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Mathematical Modeling of Inventory Dynamics in a Two-Level Supply Chain with Freshness Constraints

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ABSTRACT: This paper explores the inventory dynamics of a two-level supply chain, where distribution centers supply fresh products to supermarkets while considering the impact of freshness degradation. We develop a mathematical model to analyze the inventory levels of a product at a distribution center over a supply interval, incorporating the coefficient of freshness decay, θ. The study derives equations governing inventory changes, initial stock levels, and replenishment quantities. The results indicate how freshness degradation influences inventory replenishment requirements, helping optimize stock levels at supermarkets. A graphical analysis further illustrates the impact of key parameters on inventory management and profit margins.

Key Words: Supermarket supply, freshness coefficient, inventory replenishment, stock level analysis, distribution center.

Contents

1. Introduction

The inventory level management is a concern for two warehouse simultaneously. [1] The study by Yadav et al. (2012) revolves around inventory management, particularly focusing on deteriorating items, which are products that degrade or lose value over time. [2] Singh et al. (2014) proposed a model that addresses inventory management in a reliable production process with producing and holding inventory varies based on the stock levels. [3] Sharma et al. (2023) proposed an approach for predicting optimum stock levels by utilizing Artificial Neural Networks (ANNs). Their study revolves around improving inventory management through data-driven methods, specifically using machine learning techniques. [4] Evgenii et al. (2019) proposed a model that addresses inventory management with a focus on a fixed replenishment period and periodic demand. Their model offers several key innovations and features that distinguish it from traditional inventory models. [5] Verma et al. (2018) developed an inventory model incorporating time-dependent linear holding costs. Their model focuses on optimizing inventory management by considering how holding costs change over time [6] Kumar et al. (2017) developed a two-warehouse inventory model designed to handle non-instantaneous deteriorating items. This type of model is particularly relevant in industries dealing with goods that deteriorate over time but do not spoil immediately. [7] Bhuniaa et al. (2013) focused on a deterministic inventory model for deteriorating items with two separate storage facilities: an owned warehouse (with limited capacity) and a rented warehouse. [8] R Susanto (2018) proposed a method aimed at minimizing the total inventory cost, which consists of two key components: ordering cost and carrying cost. The method focuses on optimizing inventory levels and aligning the number of raw materials ordered with the actual production needs, making the inventory management process more economically efficient. [9] Yadav et al. (2013) developed a two-warehouse inventory model specifically designed for deteriorating items, which incorporates several key

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assumptions and features to address the complexities of managing perishable goods. [10] Tripathia et al. (2014) proposed an order level inventory model focusing on a single warehouse where shortages are allowed and all shortages are completely backlogged. The model operates within a finite planning horizon and aims to identify an optimal cycle time that minimizes the total inventory cost per cycle. [11] Ban et al. (2018) investigate the data-driven news vendor problem when one has n observations of p features related to the demand as well as historical demand data. [12] Jackson et al. (2020) delved into the timeline of inventory control models and the various methodologies employed to derive optimal control parameters. Their analysis highlights how different methodologies contribute to improving inventory management systems, enhancing efficiency, and minimizing costs. [13] Tundura et al.(2016) explored three key inventory control strategies: cycle counting, inventory coding, and computerized inventory accuracy. [14] Aamir et al. (2020) investigated the dimensions of disruptive factors affecting inventory control in public healthcare facilities. Their research aims to identify the challenges that these facilities face in managing their inventory effectively, which is crucial for ensuring the availability of medical supplies and equipment. [15] Valmiki et al. (2024). presented a deterministic model for the inventory control of blood bank storage systems, accounting for item deterioration with increasing demand, and the effects of inbound and outbound logistics, ramp time demand, and inflation. [16] Alderremy et al.(2020) studied an accurate numerical method for solving the multi space-fractional Gardner equation (MSFGE) with the Caputo–Fabrizio (CF) and Atangana–Baleanu (AB) fractional derivatives where the space-fractional terms are under the sense of Caputo. [17] Shi.Lei et al. (2022) expressed inference about a general class of time series models including fractional Brownian motion. This paper is organized as follows, section 2, describe about the symbols which are using for formation of model. Here we defined the weibull distribution and demand of material which proportional to sells price and exponential with time. In Section2, we prepare model to maximize the profit and calculate the total cost. In section 3, we present the sensitive analysis and present in graphical manner. These graph prepare with the MATLAB and showing the relation between different parameters like Q, p, t. In section 4, we put the observation of the graph which help to get the results. section5 conclude the paper, here the total cost of inventory HC is vary with the inventory cost I_1 and I_2.

