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# On Fixed Point Results of Generalized Contractions in Dislocated Quasi B-Metric Spaces

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ABSTRACT: This research article aims to present novel contractive conditions and explore the existence and uniqueness of fixed points of self-mappings within dislocated quasi b-metric spaces, utilizing these new contractive conditions. Additionally, examples are provided to demonstrate the validity and superiority of the approach. Our findings broaden and enhance several established results in exiting literature.

Key Words: quasi b-metric space, dislocated b-metric space, dislocated quasi b-metric space, generalized contraction, fixed point.

## Contents



## 1. Introduction and Preliminaries

In recent decades, there has been considerable research in the field of fixed point theory. Banach [1] introduced the renowned fixed point theorem for contraction mappings in metric spaces, which has since been generalized in various ways within the literature. Numerous researchers have established the contraction mapping theorem and its generalizations for metric spaces using different approaches. Generalized metric spaces include, but are not limited to, dislocated metric spaces, b-metric spaces, dislocated quasi-metric spaces, cone metric spaces, G-metric spaces, and quasi-metric spaces. By creating new contraction principles within metric spaces and generalized structures, Banach's contraction principle has been further expanded along similar lines. Notable examples of these expansions include the Ciric contraction, Kannan contraction, Chetterjea-type contraction, T-Kannan contraction, T-Banach contraction, cyclic contraction, $\alpha - \phi$-contractive mappings, among others.

The idea of b-metric spaces was initially presented by Bakhtin [2]. Shah and Hussain [3] later extended this framework by introducing quasi-b-metric spaces to further generalize b-metric spaces and derive a range of fixed point theorems. In addition, Alghamdi, Hussain, and Salimi [4] introduced b-metric-like spaces, or dislocated b-metric spaces, as a generalization of metric-like spaces.

The idea of dislocated quasi metric space was first proposed by F. M. Zeyada, G. H. Hassan, and M. A. Ahmed [5], which is a generalization of dislocated metric by P. Hitzler and A. K. Seda [6,7]. Chakkrid and Cholatis [8] recently introduced the idea of dislocated quasi b-metric space, in which they proved some fixed point theorems for cyclic contractions, as well as discussing the topological structure of dislocated quasi b-metric spaces and examining their properties. Mujeeb Ur Rahman and Muhammad Sarwar [9] created the idea of dislocated quasi b-metric spaces and established fixed point theorems for Kannan and Chetterjea type contractions. We also examine recent developments in dislocated metric spaces in [19,20,21,22,23,24].

In [10], Zada, Shah, and Li expanded upon integral-type contractions by introducing coupled coincidence fixed point theorems in G-metric spaces. Later, in [11], Wang, Zada, Shah, and Li developed common fixed point results for pairs of self-maps in dislocated metric spaces, and in [12], they extended these results to dislocated quasi-metric spaces, thereby enhancing our understanding of fixed points in these frameworks. Shah, Zada, and Li [13] further advanced the field by establishing new common coupled fixed point results for integral-type contractions in generalized metric spaces. Additionally, Zada,

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Shah, and Li [14] introduced fixed point theorems for ordered cone b-metric spaces, broadening the scope of the theory. In [15], Shah and Zada focused on common fixed point theorems for compatible maps with integral-type contractions in G-metric spaces, while Shah [16] proposed new fixed point results in b-metric-like spaces.

On the applied side, Turab and Sintunavarat [17] and [18] demonstrated the use of fixed point theory in modeling biological and psychological systems, showing how the Banach fixed point theorem can be applied to real-world problems. Collectively, these contributions advance fixed point theory, providing both theoretical insights and practical applications, with potential for future exploration.

The objective of this study is to establish a fixed point theorem for a new class of generalized contractions in dislocated quasi-b-metric spaces. The fixed point results presented here generalize, extend, and refine previous results of a similar nature. Additionally, we derive the well-known Banach fixed point theorem as a corollary of one of the contractions. We will validate our results through the use of various examples.

We'll begin with a few definitions that can be found in literature.

