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The gaps between topological indices of dominating David derived networks

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ABSTRACT: The application of graph theory has been shown to be a valuable tool in the discipline of chemistry. It provides a valuable means of understanding the molecular structures commonly used in chemistry. For this reason, topological indices are employed which are numerical invariants and play an important role in the analysis of graph structures. In this study, we investigate topological properties of dominating David derived networks for the first, second and third types, namely the gaps between arithmetic-geometric and Randić as well as geometric-arithmetic and Randić indices. These results make easier to understand the underlying topologies of dominating David derived networks.

Key Words: Dominating David derived networks, topological index, Randić index, arithmetic-geometric index, geometric-arithmetic index.

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1. Introduction

Graph theory has become an active research area in chemistry. The utilization of graph theory in the comprehension of chemical molecule structure is a highly advantageous approach to understand the structure of a chemical molecule. Representing a chemical molecule as a molecular graph means that the vertices represent the atoms with the edges representing the connections between the atoms.

Topological indices are numerical invariants that capture the structural characteristics of graphs. These indices are also known as connectivity indices, represent a category of molecular descriptor. They are derived from the molecular graph of a chemical compound. The calculation and comparison of topological indices has a significant place in research ([1], [2]). A variety of topological indices of dominating David derived networks (DDNs) have already been studied ([3], [4], [5]). In the present study, an investigation is conducted into the topological indices of DDNs, with the objective of achieving a more profound comprehension of their mathematical characteristics and prospective applications. The focus of this study is directed towards the gaps between arithmetic-geometric index AG and Randić index R, as well as geometric-arithmetic index GA and Randić index R. To be more precise, the present work focuses on investigating $AG - R$ and $GA - R$ for DDNs.

Throughout the article we consider G as a network with vertex set $V(G)$ and edge set $E(G)$. An edge from $E(G)$ is between two vertices u and v, denoted by $uv \in E(G)$. The adjacent vertices are connected together by an edge and the degree of a vertex u_i is the number of edges connected to the vertex, denoted by d_i ([6], [7]).

The Randić index, also referred to as the connectivity index, is a degree-based topological index that has been the subject of extensive research. It is considered to be one of the most widely applied in chemistry and pharmacology, in particular for the purpose of designing quantitative structure property and structure activity relations ([8], [9], [10]). It was developed in 1976 by Milan Randić ([11]) and is defined as follows:

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Definition 1.1 *The Randić index of a graph G is given by*

$$R(G) = \sum_{u_iu_j \in E(G)} \frac{1}{\sqrt{d_i d_j}}.$$

The arithmetic-geometric index is given in [13], where its value for path graphs was calculated by the authors.

Definition 1.2 *The arithmetic-geometric index of a graph G is given by*

$$AG(G) = \sum_{u_iu_j \in E(G)} \frac{d_i + d_j}{2\sqrt{d_i d_j}}.$$

The geometric-arithmetic index is introduced in [14] to surpass the predictive capabilities of the Randić index. It has applied succesfully in QSPR and associated domains. The formal definition is given as follows:

Definition 1.3 *The geometric-arithmetic index of a graph G is given by*

$$GA(G) = \sum_{u_iu_j \in E(G)} \frac{2\sqrt{d_i d_j}}{d_i + d_j}.$$

In [12], Chandra Das et al. defined $AG - R$ and $GA - R$ for a graph G as follows:

$$M_{AG}(G) = AG(G) - R(G) = \sum_{u_iu_j \in E(G)} \frac{d_i + d_j - 2}{2\sqrt{d_i d_j}}. \tag{1.1}$$

$$M_{GA}(G) = GA(G) - R(G) = \sum_{u_iu_j \in E(G)} \frac{2d_i d_j - d_i - d_j}{(d_i + d_j)\sqrt{d_i d_j}}. \tag{1.2}$$

2. Dominating David derived networks

Dominating David derived networks can be constructured with the help of honeycomb networks ([3], [5]). Consider an n-dimensional honeycomb network and divide each edge into two by inserting a new vertex. In each hexagonal cell, connect the new vertices if the distance between them is four units within the cell. Add new vertices at the intersection of new edge crossings. Then remove the vertices and edges of the honeycomb. By inserting a new vertex to the horizontal edges, split the edges into two. The graph that results from the proces is the dominating David derived network of dimension n (Figure 1). The figures of the each step of the algorithm can be found in [5].

