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Applying Some Dominating Parameters on Certain Topological Graphs

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ABSTRACT: The idea of construct a graph from special topological space is a new approach in graph theory. In this paper, general domination numbers and dominating sets are evaluated for the special topological graph G_τ that constructed from the discrete topological space. Also, the dominating set of corona and join operations between two topological graphs are given. The inverse dominating set and inverse domination number are studied for G_τ. Then, the inverse dominating set of join and corona operations between two topological graphs are introduced. All results are applied in several topological graphs.

Key Words: Dominating set; discrete topology; topological graphs.

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1. Introduction

Let $G = (V, E)$ be a graph has a vertex set V and edge set E . The subgraph M of G is induced subgraph denoted by $G[M]$, if contains all vertices of $V(M)$, and all edges that incident to they. The join operation between any two graphs $G+M$, such that $V(G)\cup V(M)$ is all vertices of it and $E(G)\cup E(M)$ is all edges of it, which are each vertex of G is adjacent with all vertices of M. The corona operation between two graphs $G \odot M$has one copy of G (that have r vertices) and r copies of M, such that joining the j^{th} vertices of G with each vertex in the j^{th} copy of M. All of the above concepts and for more information can be found, see [26]. A subset D is dominating set if each vertex in V is either belong to D or adjacent with a vertex in D. The domination number $\gamma(G)$ is the number of all vertices of minimum dominating set D. The inverse dominating set D^{-1} is a dominating set exist in $V - D$. The inverse domination number and denoted by $\gamma^{-1}(G)$ is a number of all vertices in D^{-1}. For more detailed and information about domination, see [18]. There are many types of dominating sets applied in many different graphs. Some parameters of domination putted a conditions on the dominating set such as [5,7,9,11,13,15,16,17,25]. Another dominating parameters suggested a conditions on the complement dominating set $V - D$ such as [8]. So that, more papers discussed the stability of domination parameters [1,3,14]. Other papers studied the domination on the edge set, such as [27,28]. While, some types of dominating definitions assess a conditions on both sets, the dominating set D and its complement set $V - D$, such as [2,4,6]. Some papers deals with relations between domination of graph and another branches of Mathematics such as formed a graph from certain modules [12,29] or transform special kinds of topological spaces into graphs say "topological graphs" with distinct properties and applied several dominating results on that topological graphs [10,19,20,21,22,23]. In our previous paper [24], we construct a new graph called topological graph (G_τ) by applied special relations on the discrete topological space. More properties and results are proved for this graph. In this paper, we continue to give some new properties by prove the general dominating set and domination number for G_τ. Also, by prove the dominating set of corona and join operations between two topological graphs. The inverse dominating set and inverse domination number are studied for G_τ. So that, the inverse dominating set of corona and join operations between two topological graphs are proved.

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2. Certain Topological Graphs

In this section, many properties for the certain topological graph G_τ are proved. New definition and examples with some properties will discuss.

Definition 2.1 *Let (X, τ) be a discrete topological space. The discrete topological graph denoted by $G_\tau = (V, E)$ is a graph with vertex set $V = \{A; A \in \tau$ and $A \neq \phi, X\}$ and edge set $E = \{AB; A \nsubseteq B$ and $B \nsubseteq A\}$.*

Proposition 2.1 *let (X, τ) be a discrete topological space, if $n = 2$, then $G_\tau \cong K_2$.*

Proof: *Let$X = \{1, 2\}$, then $\tau = \{\phi, X, \{1\}, \{2\}\}$ so $V = \{\{1\}, \{2\}\}$. Since $\{1\}$ is not subset of $\{2\}$ and $\{2\}$ is not subset of $\{1\}$. Then, there is an edge between them. Then, the graph G_τ isomorphic to K_2.* □

Proposition 2.2 *Let $|X| = n$, and (X, τ) be a discrete topological graph, then the graph (X, τ) has $n-1$ complete induced subgraphs K_t such that $t \geq n$.*

