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On Multi-sequence Space Related to the p-Absolutely Summable Sequences

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ABSTRACT: In this article we introduce the concept of multi-sequence spaces of real numbers related to p-absolutely summable spaces. We have investigated its different algebraic and topological properties. These includes the solidness, symmetry, convergence free etc. Some geometric properties of the space has also been investigated.

Key Words: Banach space, Multi-sequence, Solid space; Symmetic space, ℓ_p-space of multi-sequences.

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Contents

1. Introduction

A multiset is a collection of objects (called elements) in which objects may occur more than once. The number of times of an element occurs in a multiset is called its multiplicity. The cardinality of a multiset is the sum of multiplicities of its elements. Multisets are of interest in a certain area of mathematics, computer science and physics. Therefore, a set is a multiset (shortly, mset) in which the multiplicity of each distinct element is one. The multiset theory which contains set theory as a special case was introduced by Cerf et al. [4] in 1971. The prime factorization of an integer $n > 0$, repeated roots of polynomials etc. are examples of multiset. We formalize it by defining a multiset as a collection of elements, each considered with certain multiplicity. For the sake of convenience, a multiset is written as $\{x_1/k_1, x_2/k_2, x_3/k_3, ...x_n/k_n, ...\}$ in which the element x_i occurs k_i times. We observe that each multiplicity k_i is a positive integer. For the details of research work on ℓ_p spaces, one may refer to [5,7,8,9] and [10].

Blizard [1] started the works on multiset in 1989. From 1989 to 1991, he made a through study of multiset theory, real valued multisets ([2], [3]). Roy et al. [6] studied multipoint, multi metric, multi open ball, multi closed ball, limit point in M-metric space, convergence of sequence of multipoints in M-metric space.

A multi-sequence is a sequence whose terms may occur more than one. Here the multiplicity of an term in a multi-sequence is always under some restriction, it may be identical or less than a finite number. Let X be a set, then X^s denotes a multiset, created by the elements of X, where multiplicity of each element $\leq s$, where $s \in \mathbb{N}$.

Throughout the article w, ℓ_∞ , ℓ_p denote the spaces of all, bounded, and p-absolutely summable sequences respectively. Further, by ℓ_p we denote the sequences space of all p-absolutely summable sequences, i.e.,

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$$\ell_p = \left\{ x = (x_n) : \left(\sum_{n=1}^{\infty} |x|^p \right)^{\frac{1}{p}} < \infty \right\}, \text{ for } p \geq 1$$

and its norm is given by

$$||x_n|| = \left(\sum_{n=1}^{\infty} |x|^p \right)^{\frac{1}{p}}$$

The zero sequence is denoted by $\overline{\theta} = (\theta, \theta, \theta, \theta, ...)$.

2. Definitions and Preliminaries

In this section we procure some basic definitions and notations those will be used throughout the paper.

Definition 2.1. A subset E of w is said to be *solid* if $(x_n) \in E$ implies $(y_n) \in E$ for all sequences (y_n) such that $|y_n| \leq |x_n|$, for all $n \in \mathbb{N}$

Definition 2.2. A sequence space E is said to be *symmetric* if $(x_n) \in E$ implies $\left(x_{\pi(n)}\right) \in E$, where π is a permutation of $\mathbb{N}$.

Note 2.1. If all the rearrangements of the terms of the sequence (x_n) belongs to E, then we say that the the sequence space E is symmetric.

Definition 2.3. A subset E of w is said to be *convergence free*, if $(x_n) \in E$ and $x_n = 0 \Rightarrow y_n = 0$ together with $(y_n) \in E$.

Definition 2.4. Let E be a sequence space. Then E is said to be a *sequence algebra* if there is defined a product $\star$ on E such that $x, y \in E \Rightarrow x \star y \in E$.

