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On First and Second Stress Polynomials of Graphs

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ABSTRACT: Shimbel (1953) introduced the node centrality index, which is the stress of a vertex. The number of geodesics (shortest paths) that pass through a vertex in a graph is its stress. A number that links a chemical structure to physical characteristics or chemical reactivity is called a topological index of the chemical structure (graph). This paper presents the first and second stress polynomials, which are new stress polynomials for graphs based on vertices stresses. Additionally, we compute stress polynomials for a few standard graphs, prove a few results, and establish a few inequalities.

Key Words: Graph, geodesic, graph polynomial, stress of a vertex, path, stress polynomial.

Contents

1. Introduction

For standard terminology and notion in graph theory, we follow the text-book of Harary [5]. The non-standard will be given in this paper as and when required.

Let $G = (V, E)$ be a graph (finite and undirected). The distance between two vertices u and v in G, denoted by $d(u, v)$ is the number of edges in a shortest path (also called a graph geodesic) connecting them. We say that a graph geodesic P is passing through a vertex v in G if v is an internal vertex of P (i.e., v is a vertex in P, but not an end vertex of P). The degree of a vertex v in G is denoted by $d(v)$.

The concept of stress of a node (vertex) in a network (graph) has been introduced by Shimbel as centrality measure in 1953 [23]. This centrality measure has applications in biology, sociology, psychology, etc., (See [7,21]). The stress of a vertex v in a graph G, denoted by $\mathrm{str}_G(v)$ $\mathrm{str}(v)$, is the number of geodesics passing through it. We denote the maximum stress among all the vertices of G by Θ_G and minimum stress among all the vertices of G by θ_G. Further, the concepts of stress number of a graph and stress regular graphs have been studied by K. Bhargava, N.N. Dattatreya, and R. Rajendra in their paper [2]. A graph G is k-stress regular if $\mathrm{str}(v) = k$ for all $v \in V(G)$.

The Zagreb indices have been defined using degrees of vertices in a graph to explain some properties of chemical compounds at molecular level [3,4]. The first Zagreb index $M_1(G)$ and the second Zagreb index $M_2(G)$ of a simple graph G are defined as:

$$M_1(G) = \sum_{v \in V(G)} d(v)^2 \tag{1.1}$$

$$M_2(G) = \sum_{uv \in E(G)} d(u)d(v). \tag{1.2}$$

The first Zagreb polynomial of G is defined as:

$$M_1(G, x) = \sum_{uv \in E(G)} x^{d_u + d_v}.$$

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The second Zagreb polynomial of G is defined as:

$$M_2(G,x) = \sum_{uv \in E(G)} x^{d_u d_v}.$$

By the motivation of these indices, Rajendra et al. [14] have introduced two stress polynomial for graphs called first stress index and second stress index, using stresses of vertices. The first stress index $\mathcal{S}_1(G)$ and the second stress index $\mathcal{S}_2(G)$ of a simple graph G are defined as

$$\mathcal{S}_1(G) = \sum_{v \in V(G)} \mathrm{str}(v)^2 \tag{1.3}$$

$$\mathcal{S}_2(G) = \sum_{uv \in E(G)} \mathrm{str}(u)\mathrm{str}(v). \tag{1.4}$$

We note that the first Zagreb index $M_1(G)$ satisfies the identity

$$M_1(G) = \sum_{uv \in E(G)} d(u) + d(v) \tag{1.5}$$

but $\mathcal{S}_1(G)$ does not satisfy such identity. For instance, consider the path P_3 on 3 vertices.

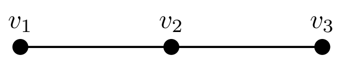


Figure 1: The path P_3.

