Bol. Soc. Paran. Mat.
©SPM – E-ISSN-2175-1188
SPM: www.spm.uem.br/bspm

(3s.) **v. 2025 (43)** : 1–10.
ISSN-0037-8712
doi:10.5269/bspm.75996

Optimal Parameter Identification in Soft Set Frameworks: A Decision Support Model

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ABSTRACT: This paper addresses the critical issue of decision-making under uncertainty, with a particular focus on soft set theory. Soft sets offer robust mathematical tools for handling uncertainty, making them highly suitable for solving real-world problems characterized by incomplete or imprecise information. In this study, a novel algorithm is proposed to handle specific types of uncertain situations, effectively targeting a distinct class of uncertainty-related problems. The research also highlights the significance of collaborative evaluation processes in group decision-making, emphasizing the role of collective input in identifying optimal solutions in uncertain environments.

The key contribution of this study lies in enhancing decision-making processes under uncertainty, with broad applicability to complex, real-life scenarios influenced by ambiguous or incomplete data. The findings are expected to benefit both researchers and practitioners dealing with uncertainty-driven challenges, providing practical and applicable solutions. Moreover, this study serves as a foundation for future research and innovation in the field, offering valuable insights into the development of advanced decision-making frameworks under uncertainty.

Key Words: soft set, best possible parameter, distance function, algorithm, decision making.

Contents

1. Introduction

In classical mathematics, the primary objective has traditionally been to find exact solutions for all mathematical models. Mathematicians aimed to precisely determine solutions for various problems and equations. However, many mathematical models are inherently complex, making it extremely difficult—if not impossible—to derive exact solutions. These complexities often stem from intricate equations, dynamic variables, or interactions between multiple parameters. In such cases, approximate solutions offer practical alternatives, providing valuable insights even if they lack complete precision.

A related challenge arises when working with sets of objects under conditions of vagueness or imprecision, which introduces significant questions and challenges in mathematics, logic, and philosophy. In contemporary fields such as computer science and artificial intelligence, these issues have become increasingly relevant. Modern AI algorithms frequently encounter incomplete, noisy, or ambiguous data, requiring researchers to apply mathematical tools such as fuzzy logic and probabilistic reasoning to manage uncertainties and enhance intelligent systems' ability to make decisions under ambiguity.

The formal treatment of uncertainty in mathematics can be traced back to the pioneering work of Lotfi Zadeh, who introduced fuzzy set theory [23]. Over time, several theoretical frameworks have been developed to handle uncertainty, including fuzzy set theory, rough set theory, probability theory, interval mathematics, and soft set theory. Among these, *soft set theory*, first introduced by Molodtsov [16], has emerged as a particularly flexible and effective approach. Soft set theory models uncertainty by associating each parameter with a crisp subset of the universal set, thus providing a parameterized framework that adapts well to contexts where criteria are subjective, dynamic, or evolving.

Submitted March 05, 2025. Published August 10, 2025
2010 *Mathematics Subject Classification:* 03E75, 90B50, 91B06 .

Molodtsov's foundational work has not only enriched the theoretical landscape but has also demonstrated practical applicability across diverse fields such as economics, social sciences, medical diagnosis, computer science, engineering, and education, especially in decision-making scenarios influenced by human judgment [5,14,6,24,7,8,11,9,21]. Building on Molodtsov's framework, numerous extensions have been proposed, including hybrid models that integrate soft sets with fuzzy sets, intuitionistic fuzzy sets, and rough sets. These hybrid approaches, as well as combinations with multi-criteria decision-making (MCDM) methods such as TOPSIS and VIKOR, have proven effective in addressing real-world decision-making problems involving incomplete or conflicting information [6,24,8,11,22,21,17,4,19].

The continuous diversification of set-theoretic approaches to handling uncertainty is primarily driven by two key motivations. The first motivation is the pursuit of identifying the most suitable or optimal solution to specific uncertainty-related problems. Different approaches offer distinct advantages in representing vagueness, capturing incomplete information, or balancing conflicting criteria, making it necessary to develop a broad set of tools tailored to different types of uncertainty. The second motivation is the search for a more elegant and generalized theoretical framework that can unify or consistently encompass the multitude of existing approaches. This quest for theoretical elegance drives researchers to refine existing models, identify overlaps, and propose hybrid or composite frameworks.

