



Estimate The Fractional Reliability and The Fractional Entropy of Computer Server

Abeer A. Abdul-Razaq*, M.Jasim Mohammed and Zahraa A. Nema

ABSTRACT: Reliability is a critical aspect of systems and processes, reflecting their ability to perform consistently over time without failure. Fractional entropy extends traditional entropy measures by accommodating non-integer orders of differentiation, allowing for a more nuanced understanding of uncertainty and complexity in systems. By systems integrating fractional entropy into reliability assessments, researchers can better model the dynamics of complex systems, offering improved predictions and strategies for maintenance and optimization so, in this paper explores the interrelationship between reliability And fractional entropy, two pivotal concepts in the analysis of complex systems . so, in this paper we will compute the reliability , entropy and fractional entropy of computer server.The MATLAB program used in this paper.

Key Words: Reliability , minimal path , fractional entropy, entropy.

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1. Introduction

Reliability and fractional entropy are fundamental concepts that provide valuable insights into the behavior of complex systems across various disciplines, including engineering, information theory, and statistical mechanics. Reliability is defined as the probability that a system will perform its intended function without failure under specified conditions for a designated period. It is a critical measure in ensuring that products, processes, and systems meet performance expectations and safety standards. High reliability is essential in sectors such as aerospace, automotive, and healthcare, where failures can lead to catastrophic consequences. Reliability engineering employs various statistical methods and models to assess and enhance the dependability of systems, focusing on failure rates, maintenance strategies, and life cycle analyses [2,5]. In contrast, fractional entropy emerges from the study of complex systems that exhibit self-similar patterns and scale-invariance, often described by fractal geometry. [6], measures the uncertainty or information content in a system. However, many real-world phenomena do not adhere to classical assumptions, necessitating a more flexible approach. Fractional entropy captures the intricacies of such systems by quantifying their disorder and complexity, thereby extending the applicability of entropy in contexts where traditional measures may be inadequate [3]. The relationship between reliability and fractional entropy is increasingly recognized as essential for understanding the dynamics of complex

* Corresponding author

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systems. By integrating the reliability of a system with its fractional entropy, researchers can gain a deeper understanding of how uncertainty and complexity influence performance and failure [13,14]. This introduction sets the stage for a detailed exploration of these concepts, examining their definitions, applications, and the potential synergies that arise from their intersection .so in this section will compute the reliability and Shannon entropy and feactional entropy of computer server as follows. For more information see [18,19,8,20].

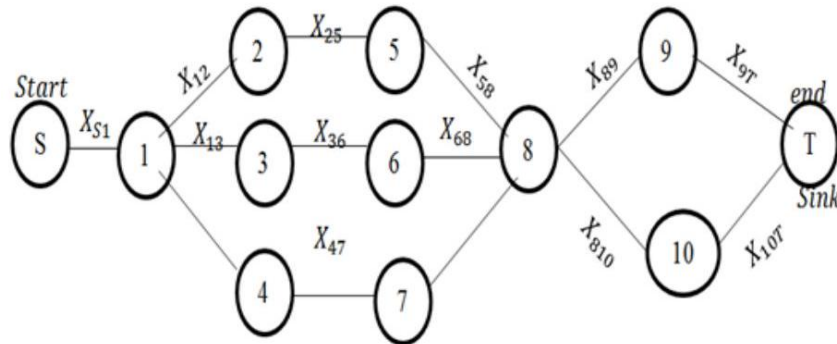


Figure 1: computer server

2. Basic Concept

1. Parallel. System. Reliability: refers to a configuration in which the system succeeds as long as at least one of its components functions properly. To evaluate the reliability of such a system, specific probabilistic methods are employed.