The stresses of the vertices of P_3 are as follows: $\mathrm{str}(v_1) = \mathrm{str}(v_3) = 0$ and $\mathrm{str}(v_2) = 1$. The first stress index of P_3 is,

$$\mathcal{S}_1(P_3) = \mathrm{str}(v_1)^2 + \mathrm{str}(v_2)^2 + \mathrm{str}(v_3)^2 = 0^2 + 1^2 + 0^2 = 1.$$

But

$$\sum_{uv \in E(P_3)} \mathrm{str}(u) + \mathrm{str}(v) = \mathrm{str}(v_1) + \mathrm{str}(v_2) + \mathrm{str}(v_2) + \mathrm{str}(v_3) = 0 + 1 + 1 + 0 = 2.$$

Therefore there is a scope for introducing a new stress polynomial using stress on vertices which is motivated by the identity (1.3) and (1.4). Using stress on vertices, we present the first and second stress polynomials for graphs in this paper. Additionally, for a few standard graphs, we compute the first and second stress polynomials and establish some inequalities.

In [12], the authors introduced the concept of vertex stress polynomial of a graph and obtained some results including a characterization of graphs with vertex stress polynomial is a non-zero constant. For new stress/degree based topological indices, we suggest the reader to refer the papers [1,6,8-20,22,24-26].

2. First and Second Stress Polynomials for Graphs

Definition 2.1 *The first and second stress polynomials are defined by;*

$$S_1(G,x) = \sum_{v \in V(G)} x^{str(v)^2},$$

$$S_2(G,x) = \sum_{uv \in E(G)} x^{str(u)str(v)}.$$

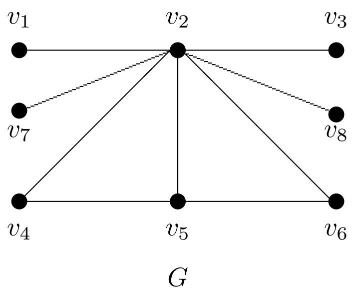


Figure 2: A graph G

Example 2.1 *Consider the graph G given in Figure 2.*
The stresses of the vertices of G are as follows:
$str(v_1) = str(v_3) = str(v_7) = str(v_8) = 0$,
$str(v_2) = 19$,
$str(v_5) = 1$,
$str(v_4) = str(v_6) = 0$.
The first and second stress polynomial of G is given by:

$$S_1(G, x) = 6 + x + x^{361},$$

$$S_2(G, x) = 8 + x^{19}.$$

Proposition 2.1 *For the complete bipartite graph $K_{m,n}$*

$$S_1(K_{m,n}, x) = m \cdot x^{\frac{n^2(n-1)^2}{4}} + n \cdot x^{\frac{m^2(m-1)^2}{4}}$$

$$S_2(K_{m,n}, x) = mn \cdot x^{\frac{mn(n-1)(m-1)}{4}}.$$

Proof: Let $V_1 = \{v_1, \ldots, v_m\}$ and $V_2 = \{u_1, \ldots, u_n\}$ be the partite sets of $K_{m,n}$. We have,

$$\mathrm{str}(v_i) = \frac{n(n-1)}{2} \text{ for } 1 \leq i \leq m \tag{2.1}$$

and

$$\mathrm{str}(u_j) = \frac{m(m-1)}{2} \text{ for } 1 \leq j \leq n. \tag{2.2}$$

Using (2.1) and (2.2) in the Definition 2.1, we have

$$\begin{aligned} S_1(K_{m,n}, x) &= \sum_{v \in V_1} x^{str(v)^2} + \sum_{v \in V_2} x^{str(v)^2} \\ &= \sum_{v \in V_1} x^{\left[\frac{n(n-1)}{2}\right]^2} + \sum_{v \in V_2} x^{\left[\frac{m(m-1)}{2}\right]^2} \\ S_1(K_{m,n}, x) &= m \cdot x^{\frac{n^2(n-1)^2}{4}} + n \cdot x^{\frac{m^2(m-1)^2}{4}}. \end{aligned}$$

$$\begin{aligned} S_2(K_{m,n}, x) &= \sum_{uv \in E(G)} x^{str(u)str(v)} \\ &= \sum_{uv \in E(G)} x^{\left[\frac{n(n-1)}{2}\right]\left[\frac{m(m-1)}{2}\right]} \\ S_2(K_{m,n}, x) &= mn \cdot x^{\frac{mn(n-1)(m-1)}{4}}. \end{aligned}$$