Definition 2.5. Let $K = \{k_1 < k_2 < k_3 < .... < k_n\} \subset N$ (set of natural numbers). Let $(x_n) \in w$. Then the K-step space of the sequence space E is defined by
$\lambda_K^E = \{(x_{k_i}) \in w : (x_n) \in E\}$.

Definition 2.6. A canonical pre-image (y_n) of a sequence $(x_n) \in E$, where K-step space λ_K^E is considered, is defined by

$$y_n = \begin{cases} x_n, & n \in K \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.7. A sequence space E is said to be *monotone*, if it contains all its step spaces.

Definition 2.8. A collection of elements which are allowed to repeat is called a *multiset*. Formally, if X is a set of elements, a multiset A drawn from the set X is represented by a function $C_A : X \longrightarrow \mathbb{N}_\theta$, where $\mathbb{N}_\theta$ represents the set of non-negative integers.

For each $x \in X, C_A(x)$ is the characteristic value or multiplicity of x in A. A multiset is a set if $C_A(x)$= 0 or 1, $\forall x \in X$.

Definition 2.9. Let $\mathbb{R}$ be the set of all real numbers. Then a set of real numbers where repetition of real numbers is allowed, is called *multiset of real numbers*, denoted by $m\mathbb{R}$, defined by

$$m\mathbb{R} = \{x_n/c_n : x_n \in \mathbb{R}, c_n \in \mathbb{N}\}$$

Here, x_n/c_n represents real number x_n appears c_n times and $\mathbb{N}$ denotes the set of natural numbers.

In this article we introduce the following definition of multisequence relating p-absolutely summable for multiplicity c and the space of this multisequences.

Definition 2.10. A function whose domain is the set $\mathbb{N}$ of natural numbers and range set is the set $m\mathbb{R}$ (multiset of real numbers) is called a *Multi-sequence*. Thus a multi-sequence is denoted symbolically as $mx : \mathbb{N} \longrightarrow m\mathbb{R}$, defined by

$$(x_n/c_n) = (x_1/c_1, x_2/c_2, x_3/c_3, ..., x_n/c_n, ...)\,, \text{where } n \in \mathbb{N}.$$

Definition 2.11. Let $p \geq 1$. A multi-sequence $mx = (x_n/c_n)$ of mX^s is said to be p-absolutely summable for multiplicity c if

$$\sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}} < \infty$$

where $mX^s = \{(x_n/c_n) \in m\mathbb{R} : c_n = card\,(x_n) \leq s, s \in \mathbb{N}\}$ denotes the set of all multi-sequences whose elements drawn from the sequence $X = (x_n)$ and no element in the multi-sequence occurs more than s times.

Example 2.1. Consider the sequence (x_n/c_n) defined by

$$x_n = \begin{cases} \frac{1}{n^2}/5, & \text{for } n \leq 100, \\ \frac{1}{n^2}/4, & \text{for } n > 100, \end{cases}$$

for all $n \in \mathbb{N}$.
This multi-sequence is 2-absolutely summable for multiplicity 4. For,

$$\sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}}$$

$$= \sum_{n=1}^{100} \left\{ \left(\frac{1}{n^2}\right)^2 + (5-4)^2 \right\}^{\frac{1}{2}} + \sum_{n=101}^{\infty} \left\{ \left(\frac{1}{n^2}\right)^2 + (4-4)^2 \right\}^{\frac{1}{2}}$$

$$= \sum_{n=1}^{100} \left\{ \left(\frac{1}{n^2}\right)^2 + 1 \right\}^{\frac{1}{2}} + \sum_{n=101}^{\infty} \frac{1}{n^2},$$

which is convergent.

Example 2.2. Let $p = 1$ and multisequence (x_n/c_n) be defined by

$$x_n = \begin{cases} 1/4, & \text{for n is odd}, \\ 2/6, & \text{for n is even}. \end{cases}$$

Now,

$$\sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}}$$

$$= \sum_{n=1}^{\infty} \{|x_n| + |c_n - c|\}$$

$$= \sum_{n=odd} \{1 + |4 - c|\} + \sum_{n=even} \{2 + |6 - c|\},$$

which is not a finite sum for any value of c.