$$R_{sys} = 1 - \prod_{i=1}^n (1 - R_i) \quad (2.1)$$

2. A minimal path consists of the smallest possible set of components that ensures system success. The removal of any component from this set results in the failure of the path .
3. A cut set represents a specific combination of components whose simultaneous failure results in overall system failure, regardless of the operational state of the remaining components.
4. A minimal cut is a set of elements that together form a cut; however, if any single element is removed, the remaining set no longer constitutes a cut.
5. Reliability system is a probability that the system will work for aperiodic time.
6. Fractional differential term:

The most commonly seen definition of a fractional order integral is via an integral transform known as the Riemann Liouville integral. As a result, the generalization to non-integer q is as follows [16,1,9]:

$$\frac{d^q f}{dx^q} = \frac{1}{\Gamma(-q)} \int_0^x (x-y)^{-q-1} f(y) dy, q < 0 \quad (2.2)$$

$$\frac{d^q f}{dx^q} = \frac{1}{\Gamma(n-q)} \frac{d^n}{dx^n} \int_0^x (x-y)^{n-q-1} f(y) dy, q < 0 \quad (2.3)$$

3. Generating of Minimal Cuts of Computer Server

This method utilizes the set of all minimal paths to construct an incidence matrix. Assuming that n minimal paths exist, labeled P_1, P_2, \dots, P_n , Algorithm (2.1) is applied to systematically generate the incidence matrix (IM) representing the relationships among the system components and the minimal paths.

$$IM = \begin{matrix} & & \begin{matrix} \text{Edges} \\ x_1 & x_2 & \cdots & x_n \end{matrix} \\ \begin{matrix} P_1 \\ P_2 \\ \vdots \\ P_m \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \end{matrix}$$

Where $i = 1, 2, \dots, m, j = 1, 2, \dots, n$, and $a_{ij} \in \{0, 1\}$. A state is defined as an n -dimensional row vector consisting of 0s and 1s. The element $a_{ij} = 1$ if and only if $x_j \in P_i$; otherwise, $a_{ij} = 0$. To generate the minimal cut sets, three steps must be followed. Step 1 : A component x_j produces a first-order diagonal cut if $a_{ij} = 0$ for all i in column x_j of the incidence matrix (IM).

Step 2 : Instantaneously merge two columns of the incidence matrix (IM). Assume that the combination $x_j x_{(j+k)}$ represents a second-order cut if and only if $a_{ij} + a_{i(j+k)} = 0$ for all i , and for all $k = 1, \dots, n$. Continue this process to identify all possible second-order cut sets.

Step 3 : To obtain the third-order cuts, repeat the process described in Step 2 by selecting combinations of three columns at a time. Ensure that any resulting cuts which overlap with previously identified first- or second-order cuts are excluded. Proceed iteratively to identify only the unique third-order minimal cut sets. The procedure is repeated until the highest order of cut sets is reached [9, 10]. The minimal path method is then applied and executed using MATLAB, a digital computing tool, to determine the minimal paths associated with the computer server.

$$\begin{aligned} (P1) &= X_{S1} X_{12} X_{25} X_{58} X_{89} X_{9T} \\ (P2) &= X_{S1} X_{12} X_{25} X_{58} X_{810} X_{10T} \\ (P3) &= X_{S1} X_{13} X_{36} X_{68} X_{89} X_{9T} \\ (P4) &= X_{S1} X_{13} X_{36} X_{68} X_{810} X_{10T} \\ (P5) &= X_{S1} X_{14} X_{47} X_{78} X_{810} X_{10T} \\ (P6) &= X_{S1} X_{14} X_{47} X_{78} X_{89} X_{9T} \end{aligned}$$

Following the application of the minimal cut method to the identity matrix representing the computer server system, the resulting incidence matrix IM = is obtained. as follows:

$$\begin{matrix} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{matrix} \begin{bmatrix} x_{s1} & x_{12} & x_{25} & x_{58} & x_{13} & x_{36} & x_{68} & x_{14} & x_{47} & x_{78} & x_{89} & x_{81} & x_{9T} & x_{10} \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