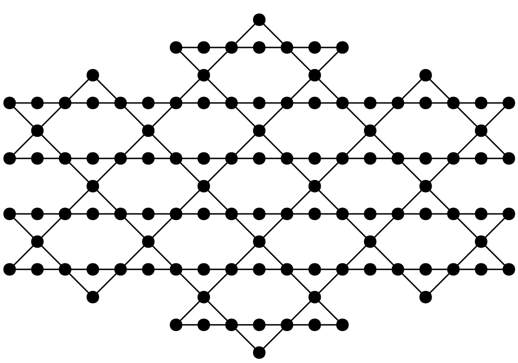

Figure 1: Dominating David derived network of dimension 2.

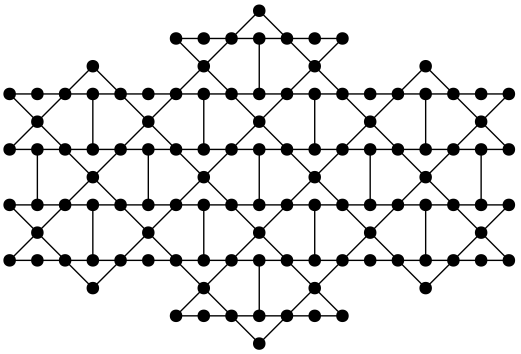

Figure 2: Dominating David derived network of the first type $D_1(2)$.

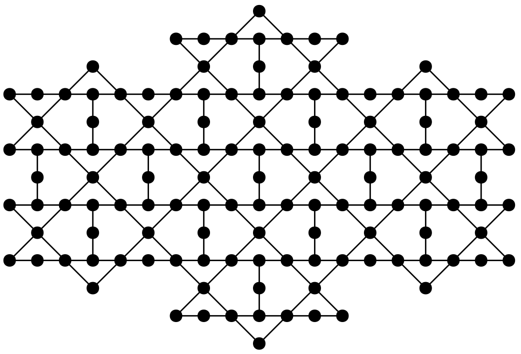

Figure 3: Dominating David derived network of the second type $D_2(2)$.

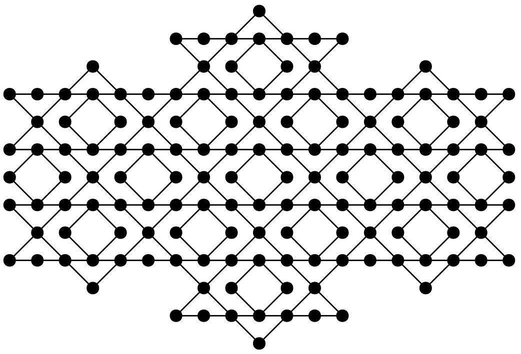

Figure 4: Dominating David derived network of the third type $D_3(2)$.

3. Topological indices of DDNs and the gaps between the indices

Randić and geometric-arithmetic indices for the first, second and third type of DDNs are given in [3]; however, we provide the theorems and the proofs to ensure integrity.