□

Corollary 2.1 *For the complete graph K_n*

$$S_1(K_n, x) = n = S_2(K_n, x).$$

Proposition 2.2 *If $G = (V, E)$ is a k-stress regular graph, then*

$$S_1(G, x) = x^{k^2} \cdot |V|;$$

$$S_2(G, x) = x^{k^2} \cdot |E|.$$

Proof: Suppose that G is a k-stress regular graph. Then

$$\text{str}(v) = k \text{ for all } v \in V(G).$$

By the Definition 2.1, we have

$$\begin{aligned} S_1(G, x) &= \sum_{v \in V(G)} x^{str(v)^2} \\ &= \sum_{v \in V(G)} x^{k^2} \\ &= x^{k^2} \cdot |V|. \end{aligned}$$

$$\begin{aligned} S_2(G, x) &= \sum_{uv \in E(G)} x^{str(u)str(v)} \\ &= \sum_{v \in V(G)} x^{k^2} \\ &= x^{k^2} \cdot |E|. \end{aligned}$$

□

Corollary 2.2 *For a cycle C_n,*

$$S_1(C_n, x) = S_2(C_n, x) = \begin{cases} n \cdot x^{\frac{(n-1)^2(n-3)^2}{64}}, & \text{if } n \text{ is odd} \\ n \cdot x^{\frac{n^2(n-2)^2}{64}}, & \text{if } n \text{ is even.} \end{cases}$$

Proof: For any vertex v in C_n, we have,

$$\text{str}(v) = \begin{cases} \dfrac{(n-1)(n-3)}{8}, & \text{if } n \text{ is odd} \\ \dfrac{n(n-2)}{8}, & \text{if } n \text{ is even.} \end{cases}$$

Hence C_n is

$$\begin{cases} \dfrac{(n-1)(n-3)}{8}\text{-stress regular}, & \text{if } n \text{ is odd} \\ \dfrac{n(n-2)}{8}\text{-stress regular}, & \text{if } n \text{ is even.} \end{cases}$$

Since C_n has n vertices and n edges, by the Proposition 2.5, we have

$$\begin{aligned} S_1(C_n, x) = S_2(C_n, x) &= n \times \begin{cases} x^{\frac{(n-1)^2(n-3)^2}{64}}, & \text{if } n \text{ is odd} \\ x^{\frac{n^2(n-2)^2}{64}}, & \text{if } n \text{ is even.} \end{cases} \\ &= \begin{cases} n \cdot x^{\frac{(n-1)^2(n-3)^2}{64}}, & \text{if } n \text{ is odd} \\ n \cdot x^{\frac{n^2(n-2)^2}{64}}, & \text{if } n \text{ is even.} \end{cases} \end{aligned}$$

□

Proposition 2.3 *Let T be a tree on n vertices. Then*

$$S_1(G,x) = \sum_{v\in I} x^{\left(\sum_{1\le i<j\le m} |C_i^v||C_j^v|\right)^2}$$
$$S_2(G,x) = \sum_{uv\in J} x^{\left[\sum_{1\le i<j\le m(u)} |C_i^u||C_j^u|\right]\left[\sum_{1\le i<j\le m(v)} |C_i^v||C_j^v|\right]}$$

where I is the set of internal(non-pendant) edges in T, J denotes the set of all vertices adjacent to pendent vertices in T, and the sets $C_1^v,\dots,C_m^v$ denotes the vertex sets of the components of $T-v$ for an internal vertex v of degree $m=m(v)$.