**Definition 1.1** [3] Suppose that $Q$ is a non-empty set. Let's say that the mapping $\rho : Q \times Q \to [0, \infty)$ such that constant $k \geq 1$ meets some conditions as below;
1. $\rho(s, w) = \rho(w, s) = 0 \iff s = w$ for all $s, w \in Q$;
2. $\rho(s, w) \leq k[\rho(s, z) + \rho(z, w)]$ for all $s, w, z \in Q$.

The pair $(Q, \rho)$ is then called a quasi b-metric space.

**Definition 1.2** [4] Suppose that $Q$ is a non-empty set. Let's say that the mapping $\rho : Q \times Q \to [0, \infty)$ such that constant $k \geq 1$ meets some conditions as below;
1. $\rho(s, w) = 0 \iff s = w$ for all $s, w \in Q$;
2. $\rho(s, w) = \rho(w, s)$ for all $s, w \in Q$;
3. $\rho(s, w) \leq k[\rho(s, z) + \rho(z, w)]$ for all $s, w, z \in Q$.

The pair $(Q, \rho)$is then called a b-metric like space (or a dislocated b-metric space).

The following definitions are necessary and can be found in [9].

**Definition 1.3** Consider the non-empty set $Q$. Assume that the mapping $\rho : Q \times Q \to [0, \infty)$ such that constant $k \geq 1$ meets some conditions as below;
1. $\rho(s, w) = \rho(w, s) = 0$ implies $s = w$ for all $s, w \in Q$;
2. $\rho(s, w) \leq k[\rho(s, z) + \rho(z, w)]$ for all $s, w, z \in Q$.

Then, $(Q, \rho)$ is referred to as a dislocated quasi b-metric space (or just dqb-metric). The constant $k$ is called the co-efficient of $(Q, \rho)$.

**Example 1.1** *Let $Q = R$ and suppose*

$$\rho(s, w) = |2s - w|^2 + |2s + w|^2.$$

*This means that $(Q, \rho)$ is a dislocated quasi b-metric space with coefficient $k = 2$. It is neither b-metric space nor dislocated quasi-metric space, however.*

**Remark 1.1** As illustrated by the example above, the distance between similar points in a dislocated quasi-b-metric space does not necessarily need to be zero, similar to what is observed in dislocated quasi-metric spaces.

**Definition 1.4** A sequence $\{s_n\}$ is referred to as dislocated quasi b-convergent in $(Q, \rho)$ if $n \geq N$ has $\rho(s_n, s) < \epsilon$ and $\epsilon > 0$, then $s$ is the dislocated quasi b-limit of the sequence $\{s_n\}$.

**Definition 1.5** In dislocated quasi b-metric space $(Q, \rho)$, a sequence $\{s_n\}$ is referred to as a Cauchy sequence if, for $\epsilon > 0$, there exists $n_0 \in N$, such that, for $m, n \geq n_0$, we have $\rho(s_m, s_n) < \epsilon$.

**Definition 1.6** If all of the Cauchy sequences in $Q$ converge to a point of $Q$, then $(Q, \rho)$ is a dislocated quasi b-metric space.

The following well-known results will be utilized in main results.

**Lemma 1.1** *[9] In a dislocated quasi-b-metric space, the limit of a convergent sequence is unique.*

**Lemma 1.2** *[9] Assume that $(Q, \rho)$ is a dislocated quasi-b-metric space and that $\{s_n\}$ is a sequence in that space such that*

$$\rho(s_n, s_{n+1}) \leq \alpha\rho(s_{n-1}, s_n), \tag{1.1}$$

*$n = 1, 2, 3 \ldots,$ and $0 \leq \alpha k < 1, \alpha \in [0, 1)$, and $k$ is defined in dislocated quasi b-metric space. Then $\{s_n\}$ is a Cauchy sequence in $Q$.*

**Theorem 1.1** *[9] Let $(Q, \rho)$ be a complete dislocated quasi-b-metric space. Suppose that $\tau : Q \to Q$ is a continuous contraction with $\alpha \in [0, 1]$ and $0 \leq k\alpha < 1$, where $k \geq 1$. Hence, $\tau$ has a single fixed point in $Q$.*

## 2. Main Results

In this section, we will utilize a new type of generalized contractions to prove several fixed point theorems in complete dislocated quasi-b-metric spaces.