Theorem 3.1 *[3] Let $D_1(n)$ be the dominating David derived network of the first type. The Randić index for $D_1(n)$ is given by*

$$R(D_1(n)) = (12+6\sqrt{3})n^2 + \Big(\frac{2\sqrt{6}}{3} + 7\sqrt{2} - \frac{28\sqrt{3}}{3} - \frac{46}{3}\Big)n + \frac{20-2\sqrt{6}}{3} - 4\sqrt{2} + 4\sqrt{3}.$$

Proof:

Table 1: The partition of the edge set of $D_1(n)$

(d_u, d_v)	$(2,2)$	$(2,3)$	$(2,4)$	$(3,3)$	$(3,4)$	$(4,4)$
number of edges	$4n$	$4n-4$	$28n-16$	$9n^2-13n+5$	$36n^2-56n+24$	$36n^2-52n+20$

Table 2: The partition of the edge set of $D_2(n)$

(d_u, d_v)	$(2,2)$	$(2,3)$	$(2,4)$	$(3,4)$	$(4,4)$
number of edges	$4n$	$18n^2-22n+6$	$28n-16$	$36n^2-56n+24$	$36n^2-52n+20$

Using Definition 1.1 and Table 1, we get the following:

$$\begin{aligned} R(D_1(n)) =& 4n\frac{1}{\sqrt{2.2}} + (4n-4)\frac{1}{\sqrt{2.3}} + (28n-16)\frac{1}{\sqrt{2.4}} + (9n^2-13n+5)\frac{1}{\sqrt{3.3}} \\ &+ (36n^2-56n+24)\frac{1}{\sqrt{3.4}} + (36n^2-52n+20)\frac{1}{\sqrt{4.4}}, \end{aligned}$$

after simplification, the desired result is obtained:

$$R(D_1(n)) = (12+6\sqrt{3})n^2 + \Big(\frac{2\sqrt{6}}{3} + 7\sqrt{2} - \frac{28\sqrt{3}}{3} - \frac{46}{3}\Big)n + \frac{20-2\sqrt{6}}{3} - 4\sqrt{2} + 4\sqrt{3}.$$

□

Theorem 3.2 *The arithmetic-geometric index and geometric-arithmetic index of $D_1(n)$ are given by*

$$AG(D_1(n)) = (45+21\sqrt{3})n^2 + \Big(2\sqrt{6} + 21\sqrt{2} - 61 - \frac{98\sqrt{3}}{3}\Big)n + 25 - 2\sqrt{6} - 12\sqrt{2} + 14\sqrt{3},$$

and

$$GA(D_1(n)) = \Big(45 + \frac{144\sqrt{3}}{7}\Big)n^2 + \Big(\frac{8\sqrt{6}}{5} + \frac{56\sqrt{2}}{3} - 61 - 32\sqrt{3}\Big)n - \frac{8\sqrt{6}}{5} - \frac{32\sqrt{2}}{3} + \frac{96\sqrt{3}}{7} + 25.$$

Proof: Using Definition 1.2, Definition 1.3 and Table 1, we get the following:

$$\begin{aligned} AG(D_1(n)) =& 4n\Big(\frac{2+2}{2\sqrt{2.2}}\Big) + (4n-4)\Big(\frac{2+3}{2\sqrt{2.3}}\Big) + (28n-16)\Big(\frac{2+4}{2\sqrt{2.4}}\Big) + (9n^2-13n+5)\Big(\frac{3+3}{2\sqrt{3.3}}\Big) \\ &+ (36n^2-56n+24)\Big(\frac{3+4}{2\sqrt{3.4}}\Big) + (36n^2-52n+20)\Big(\frac{4+4}{2\sqrt{4.4}}\Big), \end{aligned}$$

and

$$\begin{aligned} GA(D_1(n)) =& 4n\Big(\frac{2\sqrt{2.2}}{2+2}\Big) + (4n-4)\Big(\frac{2\sqrt{2.3}}{2+3}\Big) + (28n-16)\Big(\frac{2\sqrt{2.4}}{2+4}\Big) + (9n^2-13n+5)\Big(\frac{2\sqrt{3.3}}{3+3}\Big) \\ &+ (36n^2-56n+24)\Big(\frac{2\sqrt{3.4}}{3+4}\Big) + (36n^2-52n+20)\Big(\frac{2\sqrt{4.4}}{4+4}\Big). \end{aligned}$$