Proof: *Let $S^{'}$ be a set of all vertices of one element, where $\left|S^{'}\right| = n$, let $u,\ v \in S^{'}$. Since u is not subset of v and v is not a subset of u for all elements of $S^{'}$, then u is adjacent with v. Hence, $G\left[S^{'}\right]$ be a complete subgraph of order n, so that $G\left[S^{'}\right] = K_n$. Let $S^{''}$ be a set of all vertices that having two elements such that $|S^{''}| = \binom{n}{2}$, let $u_1,\ u_2 \in S^{''}$. Since u_1 is not subset of u_2 and u_2 is not subset of u_1 for all elements of $S^{''}$. Thus, u_1 is adjacent with u_2. Then, $G\left[S^{''}\right]$ be a complete induced subgraph of order $\binom{n}{2}$, so $G\left[S^{''}\right] = K_{\binom{n}{2}}$ and so on. Also, let S^{n-1} be a set of all vertices of $n-1$ elements and for any $v_1, v_2 \in S^{n-1}$. In similar prove above we get as $G\left[S^{n-1}\right] = K_{\binom{n}{n-1}} = K_n$. Therefore, the graph G_τ has $n-1$ complete induced subgraphs.* □

Proposition 2.3 *Let $|X| = n$, then the order of discrete topological graph G_τ is $2^n - 2$.*

3. Domination in Topological Graphs

Many definitions and theorems of domination are applied on the topological graphs. The domination number and dominating sets are proved.

Theorem 3.1 *Let G_τ be a discrete topological graph, where $|X| = n$, then*

$$\gamma\left(G_\tau\right) = \gamma^{-1}\left(G_\tau\right) = \begin{cases} 1, & if\ n = 2 \\ 2, & if\ n > 2. \end{cases}$$

Proof: *If $n = 2$, since $G_\tau \cong K_2$ according to Proposition 2.2. Then, the minimum dominating set of a graph G_τ has only one vertex say v such that $D = \{v\}$. Therefore, $\gamma\left(G_\tau\right) = 1$, as an example, see Figure 1 (a). Now, if $n > 2$, let u be any vertex in a graph G_τ, thus u is adjacent to all vertices in G_τ are not subset of u and u is not subset of them. Thus, the vertex u dominates all these vertices but not all vertices of G_τ. To get another vertex in a graph G_τ is adjacent to the remaining vertices which are least not adjacent with u. This vertex is only u^c. Since the vertex u^c is adjacent to the remaining vertices in a graph G_τ such as w because w is either $w \subseteq u$ or $u \subseteq w$. This get $w \nsubseteq u^c$ and $u^c \nsubseteq w$, so $u^c\ w$ is an edge in G_τ. Then, the vertex u^c dominates all these vertices that is not subset of u^c and u^c is not subset of them. If we choose with a vertex u any vertex other than u^c for example say v such that $v \neq u^c$, then there is at least one vertex say w is not adjacent with u and v, so that u, v not dominates w. So, to achieve the domination of all vertices in G_τ, we must choose more vertices in the dominating set. Thus, the graph G_τ has at least two vertices dominate all vertices in it. Then, $D = \{u, u^c\}$ is a minimum dominating set in G_τ. Therefore, $\gamma\left(G_\tau\right) = 2$. As an example, see Fig.1. Now, by the same technique of proof above, let $D^{-1} = \{v_1\}$ for $n = 2$ and let $D^{-1} = \{w,\ w^c\}$ for $n > 2$. Therefore, D^{-1} is an inverse dominating set. As an example, see 2.* □

Theorem 3.2 *Let G_τ and H_τ be a discrete topological graphs defined on a sets X and Y respectively such that $|X| = n$ and $|Y| = m$. Then, $G_\tau \bigodot H_\tau$ has dominating set and domination number where:*
$\gamma (G_\tau \bigodot H_\tau) = 2^n - 2$.

Proof: *By definition of corona operation $G_\tau \bigodot H_\tau$, every vertex of G_τ is adjacent with all vertices of only one copy of a graph H_τ. This mean if $v_i \in V(G_\tau)$, then v_i is adjacent to all vertices of the i^{th} copy of H_τ. Thus, v_i dominates all vertices of one copy of H_τ, so that $v_i \in D$. Then, $D = V(G_\tau)$ is the minimum dominating set of a graph $G_\tau \bigodot H_\tau$. Since the number of all vertices in a graph G_τ is $2^n - 2$ according to Proposition 2.4. Thus, the order of minimum dominating set D is $2^n - 2$. Therefore, $\gamma (G_\tau \bigodot H_\tau) = 2^n - 2$. As an example, see 3.* □