This process identifies all minimal cuts of the computer server.

$$\begin{aligned} &\{[x_{51}], [x_{12}, x_{13}, x_{14}], [x_{25}, x_{36}, x_{47}], [x_{58}, x_{68}, x_{78}], [x_{89}, x_{810}], [x_{97}, x_{107}], [x_{12}, x_{36}, x_{47}], [x_{12}, x_{36}, x_{78}], \\ &[x_{12}, x_{36}, x_{810}], [x_{12}, x_{36}, x_{10T}], [x_{14}, x_{36}, x_{58}], [x_{14}, x_{36}, x_{89}], [x_{14}, x_{36}, x_{9T}], [x_{14}, x_{13}, x_{25}], [x_{14}, x_{68}, x_{58}] \\ &[x_{14}, x_{68}, x_{89}], [x_{13}, x_{25}, x_{810}], [x_{13}, x_{25}, x_{10T}], [x_{13}, x_{78}, x_{89}], [x_{13}, x_{78}, x_{97}], [x_{25}, x_{68}, x_{78}], [x_{25}, x_{810}, x_{89}] \\ &[x_{25}, x_{36}, x_{78}], [x_{89}, x_{10T}], [x_{810}, x_{97}], [x_{47}, x_{68}, x_{89}], [x_{47}, x_{68}, x_{97}], [x_{58}, x_{68}, x_{810}], [x_{58}, x_{68}, x_{10T}], \\ &[x_{68}, x_{78}, x_{89}], [x_{68}, x_{78}, x_{97}], [x_{78}, x_{89}, x_{810}]\} \end{aligned}$$

4. Evaluation The Reliability of Computer Server

To achieve the minimum number of cuts using the minimal cut method, the system is modeled as a combination of parallel and series configurations [7,11]. The reliability of the computer system is then determined using Equation (4.1).

$$R_{yys} = 1 - \prod_{i=1}^n (1 - R_i) \quad (4.1)$$

By using Eq.(4.1), obtain the reliability of the computer server

$$\begin{aligned} R_{sys} = & R_{12}R_{25}R_{58}R_{10T}R_{810}R_{s1} + R_{12}R_{36}R_{68}R_{10T}R_{810}R_{s1} + R_{14}R_{47}R_{78}R_{10T}R_{810}R_{s1} + R_{12}R_{25} \\ & R_{58}R_{84}R_{9T}R_{s1} + R_{12}R_{36}R_{36}R_{68}R_{84}R_{9T}R_{s1} + R_{14}R_{47}R_{78}R_{84}R_{9T}R_{s1} - R_{12}R_{25}R_{36}R_{58}R_{68}R_{10T}R_{810} \\ & R_{s1} - R_{12}R_{25}R_{36}R_{58}R_{68}R_{84}R_{9T}R_{s1} - R_{12}R_{25}R_{58}R_{84}R_{10T}R_{810}R_{9T}R_{s1}R_{12}R_{36}R_{68}R_{84}R_{10T}R_{810} \\ & R_{9T}R_{s1} - R_{14}R_{47}R_{78}R_{84}R_{10T}R_{810}R_{9T}R_{s1} - R_{12}R_{14}R_{25}R_{47}R_{58}R_{78}R_{10T}R_{810}R_{s1} - R_{12}R_{14}R_{36} \\ & R_{47}R_{68}R_{78}R_{10T}R_{810}R_{s1} - R_{12}R_{14}R_{25}R_{47}R_{58}R_{78}R_{84}R_{9T}R_{s1} - R_{12}R_{14}R_{36}R_{47}R_{68}R_{78}R_{84}R_{9T}R_{s1} \\ & + R_{12}R_{25}R_{36}R_{58}R_{68}R_{84}R_{10T}R_{810}R_{9T}R_{s1} + R_{12}R_{14}R_{25}R_{36}R_{47}R_{58}R_{68}R_{78}R_{10T}R_{810}R_{s1}R_{12}R_{14}R_{25} \\ & R_{36}R_{47}R_{58}R_{68}R_{78}R_{84}R_{9T}R_{s1} + R_{12}R_{14}R_{25}R_{47}R_{58}R_{78}R_{84}R_{10T}R_{810}R_{9T}R_{s1} + R_{12}R_{14}R_{36}R_{47}R_{68}R_{78} \\ & R_{84}R_{10T}R_{810}R_{9T}R_{s1} - R_{12}R_{14}R_{25}R_{36}R_{47}R_{58}R_{68}R_{78}R_{84}R_{10T}R_{810}R_{9T}R_{s1} \end{aligned} \quad (4.2)$$