After simplification, the desired results are obtained:

$$AG(D_1(n)) = (45+21\sqrt{3})n^2 + \Big(\frac{5\sqrt{6}}{3} + 21\sqrt{2} - 61 - \frac{98\sqrt{3}}{3}\Big)n + 25 - \frac{5\sqrt{6}}{3} - 12\sqrt{2} + 14\sqrt{3},$$

and

$$GA(D_1(n)) = \Big(45 + \frac{144\sqrt{3}}{7}\Big)n^2 + \Big(\frac{8\sqrt{6}}{5} + \frac{56\sqrt{2}}{3} - 61 - 32\sqrt{3}\Big)n - \frac{8\sqrt{6}}{5} - \frac{32\sqrt{2}}{3} + \frac{96\sqrt{3}}{7} + 25.$$

□

Table 3: The partition of the edge set of $D_3(n)$

(d_u, d_v)	$(2,2)$	$(2,4)$	$(4,4)$
number of edges	$4n$	$36n^2 - 20n$	$72n^2 - 108n + 44$

Theorem 3.3 *The indices $M_{AG} = AG - R$ and $M_{GA} = GA - R$ of $D_1(n)$ are given by*

$$M_{AG}(D_1(n)) = (33 + 15\sqrt{3})n^2 + \Big(\sqrt{6} + 14\sqrt{2} - \frac{70\sqrt{3}}{3} - \frac{137}{3}\Big)n - \sqrt{6} - 8\sqrt{2} + 10\sqrt{3} + \frac{55}{3},$$

and

$$M_{GA}(D_1(n)) = \Big(33 + \frac{102\sqrt{3}}{7}\Big)n^2 + \Big(\frac{14\sqrt{6}}{15} + \frac{35\sqrt{2} - 68\sqrt{3} - 137}{3}\Big)n - \frac{14\sqrt{6}}{15} + \frac{55 - 20\sqrt{2}}{3} + \frac{68\sqrt{3}}{7}.$$

Proof: Theorem 3.1 and Theorem 3.2 provide a straightforward method for obtaining the desired result. As a second way, by using Equation 1.1, Equation 1.2 and Table 1, the following formulae are obtained as follows:

$$\begin{aligned} M_{AG}(D_1(n)) =& 4n\Big(\frac{2+2-2}{2\sqrt{2.2}}\Big) + (4n-4)\Big(\frac{2+3-2}{2\sqrt{2.3}}\Big) + (28n-16)\Big(\frac{2+4-2}{2\sqrt{2.4}}\Big) \\ &+ (9n^2 - 13n + 5)\Big(\frac{3+3-2}{2\sqrt{3.3}}\Big) + (36n^2 - 56n + 24)\Big(\frac{3+4-2}{2\sqrt{3.4}}\Big) \\ &+ (36n^2 - 52n + 20)\Big(\frac{4+4-2}{2\sqrt{4.4}}\Big), \end{aligned}$$

and

$$\begin{aligned} M_{GA}(D_1(n)) =& 4n\Big(\frac{2.2.2-2-2}{(2+2)\sqrt{2.2}}\Big) + (4n-4)\Big(\frac{2.2.3-2-3}{(2+3)\sqrt{2.3}}\Big) + (28n-16)\Big(\frac{2.2.4-2-4}{(2+4)\sqrt{2.4}}\Big) \\ &+ (9n^2 - 13n + 5)\Big(\frac{2.3.3-3-3}{(3+3)\sqrt{3.3}}\Big) + (36n^2 - 56n + 24)\Big(\frac{2.3.4-3-4}{(3+4)\sqrt{3.4}}\Big) \\ &+ (36n^2 - 52n + 20)\Big(\frac{2.4.4-4-4}{(4+4)\sqrt{4.4}}\Big). \end{aligned}$$