Proof: We know that a pendant vertex in T has zero stress. Let v be an internal vertex of T of degree $m=m(v)$. Let $C_1^v,\dots,C_m^v$ be the components of $T-v$. Since there is only one path between any two vertices in a tree, it follows that,

$$\mathrm{str}(v) = \sum_{1\le i<j\le m} |C_i^v||C_j^v| \tag{2.3}$$

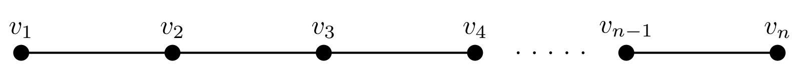
Let I denotes the set of internal(non-pendant) edges, and P denotes pendant edges and Q denotes the set of all vertices adjacent to pendent vertices in T. Then using (2.3) in the Definition 2.1 , we have

$$S_1(G,x) = \sum_{v\in I} x^{\mathrm{str}(v)^2}$$
$$S_1(G,x) = \sum_{v\in I} x^{\left(\sum_{1\le i<j\le m} |C_i^v||C_j^v|\right)^2}$$
$$S_2(G,x) = \sum_{uv\in J} x^{\mathrm{str}(u)\mathrm{str}(v)}$$
$$S_2(G,x) = \sum_{uv\in J} x^{\left[\sum_{1\le i<j\le m(u)} |C_i^u||C_j^u|\right]\left[\sum_{1\le i<j\le m(v)} |C_i^v||C_j^v|\right]}. \qquad \square$$

Corollary 2.3 *For the path P_n on n vertices*

$$S_1(P_n,x) = x^{\frac{n(2n-1)(n-1)^3}{6}}.$$
$$S_2(P_n,x) = x^{\frac{n(n+1)(n-1)(n-2)(n-3)}{30}}.$$

Proof: Let P_n be the path with vertex sequence $v_1, v_2, \dots, v_n$ (shown in Figure 3).



P_n

Figure 3: The path P_n on n vertices.

We have,

$$\mathrm{str}(v_i) = (i-1)(n-i), \qquad 1\le i\le n.$$

Then

$$S_1(P_n,x) = \sum_{v\in V(P_n)} x^{\mathrm{str}(v)^2}$$
$$= \sum_{i=1}^{n} x^{str(v_i)^2}$$

$$= \sum_{i=1}^{n} x^{[(i-1)(n-i)]^2}$$

$$S_1(P_n, x) = x^{\frac{n(2n-1)(n-1)^3}{6}}.$$

$$S_2(P_n, x) = \sum_{uv \in E(P_n)} x^{str(u)str(v)}$$

$$= \sum_{i=1}^{n-1} x^{str(v_i)str(v_{i+1})}$$

$$= \sum_{i=1}^{n-1} x^{[(i-1)(n-i)i(n-i-1)]}$$

$$S_2(P_n, x) = x^{\frac{n(n+1)(n-1)(n-2)(n-3)}{30}}.$$

□

Proposition 2.4 *Let $Wd(n,m)$ denotes the windmill graph constructed for $n \geq 2$ and $m \geq 2$ by joining m copies of the complete graph K_n at a shared universal vertex v. Then*

$$S_1(Wd(n,m), x) = x^{\frac{m^2(m-1)^2(n-1)^4}{4}} + m(n-1)$$

$$S_2(Wd(n,m), x) = m(n-1) + \frac{m(n-1)(n-2)}{2}$$

Hence, for the friendship graph F_k on $2k+1$ vertices,

$$S_1(F_k, x) = x^{4k^2(k-1)^2} + 2k$$

$$S_2(F_k, x) = 3k.$$

Proof: Clearly the stress of any vertex other than universal vertex is zero in $Wd(n,m)$, because neighbors of that vertex induces a complete subgraph of $Wd(n,m)$. Also, since there are m copies of K_n in $Wd(n,m)$ and their vertices are adjacent to v, it follows that, the only geodesics passing through v are of length 2 only. So, $\text{str}(v) = \dfrac{m(m-1)(n-1)^2}{2}$. Note that there are $m(n-1)$ edges incident on v and the edges that are not incident on v have end vertices of stress zero. Hence by the Definition 2.1, we have

$$S_1(Wd(n,m), x) = \sum_{v \in V(G)} x^{str(v)^2} + m(n-1)$$

$$= x^{\frac{m^2(m-1)^2(n-1)^4}{4}} + m(n-1)$$

Let E_1 be the set of all edges that are incident with the center vertex, and E_2 be the set of all edges of the complete graph.