**Theorem 2.1** *Let $(Q, v)$ be a complete dislocated quasi b-metric space, and let $\tau : Q \to Q$ be a self-satisfying continuous mapping such that*

$$\rho(\tau s, \tau w) \leq \alpha\rho(s, w) + \beta\rho(s, \tau s)\rho(w, \tau w) + \gamma\rho(s, \tau w)\rho(w, \tau s), \tag{2.1}$$

*for all $s, w \in Q$ and $\alpha, \beta, \gamma \geq 0$ with $k\alpha + k\beta + \gamma < 1$ where $k \geq 1$. Then $\tau$ has a unique fixed point in $Q$.*

**Proof:** Assuming $s_0$ in $Q$ is arbitrary, we define a sequence $\{s_n\}$ in $Q$ as follows

$$s_0 = s_1 = \tau s_0, s_2 = \tau s_1 \ldots s_{n+1} = \tau s_n.$$

In order to demonstrate that $\{s_n\}$ is a Cauchy sequence, consider

$$\rho(s_n, s_{n+1}) = \rho(\tau s_{n-1}, \tau s_n).$$

Using condition 2.1 we have

$$\begin{aligned}
\rho(s_n, s_{n-1}) &= \rho(\tau s_n, \tau s_n). \\
&\leq \alpha\rho(s_n, s_n) + \beta\frac{\rho(s_{n-1}, \tau s_{n-1})\rho(s_n, \tau s_n)}{\rho(s_{n-1}, s_n)} + \gamma\rho(s_{n-1}, \tau s_n)\rho(s_n, \tau s_{n-1}). \\
&= \alpha\rho(s_{n-1}, s_n) + \beta\frac{\gamma(s_{n-1}, s_n)\rho(s_n, s_{n+1})}{\rho(s_{n-1}, s_n)}\gamma\rho(s_{n-1}, s_{n+1})\gamma(s_n, s_n).
\end{aligned}$$

Following the simplifications, we obtain:

$$\rho(s_n, s_{n+1}) \leq \frac{\alpha}{1-\beta}\rho(s_{n-1}, s_n). \tag{2.2}$$

Let $h = \frac{\alpha}{1-\beta} < \frac{1}{k}$. So the inequality (2.2) becomes

$$\rho(s_n, s_{n+1}) \leq h\rho(s_{n-1}, s_n). \tag{2.3}$$

Also,

$$\rho(s_{n-1}, s_n) \leq h\rho(s_{n-2}, s_{n-1}).$$

Thus,

$$\rho(s_n, s_{n+1}) \leq h^2\rho(s_{n-2}, s_{n-1}).$$

By applying the same process, we get

$$\rho(s_n, s_{n+1}) \leq h^n\rho(s_0, s_1). \tag{2.4}$$

Since $h < \frac{1}{k}$. Taking limit $n \to \infty$, so $h^n \to 0$ and

$$\lim_{n\to\infty} \rho(s_n, s_{n+1}) = 0. \tag{2.5}$$

So by lemma 1.2 $\{s_n\}$ is a Cauchy sequence in complete dislocated quasi b-metric space, so there must exist $s' \in Q$ such that

$$\lim_{n\to\infty} (s_0, s') = 0. \tag{2.6}$$

Now we will show that $s'$ is the fixed point of $\tau$. Since $s_n \to s'$ as $n \to \infty$ using the continuity of $\tau$ we have

$$\lim_{n\to\infty} \tau s_n = \tau s',$$

which implies that

$$\lim_{n\to\infty} s_{n+1} = \tau s'. \tag{2.7}$$

Thus $\tau s' = s'$. So, $s'$ is the fixed point of $\tau$. For the uniqueness of fixed point, let $\tau$ has two fixed points i.e., $s', w'$ with $s' \neq \rho'$ then using 2.1 we have

$$\begin{aligned}\rho(s', w') &= \rho(\tau s', \tau w'). \\ &\leq \alpha\rho(s', w') + \beta\frac{\rho(s', \tau s')\rho(w', \tau w')}{\rho(s', w')} + \gamma\rho(s', \tau w')\rho(w', \tau s') \\ &\leq \alpha\rho(s', w') + \beta\frac{\rho(s', s')\rho(w', w')}{\rho(s', w')} + \gamma\rho(s', w')\rho(w', s'),\end{aligned}$$

putting $s' = w'$ in condition 2.1 one can easily shows that $\rho(s', w') = 0$ and $\rho(w', s') = 0$. So we get that $s' = w'$ and thus fixed point of $\tau$ is a unique. □