So, this multi-sequence is not p-absolutely summable for any multiplicity c.

Definition 2.12. Let $p \geq 1$. Then the class of p-absolutely summable multi-sequences of real numbers with multiplicity c , denoted by $\ell_p^{M^c}$, is defined by

$$\ell_p^{M^c} = \left\{ (x_n/c_n) \in mX^s : \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}} < \infty \right\}.$$

The norm on the mspace is given by :

$$\| \, (x_n/c_n) \, \| = \left\{ \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}}.$$

Definition 2.13. Let $(x_n/c_n) \in mX^s$ be a multi-sequence and $\alpha \in \mathbb{R}$. Then the scalar multiplication of this multi-sequence, denoted by $\alpha\,(x_n/c_n)$, whose norm is given by,

$$\|\alpha(x_n/c_n)\| = \alpha \left\{ \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}}, \text{for } 1 \leq p < \infty .$$

Definition 2.14. Let (x_n/c_n) and $(y_n/c_n) \in mX^s$ be two multi-sequences having same multiplicity of each element both of the multi-sequences. Then the sum of the multi-sequences is defined by

$$(x_n/c_n) + (y_n/c_n) = (x_n + y_n)/c_n, \ \forall n \in \mathbb{N}.$$

Definition 2.15. Let (x_n/c_n) and $(y_n/c_n) \in mX^w$ be two multi-sequences having same multiplicity of each element both of the multi-sequences and let $\alpha, \beta \in \mathbb{R}$. Then the linear combination of these two multi-sequences is given by

$$\alpha\,(x_n/c_n) + \beta\,(y_n/c_n) = (\alpha x_n + \beta y_n)/c_n.$$

Definition 2.16: Let $(x_n/c) \in mX^s$ be a multi-sequence. Then the modulus of this multi-sequence, denoted by, $|(x_n/c)|$, defined by,

$$|(x_n/c)| = \sqrt{x_n^2 + (c-1)^2}.$$

3. Main Results

In this section we establish some results involving the multisequence space $\ell_p^{M^c}$.

Theorem 3.1.. The class of p-absolutely summable multi-sequences of multiset real numbers with multiplicity c, i.e., $\ell_p^{M^c}$ is a linear space, where $p \geq 1$.

Proof. Let $(x_n/c_n), (y_n/c_n) \in \ell_p^{M^c}$ and Let $\alpha, \beta \in \mathbb{R}$, where $p \geq 1$.

Then, we have

$$\| (x_n/c_n) \| = \left\{ \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}} < \infty$$

and

$$\| (y_n/c_n) \| = \left\{ \sum_{n=1}^{\infty} \{|y_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}} < \infty$$

.

Now,

$$\| (\alpha x_n + \beta y_n) / c_n \| = \left\{ \sum_{n=1}^{\infty} \{|\alpha x_n + \beta y_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}}$$

$$\leq \left\{ \sum_{n=1}^{\infty} \{|\alpha x_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}} + \left\{ \sum_{n=1}^{\infty} \{|\beta y_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}}$$

$$\leq \left\{ \alpha \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}} + \left\{ \beta \sum_{n=1}^{\infty} \{|y_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}} < \infty$$

i.e.,

$$\| (\alpha x_n + \beta y_n) / c_n \| = \left\{ \sum_{n=1}^{\infty} \{|\alpha x_n + \beta y_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}} < \infty.$$

Hence, the class of p-absolutely summable multi-sequences of multiset real numbers, i.e., $\ell_p^{M^c}$ is a linear space.