$$S_2(Wd(n,m), x) = \sum_{uv \in E_1(G)} x^{str(u)str(v)} + \sum_{uv \in E_2(G)} x^{str(u)str(v)}$$

$$= m(n-1) + \frac{m(n-1)(n-2)}{2}$$

Since the friendship graph F_k on $2k+1$ vertices is nothing but $Wd(3,k)$, it follows that

$$S_1(F_k, x) = x^{\frac{2^4 k^2 (k-1)^2}{4}} + 2k = x^{4k^2(k-1)^2} + 2k.$$

$$S_2(F_k, x) = 3k.$$

□

Proposition 2.5 *Let W_n denotes the wheel graph constructed on $n \geq 4$ vertices. Then*

$$S_1(W_n, x) = x^{\frac{(n-1)(n-4)}{2}} + (n-1)x;$$
$$S_2(W_n, x) = (n-1) \cdot x^{\frac{(n-1)(n-4)}{2}} + (n-1) \cdot x.$$

Proof: In W_n with $n \geq 4$, there are $(n-1)$ peripheral vertices and one central vertex, say v. It is easy to see that

$$\mathrm{str}(v) = \frac{(n-1)(n-4)}{2} \tag{2.4}$$

Let p be a peripheral vertex. Since v is adjacent to all the peripheral vertices in W_n, there is no geodesic passing through p and containing v. Hence contributing vertices for $\mathrm{str}(p)$ are the rest peripheral vertices. So, by denoting the cycle $W_n - p$ (on $n-1$ vertices) by C_{n-1}, we have

$$\begin{aligned} \mathrm{str}_{W_n}(p) &= \mathrm{str}_{W_n - v}(p) \\ &= \mathrm{str}_{C_{n-1}}(p) \\ &= 1. \end{aligned} \tag{2.5}$$

Let us denote the set of all radial edges in W_n by R, and the set of all peripheral edges by Q. Note that there are $(n-1)$ radial edges and $(n-1)$ peripheral edges in W_n. Using (2.4) and (10) in the Definition 2.1, we have

$$\begin{aligned} S_1(W_n, x) &= \sum_{v \in V(W_n)} str(v)^2 \\ &= x^{\frac{(n-1)(n-4)}{2}} + (n-1)x \\ S_2(W_n, x) &= \sum_{xy \in R} x^{\mathrm{str}(x)\mathrm{str}(y)} + \sum_{xy \in Q} x^{\mathrm{str}(x)\mathrm{str}(y)} \\ &= (n-1) \cdot x^{\mathrm{str}(v)\mathrm{str}(p)} + (n-1) \cdot x^{\mathrm{str}(p)^2} \\ S_2(W_n, x) &= (n-1) \cdot x^{\frac{(n-1)(n-4)}{2}} + (n-1) \cdot x. \end{aligned}$$

□

Conclusion

Using the stresses of vertices, we have presented a new stress polynomial for graphs known as the first and second stress polynomial. Additionally, we calculated the stress polynomial for a few standard graphs, established some inequalities, and proved some results. Numerous polynomials based on vertex degrees have been defined. However, we have defined new graph polynomials in this paper without utilizing vertex degrees. Results in this direction will be reported in a future paper. This polynomial can be used to determine $S_1(G, x)$ and $S_2(G, x)$ for other classes of graphs.

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