After simplification, we get the following results:

$$M_{AG}(D_1(n)) = (33 + 15\sqrt{3})n^2 + \Big(\sqrt{6} + 14\sqrt{2} - \frac{70\sqrt{3}}{3} - \frac{137}{3}\Big)n - \sqrt{6} - 8\sqrt{2} + 10\sqrt{3} + \frac{55}{3},$$

and

$$M_{GA}(D_1(n)) = \Big(33 + \frac{102\sqrt{3}}{7}\Big)n^2 + \Big(\frac{14\sqrt{6}}{15} + \frac{35\sqrt{2} - 68\sqrt{3} - 137}{3}\Big)n - \frac{14\sqrt{6}}{15} + \frac{55 - 20\sqrt{2}}{3} + \frac{68\sqrt{3}}{7}.$$

□

Theorem 3.4 *[3] The Randić index for $D_2(n)$ is given by*

$$R(D_2(n)) = (3\sqrt{6} + 6\sqrt{3} + 9)n^2 - \Big(\frac{11\sqrt{6} + 28\sqrt{3}}{3} + 11 - 7\sqrt{2}\Big)n + \sqrt{6} - 4\sqrt{2} + 4\sqrt{3} + 5.$$

Proof: Using Definition 1.1 and Table 2, we get the following:

$$\begin{aligned} R(D_2(n)) =& 4n\frac{1}{\sqrt{2.2}} + (18n^2 - 22n + 6)\frac{1}{\sqrt{2.3}} + (28n-16)\frac{1}{\sqrt{2.4}} + (36n^2 - 56n + 24)\frac{1}{\sqrt{3.4}} \\ &+ (36n^2 - 52n + 20)\frac{1}{\sqrt{4.4}}, \end{aligned}$$

after simplification, the desired result is obtained:

$$R(D_2(n)) = (3\sqrt{6}+6\sqrt{3}+9)n^2 - \Big(\frac{11\sqrt{6}+28\sqrt{3}}{3}+11-7\sqrt{2}\Big)n+\sqrt{6}-4\sqrt{2}+4\sqrt{3}+5.$$

□

Theorem 3.5 *The arithmetic-geometric index and geometric-arithmetic index of $D_2(n)$ are given by*

$$\begin{aligned}AG(D_2(n)) &= \Big(\frac{15\sqrt{6}}{2}+21\sqrt{3}+36\Big)n^2+\Big(21\sqrt{2}-\frac{55\sqrt{6}+196\sqrt{3}}{6}-48\Big)n\\ &\quad+\frac{5\sqrt{6}}{2}-12\sqrt{2}+14\sqrt{3}+20,\end{aligned}$$

and

$$\begin{aligned}GA(D_2(n)) &= \Big(\frac{36\sqrt{6}}{5}+\frac{144\sqrt{3}}{7}+36\Big)n^2+\Big(\frac{56\sqrt{2}}{3}-\frac{44\sqrt{6}}{5}-32\sqrt{3}-48\Big)n\\ &\quad+\frac{12\sqrt{6}}{5}-\frac{32\sqrt{2}}{3}+\frac{96\sqrt{3}}{7}+20.\end{aligned}$$

Proof: Using Definition 1.2, Definition 1.3 and Table 2, we get the following:

$$\begin{aligned}AG(D_2(n)) &= 4n\Big(\frac{2+2}{2\sqrt{2.2}}\Big)+(18n^2-22n+6)\Big(\frac{2+3}{2\sqrt{2.3}}\Big)+(28n-16)\Big(\frac{2+4}{2\sqrt{2.4}}\Big)\\ &\quad+(36n^2-56n+24)\Big(\frac{3+4}{2\sqrt{3.4}}\Big)+(36n^2-52n+20)\Big(\frac{4+4}{2\sqrt{4.4}}\Big),\end{aligned}$$

and

$$\begin{aligned}GA(D_2(n)) &= 4n\Big(\frac{2\sqrt{2.2}}{2+2}\Big)+(18n^2-22n+6)\Big(\frac{2\sqrt{2.3}}{2+3}\Big)+(28n-16)\Big(\frac{2\sqrt{2.4}}{2+4}\Big)\\ &\quad+(36n^2-56n+24)\Big(\frac{2\sqrt{3.4}}{3+4}\Big)+(36n^2-52n+20)\Big(\frac{2\sqrt{4.4}}{4+4}\Big)\end{aligned}$$