2. Notation and Model Formulation

In this article we discuss the $k = 1, 2$ distribution center supply the product to $j = 1, 2, 3$ supermarkets according to demand and quantitative change in coefficient of freshness θ_2 at distribution center.The inventory level of product j of the distribution center k at time t during the supply interval $[(z_{ji} - 1)t_{ji}, z_{ji}t_{ji}]$ is

$$\frac{dq_{ji}(t)}{dt} = -(\theta)q_{ji}(t) \tag{2.1}$$

Where $q_{ji}(t)$ = During the supply interval z_{ji}, the inventory level at time t. At the end of supply interval $[(z_{ji} - 1)t_{ji}, z_{ji}t_{ji}]$ that is when $t = z_{ji}t_{ji}$, the closing inventory of the fresh medicine product j at distribution center k is equal to the sum of the initial inventory g_{ji} in interval $[z_{ji}t_{ji}, (z_{ji} + 1)t_{ji}]$.

$$q_{ji}(z_{ji}.t_{ji}) = g_{ji}^{z_{ji+1}} + g_{ji} \tag{2.2}$$

Combining both

$$q_{ji}^{z_{ji}}(t) = (g_{ji}^{z_{ji}+1} + g_{ji})e^{-\theta.(t - z_{ji}t_{ji})} \tag{2.3}$$

When $t = (z_{ji} - 1)t_{ji}$ can get the initial inventory of the supply interval

$$q_{ji}((z_{ji} - 1)t_{ji}) = (g_{ji}^{z_{ji}+1} + g_{ji})e^{\theta t_{ji}} \tag{2.4}$$

According to the analysis of inventory level changes of distribution center closing inventory, when $t = b_{ji}.t_{ji}$ of the product in distribution center at the supply interval

$$\begin{aligned} g_{ji} &= q_{ji}(b_{ji}t_{ji}) \\ &= (g_{ji}^{b_{ji}+1} + g_{ji})e^{-\theta_2}(b_{ji}t_{ji} - b_{ji}t_{ji}) \\ &= g_{ji} + g_{ji}^{b_{ji}+1} \end{aligned} \tag{2.5}$$

From the above, we get $g_{ji}^{b_{ji}+1} = 0$ when $t = (z_{ji} - 1)t_{ji}$, the initial inventory of supply interval $[(z_{ji} - 1)t_{ji}, z_{ji}t_{ji}]$ is

$$\begin{aligned} g_{ji}^{z_{ji}} &= q_{ji}^{z_{ji}}[(z_{ji} - 1)t_{ji}] \\ &= (g_{ji} + g_{ji}^{z_{ji}+1})e^{(\theta_2.t_{ji})} \end{aligned} \tag{2.6}$$

So the initial inventory $g_{ji}^{b_{ji}}$ of the supply interval b_{ji} can be obtained. That is

$$g_{ji}^{b_{ji}-1} = (g_{ji} + g_{ji}^{b_{ji}})e^{(\theta_2.t_{ji})} \tag{2.7}$$

by the equation 2.8 and 2.7, we get

$$g_{ji}^{b_{ji}-1} = g_{ji}(e^{\theta_2.t_{ji} + e^{2\theta_2 t_{ji}}}) \tag{2.8}$$

again continue with this

$$g_{ji}^{b_{ji}-1} = g_{ji}(e^{\theta_2.t_{ji}} + e^{2\theta_2 t_{ji}} + e^{3\theta_2 t_{ji}}) \tag{2.9}$$

We can get the initial inventory $g_{ji}^{z_{ji}}$ of the product j at the distribution center of the supply intervalz_{ji} when $t = (z_{ji} - 1).t_{ji}$ which can

$$g_{ji}^{z_{ji}+1} = g_{ji}(e^{\theta_2.t_{ji} + e^{2\theta_2 t_{ji}}} + e^{3\theta_2 t_{ji}} \ldots\ldots\ldots + e^{(b_{ji} - z_{ji} + 1).(\theta_2.t_{ji})}) \tag{2.10}$$

Now put the value from equation 2.11 to 2.2

$$q_{ji}^{(z_{ji}.t_{ji})} = g_{ji}\left[1 + e^{\theta_2.t_{ji} + e^{2\theta_2 t_{ji}}} + e^{3\theta_2 t_{ji}} \ldots\ldots\ldots + e^{(b_{ji} - z_{ji})(\theta_2.t_{ji})}\right] + e^{-\theta_2(t - z_{ji}t_{ji})} \tag{2.11}$$

The total inventory of the product j supplied to supermarket i

$$q_{ji}^{z_{ji}} = \int_{(z_{ji}-1)t_{ji}}^{(z_{ji}t_{ji})} q_{ji}^{(z_{ji}.t_{ji})}(t)dt \tag{2.12}$$

By put the value of $q_{ji}^{(z_{ji}.t_{ji})}$ from equation 2.12 to 2.13, we get

$$= -\frac{1}{\theta_2}\left[1 - e^{(b_{ji} - z_{ji} + 1)(\theta_2.t_{ji})}\right].g_{ji} \tag{2.13}$$

This is quantity requirement to fill the inventory stock in time interval $[(z_{ji} - 1)t_{ji}, z_{ji}t_{ji}]$ for item i by distribution center to supermarket.