As a result of these efforts, the field has evolved into a rich research domain where multiple theoretical perspectives coexist and complement each other. This naturally raises a fundamental question: Can a single, unified set theory comprehensively address all types of uncertainty problems? While some researchers have pursued this goal, others advocate for a modular or hybrid approach that blends different set theories based on the characteristics of the specific problem at hand. Moreover, emerging domains such as quantum computing, big data analytics, and real-time decision support systems continue to introduce new forms of uncertainty, necessitating further theoretical advancements.

In this study, we aim to contribute to the growing body of research on uncertainty modeling by focusing on the identification of the optimal parameter from a predefined set within soft sets defined over a universal set U. This task is fundamental to soft set-based decision-making, as it enables decision-makers to determine the most relevant parameter for a given set of objects. Such a process enhances both the flexibility and the adaptability of decisions in environments characterized by uncertainty and evolving criteria.

To achieve this, we propose a novel distance-based algorithm that overcomes limitations commonly associated with traditional decision-making methods, such as reliance on fixed weights or rigid scoring schemes. By decoupling object evaluation from these constraints, our method allows for a more dynamic and context-sensitive analysis—one that naturally accommodates human judgment and the nuanced nature of real-world decision scenarios.

The effectiveness of the proposed approach is illustrated through its application to a practical decision-making problem, detailed in the "Decision Making" section. We also employ visual representations to improve the interpretability of our results and facilitate broader comprehension. We believe that this study not only introduces a valuable framework for parameter selection in uncertain settings but also contributes to the ongoing academic dialogue on soft set theory and its application in intelligent decision support systems.

2. Preliminaries

In this section, we recall some basic notions in fuzzy set and soft set. Let E be a set of parameters and U be a set of objects associated with those parameters. Here, each parameter is a word or a sentence. Also, let $P(U)$ denote the power set of U.

Definition 1 *[23] A fuzzy set $\mathcal{F}$ over U is a set defined by a function $\mu_{\mathcal{F}} : U \to [0,1]$. Thus, a fuzzy set $\mathcal{F}$ over U can be represented as follows:*

$$\mathcal{F} = \{u/\mu_{\mathcal{F}}(u) : u \in U\}$$

$\mu_{\mathcal{F}}$ is called the membership function of $\mathcal{F}$, and the value $\mu_{\mathcal{F}}(u)$ is called the grade of membership of $u \in U$. The value represents the degree of u belonging to the fuzzy set $\mathcal{F}$.

Example 1 *Let $U = \{u_1 : grey, u_2 : yellow, u_3 : black, u_4 : red, u_5 : blue\}$ be a set of colors. If the membership degrees of grey, yellow, black, red and blue are defined as* 0.57, 0, 0.25, 0.78, *and* 1, *respectively; then the fuzzy set $\mathcal{F}$ on U can be written as*

$$\mathcal{F} = \{u_1/0.57, u_2/0, u_3/0.25, u_4/0.78, u_5/1\}$$

or

$$\mathcal{F} = \{u_1/0.57, u_3/0.25, u_4/0.78, u_5\}.$$

Definition 2 *[16] A pair (F, E) is called a soft set over U, where F is a mapping given by $F : E \to P(U)$. Thus, a soft set (F, E) over U can be represented as follows:*

$$(F, E) = \{(e, F(e)) : e \in E, F(e) \in P(U)\}$$

In other words, the soft set is a parametrized family of subsets of the set U. For $e \in E$, $F(e)$ may be considered as the set of e-approximate elements of the soft set (F, E).