Theorem 3.2. For $1 \leq p < \infty$, the class of p-absolutely summable multi-sequences of multiset real numbers with multiplicity c, i.e., $\ell_p^{M^c}$ is a p-normed linear space with respect to the norm

$$\| (x_n/c_n) \| = \left\{ \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}} .$$

Proof: Let $(x_n/c_n), (y_n/c_n) \in \ell_p^{M^c}$ and Let $\lambda \in \mathbb{R}$. Then we have

$$(i)\ \|\,(x_n/c_n)\,\| = \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}} = 0$$

.

$$\Rightarrow \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\} = 0$$

.

$$\Rightarrow \{|x_1|^p + |c_1 - c|^p\} + \{|x_2|^p + |c_2 - c|^p\} + \{|x_3|^p + |c_3 - c|^p\} + ... = 0$$

.

$$\Rightarrow \{|x_1|^p + |c_1 - c|^p\} = \{|x_2|^p + |c_2 - c|^p\} = \{|x_3|^p + |c_3 - c|^p\} = ... = 0$$

.
$\Rightarrow |x_1|^p = |x_2|^p = |x_3|^p = ... = 0$
and $|c_1 - c|^p = |c_1 - c|^p = |c_1 - c|^p = ... = 0.$
$\Rightarrow x_1 = x_2 = x_3 = ... = 0$ and $x_1 = x_2 = x_3 = ... = 0.$

Therefore, $\|(x_i/c_i)\| = 0 \Rightarrow (x_n/c_n) = (0/c) = m\overline{\theta}$, the zero multi-sequence.
(ii) Let $\lambda \in \mathbb{R}$ and (x_n/c_n) be a multi sequence, then we have

$$\|\lambda(x_n/c_n)\|^p = \left\{\left\{\sum_{n=1}^{\infty} \{|\lambda x_n|^p + |\lambda(c_n - c)|^p\}\right\}^{\frac{1}{p}}\right\}^p = \lambda\|(x_n/c_n)\|^p.$$

(iii) $\|(x_n/c_n) + (y_n/c_n)\| = \|(x_n + y_n)/c_n\|$

$$= \left\{\sum_{n=1}^{\infty} \{|x_n + y_n|^p + |c_n - c|^p\}\right\}^{\frac{1}{p}}$$

$$\leq \left\{\sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}\right\}^{\frac{1}{p}} + \left\{\sum_{n=1}^{\infty} \{|y_n|^p + |c_n - c|^p\}\right\}^{\frac{1}{p}}$$

$$= \|(x_n/c_n)\| + \|(y_n/c_n)\|$$

i.e.,

$$\|(x_n/c_n) + (y_n/c_n)\| \leq \|(x_n/c_n)\| + \|(y_n/c_n)\|$$

Hence, $\ell_p^{M^c}$ is a p-normed linear space, where $p \geq 1$.

Theorem 3.3. The class of p-absolutely summable multi-sequences of multiset real numbers with multiplicity c, i.e., $\ell_p^{M^c}$ is solid, where $p \geq 1$.

Proof. Let $(x_n/c_n) \in \ell_p^{M^c}$, where $p \geq 1$.

Then, we have $(x_n/c_n) \in mX^s$, and

$$\sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}} < \infty$$

where $mX^s = \{(x_i/c) \in m\mathbb{R} : c = card\,(x_i) \leq s, s \in \mathbb{N}\}$.

Let (y_n/c_n) be such that $|(y_n/c_n)| \leq |(x_n/c_n)|$, i.e., $\sqrt{y_n^2 + (c_n-1)^2} \leq \sqrt{x_n^2 + (c_n-1)^2}$

$\Rightarrow\ y_n^2 + (c_n-1)^2 \leq x_n^2 + (c_n-1)^2$

$\Rightarrow |y_n| \leq |x_n|$

$\Rightarrow |y_n|^p \leq |x_n|^p$

$\Rightarrow \{|y_n|^p + |c_n - c|^p\}^{\frac{1}{p}} \leq \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}}$

$\Rightarrow \sum_{n=1}^{\infty} \{|y_n|^p + |c_n - c|^p\}^{\frac{1}{p}} \leq \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}} < \infty$

$\Rightarrow \|y_n/c_n\| \leq \|(x_n/c_n)\| < \infty$.