Theorem 3.3 *Let H_τ and G_τ are two discrete topological graphs. Such that G_τ defined on a set X and H_τ defined on a set Y. Where $|X| = n$ and $|Y| = m$. Then, $G_\tau \bigodot H_\tau$ has inverse dominating set and domination number where:*

$$\gamma^{-1} (G_\tau \bigodot H_\tau) = \begin{cases} 2^n - 2 & if\ n \geq 2 \wedge\ m = 2 \\ 2(2^n - 2), & if\ n \geq 2 \wedge\ m > 2 \end{cases}.$$

Proof: *Since $D = V(G_\tau)$ in $G_\tau \bigodot H_\tau$ from proof of Theorem 3.2. Then, the subset D^{-1} contains in each copy of H_τ. So, if $n \geq 2$ and $m = 2$. Thus, $H_\tau \cong K_2$ by Proposition 2.2. So, we take one vertex from each copy of K_2 belong to D^{-1}. Such that this vertex dominates one copy of H_τ and dominates only one vertex of G_τ, and the number of all these vertices that belong to D^{-1} equal to order of G_τ. Hence, $\gamma^{-1} (G_\tau \bigodot H_\tau) = 2^n - 2$, as an example, see Figure 4 (b). Now, if $n \geq 2$ and $m > 2$, by similar to proof of Theorem 3.1, let $D^{-1} = \{u_i,\ u_i^c\}$, $i = 1,\ 2,\ \ldots,\ 2^n - 2$. Such that D^{-1} containing two vertices from each copy of H_τ. Where u_i and u_i^c dominate all vertices of i^{th} copy of H_τ and dominate the i^{th} vertex of G_τ. Since in the graph $G_\tau \bigodot H_\tau$ the number of all copies of H_τ is equal to the order of a graph G_τ which is $2^n - 2$ by Proposition 2.4. Then, the two vertices of D^{-1} are duplicate $2^n - 2$ times. Now, to prove that D^{-1} is a minimum inverse dominating set of a graph $G_\tau \bigodot H_\tau$. Let D^{-1} is not minimum inverse dominating set. So, there exist a minimum dominating set D' where $\left|D'\right| < \left|D^{-1}\right|$. Thus, there is one or more vertices in $V - D^{-1}$ do not dominated by any vertex of D'. Hence, D^{-1} is a minimum inverse dominating set of a graph $G_\tau \bigodot H_\tau$. Therefore, $D^{-1} = \left\{u_1,\ u_1^c, u_2,\ u_2^c, u_3,\ u_3^c, \ldots,\ u_{2^n-2},\ u_{2^n-2}^c\right\}$ and $\gamma^{-1} (G_\tau \bigodot H_\tau) = 2(2^n - 2)$. As an example, see 6a.*

□

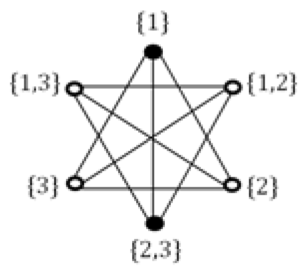
Theorem 3.4 *Let and H_τ and G_τ be two discrete topological graphs defined on X and Y. If $|X| = n$ and $|Y| = m$, then* $\gamma (G_\tau + H_\tau) = \gamma^{-1} (G_\tau + H_\tau) = \begin{cases} 1, & if\ n = 2\ or\ m = 2 \\ 2, & if\ n,\ m > 2 \end{cases}.$

Proof: *In the graph $G_\tau + H_\tau$ each vertex from a graph G_τ is adjacent to all vertices of a graph H_τ. Then, if $n = 2$ and from proof of Theorem 3.1. The graph G_τ contains one dominating vertex say u and this vertex will dominates all vertices of a graph H_τ and one vertex of G_τ (the proof is similar if $m = 2$). Then, $D = \{u\}$ is a minimum dominating set of a graph $G_\tau + H_\tau$ and $\gamma (G_\tau + H_\tau) = 1$. Now, if $n, m > 2$ and from proof of Theorem 3.1. The graph G_τ has two dominating vertices say u and u^c where $\gamma (G_\tau) = 2$. Also, since $u, u^c \in V(G_\tau)$. Then, each vertex of a set $D = \{u, u^c\}$ is adjacent to all the vertices in H_τ. So that, each vertex of D dominates all vertices of a graph H_τ. Thus, $D = \{u, u^c\}$ be a minimum dominating set in a graph $G_\tau + H_\tau$. Thus, $\gamma (G_\tau + H_\tau) = 2$. As an example, see 5. Now, by the same technique of proof above, let $D^{-1} = \{w\}$ for $n = 2$ or $m = 2$ and $D^{-1} = \{t,\ t^c\}$ for $n,\ m > 2$. Then, D^{-1} is a minimum inverse dominating set. As an example, see 6.* □