Definition 3 *[2] Let $P(E)$ be the power set of E. A pair (F, U) is called an inverse soft set over E, where F is a mapping given by $F : U \to P(E)$. Thus, an inverse soft set (F, U) over E can be represented as follows:*

$$(F, U) = \{(u, F(u)) : u \in U, F(u) \in P(E)\}$$

Definition 4 *[15] Let I^U denotes the set of all fuzzy sets on U. A pair (F, E) is called a fuzzy soft set over U, where F is a mapping from E into I^U. That is, for each $e \in E$, $F : E \to I^U$, is a fuzzy set on U (where $I = [0, 1]$). Thus, a fuzzy soft set (F, E) over U can be represented as follows:*

$$(F, E) = \{(e, F(e)) : e \in E, F(e) \in I^U\}$$

Example 2 *Consider the universal set $U = \{u_1, u_2, u_3\}$ which represents the set of cars and the parameter set $E = \{e_1 : expensive, e_2 : comfortable, e_3 : commercial\}$. Then the fuzzy soft set (F, E) is defined as follows:*

$$(F, E) = \left\{ \begin{array}{c} (e_1, \{u_1/0.25, u_2/0.50, u_3/0.75\}), (e_2, \{u_1/0.65, u_2/0.75, u_3/0.95\}), \\ (e_3, \{u_1/0.75, u_2/0.50, u_3/0.25\}) \end{array} \right\}$$

where

$$F(e_1) = \{u_1/0.25, u_2/0.50, u_3/0.75\},$$

$$F(e_2) = \{u_1/0.65, u_2/0.75, u_3/0.95\},$$

$$F(e_3) = \{u_1/0.75, u_2/0.50, u_3/0.25\}.$$

Definition 5 *[2] A mapping $F : U \to I^E$ is called an inverse fuzzy soft set on U. Thus, an inverse fuzzy soft set (F, E) over U can be represented as follows:*

$$(F, U) = \{(u, F(u)) : u \in U, F(u) \in I^E\}$$

Note that, the elements of inverse fuzzy soft sets are fuzzy sets on the parameter set E, i.e., for each $u \in U$, $F(u)$ is a fuzzy set on E.

Example 3 *Consider Example 2. We define the inverse fuzzy soft set by Definition 5 as follows:*

$$(F, U) = \left\{ \begin{array}{c} \big(u_1, \{e_1/0.25, e_2/0.65, e_3/0.75\}\big), \big(u_2, \{e_1/0.50, e_2/0.75, e_3/0.50\}\big), \\ \big(u_3, \{e_1/0.75, e_2/0.95, e_3/0.25\}\big) \end{array} \right\}$$

where

$$F(u_1) = \{e_1/0.25, e_2/0.65, e_3/0.75\},$$

$$F(u_2) = \{e_1/0.50, e_2/0.75, e_3/0.50\},$$

$$F(u_3) = \{e_1/0.75, e_2/0.95, e_3/0.25\}.$$

3. Technical Details

In this section, we describe the process of identifying the most suitable parameter for a given set of objects selected by the decision-maker. First, we formalize the soft sets that capture the current state of uncertainty and consolidate them into a unified mathematical model. Then, we systematically establish the relationships between objects and parameters and quantify the degree of interaction between the objects and the parameter-associated sets.

Throughout this section, we denote by E the set of parameters, by U the universal set of objects, and by $X \subseteq U$ the subset of objects chosen by the decision-maker. The cardinalities of these sets are denoted by $s(U) = a$, $s(X) = b$, and $s(E) = l$. In our context, each parameter in E is expressed as a word or sentence.

Definition 6 *Let $\{(F_i, E)\}_{i=1}^n$ be n soft sets defined on the universe U, where each mapping $F_i : E \to \mathcal{P}(U)$ assigns to every parameter $e \in E$ a subset $F_i(e) \subseteq U$. For each $x_k \in U$ and $e_j \in E$, define the indicator function*

$$A_{ij}(x_k) = \begin{cases} 1, & \text{if } x_k \in F_i(e_j), \\ 0, & \text{if } x_k \notin F_i(e_j). \end{cases}$$

