.*

Set $\mathcal{C}$ as the family of C-class function.

Few more properties of C-class functions are given in [7].

Example 1.1 [7] Some examples of C class function, for all $b_1, b_2 \in [0, \infty)$, are:

1. $H(b_1, b_2) = b_1 - b_2$, $H(b_1, b_2) = b_1 \Rightarrow b_2 = 0$;
2. $H(b_1, b_2) = mr$, $0<m<1$, $H(b_1, b_2) = b_1 \Rightarrow b_1 = 0$;
3. $H(b_1, b_2) = \frac{b_1}{(1+b_2)^h}$; $h \in (0, \infty)$, $H(b_1, b_2) = b_1 \Rightarrow b_1 = 0$ or $b_2 = 0$;
4. $H(b_1, b_2) = b_1 \log_{b_2+a} a$, $a > 1$, $H(b_1, b_2) = b_1 \Rightarrow b_1 = 0$ or $b_2 = 0$;
5. $H(b_1, b_2) = b_1 - (\frac{1+b_1}{2+b_1})(\frac{b_2}{1+b_2})$, $H(b_1, b_2) = b_1 \Rightarrow b_2 = 0$;
6. $H(b_1, b_2) = b_1\beta(b_1)$, $\beta : [0, \infty) \to \{0, 1)$, $H(b_1, b_2) = b_1 \Rightarrow b_1 = 0$;
7. $H(b_1, b_2) = b_1 - \frac{b_2}{b_1+b_2}$, $H(b_1, b_2) = b_1 \Rightarrow b_2 = 0$;
8. $H(b_1, b_2) = b_1 - (\frac{2+b_2}{1+b_2})b_2$, $H(b_1, b_2) = b_1 \Rightarrow b_2 = 0$.

9. $H(b_1, b_2) = \frac{b_1}{(1+b_1)^\lambda}$; $\lambda \in (0, \infty)$, $H(b_1, b_2) = b_1$ implies $b_1 = 0$.

Recently, Gupta, Mani and Ansari [21] for a C -class mapping, gave a sufficient condition for the existence and uniqueness of fixed points satisfied generalized contraction. Saini, Gupta and Mani [22] develops and deduced a result for two maps using C -class function satisfying weak rational expression. Over the past decade, numerous researchers have presented a variety of results employing diverse methodologies; some of these includes [24]

Following lemma is useful in deriving our main result.

Lemma 1.1 *[23] Let (W, Ω, λ) be a b-metric space, and let $\{s_n\}$ be a sequence in W. If there exists $r \in [0, 1)$ satisfying*

$$\Omega(s_n, s_{n+1}) \leq r\Omega(s_{n-1}, s_n) \quad \forall \quad n \in \mathbb{N},$$

then $\{s_n\}$ is a Cauchy sequence.

In this article, first a C -class cyclic contractive mappings defined and then a fixed point theorem in b-metric spaces with the help of monotone triplet (ψ, φ, H) is proved. In next section, as an applications, we have deduce corollaries.

2. Main Results

Throughout assume that $\lambda \geq 1$. First start with a definition and supported examples.

Definition 2.1 *We say 3 -tuples (ψ, φ, H) are monotone, if for any $s, v \in [0, \infty)$*

$$s \leqslant v \Longrightarrow H(\psi(s), \varphi(s)) \leqslant H(\psi(v), \varphi(v)).$$

where ψ is a control function, $H \in \mathcal{C}$ and $\varphi \in \Phi_u$.

Example 2.1 Let $H(b_1, b_2) = b_1 - b_2, \varphi(s) = \sqrt{s}$,

$$\psi(s) = \begin{cases} \sqrt{s}, & \text{if } 0 \leq s \leq 1, \\ s^2, & \text{if s} > 1 \end{cases},$$

then triplet (ψ, φ, H) is monotone.

Example 2.2 Let $H(b_1, b_2) = b_1 - b_2, \varphi(s) = s^2$

$$\psi(s) = \begin{cases} \sqrt{s}, & \text{if } 0 \leq s \leq 1, \\ s^2, & \text{if s} > 1 \end{cases},$$

then triplet (ψ, φ, H) is not monotone.

We introduce the following definition of C -class cyclic contraction.

Definition 2.2 *Let (W, Ω, λ) be a b-metric-space, and let $S \neq \phi, T \neq \phi \subset W$. Let $Y = S \cup T$ and $P : Y \to Y$ be a map. We say P is C -class cyclic generalized contractive mapping if:*

(A1) $Y = S \cup T$ is a cyclic representation of Y w.r.t. P, that is, $P(S) \subset T$ and $P(T) \subset S$.