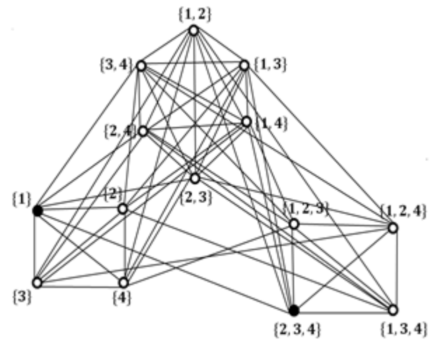
4. Conclusion

Several dominating parameters can be studied on special topological graphs which was constructed from discrete topological space. Also, the dominating set of corona and join operations between two topological graphs are applied. Some examples are given to explain the properties of the topological graphs.

(a) $G_\tau \cong K_2$



(b) $G_\tau \cong \overline{C_2}$



(c) $|X| = 4$

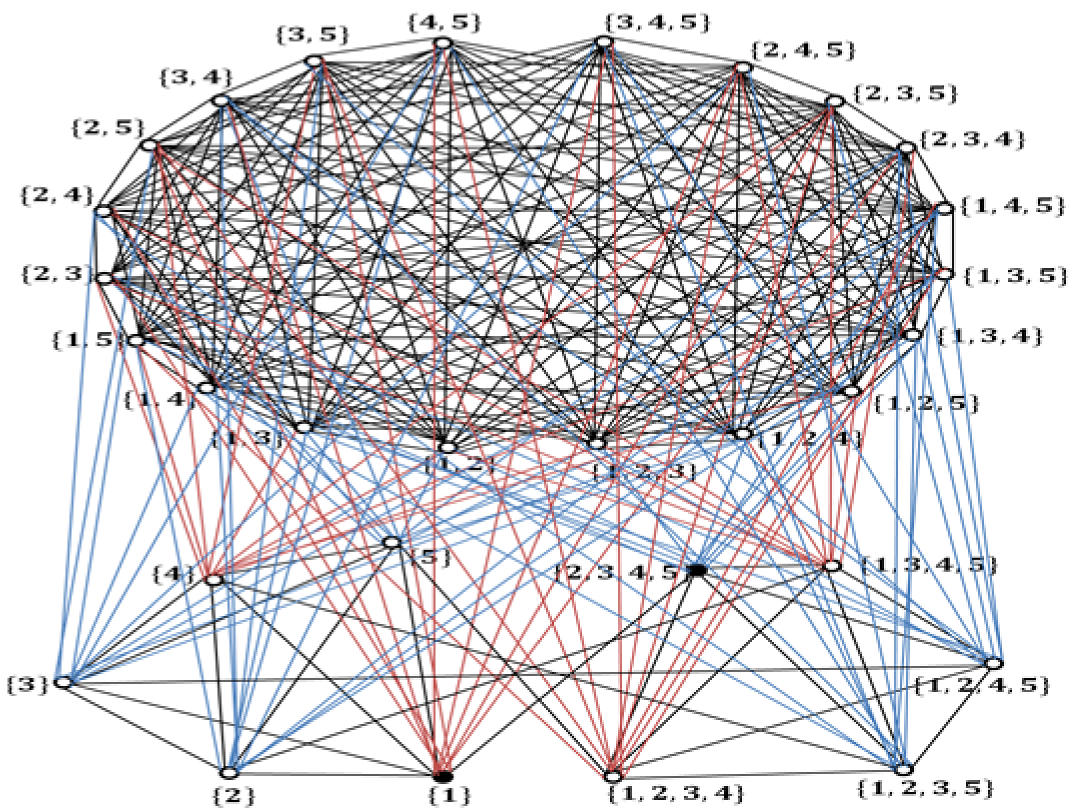
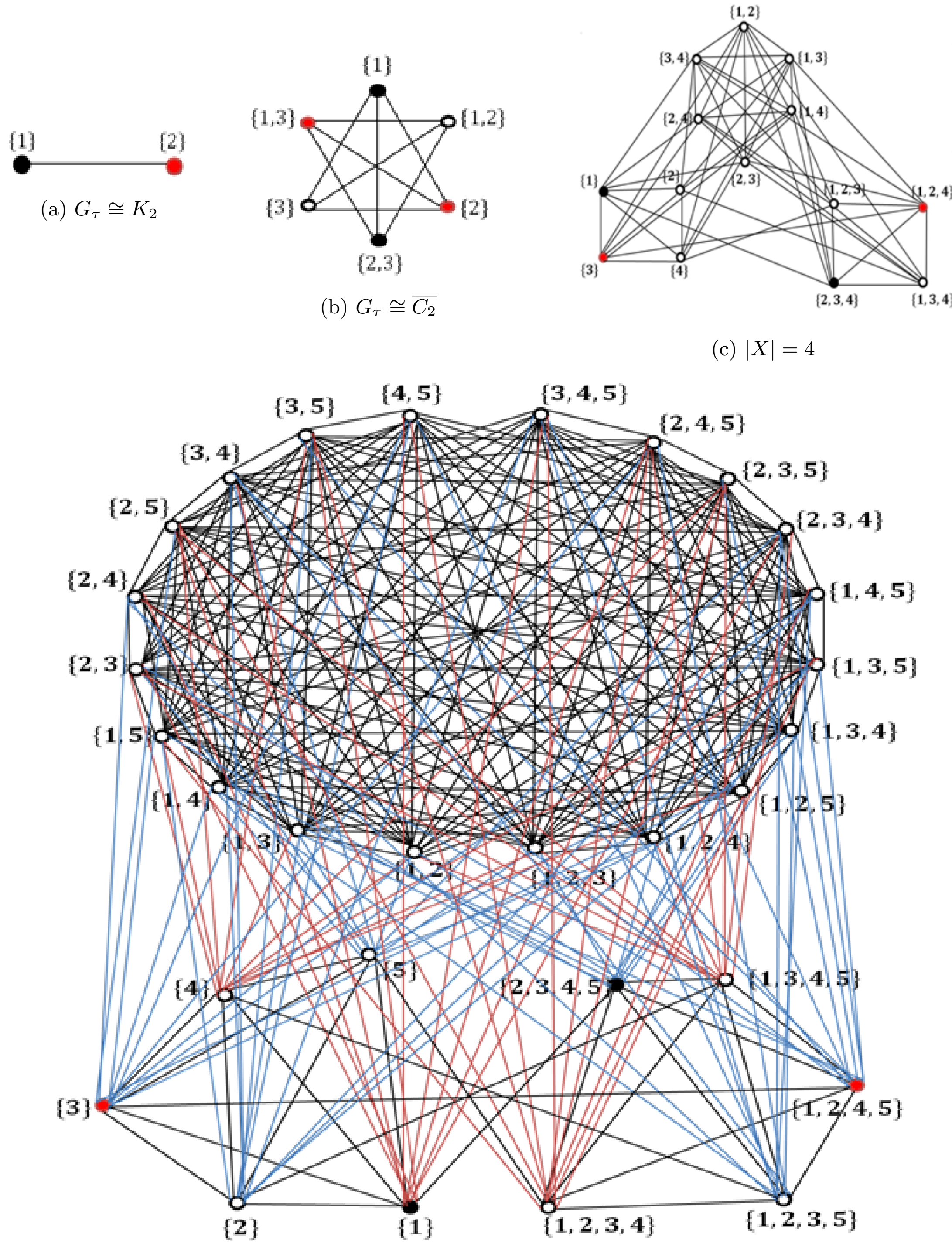