Using these n soft sets, we define the aggregated soft set over E as

$$(\widetilde{F}, E) = \left\{ \left(e_j, \left\{ \frac{1}{n} \sum_{i=1}^{n} A_{ij}(x_k) \;/\; x_k \right\} : 1 \le k \le s(U) \right) : 1 \le j \le s(E) \right\}, \tag{3.1}$$

and the corresponding aggregated soft set over U as

$$(\widetilde{F}, U) = \left\{ \left(x_k, \left\{ \frac{1}{n} \sum_{i=1}^{n} A_{ij}(x_k) \;/\; e_j \right\} : 1 \le j \le s(E) \right) : 1 \le k \le s(U) \right\}. \tag{3.2}$$

Example 4 *Let $U = \{x_1, x_2, x_3, x_4\}$ and $E = \{e_1, e_2, e_3\}$. Consider three soft sets defined as follows:*

$$F_1(e_1) = \{x_2, x_4\}, \quad F_1(e_2) = \{x_1, x_3, x_4\}, \quad F_1(e_3) = \{x_1, x_4\},$$

$$F_2(e_1) = \{x_1, x_2, x_4\}, \quad F_2(e_2) = \{x_2, x_3, x_4\}, \quad F_2(e_3) = \{x_1, x_2\},$$

$$F_3(e_1) = U, \quad F_3(e_2) = \{x_1, x_2, x_3\}, \quad F_3(e_3) = \{x_2, x_3\}.$$

Then, using Definition 6, the aggregated soft sets $(\widetilde{F}, E)$ *and* $(\widetilde{F}, U)$ *are given by*

$$(\widetilde{F}, E) = \left\{ \begin{array}{l} \left(e_1, \left\{\frac{2/3}{x_1}, \frac{1}{x_2}, \frac{1/3}{x_3}, \frac{1}{x_4}\right\}\right), \\ \left(e_2, \left\{\frac{2/3}{x_1}, \frac{2/3}{x_2}, \frac{1}{x_3}, \frac{2/3}{x_4}\right\}\right), \\ \left(e_3, \left\{\frac{2/3}{x_1}, \frac{2/3}{x_2}, \frac{1/3}{x_3}, \frac{1/3}{x_4}\right\}\right) \end{array} \right\}$$

and

$$(\widetilde{F}, U) = \left\{ \begin{array}{l} \left(x_1, \left\{\frac{2/3}{e_1}, \frac{2/3}{e_2}, \frac{2/3}{e_3}\right\}\right), \\ \left(x_2, \left\{\frac{1}{e_1}, \frac{2/3}{e_2}, \frac{2/3}{e_3}\right\}\right), \\ \left(x_3, \left\{\frac{1/3}{e_1}, \frac{1}{e_2}, \frac{1/3}{e_3}\right\}\right), \\ \left(x_4, \left\{\frac{1}{e_1}, \frac{2/3}{e_2}, \frac{1/3}{e_3}\right\}\right). \end{array} \right\}$$

For instance, for $j = 1$ *and* x_3*, we have*

$$\frac{1}{3}\sum_{i=1}^{3} A_{i1}(x_3) = \frac{0+0+1}{3} = \frac{1}{3}.$$

Remark 1 *Note that in the construction of* $(\widetilde{F}, E)$ *and* $(\widetilde{F}, U)$*, we do not employ a membership function; rather, the concept of aggregating* n *soft sets is used.*

Remark 2 *The use of multiple soft sets enhances the accuracy of the results; as the number of soft sets increases, the reliability of the derived parameter evaluation improves.*

Definition 7 *Let* $\{(F_i, E)\}_{i=1}^n$ *be* n *soft sets defined on* U*. We define a mapping* $f : U \times U \to \mathbb{R}$ *to measure the distance between any two objects as*

$$f(x_k, x_t) = \frac{1}{n}\sum_{j=1}^{s(E)} \left| \sum_{i=1}^{n} A_{ij}(x_k) - \sum_{i=1}^{n} A_{ij}(x_t) \right|.$$

This function, referred to as the Distance Function, *quantifies the dissimilarity between objects.*

Table 1 provides a schematic representation of the distance relationships among objects in U (or a subset $X \subseteq U$):

Table 1: Distance Function values for objects in U and a subset X

$U \setminus X$	x_1	x_2	$\dots$	x_b
x_1	0	$f(x_1, x_2)$	$\dots$	$f(x_1, x_b)$
x_2	$f(x_2, x_1)$	0	$\dots$	$f(x_2, x_b)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_a	$f(x_a, x_1)$	$f(x_a, x_2)$	$\dots$	$f(x_a, x_b)$

Next, we introduce mappings to measure the distance between a selected subset $X \subseteq U$ and the parameter-associated sets.

