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A New Investgation on Steinberg Groups $^3D_4(q)$

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ABSTRACT: In this paper, we prove that simple groups $^3D_4(q)$, where $q^4 - q^2 + 1$ is a prime number can be uniquely determined by the order of group and the second largest element order, in the group $^3D_4(q)$.

Key Words: Element order, second largest element order, prime graph.

Contents

1. Introduction

Throughout this paper G is a finite group, the set of prime divisors of $|G|$ is denoted by $\pi(G)$ and the largeset element order and the second largest element order of the set $\pi_e(G)$ of element orders of G is denoted by $k_1(G)$ and $k_2(G)$, respectively. The prime graph $\Gamma(G)$ of group G is a graph whose vertex set is $\pi(G)$, and two vertices p and q are adjacent if and only if $pq \in \pi_e(G)$. Moreover, assume that $\Gamma(G)$ has $t(G)$ connected components π_i, for $i = 1, 2, \ldots, t(G)$. In the case where $|G|$ is of even order, we assume that $2 \in \pi_1$.

Next, we say the group G is characterized by property M, if any group H with property M is isomorphic to G. However, in some papers by different methods it is proved some of groups can be characterized by some property M. One of the methods is characterization groups by using the order of group and the largest element order. In other words, we say the group G is characterizable by using the order of group and the largest element order or the second largest element order if for any group H, so that $k_i(G) = k_i(H)$,$i = 1, 2$, and $|G| = |H|$ then $G \cong H$.

In this direction, the authors in different references proved that some of groups is determined by this method. For example groups such as, Projective special linear group $PSL(3, q)$ ([1]), Chevalley group $G_2(q)$, $q \leq 11$ ([2]), sporadic groups([4]), $PSL(2, q)$, $q = p^n < 125$([5]), K_4-group type $PSL(2, p)$ where p is a prime but not 2^n-1([6]), Suzuki groups $Sz(q)$([7]), the projective special unitary group $PSU(3, 3^n)$ ([8]), symplectics groups $PSP(8, q)$([9]), symplectics groups $C_4(q)$([10]), the simple groups $^2D_8(2^n)^2$ ([11]), the symplectics groups $PSP(4, 2^n)$, where $2^{2n} + 1$, $PSP(8, q)$ ([12]), the simple groups $^2D_n(3)$ ([13]),the projective special linear groups $PSL(5, 2)$ and $PSL(4, 5)$ ([14]), On the simple K_5-groups ([16]), the simple $^2E_6(q)$ ([17]), the simple $B_4(q)$ group ([18]), $PSL(3, q)$ ($q \leq 8$) and $PSU(3, q)$ ($q \leq 11$) ([20]), $PGL(2, q)$ ([23]), Alternating group ([24] can be determined by the largest element order or the second largest element order.

In this article, motivated by the above references we prove that the steinberg groups $^3D_4(q)$, where $q^4 - q^2 + 1$ is a prime number can be characterized by by using the second largest element order and order of group. In fact, we prove the following main theorem:

Main Theorem. Let G be a group and $D = {}^3D_4(q)$ so that $k_2(G) = k_2(D)$ and $|G| = |D|$, then $G \cong D$.

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2. Title Material

In this section, we give some useful lemmas which will be used in the proof of the main Theorem.

Lemma 2.1 *[19] Let G be a Frobenius group of even order with kernel K and complement H. Then*

1. *$t(G) = 2$, $\pi(H)$ and $\pi(K)$ are vertex sets of the connected components of $\Gamma(G)$;*
2. *$|H|$ divides $|K| - 1$;*
3. *K is nilpotent.*

Definition 2.1 A group G is called a 2-Frobenius group if there is a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that G/H and K are Frobenius groups with kernels K/H and H respectively.

Lemma 2.2 *[3] Let G be a 2-Frobenius group of even order. Then*

1. *$t(G) = 2$, $\pi(H) \cup \pi(G/K) = \pi_1$ and $\pi(K/H) = \pi_2$;*
2. *G/K and K/H are cyclic groups satisfying $|G/K|$ divides $|Aut(K/H)|$.*

Lemma 2.3 *[26] Let G be a finite group with $t(G) \geq 2$. Then one of the following statements holds:*

1. *G is a Frobenius group;*
2. *G is a 2-Frobenius group;*
3. *G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ such that H and G/K are π_1-groups, K/H is a non-abelian simple group, H is a nilpotent group and $|G/K|$ divides $|Out(K/H)|$.*

Lemma 2.4 *[28] Let q, k, l be natural numbers. Then*

[(a)]

