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On Semi-weakly D-compactness

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ABSTRACT: In this paper, we introduce semi-weakly D-compactness, continuous mapping properties, and some characterization of semi-weakly D-compact space. First, we present a special type of cover called a semi-weakly-D open cover by using a difference set (D-set), and several semi-weakly-D mapping properties are studied. Then, we investigate the image of these spaces under semi-weakly-D mappings and study their relationship with other spaces. Furthermore, We derive some findings which connect some generalized D-compact spaces and we give some sufficient conditions which guarantee that any locally indiscrete weakly D-compact space is semi-weakly D-compact.

Key Words: D-sets, D-compact space, weakly D-open sets.

Contents

1. Introduction

In [12], Sundstrom explained the importance of compactness and studied the many relevant concepts of compactness. The open set plays an important role in defining a new type of set. The compactness properties have been generalized in many directions by using open sets, one of them is [11]. The urgent need for theories dealing with uncurtaining data which are not always crips. One of the recent mathematical tools to handle this problem is D-sets which was developed by Tong [8]. By connecting D-sets with an open set, we can obtain new results that will be a much more useful result than just open sets and it generalized into other topological spaces. Norman Levine [13] first developed the new concept of semi-open sets and then Adam Hassan [1] introduced semi-generalized open sets and semi-generalized closed sets in topological spaces. Sarsak [10] defined semi-compact spaces and investigated the mapping properties of semi-compact sets. Caldas [9] studied the novel concept of semi-D sets by using a semi-open set and investigated their properties. Al-shami [18] studied new types of supra semi-open sets and some applications on topological spaces. Wali and Mathad [15] defined semi-regular-weakly open sets and discussed their applications. Later other researchers worked in this direction by using different open sets and their studies were related to D-set. Qoqazeh et al. [5] started working on compactness and introduced pairwise meta D-compact space. On the other hand, the author [6] used the concept of D-sets to introduce the new concept of D-compact spaces and investigated the relationship between other topological spaces. Bani-Ahmad et al. [4] defined and discussed the new version of D-sets called D-perfect function, the author also explored some of the more advanced properties of the D-perfect functions. In recent years, the topological structure has been generalized to see the variety of applications [16,17]. In this study, we continue studying D-compact spaces by using the definition of semi-weakly-D open sets and formulating the notion of semi-weakly D-compact spaces. We also explore the concept of semi-weakly D-compact spaces and study the mapping properties of these novel concepts.

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2. Mathematical Background: Auxiliary Results

In this section, we provide the initial concepts and corollaries that are the significant relevance in the exploration of the semi-weakly D-compact spaces.

Definition 2.1 *[2] Let $(R,\ \varphi)$ be a topological space, and let K be a subset of R. Then K is called weakly D-open set if $K - K'$ is φ-open, where K' is the set of all limit points of K.*

The discrete topology on R and the subsets $\{a, b, c, d\}$ of R implies every subset of R is weakly D-open set. But if T_I is an indiscrete topology on R, then only φ is the weakly D-open set.

A collection $\{G_\alpha\}_{\alpha\in\Lambda}$ of weakly D-sets in a topological space $(R,\ \varphi)$ is said to be a weakly D-cover of R if

$$R = \bigcup_{\alpha\in\Lambda} G_\alpha, \text{ where } \Lambda \text{ is any index set.}$$

Definition 2.2 *[3] A subset K of a topological space $(R,\ \varphi)$ is said to be semi-weakly-D open sets if $K - K'$ is semi-open, where K' is the set of all limit points of K.*

Let $K \subset R$, $R = \{p,\ q,\ r,\ s,\ t\}$,

$\varphi = \{\phi,\ R,\ \{p\},\ \{p,\ q\},\ \{p,\ r,\ s\},\ \{p,\ q,\ t\},\ \{p,\ q,\ r,\ s\}\}$,
and $K = \{p,\ r,\ s,\ t\}$. So, $K' = \{q,\ r,\ s,\ t\}$.
Therefore, $K - K' = \{p\}$, which is semi-weakly-D open.