Definition 8 *For each parameter* $e_j \in E$*, define the mapping* $\widetilde{f}_j : X \to \mathbb{R}$ *by*

$$\widetilde{f}_j(x_k) = \frac{1}{n}\sum_{t=1}^{s(U)} f(x_t, x_k) \left(\sum_{i=1}^{n} A_{ij}(x_t)\right), \quad \forall\, x_k \in X.$$

Since an object $x_k \in \widetilde{F}(e_j)$ should be considered closer to the set $\widetilde{F}(e_j)$, we adjust this distance using the mapping $S_j : X \to \mathbb{R}$ defined by

$$S_j(x_k) = \begin{cases} \widetilde{f}_j(x_k)\Big(1 - \frac{1}{n}\sum_{i=1}^{n} A_{ij}(x_k)\Big), & \text{if } x_k \in \widetilde{F}(e_j), \\ \widetilde{f}_j(x_k), & \text{otherwise.} \end{cases}$$

The values $S_j(x_k)$ for all $x_k \in X$ can be organized into a table (see Table 2), and the aggregated distance of X to $\widetilde{F}(e_j)$ is given by

$$TS_j(X) = \sum_{x_k \in X} S_j(x_k).$$

A tabular representation of these aggregated distances is provided in Table 3.

Table 2: Distances of objects in X to $\widetilde{F}(e_j)$

$\widetilde{F}(e_j) \setminus X$	x_1	x_2	$\ldots$	x_b
$\widetilde{F}(e_1)$	$S_1(x_1)$	$S_1(x_2)$	$\ldots$	$S_1(x_b)$
$\widetilde{F}(e_2)$	$S_2(x_1)$	$S_2(x_2)$	$\ldots$	$S_2(x_b)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$\widetilde{F}(e_j)$	$S_j(x_1)$	$S_j(x_2)$	$\ldots$	$S_j(x_b)$

Table 3: Aggregated distances $TS_j(X)$ for X to $\widetilde{F}(e_j)$

$\widetilde{F}(e_j)$	$TS_j(X)$
$\widetilde{F}(e_1)$	$TS_1(X)$
$\widetilde{F}(e_2)$	$TS_2(X)$
$\vdots$	$\vdots$
$\widetilde{F}(e_j)$	$TS_j(X)$

Finally, the optimal parameter is identified by selecting e_k such that

$$TS_k(X) = \min\{TS_j(X) : 1 \leq j \leq s(E)\}.$$

This means that the smaller the aggregated distance between X and $\widetilde{F}(e_j)$, the stronger X aligns with the parameter e_j.

4. Decision Making

In this section, we present an algorithm to determine the optimal parameter for a given subset of objects in the universal set U. This parameter is intended to best characterize the subset X selected by the decision-maker.

To this end, we propose the following algorithm:

Algorithm 1 ***Step 1***: *Input n soft sets corresponding to a fixed parameter set E.*

Step 2: *Select a subset $X \subseteq U$ of objects for evaluation.*

Step 3: *Compute the distances between objects in X and those in U using the* Distance Function.

Step 4: *For each parameter $e_j \in E$ and each $x_k \in X$, calculate $S_j(x_k)$ to measure the distance of x_k to the aggregated set $\widetilde{F}(e_j)$.*

Step 5: *For each parameter e_j, aggregate these distances to obtain*

$$TS_j(X) = \sum_{x_k \in X} S_j(x_k).$$

***Step 6**: Identify the parameter e_k for which*

$$TS_k(X) = \min\{TS_j(X) : 1 \leq j \leq s(E)\}.$$

If multiple parameters yield the same minimum, any one of them may be selected.

Using Algorithm 1, we can determine the optimal parameter for a given object subset X. To illustrate the algorithm, we present the following example.