3. Graphical Presentation

To understand the effects of change in parameters on the profit margin, a analysis can be performed. In this we will analysis in percentage change in the profit with respect to different parameters like order quantity Q, cost HC and time duration t of inventory in both warehouses.

4. Observation

1. Figure 1 illustrates that the quantity follows an exponential growth pattern with respect to the freshness rate θ_2. In this the range of θ_2 has taken $[0, 1]$. At the level of θ_2 that is near one, the quantity of demand increases tremendously.

2. Figure 2 illustrates the behavior of quantity demand within the range of θ_2 from $-1$to 0, where it exhibits an exponential increase. However, considering the full range $[-1, 1]$, the demand remains relatively unchanged for values of θ_2 up to 0. Once θ_2 surpasses 0, the quantity demand increases drastically. This suggests that demand is highly sensitive to the freshness of items, particularly for positive values of θ_2.

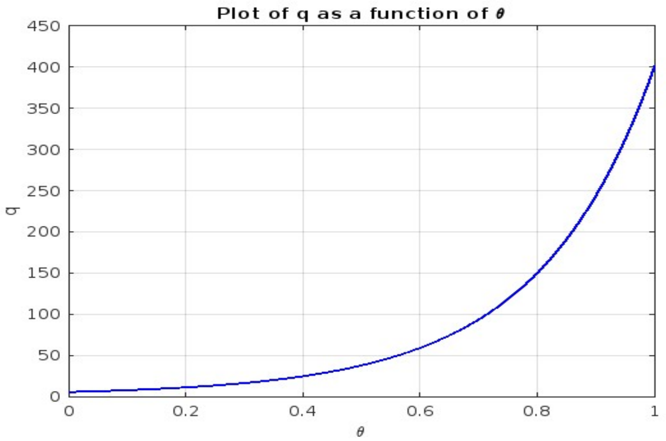

Figure 1:

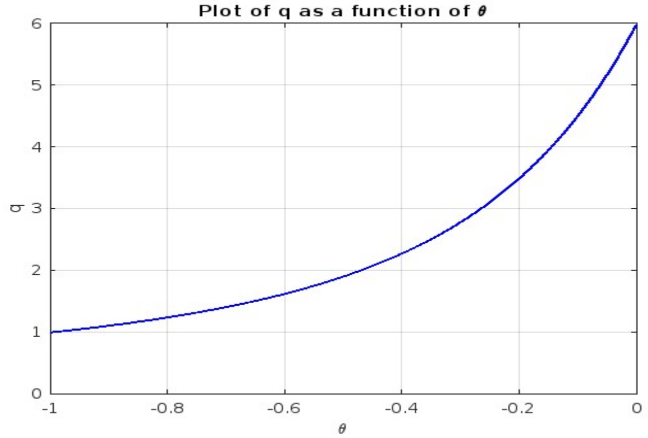

Figure 2:

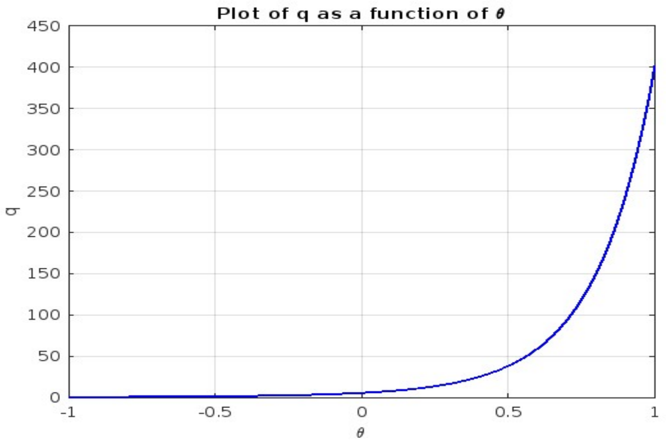

Figure 3:

5. Conclusion

This study explores the inventory dynamics of a two-level supply chain, focusing on the impact of freshness degradation on replenishment strategies. A mathematical model was developed to analyze inventory levels at distribution centers and their replenishment needs based on the freshness decay coefficient θ_2. The findings indicate that demand exhibits an exponential growth pattern with increasing freshness, significantly influencing inventory management decisions. Graphic analysis highlights the sensitivity of demand to freshness levels, particularly when θ_2 is positive, demonstrating a drastic increase in quantity demand. This insight underscores the importance of optimizing inventory replenishment to balance stock levels while minimizing waste. By understanding how freshness degradation affects supply chain efficiency, businesses can make informed decisions to improve profitability and ensure product availability at supermarkets. Future research can extend this model by incorporating real-world data and exploring advanced optimization techniques, such as machine learning and predictive analytics, to further enhance inventory management strategies.