(A2) There exists a constant $L \geq 0$ satisfying

$$\psi(\lambda\Omega(Ps, Pv)) \leq H\left(\psi(M(s, v)), \varphi(M(s, v))\right) + LN(s, v) \tag{2.1}$$

for all $(s, v) \in S \times T$ or $(s, v) \in T \times S$, where $H \in \mathcal{C}$, ψ (infinite altering distance function [18]), $\varphi \in \Phi_u$ such that (ψ, φ, H) is monotone,

$$M(s, v) = \max \left\{ \begin{array}{c} \Omega(s, v), \Omega(v, Pv), \Omega(s, Ps), \\ \frac{\Omega(s, Pv) + \Omega(v, Ps)}{\lambda}, \\ \frac{\Omega(P^2s, s) + \Omega(P^2s, Pv)}{\lambda} \\ \Omega(P^2s, Ps), \Omega(P^2s, v), \Omega(P^2s, Pv) \end{array} \right\} \tag{2.2}$$

and

$$N(s, v) = \min\{d(s, Ps), \Omega(v, Ps), \Omega(P^2s, P^2v)\}. \tag{2.3}$$

Theorem 2.1 *Let (W, Ω, λ) be a b-metric space and let $S \neq \phi, T \neq \phi \subset W$ are closed. Let $Y = S \cup T$ and selfmap P defined on Y is a C-class cyclic generalized contraction. Then P has a unique fixed point in $S \cap T$.*

Proof: Let $s_0 \in S$. We construct the sequence $\{s_n\}$ in W by $s_{n+1} = Ps_n$ for all $n \geq 0$. Since $s_0 \in S, s_1 = Ps_0 \in P(S) \subset T$. So, $s_2 = Ps_1 \in P(T) \subset S$. Continuing this process for all $n \geq 0$, we have

$$s_{2n} \in S \text{ and } s_{2n+1} \in T \tag{2.4}$$

Step -1: Claim that $\{s_n\}$ is b -Cauchy sequence.
Suppose, there exists $k \geq 0$ s.t. $s_{k+1} = s_k$, then $Ps_k = s_k$, that is, s_k is an invariant point of P.
Next assume $s_n \neq s_{n+1}$ for all $n \geq 0$. From (2.4), we have $(s_{2n-1}, s_{2n}) \in T \times S$. Since P is a C-class generalized cyclic contractive mapping, we have

$$\begin{aligned}\psi(\lambda(\Omega(s_{2n}, s_{2n+1}))) &= \psi(\lambda\Omega(Ps_{2n-1}, Ps_{2n})) \\ &\leq H(\psi(M(s_{2n-1}, s_{2n})), \varphi(M(s_{2n-1}, s_{2n}))) \\ &\quad + LN(s_{2n-1}, s_{2n}) \\ &\leq \psi(M(s_{2n-1}, s_{2n})) + LN(s_{2n-1}, s_{2n}),\end{aligned} \tag{2.5}$$

where,

$$\begin{aligned}M(s_{2n-1}, s_{2n}) &= \max\left\{\begin{array}{c}\Omega(s_{2n-1}, s_{2n}), \Omega(s_{2n-1}, Ps_{2n-1}), \Omega(s_{2n}, Ps_{2n}), \\ \frac{\Omega(s_{2n-1}, Ps_{2n}) + \Omega(s_{2n}, Ps_{2n-1})}{\lambda}, \\ \frac{\Omega(P^2 s_{2n-1}, s_{2n-1}) + \Omega(P^2 s_{2n-1}, Ps_{2n})}{\lambda}, \\ \Omega(P^2 s_{2n-1}, Ps_{2n-1}), \Omega(P^2 s_{2n-1}, s_{2n}), \\ \Omega(P^2 s_{2n-1}, Ps_{2n})\end{array}\right\} \\ &= \max\left\{\begin{array}{c}\Omega(s_{2n-1}, s_{2n}), \Omega(s_{2n-1}, s_{2n}), \Omega(s_{2n}, s_{2n+1}), \\ \frac{\Omega(s_{2n-1}, s_{2n+1}) + \Omega(s_{2n}, s_{2n})}{\lambda}, \\ \frac{\Omega(s_{2n+1}, s_{2n-1}) + \Omega(s_{2n+1}, s_{2n+1})}{\lambda}, \\ \Omega(s_{2n+1}, s_{2n}), \Omega(s_{2n+1}, s_{2n}), \Omega(s_{2n+1}, s_{2n+1})\end{array}\right\} \\ &\leq \max\left\{\begin{array}{c}\Omega(s_{2n-1}, s_{2n}), \Omega(s_{2n}, s_{2n+1}), \\ \frac{\Omega(s_{2n-1}, s_{2n}) + \Omega(s_{2n}, s_{2n+1})}{2}\end{array}\right\} \\ &= \max\{\Omega(s_{2n-1}, s_{2n}), \Omega(s_{2n}, s_{2n+1})\}\end{aligned}$$