Example 5 *Let $U = \{x_1, x_2, x_3, x_4, x_5\}$ and $E = \{e_1, e_2, e_3, e_4\}$.*

***Step 1**: Assume a single soft set is given by*

$$(F_1, E) = \{(e_1, \{x_2, x_3, x_5\}), (e_2, \{x_3, x_4\}), (e_3, \{x_1, x_2, x_5\}), (e_4, \{x_4, x_5\})\}.$$

Thus, by Definition 6, $(\widetilde{F}, E) = (F_1, E)$. Furthermore, the aggregated soft set over U is

$$(\widetilde{F}, U) = \{(x_1, \{e_3\}), (x_2, \{e_1, e_3\}), (x_3, \{e_1, e_2\}), (x_4, \{e_2, e_4\}), (x_5, \{e_1, e_3, e_4\})\}.$$

***Step 2**: Choose the subset $X = \{x_2, x_4\} \subseteq U$.*

***Step 3**: Calculate the distances between objects in X and those in U. For instance, the distance $f(x_3, x_4)$ is computed as*

$$f(x_3, x_4) = \Big|A_{11}(x_3) - A_{11}(x_4)\Big| + \Big|A_{12}(x_3) - A_{12}(x_4)\Big| + \Big|A_{13}(x_3) - A_{13}(x_4)\Big| + \Big|A_{14}(x_3) - A_{14}(x_4)\Big| = 2.$$

Table 4 summarizes all such distances.

Table 4: Distances between objects in U and $X = \{x_2, x_4\}$

$U \setminus X$	x_2	x_4
x_1	1	3
x_2	0	4
x_3	2	2
x_4	4	0
x_5	1	3

***Step 4**: For each object in X, compute $S_j(x_k)$ for all parameters. For example, to compute $S_1(x_2)$, first determine*

$$\widetilde{f}_1(x_2) = \sum_{t=1}^{5} f(x_t, x_2) A_{11}(x_t) = (1 \cdot 0) + (0 \cdot 1) + (2 \cdot 1) + (4 \cdot 0) + (1 \cdot 1) = 3.$$

Since $x_2 \in \widetilde{F}(e_1)$, we adjust the value as

$$S_1(x_2) = \widetilde{f}_1(x_2)\Big(1 - A_{11}(x_2)\Big) = 3(1-1) = 0.$$

Table 5 presents the computed $S_j(x_k)$ values for x_2 and x_4.

***Step 5**: Aggregate the distances for each parameter to obtain $TS_j(X)$, as summarized in Table 6.*

***Step 6**: Since $TS_2(X) = \min\{TS_j(X) : 1 \leq j \leq s(E)\} = 6$, the optimal parameter is identified as e_2. Thus, based on Algorithm 1, the best parameter for the subset X is e_2.*

Next, we consider an uncertainty problem where multiple soft sets are provided by different decision-makers. In this scenario, we evaluate the performance of teachers in assessing students' success across various subjects.

Table 5: Distances of objects in $X = \{x_2, x_4\}$ to $\widetilde{F}(e_j)$

$\widetilde{F}(e_j)$	$S_j(x_2)$	$S_j(x_4)$
$\widetilde{F}(e_1)$	0	9
$\widetilde{F}(e_2)$	6	0
$\widetilde{F}(e_3)$	0	10
$\widetilde{F}(e_4)$	5	0

Table 6: Aggregated distances $TS_j(X)$ for X to $\widetilde{F}(e_j)$

$\widetilde{F}(e_j)$	$TS_j(X)$	(%)
$\widetilde{F}(e_1)$	9	30
$\widetilde{F}(e_2)$	6	20
$\widetilde{F}(e_3)$	10	33.33
$\widetilde{F}(e_4)$	5	16.67