1. *$(q^k - 1, q^l - 1) = q^{(k,l)} - 1$.*
2. $(q^k + 1, q^l + 1) = \begin{cases} q^{(k,l)} + 1 & \text{if both } \frac{k}{(k,l)} \text{ and } \frac{l}{(k,l)} \text{ are odd,} \\ (2, q+1) & \text{otherwise.} \end{cases}$
3. $(q^k - 1, q^l + 1) = \begin{cases} q^{(k,l)} + 1 & \text{if } \frac{k}{(k,l)} \text{ is even and } \frac{l}{(k,l)} \text{ is odd,} \\ (2, q+1) & \text{otherwise.} \end{cases}$

In particular, for every $q \geq 2$ and $k \geq 1$ the inequality $(q^k - 1, q^k + 1) \leq 2$ holds.

Lemma 2.5 *[25] Let G be a non-abelian simple group such that $(5, |G|) = 1$. Then G is isomorphic to one of the following groups:*

1. *$PSL(n, q)$, $n = 2, 3$, $q \equiv \pm 2 \pmod 5$;*
2. *$G_2(q)$, $q \equiv \pm 2 \pmod 5$;*
3. *$PSU(3, q)$, $q \equiv \pm 2 \pmod 5$;*
4. *${}^3D_4(q)$, $q \equiv \pm 2 \pmod 5$;*
5. *${}^2G_2(q)$, $q = 3^{2m+1}$, $m \geq 1$.*

3. Figures and Tables

In the following, we list the numbers $k_1(G)$ and $k_2(G)$ of some groups.([21])

Table(I)

Group	$k_1(G)$	$k_2(G)$
$PSL(2,q)$, $q>3$ prime	q	$\frac{q+1}{2}$
$PSL(2,q)$, q composite	$\frac{q+1}{2}$	$\frac{q-1}{2}$
$PSL(3,q)$	$\frac{q^2+q+1}{(3,q-1)}$	$\frac{q^2-1}{(3,q-1)}$
$PSU(3,q)$, q prime	$\frac{q^2+q}{(3,q+1)}$	$\frac{q^2-1}{(3,q+1)}$
$PSU(3,q)$, q composite	$\frac{q^2-1}{(3,q+1)}$	$\frac{q^2-q+1}{(3,q+1)}$
$G_2(q)$, q prime	q^2+q+1	q^2+q
$G_2(q)$, q composite	q^2+q+1	q^2-1
$^3D_4(q)$, q prime	$(q^3-1)(q+1)$	$q(q^3+1)$
$^3D_4(q)$, q composite	$(q^3-1)(q+1)$	q^4-q^2+1
$^2G_2(q)$,$q=3^{2m+1}$	$q+\sqrt{3q}+1$	$q-1$

4. Mathematics

In this section, we prove the main theorem. From now on we denote the Steinberg groups $^3D_4(q)$ and prime number q^4-q^2+1 by D, p, respectively. We recall that G is a group with $k_2(G)=k_2(D)$ and $|G|=|D|$. First, we know that $|^3D_4(q)|=q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$ and $k_2(G)=q(q^3+1)$([21]).

Lemma 4.1 *p is an isolated vertex in $\Gamma(G)$.*

Proof: First we know $|G|=q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$ and $k_2(G)=q(q^3+1)$. Now we prove p is an isolated vertex in $\Gamma(G)$. Otherwise, so there is $t\in\pi(G)-p$ such that $tp\in\pi_e(G)$, we have $tp\geq 2p=2(q^4-q^2+1>q(q^3+1)=k_2(G)$, that is a contradiction, so p is an isolated vertex, it follows that $t(G)\geq 2$. □

Lemma 4.2 *The group G is neither a Frobenius group nor a 2-Frobenius group.*

Proof: First we prove that G is not a Frobenius group. Otherwise, assume G be a Frobenius group with kernel K and complement H. Then by Lemma 2.1, $t(G)=2$ and $\pi(H)$ and $\pi(K)$ are vertex sets of the connected components of $\Gamma(G)$ and $|H|$ divides $|K|-1$. Now by Lemma 4.1, p is an isolated vertex of $\Gamma(G)$. Thus we deduce that (i) $|H|=p$ and $|K|=|G|/p$ or (ii) $|H|=|G|/p$ and $|K|=p$. Since $|H|$ divides $|K|-1$. Next, first assume $|H|=p$ and $|K|=|G|/p$, hence $q^4-q^2+1\mid(q^4-q^2+1)(q^{20}+q^{18}-3q^{14}-3q^{12}+4q^8+4q^6-4q^2-4)+3$. Thus $p\mid 3$ which is impossible. Now we assume $|H|=|G|/p$ and $|K|=p$, hence since$|H|$ divides $|K|-1$ in conclude$|G|/p$ divides $p-1$, where is a contradiction.