Corollary 2.1 *[3] Let $(R,\ \varphi)$ be a topological spaces. Then*

1. *Every D-set is a semi-weakly-D sets in a topological space $(R,\ \varphi)$.*
2. *Every open set is a semi-weakly-D set.*
3. *Every weakly D-open set is semi-weakly-D open but not conversely.*
4. *Every semi-open set is semi-D set and so semi-weakly-D set but the converse need not be true.*

Definition 2.3 *[8] Let $(R,\ \varphi)$ be a topological space and let H be a subset of R called D-set, if there are two open sets F_1 and F_2 such that $F_1 \neq R$ and $H = F_1 - F_2$.*

Definition 2.4 *[13] Let $(R,\ \varphi)$ be a topological space and $K \subset R$. Then K is said to be semi-open if, $\exists$ an open set $H \in \varphi$ such that $H \subset K \subset Cl(H)$. The family of all semi-open sets is denoted by $S_H(R,\ \tau)$.*

Definition 2.5 *[10] A topological space $(R,\ \varphi)$ is said to be semi-compact if each semi-open cover of R has a finite subcover.*

Definition 2.6 *[6] A topological space $(R,\ \varphi)$ is said to be weakly D-compact if each weakly D-cover of R has a finite subcover.*

Definition 2.7 *[7] A point p is a limit point of a subset K of a topological space $(R,\ \varphi)$ if every neighbourhood of p contains a point of K other than p.*

Definition 2.8 *[13] A function $\gamma : R \to S$ is said to be semi-continuous if the inverse image of an open set is semi-open.*

Definition 2.9 *[2] A topological space $(R,\ \varphi)$ is weakly-D_2 space, if for any pair p, q of members of R, $p \neq q$, $\exists$ two disjoint weakly D-open sets G_w and H_w of R such that $p \in G_w$ and $q \in H_w$.*

3. Semi-weakly D-compact Space

Definition 3.1 *A collection $\{G_\alpha\}_{\alpha\in\Lambda}$ of semi-weakly-D open sets in a topological space $(R,\ \varphi)$ is said to be a semi-weakly D-cover of R if $R = \bigcup_{\alpha\in\Lambda} G_\alpha$, where Λ is any index set.*

Definition 3.2 *A topological space $(R,\ \varphi)$ is said to be semi-weakly D-compact if each semi-weakly-D open cover of X has a finite subcover.*

Definition 3.3 *A function $\gamma : R \to S$ is said to be semi-weakly-D continuous if the inverse image of weakly D-open sets is semi-weakly-D open.*

Theorem 3.1 *For a semi-weakly-D continuous image of a semi-weakly D-compact space is weakly D-compact.*
Proof. *Let $(R,\ \varphi)$ be a semi-weakly D-compact space and $\gamma : R \to S$ be a semi-weakly-D continuous function. Let $C = \{C_\rho : \rho \in \Delta\}$ be D-cover of S, then C is weakly D-open cover and so it is also semi-weakly-D open cover of S [see corollary 2.1 & et al.]. Let $\widetilde{C} = \{\gamma^{-1}(C_\rho) : \rho\ \in \Delta\}$ be semi-weakly-D open cover of R. Since R is semi-weakly D-compact implies that $\widetilde{C}$ has a finite sub-cover such that $\{\gamma^{-1}(C_1),\ \ \gamma^{-1}(C_2),\ \ \gamma^{-1}(C_3),\ \ .\ .\ .\ ,\ \ \gamma^{-1}(C_n)\}$. Thus $\{C_1,\ \ C_2,\ C_3,\ .\ .\ .\ ,\ \ C_n\ \}$ is cover of S. Hence the result.*

Lemma 3.2 *If J is an open subset of Z and $\gamma : (Z,\varphi) \to (W,\widetilde{\varphi})$ be a semi-weakly-D continuous, then the continuous functions $\gamma : (J,\varphi_J) \to (W,\widetilde{\varphi})$ is also semi-weakly-D continuous.*
Proof. *Let $\gamma : (Z,\varphi) \to (W,\widetilde{\varphi})$ be a semi-weakly-D continuous. Then $\gamma^{-1}(H)$ is semi-weakly-D open in (Z, φ), where $H \in \widetilde{\varphi}$. Since J is open in Z this imply that $J \cap \gamma^{-1}(H) = \gamma_J^{-1}(H)$. Then it is open in (Z, φ) so it is also in (J,φ_J). Hence $\gamma : (J,\varphi_J) \to (W,\widetilde{\varphi})$ is semi-weakly-D continuous.*