and

$$\begin{aligned}N(s_{2n-1}, s_{2n}) &= \min\{\Omega(s_{2n-1}, Ps_{2n-1}), \Omega(s_{2n}, Ps_{2n-1}), \Omega(P^2 s_{2n-1}, P^2 s_{2n})\} \\ &= \min\{\Omega(s_{2n-1}, s_{2n}), \Omega(s_{2n}, s_{2n}), \Omega(s_{2n+1}, s_{2n+2})\} = 0.\end{aligned}$$

Thus from (2.5)

$$\begin{aligned}\psi(\lambda(\Omega(s_{2n}, s_{2n+1}))) &\leq \psi\{\max\{\Omega(s_{2n-1}, s_{2n}), \Omega(s_{2n}, s_{2n+1})\}\} \\ &\quad + L(0), \\ &\leq \psi\{\max\{\Omega(s_{2n-1}, s_{2n}), \Omega(s_{2n}, s_{2n+1})\}\}.\end{aligned}$$

If there exists $n \geq 1$ such that $\max\{\Omega(s_{2n-1}, s_{2n}), \Omega(s_{2n}, s_{2n+1})\} = \Omega(s_{2n}, s_{2n+1})$, then from (2.5)

$$\begin{aligned}\psi(\Omega(s_{2n}, s_{2n+1})) &\leq \psi(\lambda\Omega(s_{2n}, s_{2n+1})) \\ &\leq H(\psi(\Omega(s_{2n}, s_{2n+1})), \varphi(\Omega(s_{2n}, s_{2n+1}))).\end{aligned}$$

which yields that $\psi(\Omega(s_{2n}, s_{2n+1})) = 0$, or , $\varphi(\Omega(s_{2n}, s_{2n+1})) = 0$. We derive $\Omega(s_{2n}, s_{2n+1}) = 0$. It is a contradiction. Thus, for all $n \geq 1$, we have $M(s_{2n-1}, s_{2n}) = \Omega(s_{2n-1}, s_{2n})$.
Hence from (2.5),

$$\begin{aligned}\psi(\Omega(s_{2n}, s_{2n+1})) &\leq \psi(\lambda\Omega(s_{2n}, s_{2n+1})) \\ &\leq H(\psi(\Omega(s_{2n-1}, s_{2n})), \varphi(\Omega(s_{2n-1}, s_{2n}))).\end{aligned}$$

Using the properties of ψ, we get

$$\lambda(\Omega(s_{2n}, s_{2n+1})) \leq \Omega(s_{2n-1}, s_{2n}).$$

Consequently, we get

$$\Omega(s_{2n}, s_{2n+1}) \leq \frac{1}{\lambda}\Omega(s_{2n-1}, s_{2n}) \tag{2.6}$$

Similarly, we get

$$\Omega(s_{2n+1}, s_{2n+2}) \quad \leq \quad \frac{1}{\lambda}\Omega(s_{2n}, s_{2n+1}) \tag{2.7}$$

Therefore, from (2.6) and (2.7) for all n, we have

$$\Omega(s_{n+1}, s_n) \quad \leq \quad \frac{1}{\lambda}\Omega(s_n, s_{n-1})$$

Since $\lambda \geq 1$, on applying Lemma 1.1 , we get that $\{s_n\}$ is a Cauchy sequence. Completeness of W implies that $\{s_n\}$ converges to some point $u \in W$.
We shall prove that $u \in S \cap T$. Since $\{s_{2n}\} \subset S$, $\{s_{2n+1}\} \subset T$ and $S \neq \phi, T \neq \phi \subset W$ are closed, thus, $u \in S \cap T$.
Step II. Claim that fixed point of P is u. i.e. $Pu = u$.
Since $(s_{2n}, u) \in S \times T$ and P is a C -class generalized cyclic contractive mapping,

$$\begin{aligned}\psi(\lambda\Omega(s_{2n+1}, Pu)) &= \psi(\lambda\Omega(Ps_{2n}, Pu)) \\ &\leq H(\psi(M(s_{2n}, u)), \varphi(M(s_{2n}, u))) + LN(s_{2n}, u) \\ &\leq \psi(M(s_{2n}, u)) + LN(s_{2n}, u)\end{aligned} \tag{2.8}$$