**Corollary 2.1** *Let $(Q, v)$ be a complete dislocated quasi b-metric space and $\tau : Q \to Q$ is a continuous self mapping satisfying*

$$\rho(\tau s, \tau w) \leq \alpha\rho(s, w), \tag{2.8}$$

*for all $s, w \in Q$ and $\alpha \geq 0$ with $0 \leq k\alpha < 1$ where $k \geq 1$. Then $\tau$ has a unique fixed point in $Q$.*

**Proof:** By taking $\beta = \gamma = 0$ in Theorem 2.1, we can obtain the desired result. □

**Remark 2.1** It is evident that Theorem 2.1 generalizes the Banach contraction principle in dislocated quasi-b-metric spaces.

**Example 2.1** *Let $Q = [0, 1]$ and $\tau : Q \to Q$ is defined as*

$$\tau s = \frac{s}{9} \quad \forall s \in Q,$$

*with complete dislocated quasi b-metric space is given by*

$$\rho(s, w) = |3s - w|^2 + |3s + w|^2 \quad \forall s, w \in Q.$$

*Then,*

$$\begin{aligned}\rho(\tau s, \tau w) = \rho\left(\frac{s}{9}, \frac{w}{9}\right) &= \left|\frac{s}{3} - \frac{w}{9}\right|^2 + \left|\frac{s}{3} + \frac{w}{9}\right|^2 \\ &= \frac{1}{81}(|3s - w|^2 + |3s + w|^2) \\ &= \frac{1}{9}(|3s - w|^2 + |3s + w|^2)\end{aligned}$$

*Hence,*

$$\rho(\tau s, \tau w) \leq \alpha\rho(s, w),$$

*with satisfying $0 \leq k\alpha < 1$ where $k = 2$. Therefore all conditions of corollary 2.1 are satisfied, so $\tau$ has a unique fixed point $0 \in [0, 1]$.*

**Theorem 2.2** *Suppose* $(Q,\rho)$ *be a complete dislocated quasi b-metric space. Let* $\tau : Q \to Q$ *be continuous function for* $k \geq 1$ *satisfying*

$$\rho(\tau s, \tau w) \leq \lambda_1 \rho(s,w) + \lambda_2 \frac{\rho(s,\tau s)\rho(s,\tau w)}{\rho(s,\tau w)+\rho(w,\tau s)} + \lambda_3 \frac{\rho(w,\tau w)\rho(w,\tau s)}{\rho(w,\tau s)+\rho(s,\tau w)}, \tag{2.9}$$

*for all* $s, w \in Q$ *and* $\lambda_1, \lambda_2, \lambda_3 \geq 0, \rho(s,\tau w)+\rho(w,\tau s) \neq 0,$ *with* $k\lambda_1 + k\lambda_2 + \lambda_3 < 1$ *where* $k \geq 1$. *Then* $\tau$ *has a unique fixed point in* $Q$.

**Proof:** Considering $s_0$ to be arbitrary in $Q$, we define a sequence $\{s_n\}$ in $Q$ as follows.

$$s_0, s_1 = \tau s_0, s_2 = \tau s_1 \ldots s_{n+1} = \tau s_n$$

for all $n \in N$. Now to show that $\{s_n\}$ is a Cauchy sequence in $Q$ consider