5. The Probability Density Function And Fractional Probability Function of Computer Server

In this section, the probability density function of computer server will be calculated. For this if the data relates to server response time, the appropriate distribution is often the exponential distribution. By the probability theory the reliability system is defined

$$R(x) = p(X > x) = 1 - P(X \leq x) = 1 - F(X) \quad (5.1)$$

Since the distribution of computer server is exponential then the cumulative Density probability (cdf) is

$$F(x) = 1 - e^{-\lambda x} \quad \text{where } \lambda \text{ greater than zero}$$

By substitute equation (5.1) in equation (4.2) gets

$$R_{sys} = 6e^{-6\lambda x} - 5e^{-8\lambda x} - 4e^{-9\lambda x} + e^{-10\lambda x} + 4e^{-11\lambda x} - e^{-13\lambda x} \quad (5.2)$$

In probability theory there is a relationship between the reliability and a pdf

$$f(x) = -dR_{sys} \quad (5.3)$$

So the probability density function of computer server is

$$f(x) = 36e^{-6\lambda x} - 40e^{-8\lambda x} - 36e^{-9\lambda x} + 10e^{-10\lambda x} + 44e^{-11\lambda x} - 13e^{-13\lambda x} \quad (5.4)$$

In this section will find the relationship between the reliability and fraction calculus by reformulating equation (5.3) as follows :