where,

$$\begin{aligned}M(s_{2n}, u) &= \max\left\{\begin{array}{c}\Omega(s_{2n}, u), \Omega(s_{2n}, Ps_{2n}), \Omega(u, Pu) \\ \frac{\Omega(s_{2n}, Pu)+\Omega(u, Ps_{2n})}{\lambda}, \\ \frac{\Omega(P^2s_{2n}, s_{2n})+\Omega(P^2s_{2n}, Pu)}{\lambda}, \\ \Omega(P^2s_{2n}, Ps_{2n}), \Omega(P^2s_{2n}, u), \Omega(P^2s_{2n}, Pu)\end{array}\right\} \\ &= \max\left\{\begin{array}{c}\Omega(s_{2n}, u), \Omega(s_{2n}, s_{2n+1}), \Omega(u, Pu), \\ \frac{\Omega(s_{2n}, Pu)+\Omega(u, s_{2n+1})}{\lambda}, \\ \frac{\Omega(s_{2n+2}, s_{2n})+\Omega(s_{2n+2}, Pu)}{\lambda}, \\ \Omega(s_{2n+2}, s_{2n+1}), \Omega(s_{2n+2}, u), \Omega(s_{2n+2}, Pu)\end{array}\right\} \\ &\leq \max\left\{\begin{array}{c}\Omega(s_{2n}, u), \Omega(s_{2n}, s_{2n+1}), \Omega(u, Pu) \\ \frac{sd(s_{2n}, u)+sd(u, Pu)+\Omega(u, s_{2n+1})}{\lambda} \\ \frac{\Omega(s_{2n+2}, s_{2n})+sd(s_{2n+2}, u)+sd(u, Pu)}{\lambda} \\ \Omega(s_{2n+2}, s_{2n+1}), \Omega(s_{2n+2}, u), \Omega(s_{2n+2}, Pu)\end{array}\right\}\end{aligned} \tag{2.9}$$

and

$$\begin{aligned}N(s_{2n}, u) &= \min\{\Omega(s_{2n}, Ps_{2n}), \Omega(u, Ps_{2n}), \Omega(P^2s_{2n}, P^2u)\} \\ &= \min\{\Omega(s_{2n}, u), \Omega(u, s_{2n+1}), \Omega(s_{2n+2}, P^2u)\}.\end{aligned} \tag{2.10}$$

From (2.9) and (2.10) on letting $n \to \infty$, we have

$$\begin{aligned}\lim_{n\to\infty} \sup M(s_{2n}, u) &= \Omega(u, Pu) \\ \lim_{n\to\infty} \sup N(s_{2n}, u) &= 0.\end{aligned} \tag{2.11}$$

Consider,

$$\begin{aligned}\psi(\Omega(u,Pu)-sd(u,s_{2n+1})) &\le \psi(sd(s_{2n+1},Pu))\\ &\le \psi(\lambda\Omega(s_{2n+1},Pu)).\end{aligned} \tag{2.12}$$

On taking the upper limit as $n\to\infty$ in (2.12) and (2.8), and using (2.11), we obtain

$$\psi(\Omega(u,Pu)) \le \psi(\Omega(u,Pu)),$$

which yields that $\Omega(u,Pu)=0$, i.e $u=Pu$.

Step III. Next we prove that u is a unique fixed point of P. Assume that v is also a fixed point of P, that is, $Pv=v$. Then, $v\in S\cap T$. Therefore, $(u,v)\in S\times T$. Since P is a C-class generalized cyclic contractive mapping, we have

$$\begin{aligned}\psi(\lambda\Omega(u,v)) &= \psi(\lambda\Omega(Pu,Pv))\\ &\le H(\psi(M(u,v)),\varphi(M(u,v)))+LN(u,v),\end{aligned} \tag{2.13}$$

where,

$$M(u,v)=\Omega(u,v), \text{ and } N(u,v)=0.$$

Then (2.13) becomes

$$\begin{aligned}\psi(\Omega(u,v)) &\le \psi(\lambda\Omega(u,v))\\ &\le H\left(\psi\left(\Omega(u,v)\right),\varphi\left(\Omega(u,v)\right)\right),\end{aligned}$$

which yields consequently that $\Omega(u,v)=0$, i.e. $u=v$. So, u is a unique fixed point of P. This accomplished our result. □

3. Applications

As an application of our main result, we give several corollaries. Some of them are novel in literature. If we take $H(b_1,b_2)=b_1\beta(b_1)$ in Theorem 2.1, we get a new result.