Figure 1: A min. dom. set in topological graph when $|X| = 2, 3, 4, 5$

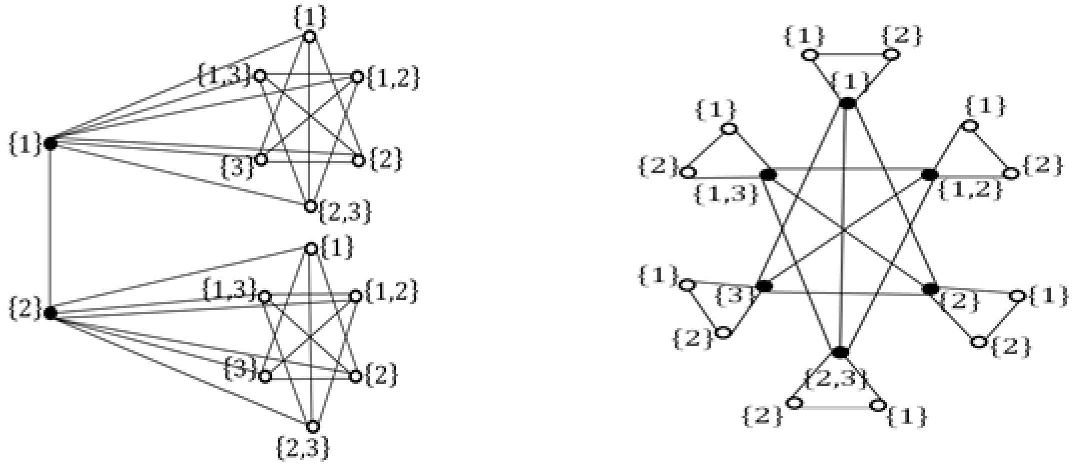


(a) $G_\tau \cong K_2$

(b) $G_\tau \cong \overline{C_2}$

(c) $|X| = 4$

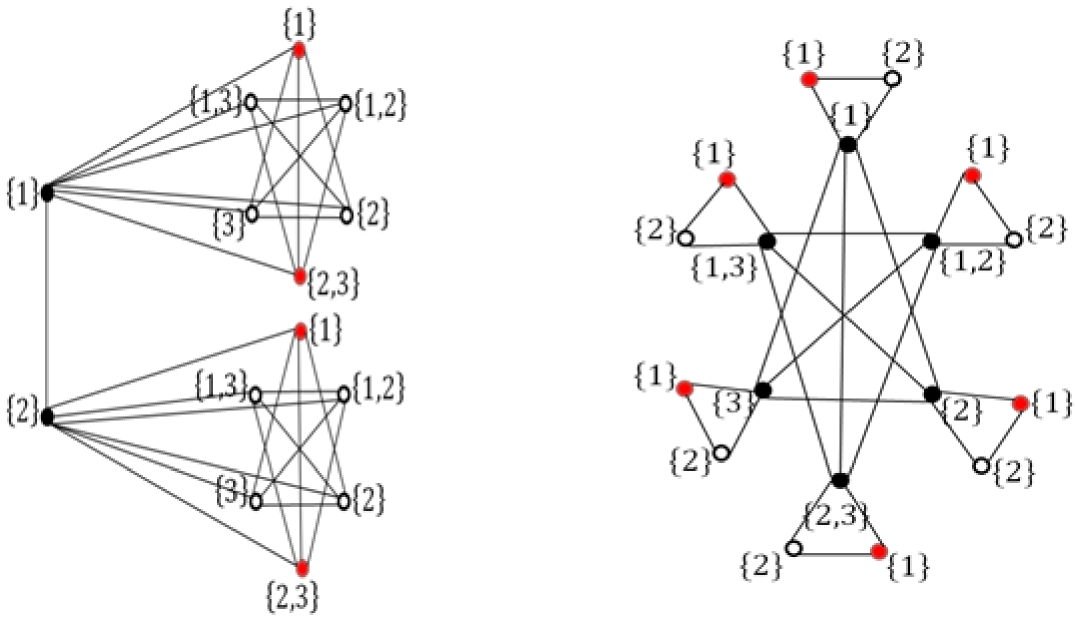
Figure 2: A minimum inverse dominating sets in topological graphs when $|X| = 2, 3, 4, 5$