Thus, $(y_n/c_n) \in \ell_p^{M^c}$.
Hence, $\ell_p^{M^c}$ is solid.

Theorem 3.4. For $1 \leq p < \infty$, the class of p-absolutely summable multi-sequences of multiset real numbers with multiplicity c, i.e., $\ell_p^{M^c}$ is Banach space with respect to the norm

$$\|\,(x_n/c_n)\,\| = \left\{\sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}\right\}^{\frac{1}{p}}.$$

Proof: Let $(x_n/c_n) \in \ell_p^{M^c}$ be Cauchy multi-sequence, $\forall n \in \mathbb{N}$. Then we have,

$\|(x_r - x_s)/c_n\| \to 0$, as $r, s \to \infty$.

i.e.,

$$\left\{\sum_{r=1}^{\infty}\sum_{s=1}^{\infty} (|x_r - x_s|^p + |c_n - c|^p)\right\}^{\frac{1}{p}} \to 0, \qquad as\ r,\ s \to \infty.$$

$$\Rightarrow \left\{\sum_{r=1}^{\infty} (|x_r - x_1|^p + |c_n - c|^p) + (|x_r - x_2|^p + |c_n - c|^p) + (|x_r - x_3|^p + |c_n - c|^p) + \ldots\ldots\ldots\right\}^{\frac{1}{p}}$$

$\to 0$, as $r, s \to \infty$.

$$\Rightarrow |x_r - x_s| \to 0\ and\ c_n \to \text{c when } r, s \to \infty.$$

So, (x_n/c_n) is convergent and since it is arbitrary multi-sequence in $\ell_p^{M^c}$, therefore, $\ell_p^{M^c}$ is Banach space.

Theorem 3.5. The class of p-absolutely summable multi-sequences of multiset real numbers with multiplicity c, i.e., $\ell_p^{M^c}$ is symmetric, where $p \geq 1$.

Proof. Let $(x_n/c_n) \in mX^s$ be any element of $\ell_p^{M^c}$, $p \geq 1$. Then we have

$$\parallel (x_n/c_n) \parallel = \left\{ \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\} \right\}^{\frac{1}{p}} < \infty.$$

We know that if a multi-sequence holds the above relation, then the multi-sequences formed by the rearrangements of the terms of the multi-sequence (x_n/c_n) also holds good. So, all the rearrangements of the terms of the multi-sequence (x_n/c_n) belong to $\ell_p^{M^c}$. Hence, $\ell_p^{M^c}$ is symmetric.

Result 3.1. The class of p-absolutely summable multi-sequences of multiset real numbers with multiplicity c, i.e., $\ell_p^{M^c}$ is not convergence free, where $p \geq 1$.

The above result follows from the following example:
Let p=2. (x_n/c_n) be defined by

$$x_n/c_n = \begin{cases} \frac{1}{n^2}/2, & \text{when } n \leq 200, \\ 0/5, & \text{when } n > 200. \end{cases}$$

Then this multi-sequence is 2-absolutely summable for multiplicity 5.
Consider (y_n/c_n) defined by

$$y_n/c_n = \begin{cases} \frac{1}{n^2}/2, & \text{when n is odd}, \\ 0/5, & \text{when n is even}. \end{cases}$$

Then, $(y_n/c_n) \notin l_p^{M^c}$, for $p = 2$.

Theorem 3.6. The class of p-absolutely summable multi-sequences of multiset real numbers with multiplicity c, i.e., $\ell_p^{M^c}$ is a sequence algebra, where $p \geq 1$.