$$f(t) = -D^{1/2}D^{1/2}R_{sys}(t) \quad (5.5)$$

by substitute equation (5.2) in equation (5.5), gets

$$\begin{aligned}
f(x) &= -D^{\frac{1}{2}} D^{\frac{1}{2}} R_{sys}(x) \\
&= -D^{\frac{1}{2}} D^{\frac{1}{2}} \left[6e^{-6\lambda x} - 5e^{-8\lambda x} - 4e^{-9\lambda x} + e^{-10\lambda x} + 4e^{-11\lambda x} - e^{-13\lambda x} \right] \\
&= D^{\frac{1}{2}} \left[6D^{\frac{1}{2}} e^{-6\lambda x} - 5D^{\frac{1}{2}} e^{-8\lambda x} - 4D^{\frac{1}{2}} e^{-9\lambda x} + D^{\frac{1}{2}} e^{-10\lambda x} + 4D^{\frac{1}{2}} e^{-11\lambda x} - D^{\frac{1}{2}} e^{-13\lambda x} \right] \\
&= D^{\frac{1}{2}} \left[6 \frac{1}{\Gamma^{\frac{1}{2}}} \frac{d}{dx} \int_0^x (x-t)^{1-\frac{1}{2}-1} e^{-6\lambda t} dt - 5 \frac{1}{\Gamma^{\frac{1}{2}}} \frac{d}{dx} \int_0^x (x-t)^{-\frac{1}{2}} e^{-8\lambda t} dt \right. \\
&\quad - 4 \frac{1}{\Gamma^{\frac{1}{2}}} \frac{d}{dx} \int_0^x (x-t)^{-\frac{1}{2}} e^{-9\lambda t} dt + \frac{1}{\Gamma^{\frac{1}{2}}} \frac{d}{dx} \int_0^x (x-t)^{-\frac{1}{2}} e^{-10\lambda t} dt \\
&\quad \left. + 4 \frac{1}{\Gamma^{\frac{1}{2}}} \frac{d}{dx} \int_0^x (x-t)^{-\frac{1}{2}} e^{-11\lambda t} dt - \frac{1}{\Gamma^{\frac{1}{2}}} \frac{d}{dx} \int_0^x (x-t)^{-\frac{1}{2}} e^{-13\lambda t} dt \right] \\
&= D^{-\frac{1}{2}} \left[\frac{6}{\sqrt{\pi}} \frac{d}{dx} \left(-\sqrt{6}\sqrt{\pi} e^{-6\lambda x} \right) - \frac{5}{\sqrt{\pi}} \frac{d}{dx} \left(-\sqrt{8}\sqrt{\pi} e^{-8\lambda x} \right) - \frac{4}{\sqrt{\pi}} \frac{d}{dx} \left(-\sqrt{9}\sqrt{\pi} e^{-9\lambda x} \right) \right. \\
&\quad \left. + \frac{1}{\sqrt{\pi}} \frac{d}{dx} \left(-\sqrt{10}\sqrt{\pi} e^{-10\lambda x} \right) + \frac{4}{\sqrt{\pi}} \frac{d}{dx} \left(-\sqrt{11}\sqrt{\pi} e^{-11\lambda x} \right) - \frac{1}{\sqrt{\pi}} \frac{d}{dx} \left(-\sqrt{13}\sqrt{\pi} e^{-13\lambda x} \right) \right] \\
&= D^{-\frac{1}{2}} \left[36\sqrt{6}\lambda e^{-6\lambda x} - 40\sqrt{8}\lambda e^{-8\lambda x} - 36\sqrt{9}\lambda e^{-9\lambda x} + \sqrt{10}\lambda e^{-10\lambda x} + 44\sqrt{11}\lambda e^{-11\lambda x} - 13\sqrt{13}\lambda e^{-13\lambda x} \right] \\
&= - \left[-216\sqrt{6}\lambda^2 e^{-6\lambda x} + 320\sqrt{8}\lambda^2 e^{-8\lambda x} + 324\sqrt{9}\lambda^2 e^{-9\lambda x} - \sqrt{4}\lambda^2 e^{-10\lambda x} - 484\sqrt{11}\lambda^2 e^{-11\lambda x} + 169\sqrt{13}\lambda^2 e^{-13\lambda x} \right] \\
&= 216\sqrt{6}\lambda^2 e^{-6\lambda x} - 320\sqrt{8}\lambda^2 e^{-8\lambda x} - 324\sqrt{9}\lambda^2 e^{-9\lambda x} + \sqrt{4}\lambda^2 e^{-10\lambda x} + 484\sqrt{11}\lambda^2 e^{-11\lambda x} - 169\sqrt{13}\lambda^2 e^{-13\lambda x}
\end{aligned}$$

So the fractional probability function is

$$\text{FC-f}(t) = 216\sqrt{6}\lambda^2 e^{-6\lambda x} - 320\sqrt{8}\lambda^2 e^{-8\lambda x} - 324\sqrt{9}\lambda^2 e^{-9\lambda x} + \sqrt{4}\lambda^2 e^{-10\lambda x} + 484\sqrt{11}\lambda^2 e^{-11\lambda x} - 169\sqrt{13}\lambda^2 e^{-13\lambda x} \quad (5.6)$$

6. Shannon Entropy of Computer Server

Shannon entropy, referred to as information entropy or Shannon information serves as a metric, for determining the level of information, uncertainty or randomness in a discrete random variable or an information source. This concept was introduced by Claude Shannon within the realm of information theory

Shannon entropy is defined by

$$H = -E(\log(p(x)))$$

Where $p(x)$ is a probability of a system being in cell x of its phase [15].