Example 6 *Let the parameter set be*

$$E = \{e_1, e_2, e_3, e_4, e_5\} = \Big\{\textit{successful in maths, successful in geometry,}\\ \textit{successful in physics, successful in chemistry,}\\ \textit{successful in biology}\Big\},$$

and let $U = \{x_1, x_2, x_3, x_4, x_5\}$ denote the set of students. Suppose three teachers provide their evaluations as follows:

$$\begin{aligned}(F_1, E) &= \{(e_1, \{x_1, x_3, x_4, x_5\}), (e_2, \{x_2, x_4\}), (e_3, \{x_1, x_5, x_6\}),\\ &\quad (e_4, \{x_2, x_3, x_6\}), (e_5, \{x_1, x_3, x_5\})\},\\ (F_2, E) &= \{(e_1, \{x_2, x_3, x_5\}), (e_2, \{x_1, x_2\}), (e_3, \{x_1, x_4, x_5\}),\\ &\quad (e_4, \{x_2, x_3, x_4\}), (e_5, \{x_2, x_4, x_6\})\},\\ (F_3, E) &= \{(e_1, \{x_1, x_2, x_6\}), (e_2, \{x_2, x_4\}), (e_3, \{x_3, x_5\}),\\ &\quad (e_4, \{x_1, x_3, x_6\}), (e_5, \{x_2, x_4\})\}.\end{aligned}$$

From these soft sets, the aggregated soft sets $(\widetilde{F}, E)$ and $(\widetilde{F}, U)$ are constructed as in Definition 6.

Assume that students x_1, x_2, and x_4 wish to collaborate, so we choose $X = \{x_1, x_2, x_4\}$. The subsequent steps (Steps 3–6) are then applied to compute the distances and aggregate them for each parameter. Ultimately, the analysis indicates that the parameter corresponding to the smallest aggregated distance (i.e., the optimal parameter) is, for example, e_2, representing "successful in geometry." This suggests that the group of students should select geometry as the subject for their joint project.

5. Conclusion

This study presents a parameter selection algorithm based on soft set theory to address decision-making problems under uncertainty. Building upon the foundational work of Molodtsov, our approach aims to identify the optimal parameter for a given subset of objects within a soft set environment. By incorporating multiple soft sets provided by different decision-makers, we developed a method to aggregate and evaluate their opinions, offering a more flexible and collaborative decision-making framework.

The proposed method offers several advantages: it frees decision-making processes from rigid rule-based structures, allows for flexibility in incorporating expert opinions, and provides a systematic approach to parameter selection based on distance measures between object sets and parameter-associated sets. These features make it particularly suitable for group decision-making settings where subjective judgments and incomplete information play a role.

To demonstrate the practical applicability of the proposed algorithm, we applied it to an illustrative real-world example involving student project selection based on teacher evaluations across multiple subjects. The example highlights how the algorithm aggregates expert opinions and systematically identifies the most suitable subject for a group of students working together. This type of collaborative decision-making process is particularly relevant in educational, medical, and industrial contexts where multi-expert evaluations are common.

While the proposed approach builds upon existing soft set methodologies, it offers a distinct contribution by focusing on collaborative decision-making under uncertainty, an area that remains underexplored in the existing literature. To strengthen the scientific positioning of the study, we have expanded the theoretical foundation to establish clearer links with classical soft set theory, fuzzy soft sets, and multi-criteria decision-making techniques. Additionally, we emphasize the importance of systematically comparing the proposed algorithm with existing soft set-based decision-making methods, which will be a focus of future work.

We recognize that the current study has certain limitations. For example, while the algorithm has been demonstrated with illustrative examples, future research will focus on validating its performance on real datasets with known outcomes. Additionally, a comparative performance analysis with other well-established uncertainty handling techniques, such as fuzzy soft sets, rough soft sets, and hybrid models, will be conducted to further clarify the advantages and potential application domains of the proposed approach.

Despite these limitations, we believe the proposed method contributes to the ongoing discourse on uncertainty modeling and collaborative decision-making. By offering a flexible framework that accommodates expert input, incorporates parameter-object relationships, and systematically identifies optimal parameters, this study provides a solid foundation for future extensions and applications in a wide range of domains, including education, healthcare, and engineering.

We hope that this study will encourage further research into collaborative soft set-based decision-making and inspire the development of enhanced algorithms that address the complexities and evolving challenges of real-world uncertainty.

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