After simplification, the desired results are obtained:

$$\begin{aligned}AG(D_2(n)) &= \Big(\frac{15\sqrt{6}}{2}+21\sqrt{3}+36\Big)n^2+\Big(21\sqrt{2}-\frac{55\sqrt{6}+196\sqrt{3}}{6}-48\Big)n\\ &\quad+\frac{5\sqrt{6}}{2}-12\sqrt{2}+14\sqrt{3}+20,\end{aligned}$$

and

$$\begin{aligned}GA(D_2(n)) &= \Big(\frac{36\sqrt{6}}{5}+\frac{144\sqrt{3}}{7}+36\Big)n^2+\Big(\frac{56\sqrt{2}}{3}-\frac{44\sqrt{6}}{5}-32\sqrt{3}-48\Big)n\\ &\quad+\frac{12\sqrt{6}}{5}-\frac{32\sqrt{2}}{3}+\frac{96\sqrt{3}}{7}+20.\end{aligned}$$

□

Theorem 3.6 *The indices $M_{AG} = AG - R$ and $M_{GA} = GA - R$ of $D_2(n)$ are given by*

$$\begin{aligned}M_{AG}(D_2(n)) &= \Big(\frac{9\sqrt{6}}{2}+15\sqrt{3}+27\Big)n^2+\Big(14\sqrt{2}-\frac{11\sqrt{6}}{2}-\frac{70\sqrt{3}}{3}-37\Big)n\\ &\quad+\frac{3\sqrt{6}}{2}-8\sqrt{2}+10\sqrt{3}+15,\end{aligned}$$

and

$$\begin{aligned}M_{GA}(D_2(n)) =&\Big(\frac{21\sqrt{6}}{5}+\frac{102\sqrt{3}}{7}+27\Big)n^2+\Big(\frac{35\sqrt{2}}{3}-\frac{77\sqrt{6}}{15}-\frac{68\sqrt{3}}{3}-37\Big)n\\&+\frac{7\sqrt{6}}{5}-\frac{20\sqrt{2}}{3}+\frac{68\sqrt{3}}{7}+15.\end{aligned}$$

Proof: Theorem 3.4 and Theorem 3.5 provide a straightforward method for obtaining the desired result. As a second way, by using Equation 1.1, Equation 1.2 and Table 2, the following formulae are obtained as follows:

$$\begin{aligned}M_{AG}(D_2(n)) =&4n\Big(\frac{2+2-2}{2\sqrt{2.2}}\Big)+(18n^2-22n+6)\Big(\frac{2+3-2}{2\sqrt{2.3}}\Big)+(28n-16)\Big(\frac{2+4-2}{2\sqrt{2.4}}\Big)\\&+(36n^2-56n+24)\Big(\frac{3+4-2}{2\sqrt{3.4}}\Big)+(36n^2-52n+20)\Big(\frac{4+4-2}{2\sqrt{4.4}}\Big),\end{aligned}$$

and

$$\begin{aligned}M_{GA}(D_2(n)) =&4n\Big(\frac{2.2.2-2-2}{(2+2)\sqrt{2.2}}\Big)+(18n^2-22n+6)\Big(\frac{2.2.3-2-3}{(2+3)\sqrt{2.3}}\Big)\\&+(28n-16)\Big(\frac{2.2.4-2-4}{(2+4)\sqrt{2.4}}\Big)+(36n^2-56n+24)\Big(\frac{2.3.4-3-4}{(3+4)\sqrt{3.4}}\Big)\\&+(36n^2-52n+20)\Big(\frac{2.4.4-4-4}{(4+4)\sqrt{4.4}}\Big).\end{aligned}$$