We now show that G is not a 2-Frobenius group. In contrary, we assume G be a 2-Frobenius group. Then G has a normal series $1\trianglelefteq H\trianglelefteq K\trianglelefteq G$ such that G/H and K are Frobenius groups with kernels K/H and H, respectively. Now, we set $|G/K|=x$. Since p is an isolated vertex of $\Gamma(G)$, so we have $|K/H|=p$ and $|H|=|G|/(xp)$. By Lemma 2.2, $|G/K|$ divides $|Aut(K/H)|$. Thus $|G/K|\mid q^4-q^2$ now by Lemma 2.4, $(q^4-q^2,p)=1$, hence $p\mid |H|$. Therefore $H_t\rtimes K/H$ is a Frobenius group with kernel H_t and complement K/H, where t is p. So $|K/H|$ divides $|H_t|-1$. It implies that $q^4-q^2+1\mid q^4-q^2$, but this is a contradiction. Hence G is not a 2-Frobenius group. □

Lemma 4.3 *The group G is isomorphic to the group D.*

Proof: By Lemma 4.1, p is an isolated vertex of $\Gamma(G)$. Thus $t(G)>1$ and G satisfies one of the cases of Lemma 2.3. Now Lemma 4.2 implies that G is neither a Frobenius group nor a 2-Frobenius group. Thus only the case (c) of Lemma 2.3 occures. So G has a normal series $1\trianglelefteq H\trianglelefteq K\trianglelefteq G$ such that H

and G/K are π_1-groups, K/H is a non-abelian simple group. Since p is an isolated vertex of $\Gamma(G)$, we have $p \mid |K/H|$. On the other hand, we know by Lemma 2.5, since that 5 does not divide $|K/H|$. So, by classification of finite simple groups and table(I), K/H is isomorphic to one of the following groups. □

Case(1).$K/H \ncong PSL(2, q')$, where $q' \equiv \pm 2(\mod 5)$. Suppose that $K/H \cong PSL(2, q')$. Now, by [21], $k_2(PSL(2, q')) = \frac{q \pm 1}{2}$. On the other hand, we know $|PSL(2, q')|$ divided $|G|$ as $\frac{q'(q'^2-1)}{(2,q'-1)} \mid q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$. First, assume $(2, q'-1) = 1$ and q' is prime number so we deduce $q'(q'^2-1) \mid q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$. Now, we consider $k_2(PSL(2, q')) \mid k_2(G)$ so $q(q^3+1) = \frac{q'+1}{2}$ it follows that $q' = 2q^4+2q-1$. Since that $|PSL(2, q')| \nmid |G|$, which is a contradiction. Now, if $(2, q'-1) = 2$, q' be composite then we deduce $\frac{q'(q'^2-1)}{2} \mid q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$. Now, we consider $q(q^3+1) = \frac{q'-1}{2}$ it follows that $q' = 2q^4+2q+1$. Since that $|PSL(2, q')| \nmid |G|$, which is a contradiction

Case(2).$K/H \ncong PSL(3, q')$, where $q' \equiv \pm 2(\mod 5)$. Suppose that $K/H \cong PSL(3, q')$. Now, by [21], $k_2(PSL(3, q')) = \frac{q'^2-1}{(3,q'-1)}$. On the other hand, we know $|PSL(3, q')|$ divided $|G|$ as $\frac{q'^3(q'^3-1)(q'^2-1)}{(3,q'-1)} \mid q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$. First, assume $(3, q'-1) = 1$ so we deduce $q'^3(q'^3-1)(q'^2-1) \mid q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$. Now, we consider $k_2(PSL(3, q')) \mid k_2(G)$ so $q(q^3+1) = q'^2-1$ it follows that $(q'-1)(q'+1) = q(q^3+1)$. Now, we know $(q'+1, q'-1) = 1$ or 2. First, assume $(q'+1, q'-1) = 1$ so $q'-1 = q$ and $q'+1 = q^3+1$ it follows that $q' = q+1$, $q' = q^3$. Since that $|PSL(3, q')| \nmid |G|$, which is a contradiction. For other case, we have a contradiction, similarly. Now, if $(3, q'-1) = 3$ then $\frac{q'^3(q'^3-1)(q'^2-1)}{3} \mid q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$ on the other hand, we consider $q(q^3+1) = \frac{q'^2-1}{3}$. Hence, $3q(q^3+1) = (q'-1)(q'+1)$ but we know $(q'+1, q'-1) = 1$ or 2. So, assume $(q'+1, q'-1) = 1$ so $q'-1 = 3q$ and $q'+1 = q^3+1$ it follows that $q' = 3q+1$, $q' = q^3$. Since that $|PSL(3, q')| \nmid |G|$, which is a contradiction. For the other case we have a contradiction, similarly.