Proof. Let $(x_n/c_n), (y_n/c_n) \in mX^s$ be any two multi-sequences in $\ell_p^{M^c}$, where $p \geq 1$. Then we have,

$$\sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}} < \infty$$

and

$$\sum_{n=1}^{\infty} \{|y_n|^p + |c_n - c|^p\}^{\frac{1}{p}} < \infty.$$

Consider the term wise product of these two multi-sequences, i.e., $(x_n/c_n)(y_n/c_n) = (x_n y_n/c_n)$.
Now,

$$\sum_{n=1}^{\infty} \{|x_n y_n|^p + |c_n - c|^p\}^{\frac{1}{p}} \leq \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}} \sum_{n=1}^{\infty} \{|y_n|^p + |c_n - c|^p\}^{\frac{1}{p}} < \infty, \text{ by equation (3) and (4)}$$

This implies

$$(x_n/c_n), (y_n/c_n) \in \ell_p^{M^c} \Rightarrow (x_n y_n/c) \in \ell_p^{M^c},$$

where $p \geq 1$.

Therefore, $\ell_p^{M^c}$ is a sequence algebra.

Theorem 3.7. Let $1 \leq p < q < \infty$. Then $\ell_q^{M^c} \subset \ell_p^{M^c}$, where $p \geq 1$, where $q > 1$.

Proof. Let $(x_n/c_n) \in \ell_q^{M^c}$. Then we have

$$\sum_{n=1}^{\infty} \{|x_n|^q + |c_n - c|^q\} < \infty$$

Since , $p < q$
$\Rightarrow |x_n|^p < |x_n|^q$

$\Rightarrow \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}} < \{|x_n|^q + |c_n - c|^q\} \frac{1}{q}$

$\Rightarrow \sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\}^{\frac{1}{p}} < \sum_{n=1}^{\infty} \{|x_n|^q + |c_n - c|^q\}^{\frac{1}{q}}$

i.e., $\sum_{n=1}^{\infty} \{|x_n|^p + |c_n - c|^p\} < \infty$

Therefore, $(x_n/c_n) \in \ell_p^{M^c}$

So, $\ell_q^{M^c} \subset \ell_p^{M^c}$.

4. Conclusion

In this article we have introduced the multi sequences of p-absolutely summable sequences. We have investigated its different algebraic and topological properties. We have also investigated some of its geometrical properties.

5. Future works

We have planned to introduce W.L.C. Sargent type multi sequence spaces and investigate their different properties. Also the different sequence spaces, sequence spaces defined by Orlicz functions etc. will be introduced and their different properties will be investigated.

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References

1. W.D. Blizard, *Multiset theory.* Notre Dame Journal of Formal Logic, 30(1), 36-66, (1989).
2. W.D. Blizard, *Real-valued multisets and fuzzy sets.* Fuzzy Sets and Systems, 33(1), 77-97, (1989).
3. W.D. Blizard, *The development of multiset theory.* Modern Logic,1(4), 319-352, (1991).
4. V. Cerf, E. Fernandez, K. Gostelow, S. Volausky, *Formal control and low properties of a model computation.* Report ENG 7178, Computer Science Department, University of California, Los Angeles, CA, p. 81, (1971).
5. H. Kizmaz, *On certain sequence spaces.* Canad. Math. Bull. 24(2), 169–176,(1981).
6. R. Roy, S. Das and S.K. Sumanta, *On multi normed linear spaces.* International Journal of Mathematics, Trends and Technology, 48(2), 111-119, (2017).
7. W.L.C. Sargent, *Some sequence spaces related to the ℓ_p spaces.* Journal of the London Mathematical Society, 35(2), 161-171, (1960).

8. B.C. Tripathy and M. Sen, *On a new class of sequences related to the space ℓ_p*. Tamkang Journal of Mathematics, 33(2), 167-172, (2002).

9. B. C. Tripathy and S. Mahanta, *On a class of generalized lacunary difference sequence spaces defined by Orlicz function*. Acta Math. Applicata Sinica (Eng. Ser.), 20(2), 231-238, (2004).

10. B.C. Triapthy and S. Mahanta, *On a class of vector valued sequences associated with multiplier sequences*. Acta Math. Applicata Sinica(Eng. Ser.); 20(3),487-494, (2004).

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