After simplification, the desired results are obtained:

$$\begin{aligned}M_{AG}(D_2(n)) =&\Big(\frac{9\sqrt{6}}{2}+15\sqrt{3}+27\Big)n^2+\Big(14\sqrt{2}-\frac{11\sqrt{6}}{2}-\frac{70\sqrt{3}}{3}-37\Big)n\\&+\frac{3\sqrt{6}}{2}-8\sqrt{2}+10\sqrt{3}+15,\end{aligned}$$

and

$$\begin{aligned}M_{GA}(D_2(n)) =&\Big(\frac{21\sqrt{6}}{5}+\frac{102\sqrt{3}}{7}+27\Big)n^2+\Big(\frac{35\sqrt{2}}{3}-\frac{77\sqrt{6}}{15}-\frac{68\sqrt{3}}{3}-37\Big)n\\&+\frac{7\sqrt{6}}{5}-\frac{20\sqrt{2}}{3}+\frac{68\sqrt{3}}{7}+15.\end{aligned}$$

□

Theorem 3.7 *[3] The Randić index for $D_3(n)$ is given by*

$$R(D_3(n)) = (9\sqrt{2}+18)n^2-(25+5\sqrt{2})n+11.$$

Proof: Using Definition 1.1 and Table 3, we get the following:

$$R(D_3(n)) = 4n\frac{1}{\sqrt{2.2}}+(36n^2-20n)\frac{1}{\sqrt{2.4}}+(72n^2-108n+44)\frac{1}{\sqrt{4.4}},$$

after simplification, the desired result is obtained:

$$R(D_3(n)) = (9\sqrt{2}+18)n^2-(25+5\sqrt{2})n+11.$$

□

Theorem 3.8 *The arithmetic-geometric index and geometric-arithmetic index of $D_3(n)$ are given by*

$$AG(D_3(n)) = (72+27\sqrt{2})n^2 - (15\sqrt{2}+104)n + 44,$$

and

$$GA(D_3(n)) = (72+24\sqrt{2})n^2 - \Big(104 + \frac{40\sqrt{2}}{3}\Big)n + 44.$$

Proof: Using Definition 1.2, Definition 1.3 and Table 3, we get the following:

$$AG(D_3(n)) = 4n\Big(\frac{2+2}{2\sqrt{2.2}}\Big) + (36n^2-20n)\Big(\frac{2+4}{2\sqrt{2.4}}\Big) + (72n^2-108n+44)\Big(\frac{4+4}{2\sqrt{4.4}}\Big),$$

and

$$GA(D_3(n)) = 4n\Big(\frac{2\sqrt{2.2}}{2+2}\Big) + (36n^2-20n)\Big(\frac{2\sqrt{2.4}}{2+4}\Big) + (72n^2-108n+44)\Big(\frac{2\sqrt{4.4}}{4+4}\Big).$$

After simplification, the desired results are obtained:

$$AG(D_3(n)) = (72+27\sqrt{2})n^2 - (15\sqrt{2}+104)n + 44,$$

and

$$GA(D_3(n)) = (72+24\sqrt{2})n^2 - \Big(104 + \frac{40\sqrt{2}}{3}\Big)n + 44.$$

□

Theorem 3.9 *The indices $M_{AG} = AG - R$ and $M_{GA} = GA - R$ of $D_3(n)$ are given by*

$$M_{AG}(D_3(n)) = (54+18\sqrt{2})n^2 - (10\sqrt{2}+79)n + 33,$$

and

$$M_{GA}(D_3(n)) = (54+15\sqrt{2})n^2 - \Big(79 + \frac{25\sqrt{2}}{3}\Big)n + 33.$$