Case(3).$K/H \ncong PSU(3, q')$, where $q' \equiv \pm 2(\mod 5)$. Suppose that $K/H \cong PSU(3, q')$. Now, by [21], $k_2(PSU(3, q')) = \frac{q'^2-q'+1}{(3,q'+1)}$. On the other hand, we know $|PSU(3, q')|$ divided $|G|$ as $\frac{q'^3(q'^3+1)(q'^2-1)}{(3,q'+1)} \mid q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$. First, assume $(3, q'+1) = 1$ so we deduce $q'^3(q'^3+1)(q'^2-1) \mid q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$. Now, we consider $k_2(PSU(3, q')) \mid k_2(G)$ so $q(q^3+1) = q'^2-q'+1$ it follows that $(q-1)(q^3+q^2+q+2) = \frac{q'-1-\sqrt{5}}{2}\frac{q'-1+\sqrt{5}}{2}$, which is a contradiction. Now, if $(3, q'+1) = 3$ then $\frac{q'^3(q'^3+1)(q'^2-1)}{3} \mid q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$ on the other hand, we consider $q(q^3+1) = \frac{q'^2-q'+1}{3}$. Hence, $3q(q^3+1) = q'^2-q'+1$ which is a contradiction, similarly.

Case(4).$K/H \ncong G_2(q')$, where $q' \equiv \pm 2(\mod 5)$. Suppose that $K/H \cong G_2(q')$. Now, by [21], $k_2(G_2(q')) = q'^2+q'$, where q' is prime. On the other hand, we know $|G_2(q')|$ divided $|G|$ as $q'^6(q'^6-1)(q'^2-1) \mid q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$. Now, we consider $k_2(G_2(q')) \mid k_2(G)$ so $q(q^3+1) = q'^2+q'$ it follows that $q(q^3+1) = q'(q'+1)$. Since that $(q', q'+1) = 1$ so $q = q'$ and $q^3+1 = q'+1$ it follows that $q' = q$, $q' = q^3$. Since $|G_2(q')| \nmid |G|$, which is a contradiction. Now, if q' be composite, then we consider $q(q^3+1) = q'^2-1$ it follows that $(q'-1)(q'+1) = q(q^3+1)$. Now, we know $(q'+1, q'-1) = 1$ or 2. First, assume $(q'+1, q'-1) = 1$ so $q'-1 = q$ and $q'+1 = q^3+1$ it follows that $q' = q+1$, $q' = q^3$. Since that $|G_2(q')| \nmid |G|$, which is a contradiction.

Case(5).$K/H \ncong^2 G_2(q')$, where $q' = 3^{2m+1}, m \geq 1$. Suppose that $K/H \cong^2 G_2(q')$. Now, by [21], $k_2(^2G_2(q')) = q'-1$. On the other hand, we know $|^2G_2(q')|$ divided $|G|$ as $q'^3(q'^3+1)(q'-1) \mid q^{12}(q^8+q^4+1)(q^6-1)(q^2-1)$. Now, we consider $k_2(^2G_2(q')) \mid k_2(G)$ so $q(q^3+1) = q'-1$ it follows that $q^4+q+1 = q'$. Since $|^2G_2(q')| \nmid |G|$, which is a contradiction.

Case(6) Hence $K/H \cong^3 D_4(q')$ it follows that $|K/H| = |G|$. First, if q' be composite, then we consider $q(q^3+1) = q'^4-q'^2+1$ it follows that $q^4+q = q'^4-q'^2+1$. So$q^4+q-2 = q'^4-q'^2-1$. Hence, $(q-1)(q^3+q^2+q+2) = \frac{q'-1-\sqrt{5}}{2}\frac{q'-1+\sqrt{5}}{2}$, which is a contradiction. It follows that q' must be prime. On the other hand $k_2(K/H) \mid k_2(G)$ as $q'(q'^3+1) = q(q^3+1)$ so $q = q'$. Next, G has a normal series $1 \trianglelefteq H \trianglelefteq K \trianglelefteq G$ so $H = 1$, then $K = G \cong D$. The proof be completed.

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