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References

1. Yadav, D., Singh, S. R., & Kumari, R. (2012). Inventory model of deteriorating items with two-warehouse and stock-dependent demand using genetic algorithm in fuzzy environment. *Yugoslav Journal of Operations Research*, **22**(1), 51-78.

2. Singh, S. R., & Rathore, H. (2014). An inventory model for deteriorating item with reliability consideration and trade credit. *Pakistan Journal of Statistics and Operations Research*, **10**(3), 349-360.

3. Sharma, G. S., & Baghel, R. (2023). Artificial neural network approach for inventory management problem. *International Journal of Mathematical and Computational Sciences*, **17**(2), 160-164.

4. Anisimov, E. G., Gapov, M. R., Rodionova, E. S., & Saurenko, T. N. (2019). The model for determining rational inventory in occasional demand supply chains. *International Journal of Supply Chain Management*, **8**(2), 86-90.

5. Verma, S. K., Rizwanullah, M., & Singh, C. (2018). An EOQ model for deteriorating items with selling price-dependent exponential demand for time varying holding and deterioration costs. *International Journal of Engineering and Technology*, **7**(3), 36-41.

6. Kumar, S., Chakraborty, D., & Malik, A. K. (2017). A two-warehouse inventory model with stock-dependent demand and variable deterioration rate. *International Journal on Future Revolution in Computer Science and Communication Engineering*, **3**(4), 20-24.

7. Bhunia, A. K., Shaikh, A. A., Maiti, A. K., & Maiti, M. (2013). A two-warehouse deterministic inventory model for deteriorating items with a linear trend in time-dependent demand over finite time horizon by elitist real-coded genetic algorithm. *International Journal of Industrial Engineering Computations*, **4**(2), 241-248.

8. Susanto, R. (2018). Raw material inventory control analysis with economic order quantity method. *Materials Science and Engineering*, **407**, 1-5.

9. Yadav, A. S., & Swami, A. (2013). A two-warehouse inventory model for decaying items with exponential demand and variable holding cost. *International Journal of Inventive Engineering and Sciences*, **1**(6), 18-22.

10. Tripathi, R. P., & Mishra, S. M. (2014). Inventory model with inventory-dependent demand for deteriorating items in a single warehouse system. *Uncertain Supply Chain Management*, **2**(4), 209-218.

11. Ban, G. Y., & Rudin, C. (2018). The big data news vendor: Practical insights from machine learning. *Production and Operations Management*, **27**(1), 90-108.

12. Jackson, I., Tolujevs, J., & Kegenbekov, Z. (2020). Review of inventory control models: A classification based on methods of obtaining optimal control parameters. *Transport and Telecommunication*, **21**(2), 191-202.

13. Tundura, L., & Wanyoike, D. (2016). Effect of inventory control strategies on inventory record accuracy in Kenya Power Company, Nakuru. *Journal of Investment and Management*, **5**(5), 82-92.

14. Hashmi, A. R., Amirah, N. A., Yusof, Y., & Zaliha, T. N. (2020). Exploring the dimensions using exploratory factor analysis of disruptive factors and inventory control. *The Economics and Finance Letters*, **7**(3), 247-254.

15. Valmiki, A. K., Sharma, G. S., Deepak, & Gour, M. M. (2024). Deterministic model for blood bank inventory control with increasing demand and logistic considerations using multi-objective optimization. *REDVET - Revista Electrónica de Veterinaria*, **25**(2).

16. Alderremy, A. A., Khaled M. Saad, Agarwal.P, Shaban Aly & Jain. S. (2020). Certain new models of the multi space-fractional Gardner equation. *Physica A: Statistical Mechanics and its Applications*, **545**.

17. Shi.lei, Jain. Shilpi, Agarwal. Praveen , Altayed. Yousif, & Momani. Shaher Ti (2022). Asymptotic Analysis About the Periodogram of a General Class of Time Series Models With Spectral Supportson Lines Not Parallel to the Main Diagonal. *fractal*, **30**(17).

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