By displaying equation(5.4) in equation(6) optine

$$\begin{aligned}
H &= -E \left(\log \left(36e^{-6\lambda x} - 40e^{-8\lambda x} - 36e^{-9\lambda x} + 10e^{-10\lambda x} + 44e^{-11\lambda x} - 13e^{-13\lambda x} \right) \right) \\
&= \int_0^{\infty} \lambda e^{-\lambda x} \left(\log \left(36e^{-6\lambda x} - 40e^{-8\lambda x} - 36e^{-9\lambda x} + 10e^{-10\lambda x} + 44e^{-11\lambda x} - 13e^{-13\lambda x} \right) \right) dx
\end{aligned}$$

So $H = 0.09892$

7. Fractional Entropy of Computer Server

Fractional entropy is a measure of the uncertainty or disorder in a system with fractional probability. It is derived using the [12,17,10].

$$Sq(x) = Sq(p) = E[-\log Pi], 0 < q \leq 1$$

Where $Sq(x)$ is the fractional entropy, P_i represents the probabilities of thith event, and q is the exponent determining the degree of fraction alization the fractional entropy is concare, positive and non additive. More over for $q = 1$, the fractional entropy reduces to the shannon Entropy.

$$Sq(x) = Sq(p) = P_i[-\log P_i]qdx, 0 < q \leq 1 \quad (7.1)$$

$$Sq(x) = Sq(p) = \int_0^\infty P[-\log P_i]^q dx, 0 < q \leq 1 \quad (7.2)$$

Where p_i is the Fractional pdf of Iraqi super grid , By substitute equation (5.6) in equation (7.2) gets

$$\begin{aligned} Sq(x) = \int_0^\infty & 216\sqrt{6}\lambda^2 e^{-6\lambda x} - 320\sqrt{8}\lambda^2 e^{-8\lambda x} - 324\sqrt{9}\lambda^2 e^{-9\lambda x} + \sqrt{4}\lambda^2 e^{-10\lambda x} + 484\sqrt{11}\lambda^2 e^{-11\lambda x} \\ & - 169\sqrt{13}\lambda^2 e^{-13\lambda x} \left[-\log 216\sqrt{6}\lambda^2 e^{-6\lambda x} - 320\sqrt{8}\lambda^2 e^{-8\lambda x} - 324\sqrt{9}\lambda^2 e^{-9\lambda x} + \sqrt{4}\lambda^2 e^{-10\lambda x} \right. \\ & \left. + 484\sqrt{11}\lambda^2 e^{-11\lambda x} - 169\sqrt{13}\lambda^2 e^{-13\lambda x} \right]^q dx \end{aligned}$$

After intgrate it by matlab get
Fractional Entropy $Sq(x) = 0.0026$

8. Conculution

Together, reliability and fractional entropy offer complementary insights. While reliability metrics help ensure the performance and stability of systems, fractional entropy helps quantify the complexity and unpredictability inherent in these systems. A thorough understanding of both concepts can lead to more robust designs and improved predictive models, ultimately enhancing system performance and resilience in the face of uncertainty. so in this paper the reliability and fractional entropy are compute . We note here that fractional entropy is better than shannon entropy, as the less the entropy is than one, the more reliable and accurate the result is, whereas the shannon entropy equale 0.098923 and fractional entropy equale 0.0026

That's mean the fractional entropy is beter than the Shannon entropy ,as it cover all the rate of component of computer server .

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Abeer A. Abdul-Razaq,
Department of Mathematics,
College of Education for pure Science University of Thi-Qar, Thi-Qar,
Iraq.
E-mail address: Abeeraladub80@gmail.com

and

M.Jasim Mohammed,
Department of Mathematics,
College of science,University Of Anbar, Ramadi,
Iraq.
E-mail address: mohadmath87@uoanbar.edu.iq

and

Zahraa A. Nema,
Ministry of Education, Thi-Qar Education Directorate
E-mail address: Zahalshabib@gmail.com