$$\rho(s_n, s_{n+1}) = \rho(\tau s_{n-1}, \tau s_n).$$

Using 2.9 we have

$$\begin{aligned}\rho(s_n, s_{n+1}) &= \rho(\tau s_{n-1}, \tau s_n)\\ &\leq \lambda_1 \rho(s_{n-1}, s_n) + \lambda_2 \frac{\rho(s_{n-1},\tau s_{n-1})\rho(s_{n-1},\tau s_n)}{\rho(s_{n-1},\tau s_n)+\rho(s_n,\tau s_{n-1})} + \lambda_3 \frac{\rho(s_n,\tau s_n)\rho(s_n,\tau s_{n-1})}{\rho(s_n,\tau s_{n-1})+\rho(s_{n-1},\tau s_n)}\\ &= \lambda_1 \rho(s_{n-1}, s_n) + \lambda_2 \frac{\rho(s_{n-1},s_n)\rho(s_{n-1},s_{n+1})}{\rho(s_{n-1},s_{n+1})+\rho(s_n,s_n)} + \lambda_3 \frac{\rho(s_n,s_{n+1})+\rho(s_n,s_n)}{\rho(s_n,s_n)+\rho(s_{n-1},s_{n+1})}\end{aligned}$$

After simplifying, we arrive at

$$\rho(s_n, s_{n+1}) \leq (\lambda_1 + \lambda_2)\rho(s_{n-1}, s_n). \tag{2.10}$$

Let $h = \lambda_1 + \lambda_2 < \frac{1}{k}$. So, inequality 2.10 becomes

$$\rho(s_n, s_{n+1}) \leq h\rho(s_{n-1}, s_n). \tag{2.11}$$

Also,

$$\rho(s_{n-1}, s_n) \leq h\rho(s_{n-2}, s_{n-1}).$$

Thus,

$$\rho(s_n, s_{n+1}) \leq h^2\rho(s_{n-2}, s_{n-1}).$$

Continuing in the same manner, we get

$$\rho(s_n, s_{n+1}) \leq h^n\rho(s_0, s_1) \tag{2.12}$$

Thus, according to lemma, sequence $\{s_n\}$ is a Cauchy sequence in complete dislocated quasi-b-metric space, $s' \in Q$ must exist such that

$$\lim_{n\to\infty} (s_n, s') = 0.$$

Now we will show that $s'$ is fixed point of $\tau$. Since $s_n \to s'$ as $n \to \infty$ using continuity of $\tau$ and from Theorem 2.1 we will get $\tau s' = s'$. So, $s'$ is the fixed point of $\tau$.
for uniqueness of fixed point, let $\tau$ has two fixed points $s'$ and $w'$ with $s' \neq w'$ then we have

$$\begin{aligned}\rho(s', w') &= \rho(\tau s', \tau w')\\ &\leq \lambda_1 \rho(s', w') + \lambda_2 \frac{\rho(s',\tau s')\rho(s',\tau w')}{\rho(s',\tau w')+\rho(w',\tau s')} + \lambda_3 \frac{\rho(w',\tau w')\rho(w',\tau s')}{\rho(w',\tau s')+\rho(s',\tau w')}.\\ &= \lambda_1 \rho(s', w') + \lambda_2 \frac{\rho(s',s')\rho(s',w')}{\rho(s',w')+\rho(w',s')} + \lambda_3 \frac{\rho(w',w')\rho(w',s')}{\rho(w',s')+\rho(s',w')},\end{aligned} \tag{2.13}$$

since, $s'$ and $\rho'$ are fixed point of $\tau$ and using given condition in the theorem one can easily get that $\rho(s', s') = 0$ and $\rho(w', w') = 0$, so Finally, we obtain

$$\rho(s', w') \leq \lambda_1 \rho(s', w'). \tag{2.14}$$

In the same way, we can show that

$$\rho(w', s') \leq \lambda_1 \rho(w', s') \tag{2.15}$$

The inequalities 2.14 and 2.15 are possible only if $\rho(s', w') = \rho(w', s') = 0$. So, we get $s' = w'$. Thus fixed point of $\tau$ is unique. □

**Corollary 2.2** *Assume that $(Q, \rho)$ is a complete quasi-b-metric space and that $\tau : Q \to Q$ is a continuous self mapping that satisfies*

$$\rho(\tau s, \tau w) \leq \lambda_1 \rho(s, w) + \lambda_2 \frac{\rho(s, \tau s)\rho(s, \tau w)}{\rho(s, \tau w) + \rho(w, \tau s)}, \tag{2.16}$$

*for all $s, w \in Q$ and $\lambda_1, \lambda_2, \geq 0, \rho(s, \tau w) + \rho(w, \tau s) \neq 0$, with $k\lambda_1 + k\lambda_2 < 1$ where $k \geq 1$. Then $\tau$ has a unique fixed point in $Q$.*