(a) $K_2 \odot \overline{C_6}$

(b) $\overline{C_6} \odot K_2$

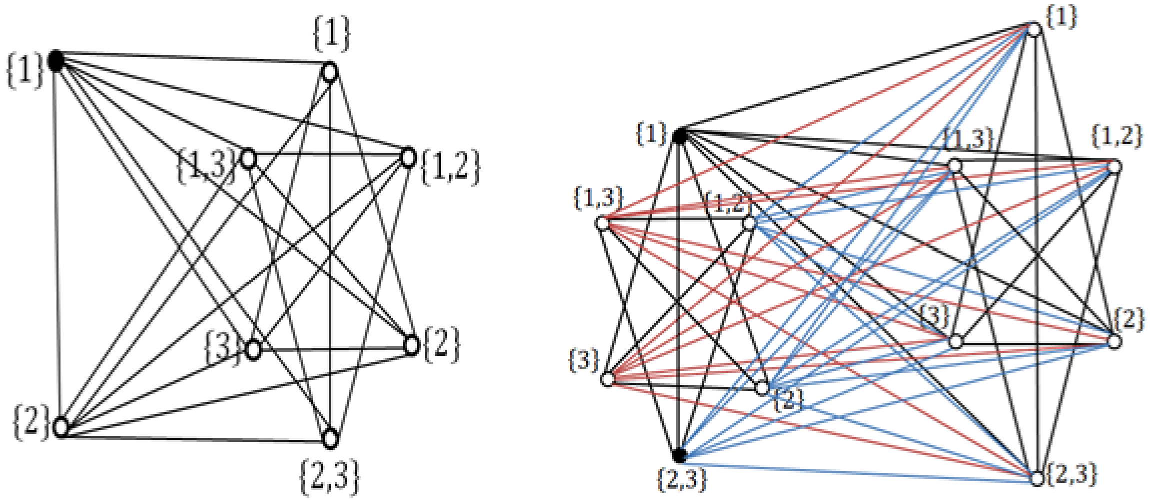
Figure 3: A minimum dominating set in $\overline{C_6} \odot K_2$ and $K_2 \odot \overline{C_6}$



(a) $K_2 \odot \overline{C_6}$

(b) $\overline{C_6} \odot K_2$

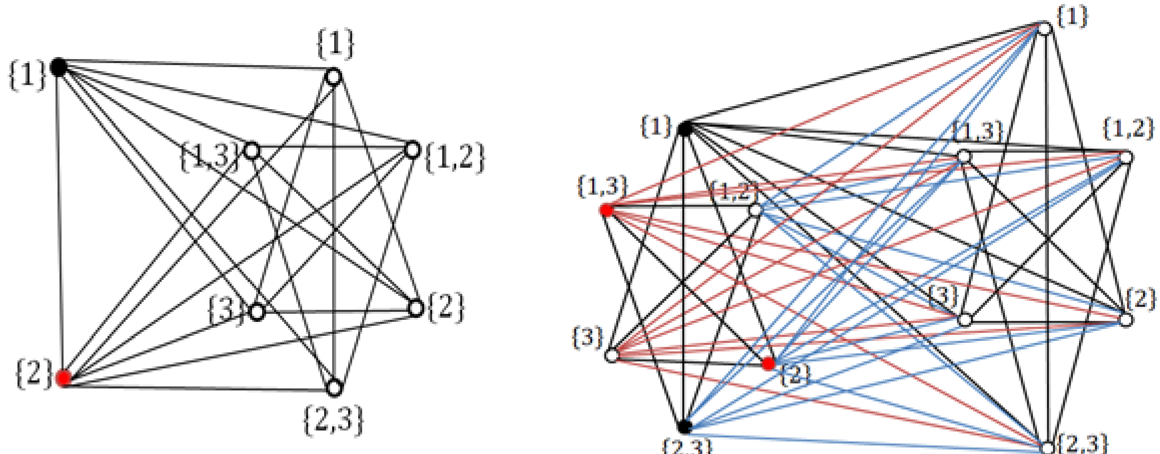
Figure 4: A minimum inverse dominating set in $\overline{C_6} \odot K_2$ and $K_2 \odot \overline{C_6}$



(a) $K_2 + \overline{C_6}$

(b) $\overline{C_6} + \overline{C_6}$

Figure 5: A minimum dominating set in $\overline{C_6} + \overline{C_6}$ and $K_2 + \overline{C_6}$



(a) $K_2 + \overline{C_6}$

(b) $\overline{C_6} + \overline{C_6}$

Figure 6: A minimum inverse dominating set in $\overline{C_6} + \overline{C_6}$ and $K_2 + \overline{C_6}$

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