Corollary 3.1 *Let triplet (W,Ω,λ) be a b-metric space, $S\neq\phi, T\neq\phi\subset W$ are closed. Let $Y=S\cup T$ and P be a selfmap defined on Y. Assume that assertion (A1) holds and there exists a constant $L\ge 0$ satisfying*

$$\psi(\lambda\Omega(Ps,Pv)) \le \psi(M(s,v))\beta(\psi(M(s,v)))+LN(s,v)$$

for all $(s,v)\in S\times T$ or $(s,v)\in T\times S$, where ψ (infinite altering distance function [18]), $M(s,v)$ and $N(s,v)$ are defined in (2.2) and (2.3) and $\beta:[0,\infty)\to[0,1)$ be function. Then P has a unique fixed point in $S\cap T$.

If we let $H(b_1,b_2)=b_1-\gamma(b_1)$ in Theorem 2.1, we find below result.

Corollary 3.2 *Let triplet (W,Ω,λ) be a b-metric space, $S\neq\phi, T\neq\phi\subset W$ are closed. Let $Y=S\cup T$ and P be a selfmap defined on Y. Assume that assertion (A1) holds and there exists a constant $L\ge 0$ satisfying*

$$\psi(\lambda\Omega(Ps,Pv)) \le \psi(M(s,v))-\gamma(\psi(M(s,v)))+LN(s,v)$$

for all $(s,v)\in S\times T$ or $(s,v)\in T\times S$, where $\gamma:[0,\infty)\to[0,\infty)$ is a continuous function such that $\gamma(t)=0$ iff $t=0$, ψ (infinite altering distance function [18]), $M(s,v)$ and $N(s,v)$ are defined in (2.2) and (2.3). Then P has a unique fixed point in $S\cap T$.

on letting $H(b_1,b_2)=kb_1$, $0<k<1$ in Theorem 2.1, we have the corollary as follow.

Corollary 3.3 *Let triplet (W, Ω, λ) be a b-metric space, $S \neq \phi, T \neq \phi \subset W$ are closed.Let $Y = S \cup T$ and P be a selfmap defined on Y. Assume that assertion (A1) holds and there exists a constant $L \geq 0$ satisfying*

$$\psi(\lambda\Omega(Ps, Pv)) \leq k\psi(M(s,v)) + LN(s,v)$$

for all $(s, v) \in S \times T$ or $(s, v) \in T \times S$, where ψ (infinite altering distance function [18]), $M(s, v)$ and $N(s, v)$ are defined in (2.2) and (2.3). Then P has a unique fixed point in $S \cap T$.

If we assume that $H(b_1, b_2) = \alpha(b_1)$ in Theorem 2.1, then we obtain one more valuable result.

Corollary 3.4 *Let triplet (W, Ω, λ) be a b-metric space, $S \neq \phi, T \neq \phi \subset W$ are closed.Let $Y = S \cup T$ and P be a selfmap defined on Y. Assume that assertion (A1) holds and there exists a constant $L \geq 0$ satisfying*

$$\psi(\lambda\Omega(Ps, Pv)) \leq \alpha(\psi(M(s,v))) + LN(s,v)$$

for all $(s, v) \in S \times T$ or $(s, v) \in T \times S$, where ψ (infinite altering distance function [18]), $M(s, v)$ and $N(s, v)$ are defined in (2.2) and (2.3) and $\alpha : [0, \infty) \to [0, \infty)$ is an upper semi-continuous function such that $\alpha(0) = 0$, and $\alpha(t) < t$ for all $t > 0$. Then P has a unique fixed point in $S \cap T$.

on setting $H(b_1, b_2) = b_1 - b_2$ in Theorem 2.1, we deduce following generalized result.

Corollary 3.5 *Let triplet (W, Ω, λ) be a b-metric space, $S \neq \phi, T \neq \phi \subset W$ are closed.Let $Y = S \cup T$ and P be a selfmap defined on Y. Assume that assertion (A1) holds and there exists a constant $L \geq 0$ satisfying*

$$\psi(\lambda\Omega(Ps, Pv)) \leq \psi(M(s,v)) - \varphi(M(s,v)) + LN(s,v)$$

for all $(s, v) \in S \times T$ or $(s, v) \in T \times S$, where ψ (infinite altering distance function [18]), $\varphi \in \Phi_u$, $M(s, v)$ and $N(s, v)$ are defined in (2.2) and (2.3). Then P has a unique fixed point in $S \cap T$.

4. Conclusion

In this finding, we have defined a new definition of C- class contractive mapping, and then derived a result Theorem 2.1. Our introduction is new. We have deduced some corollaries as a simple application of our main finding (Theorem 2.1). Some of results given here are easily derived with help of our auxiliary function, but in nature these results are proved with additional assumption or condition on mappings or on set.

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