Theorem 3.3 *If $\gamma : (Z,\ \varphi) \to (W,\ \widetilde{\varphi})$ is a semi-weakly-D continuous surjection from $(Z,\ \varphi)$ into $(W,\ \widetilde{\varphi})$, if $(Z,\ \varphi)$ and semi-weakly D-compact then $(W,\ \widetilde{\varphi})$ is so.*
Proof. *Let $\{M_\rho : \rho \in \Delta\}$ be a D-cover of $(W,\ \widetilde{\varphi})$, then it is also semi-weakly D-cover [using corollary 2.1]. Since $\gamma : (Z,\ \varphi) \to (W,\ \widetilde{\varphi})$ is semi-weakly-D continuous map therefore $\gamma^{-1}(M_\rho : \rho \in \Delta)$ is a semi-weakly-D open in $(Z,\ \varphi)$. Hence $\{\gamma^{-1}(M_\rho) : \rho \in \Delta\}$ is φ-semi-weakly-D open cover of $(Z,\ \varphi)$. By hypothesis, $(Z,\ \varphi)$ is semi-weakly D-compact and so there exists a φ-semi-weakly-D finite subcover $\{\gamma^{-1}(M_\rho) : \rho = 1,\ 2,\ 3,\ .\ .\ .\ k\}$ such that $Z = \cup_{k=1}^{n}\gamma^{-1}(M_n\ )$. So, $\{M_1,\ \ M_2,\ M_3,\ .\ .\ .\ ,\ \ M_n\ \}$ is finite sub-cover of $(W,\ \widetilde{\varphi})$. It's the result.*

Corollary 3.1 *Every D-cover is a semi-weakly-D open cover.*
Proof.*It follows from corollary 2.1 et al.*

Theorem 3.4 *If J_1 is semi-weakly D-compact relative to topological space $(R,\ \varphi)$, then J_1 is semi-weakly D-compact relative to J_2, where $J_1, J_2 \subseteq R$ and $J_1 \subseteq J_2$.*
Proof. *Assume that $P = \{P_\rho : \rho \in \Delta\}$ is a D-cover of J_1. Since every D-cover is a semi-weakly-D open cover. So, $\{P_\rho : \rho \in \Delta\}$ semi-weakly-D open cover [using corollary 2.1] concerning semi-open sets in J_2. Then $P_\rho = \{D_\rho \cap J_2 : \rho \in \Delta\}$, where D_ρ is semi-weakly-D open in R for each $\rho \in \Delta$. Thus $D = \{D_\rho : \rho \in \Delta\}$ is a cover of J_1 concerning semi-weakly-D open sets contained in R. By hypothesis J_1 is semi-weakly D-compact space of R, so there exists a finite sub-cover such that $\cup\{D_\rho : \rho = 1,\ 2,\ 3,\ .\ .\ .\ k\}$. Thus*

$$J_1 \subseteq \cup\{D_\rho \cap J_2 : \rho = 1,\ 2,\ 3,\ .\ .\ .\ k\} = \cup\{P_\rho : \rho = 1,\ 2,\ 3, .\ .\ .\ .\ k\},$$

where k is any finite integer. Hence J_1 is semi-weakly D-compact for J_2.

Theorem 3.5 *A semi-open subset S of topological space $(R,\ \varphi)$ is semi-weakly D-compact if and only if semi-open subspace $(S,\ \varphi)$ is semi-weakly D-compact.*
Proof. *Let $H = \{V_\alpha : \alpha \in \Delta\}$ be a semi-open cover of S. Since S is semi open set containing H. Therefore, the cover H of S is a semi-weakly-D open subset of R [using theorem 3.4 et al.]. By hypothesis,*

$$S \subseteq \cup\{V_\alpha : \alpha = 1,\ 2,\ .\ .\ .,k\} = \cup\{V_\alpha \cap S : \alpha = 1,\ 2,\ .\ .\ .,k\},$$

where k is finite positive integer. So, $(S,\ \varphi)$ is semi-weakly D-compact.
Conversely, suppose that $\{E_\alpha : \alpha \in \Delta\}$ be a semi-open cover of S in $(R,\ \varphi)$. Therefore $G = \{E_\alpha : \alpha \in \Delta\} \cap S$ is a semi-weakly-D open subset of $(R,\ \varphi)$. It follows that G is a semi-weakly-D open subset of $(S,\ \varphi)$. Hence proof is completed.

Theorem 3.6 *A subset E of topological space $(R,\ \varphi)$ is semi-weakly D-compact if and only if for each collection $\{w_\alpha : \alpha \in \Delta\}$ of σ-closed sets of R such that*

$$(\cap_{\alpha\in\Delta} w_\alpha) \cap (E \cap \vartheta_\sigma) = \phi,$$

where ϑ_σ is closed subsets of R and there is a finite sub-cover $\{w_1,\ w_2,\ w_3,\ .\ .\ .\ ,w_n\}$ such that

$$(\cap_{r=1}^{n} w_r) \cap (E \cap \vartheta_\sigma) = \phi.$$

Proof. *Let $\{S_\alpha : \alpha \in \Delta\}$ be a family of D-cover in R therefore $\{S_\alpha : \alpha\ \in \Delta\ \}$ is semi-weakly-D open cover of R [corollary 3.1. et al.] such that*