**Proof:** By taking $\lambda_3 = 0$ in Theorem 2.2, we obtain the desired result. □

**Theorem 2.3** *Let $(Q, \rho)$ be a complete dislocated quasi-b-metric space. Assume that $\tau : Q \to Q$ is a continuous self function on $Q$. For $k \geq 1$ fulfilling*

$$\rho(\tau s, \tau y) \leq \xi_1 \rho(s, w) + \xi_2 \frac{\rho(s, \tau s)\rho(w, \tau w)}{\rho(s, w)} + \xi_3 \frac{\rho(s, \tau w)(w, \tau s)}{\rho(s, w)} + \xi_4 \frac{\rho(s, \tau s)\rho(w, \tau s)}{\rho(s, w)}, \tag{2.17}$$

*for all $s, w \in Q$, where $\xi_1, \xi_2, \xi_3, \xi_4 \geq 0$ with $k\xi_1 + k\xi_2 + \xi_3 + \xi_4 < 1$. Then $\tau$ has a unique fixed point in $Q$.*

**Proof:** Considering $s_0$ to be arbitrary in $Q$, we define a sequence $\{s_n\}$ in $Q$ as follows

$$s_0, s_1 = \tau s_0, s_2 = \tau s_1, s_3 = \tau s_2, \ldots, s_{n+1} = \tau s_n$$

for all $n \in N$.
Now to demonstrate that $\{s_n\}$ is a Cauchy sequence in $Q$ considered

$$\rho(s_n, s_{n+1}) = \rho(\tau s_{n-1}, \tau s_n), \tag{2.18}$$

using condition 2.17 we have

$$\begin{aligned}
\rho(s_n, s_{n+1}) &\leq \xi_1 \rho(s_{n-1}, s_n) + \xi_2 \frac{\rho(s_{n-1}, \tau s_{n_1})\rho(s_n, \tau s - n)}{\rho(s_{n-1}, s_n)} + \xi_3 \frac{\rho(s_{n-1}, \tau s_n)\rho(s_n, \tau s_{n-1})}{\rho(s_{n-1}, s_n)} \\
&\quad + \xi_4 \frac{\rho(s_{n-1}, \tau s_{n-1})\rho(s_n, \tau s_{n-1})}{\rho(s_{n-1}, s_n)} \\
&= \xi_1 \rho(s_{n-1}, s_n) + \xi_2 \frac{\rho(s_{n-1}, s_n)\rho(s_n, s_{n+1})}{\rho(s_{n-1}, s_n)} + \xi_3 \frac{\rho(s_{n-1}, s_{n+1})\rho(s_n, s_n)}{\rho(s_{n-1}, s_n)} \\
&\quad + \xi_4 \frac{\rho(s_{n-1}, s_n)\rho(s_n, s_n)}{\rho(s_{n-1}, s_n)}.
\end{aligned}$$

After simplification, we obtain

$$\rho(s_n, s_{n+1}) \leq \frac{\xi_1}{1 - \xi_2} \rho(s_{n-1}, s_n). \tag{2.19}$$

Let $h = \frac{\xi_1}{1-\xi_2} < \frac{1}{k}$. So, the inequality 2.19 becomes

$$\rho(s_n, s_{n+1}) \leq h\rho(s_{n-1}, s_n). \tag{2.20}$$

Also,

$$\rho(s_{n-1}, s_n) \leq h\rho(s_{n-2}, s_{n-1}).$$

Thus,

$$\rho(s_n, s_{n+1}) \leq h^2\rho(s_{n-2}, s_{n-1}).$$

By proceeding in a similar manner, we get

$$\rho(s_n, s_{n+1}) \leq h^n\rho(s_0, s_1). \tag{2.21}$$

Thus, using the lemma, the given sequence is a Cauchy sequence in the complete dislocated quasi-b-metric space, and hence there must be $s' \in Q$ such that

$$\lim_{n\to\infty}(s_n, s') = 0.$$

The next step is to demonstrate that $s'$ is fixed point of $\tau$. Since $s_n \to s'$ as $n \to \infty$ using the continuity of $\tau$ we have