Proof: Theorem 3.7 and Theorem 3.8 provide a straightforward method for obtaining the desired result. As a second way, by using Equation 1.1, Equation 1.2 and Table 3, the following formulae are obtained as follows:

$$M_{AG}(D_3(n)) = 4n\Big(\frac{2+2-2}{2\sqrt{2.2}}\Big) + (36n^2-20n)\Big(\frac{2+4-2}{2\sqrt{2.4}}\Big) + (72n^2-108n+44)\Big(\frac{4+4-2}{2\sqrt{4.4}}\Big),$$

and

$$\begin{aligned} M_{GA}(D_3(n)) =& 4n\Big(\frac{2.2.2-2-2}{(2+2)\sqrt{2.2}}\Big) + (36n^2-20n)\Big(\frac{2.2.4-2-4}{(2+4)\sqrt{2.4}}\Big) \\ &+ (72n^2-108n+44)\Big(\frac{2.4.4-4-4}{(4+4)\sqrt{4.4}}\Big). \end{aligned}$$

After simplification, the desired results are obtained:

$$M_{AG}(D_3(n)) = (54+18\sqrt{2})n^2 - (10\sqrt{2}+79)n + 33,$$

and

$$M_{GA}(D_3(n)) = (54+15\sqrt{2})n^2 - \Big(79 + \frac{25\sqrt{2}}{3}\Big)n + 33.$$

□

4. Conclusion

In this paper, we study some topological indices and the gaps between them of dominating David derived networks of the first, second and third types. The findings of this study are expected to facilitate a more profound comprehension and examination of the structural intricacies of these networks.

References

1. Zaman, S., Jalani, M., Ullah, A., Ahmad, W., and Saeedi, G., *Mathematical analysis and molecular descriptors of two novel metal–organic models with chemical applications.* Scientific Reports, 13 (1), 5314, (2023).
2. Liu, H., *Comparison between Merrifield-Simmons index and some vertex-degree-based topological indices.* Computational and Applied Mathematics, 42 (2), 89, (2023).
3. Imran, M., Baig, A. Q., and Ali, H., *On topological properties of dominating David derived networks.* Canadian Journal of Chemistry, 94 (2), 137-148, (2016).
4. Kang, S. M., Nazeer, W., Gao, W., Afzal, D., and Gillani, S. N. *M-polynomials and topological indices of dominating David derived networks.* Open Chemistry, 16 (1), 201-213, (2018).
5. Zaman, S., Ahmed, W., Sakeena, A., Rasool, K. B., and Ashebo, M. A., *Mathematical modeling and topological graph description of dominating David derived networks based on edge partitions.* Scientific Reports, 13 (1), 15159, (2023).
6. Wilson, R. J., *Introduction to graph theory*, Pearson Education India, (1979).
7. Bondy, J. A., and Murty, U. S. R., *Graph theory with applications* (Vol. 290). London: Macmillan, (1976).
8. Randić, M., Novič, M., and Plavšić, D., *Solved and unsolved problems of structural chemistry.* Boca Raton: CRC Press, (p. 379) (2016).
9. Kier, L., *Molecular connectivity in chemistry and drug research.* Elsevier, (Vol. 14) (2012).
10. Kier, L. B., and Hall, L. H., *Molecular connectivity in structure-activity analysis* (1986).
11. Randić, M., *Characterization of molecular branching.* Journal of the American Chemical Society, 97(23), 6609-6615, (1975).
12. Das, K. C., Huh, D. Y., Bera, J., and Mondal, S., *Study on geometric–arithmetic, arithmetic–geometric and Randić indices of graphs.* Discrete Applied Mathematics, 360, 229-245, (2025).
13. Shegehalli, V. S., and Kanabur, R., *Arithmetic-geometric indices of path graph.* J. Math. Comput. Sci, 16, 19-24, (2015).
14. Vukičević, D., and Furtula, B., *Topological index based on the ratios of geometrical and arithmetical means of end-vertex degrees of edges.* Journal of mathematical chemistry, 46, 1369-1376, (2009).

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