$$(E \cap \vartheta_\sigma) \subseteq \cup_{\alpha\in\Delta} S_\alpha.$$

It follows that

$$(R - \cup_{\alpha\in\Delta} S_\alpha) \cap (E \cap \vartheta_\sigma) = \phi$$

and clearly $(R - \cup_{\alpha\in\Delta} S_\alpha)$ is σ-closed subset of R.
By hypothesis, there is a finite sub-cover $(R - \cup_{r=1}^{n} S_\alpha)$ such that

$$(\cap_{r=1}^{n} (R - \cup_{r=1}^{n} S_r)) \cap (E \cap \vartheta_\sigma) = \phi.$$

So, $E \cap \vartheta_\sigma \subseteq R - \cap_{r=1}^{n} (R - S_r) = \cup_{r=1}^{n} S_r$.
Thus $(R,\ \varphi)$ is semi-weakly D-compact.
Conversely, suppose the collection $\{w_\alpha : \alpha \in \Delta\}$ are σ-closed sets of R such that $(\cap_{\alpha\in\Delta} w_\alpha) \cap (E \cap \vartheta_\sigma) = \phi$. So,

$$(E \cap \vartheta_\sigma) \subseteq R - (\cap_{\alpha\in\Delta} w_\alpha) = \cup_{\alpha\in\Delta} (R - w_\alpha).$$

Clearly, $\{R - w_\alpha\}_{\alpha\in\Delta}$ forms a σ-open sets of R covering $(E \cap \vartheta_\sigma)$.
By hypothesis, there is a finite sub-collection $\{w_1,\ \ w_2,\ w_3,\ .\ .\ .\ ,w_n\}$ such that

$$(E \cap \vartheta_\sigma) \subseteq\ \cup_{r=1}^{n} (R - w_r).$$

Thus $R - (\cup_{r=1}^{n} (R - w_r)) \cap (E \cap \vartheta_\sigma) = \phi \Rightarrow \cap_{r=1}^{n} w_r \cap (E \cap \vartheta_\sigma) = \phi$.
Hence the result.

Definition 3.4 *Let R and S be two topological spaces. A function $\gamma : R \to S$ is called semi-weakly-D irresolute function if the inverse image of any semi-weakly-D open set in S is a weakly D-open set in R.*

Theorem 3.7 *The semi-weakly-D irresolute image of a weakly D-compact space is semi-weakly D-compact.*
Proof. *Let R be a weakly D-compact space and the function $\gamma : R \to S$ be semi-weakly-D irresolute function. Let the collection of the set $\{Q_\alpha : \alpha \in \Delta\}$ be a weakly D-cover of S then the collection $\{\gamma^{-1}(Q_\alpha) : \alpha \in \Delta\}$ is an open cover of R. So, it must be semi-weakly-D open cover of R [corollary 3.1 & 2.1 et al.]. Since R is weakly D-compact, so there must exist a finite sub-covers such that $\{\gamma^{-1}(Q_1),\ \ \gamma^{-1}(Q_2),\ \gamma^{-1}(Q_3),\ \ .\ \ .\ \ .\ ,\gamma^{-1}(Q_n)\}$. Thus, the set $\{Q_1,\ Q_2,\ Q_3,\ \ \ .\ \ .\ .,Q_n\}$ is a cover of S. This is the outcome.*

Theorem 3.8 *Let $(R,\ \varphi)$ be a topological space. Then for any locally indiscrete weakly D-compact space is semi-weakly D-compact.*
Proof. *Let λ_α be a clopen set for every $\alpha \in \Delta$ and let $\mu = \{\lambda_\alpha : \alpha \in \Delta\}$ be weakly D-cover of $(R,\ \varphi)$ then μ is weakly D-open cover of $(R,\ \varphi)$. So μ is semi-weakly-D open cover [corollary 3.1 et al.]. Since $(R,\ \varphi)$ is weakly D-compact, so μ has a finite sub-cover.*
Hence the result.

4. Conclusions

This study aims to generalize semi-weakly-D open sets in the sense of D-compact and weakly D-compact spaces and we obtained new results that will be a much more useful result than just a compact set and it generalized into other topological spaces. We have formulated a new class of semi-weakly D-compact spaces. We have investigated the relationship between these concepts and other spaces. Further, the new results of semi-weakly-D mappings are established by using these new ideas of compactness. In future works, we shall study these concepts concerning another generalization of weakly D-sets such as regular-weakly-D sets and semi-regular-weakly-D sets, and apply these ideas in biological science and engineering science.

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Conflict of Interest

The author declares that there is no conflict of interest.

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