$$\lim_{n\to\infty} \tau s_n = \tau s',$$

which implies that

$$\lim_{n\to\infty} \tau s_{n+1} = \tau s'.$$

Thus $\tau s' = s'$. So, $s'$ is the fixed point of $\tau$. Now, for the uniqueness of fixed point, let $\tau$ have two fixed points with $s' \neq w'$ then we have

$$\begin{aligned}
\rho(s', w') &= \rho(\tau s', \tau w') \\
&\leq \xi_1\rho(s', w') + \xi_2\frac{\rho(s', \tau s')\rho(w', \tau w')}{\rho(s', w')} + \xi_3\frac{\rho(s', \tau w')\rho(w', \tau s')}{\rho(s', w')} + \xi_4\frac{\rho(s', \tau s')\rho(w', \tau s')}{\rho(s', w')} \\
&= \xi_1\rho(s', w') + \xi_2\frac{\rho(s', s')\rho(w', w')}{\rho(s', w')} + \xi_3\frac{\rho(s', w')\rho(w', s')}{\rho(s', w')} + \xi_4\frac{\rho(s', s')\rho(w', s')}{\rho(s', w')}.
\end{aligned}$$

Since, $s'$ and $w'$ are fixed point of $\tau$ and using condition 2.17 in the theorem one can easily get that $\rho(s', s') = 0$ and $\rho(w', w') = 0$, so finally we get,

$$\rho(s', w') \leq \xi_1\rho(s', w') + \xi_3\rho(w', s'), \tag{2.22}$$

Similarly we can show that

$$\rho(s', w') \leq \xi_1\rho(w', s') + \xi_3\rho(s', w'), \tag{2.23}$$

Adding both 2.22 and 2.23 we get,

$$[\rho(s', w') + \rho(w', s')] \leq (\xi_1 + \xi_3)[\rho(s', w') + \rho(w', s')]. \tag{2.24}$$

The inequality 2.24 is possible only if $\rho(s', w') + \rho(w', s') = 0$, which is again possible if $\rho(s', w') = \rho(w', s') = 0$. Therefore, we get that $s' = w'$. Thus fixed point of $\tau$ is unique. □

**Corollary 2.3** *Let $(Q, \rho)$ be a complete dislocated quasi-b-metric space. Let $\tau : Q \to Q$ be a self function on $Q$ that is continuous. For $k \geq 1$ satisfying*

$$\rho(\tau s, \tau y) \leq \xi_1\rho(s, w) + \xi_2\frac{\rho(s, \tau s)\rho(w, \tau w)}{\rho(s, w)} + \xi_3\frac{\rho(s, \tau w)(w, \tau s)}{\rho(s, w)}, \tag{2.25}$$

*for all $s, w \in Q$, where $\xi_1, \xi_2, \xi_3, \geq 0, \rho(s, w) \neq 0$, with $k\xi_1 + k\xi_2 + \xi_3 < 1$. Then $\tau$ has a unique fixed point in $Q$.*

**Proof:** By choosing $\xi_4 = 0$ in Theorem 2.3, the desired result follows. □

**Example 2.2** *Let $Q = [0, 1]$, with complete dislocated quasi b-metric space defined by*

$$\rho(s, w) = |s - w| + |s|,$$

*for all $s, w \in Q$. The continuous self mapping is defined by*

$$\tau s = \frac{s}{2}, \quad \forall s \in Q.$$

*In partition, if we take $\xi_1 = \frac{1}{4}, \xi_2 = \frac{1}{8}, \xi_3 = \frac{1}{12}, \xi_4 = \frac{1}{16}$, with $k = 2$ satisfying $k\xi_1 + k\xi_2 + \xi_3 + \xi_4 < 1$. All conditions of Theorem 2.3 are satisfied and $x = 0 \in [0, 1]$ is the unique fixed point of $\tau$.*

## 3. Conclusion

In this work, we have proven several fixed point theorems for a new class of generalized contractions in dislocated quasi-b-metric spaces. The results presented are a generalization, extension, and improvement of prior similar fixed point theorems. Examples are provided to demonstrate the practical utility and relevance of our findings.

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